

Lab task #9

Heap Operations Solution

Problem Overview

Given heap with root 45:

```
      45
     / \
    30  25
   / \  / \
  18 27 13 2
 / \ / \
11 8 16 5
```

Question 1: Node at Position 4

Answer: Node 27

Array Representation (0-based indexing):

Position	0	1	2	3	4	5	6	7	8
Value	45	30	25	18	27	13	2	11	8

Node at position 4 = 27

Question 2: Arithmetic Formulas

For a node at position i (0-based indexing):

Relation	Formula
Left Child	$2i + 1$
Right Child	$2i + 2$
Parent	$\lfloor (i - 1) / 2 \rfloor$

Example with Node at Position 4 (value 27):

- **Left child:** $2(4) + 1 = 9 \rightarrow$ value 16
- **Right child:** $2(4) + 2 = 10 \rightarrow$ value 5
- **Parent:** $(4 - 1) / 2 = 1 \rightarrow$ value 30

Question 3: First remove() Call

Removed Item

45 (root - maximum value in max heap)

Replacement Process

1. Copy 2 (last element) to root position
2. Sift down to restore heap property

After First remove():

```
      30
     / \
    27  25
   / \  / \
  18 8 13 2
 / \ / \
11 5 16
```

Array: [30, 27, 25, 18, 8, 13, 2, 11, 5, 16]

Question 4: Second and Third remove() Calls

After Second remove():

- Removes **30**
- Replaces with **16** (last element)
- Sifts down

```
      27
     / \
    18  25
   / \  / \
  11 8 13 2
 / \
16  5
```

Array: [27, 18, 25, 11, 8, 13, 2, 16, 5]

After Third remove():

- Removes **27**
- Replaces with **5** (last element)
- Sifts down

```
      25
     / \
    18  13
   / \  /
  11 8  2
 / \
16  5
```

Array: [25, 18, 13, 11, 8, 2, 16, 5]

Question 5: Insert 21

Insertion Process

1. Add **21** at the end (position 8)
2. Sift up to restore heap property

Steps:

1. Initial: [25, 18, 13, 11, 8, 2, 16, 5, 21]
2. Compare 21 with parent 8 → swap
3. Compare 21 with parent 18 → swap
4. Compare 21 with parent 25 → no swap ($21 < 25$)

Final Tree After insert(21):

```
      25
     / \
    21  13
   / \  / \
  18 8 2 16
 / \
11  5
```

Final Array: [25, 21, 13, 18, 8, 2, 16, 11, 5]

Summary

Operation	Root Value	Array Length
Original	45	11
After remove #1	30	10
After remove #2	27	9
After remove #3	25	8

Operation	Root Value	Array Length
After insert(21)	25	9