MATH 3043, Numerical Analysis I
Fall 2020

Lab 2

This lab will have you implementing Newton's method to approximate solutions for several problems.

Solutions must be submitted on Canvas by **September 27** at **11:59 PM**. Please submit a single script file `Lab2Lastname.m` and the corresponding published file `Lab2Lastname.pdf` (for example, my submitted files would be `Lab2Zumbrum.m` and `Lab2Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

As part of your solution for each problem, output the error tolerance, the approximation, and the number of iterations required, formatted using the `fprintf` function as the sample output below:

```
Tolerance: 10e-8, Approximation: 1.23456789, Iterations: 23
```

Use the stopping criteria

$$\left| \frac{p_n - p_{n-1}}{p_n} \right| < \epsilon.$$

1. Use Newton's method to find a solution accurate to within 10^{-6} for $\sin x - e^{-x} = 0$ using $p_0 = 0$.
2. Use Newton's method to find an approximation of $\sqrt{3}$ accurate to within $\epsilon = 10^{-8}$ using $p_0 = 2$. Compare the number of iterations required for Newton's method and the number of iterations required for the Bisection method used in Lab 1 Problem 4 to solve the same problem.
3. The accumulated value of a savings account based on regular periodic payments can be determined from the annuity due equation

   $$A = \frac{P}{i}\left[(1+i)^n - 1\right].$$

   In this equation, A is the amount in the account, P is the amount regularly deposited, and i is the rate of interest per period for the n deposit periods. A mathematician would like to have \$1,000,000 dollars in the account upon retirement in 30 years and can afford to put \$6,000 per year toward this goal. What is the minimal interest rate at which this amount can be invested, assuming that the interest is compounded yearly? How would the minimal interest rate change, assuming the mathematician makes \$1000 monthly deposits and interest is compounded monthly?

4. A drug administered to a patient produces a concentration in the bloodstream given by $c(t) = Ate^{-t/3}$ milligrams per milliliter, t hours after A units have been injected. The maximum safe concentration is 1 mg/mL.

   (a) What amount should be injected to reach this maximum safe concentration, and when does this maximum occur? **Hint:** Do this by hand using techniques from calculus!

   (b) An additional amount of this drug is to be administered to the patient after the concentration falls to 0.25 mg/mL. Determine, to the nearest minute, when this second injection should be given.

5. Use Newton's method to approximate the zero of the function $f(x) = x^2 - 2xe^{-x} + e^{-2x}$ accurate to within 10^{-8} using $p_0 = 1$. What do you notice about the convergence of Newton's method? Repeat the problem using the modified Newton's method and compare the number of iterations required for both methods.