Tutorial 1: Resolution of linear systems

Exercise 1: Determinant

By evaluating the determinant, classify the following matrices as singular, ill-conditioned, or well-conditioned:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 0 & -1 \\ 4 & 1 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 2.0001 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Exercise 2: LU decomposition (Dolittle)

Find Dolittle's LU decomposition of $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 0 & -1 \\ 4 & 1 & 1 \end{bmatrix}$$

Exercise 3: Resolution from LU decomposition

Using the LU decomposition of $\mathbf{A}$ found previously, solve $\mathbf{Ax} = \mathbf{b}$, where:

$$\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Exercise 4: Gauss elimination

Use Gauss elimination to solve $\mathbf{AX} = \mathbf{B}$, where:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 4 \\ 2 & 0 & 7 \\ 1 & 4 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -1 & 3 \\ 6 & 2 \\ 0 & 1 \end{bmatrix}$$

Exercise 5: Condition number

Compute the condition number of $\mathbf{A}$ using the infinity norm:

$$\mathbf{A} = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 0 & -1 \\ 4 & 1 & 1 \end{bmatrix}$$

Exercice 6: Matrix Inversion

Invert the following matrix:

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 1 & 0 \\ 5 & 8 & 9 \end{bmatrix}$$

Exercice 7: LU decomposition (Cholesky)

Find the Cholesky decomposition of $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

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