MATH 3043, Numerical Analysis I
Fall 2020

Lab 8

This lab will have you approximating the solution of an integral equation by implementing composite numerical integration methods and solving linear systems.

Solutions must be submitted on Canvas by **November 8** at **11:59 PM**. Please submit a single script file `Lab8Lastname.m` and the corresponding published file `Lab8Lastname.pdf` (for example, my submitted files would be `Lab8Zumbrum.m` and `Lab8Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

1. A Fredholm integral equation of the second kind is an equation of the form

$$u(x) = f(x) + \int_a^b K(x,t)u(t)\,dt,$$

where a and b and the functions f and K are given. To approximate the function u on the interval $[a,b]$, a partition $x_0 = a < x_1 < \cdots < x_{m-1} < x_m = b$ is selected, and the equations

$$u(x_i) = f(x_i) + \int_a^b K(x_i,t)u(t)\,dt, \quad \text{for } i = 0,\ldots,m,$$

are solved for $u(x_0), u(x_1), \ldots, u(x_m)$. The integrals are approximated using quadrature formulas based on the nodes $x_0, \ldots, x_m$. In our problem, $a = 0, b = 1, f(x) = x^2$, and $K(x,t) = e^{|x-t|}$.

(a) Set up and solve the linear system that results when the Composite Trapezoidal rule is used with $m = 4$.

(b) Repeat part (a) using the Composite Simpson's rule.

(c) Repeat part (a) with $m = 10$.

Note: Your code should be general enough that the only change required for part (c) from part (a) is updating the value of m at the beginning of script.

Hint: Use the built-in function `meshgrid` to aid in performing function evaluations for $K(x,t)$; use the built-in function `\` to solve linear systems (`x = A\b` can be used to solve the system $A\mathbf{x} = \mathbf{b}$).