MATH 3043, Numerical Analysis I
Fall 2020

Lab 7

This lab will have you constructing cubic splines to approximate functions.

Solutions must be submitted on Canvas by **November 1** at **11:59 PM**. Please submit a single script file `Lab7Lastname.m` and the corresponding published file `Lab7Lastname.pdf` (for example, my submitted files would be `Lab7Zumbrum.m` and `Lab7Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,

- run independent of other cells,

- be adequately commented.

1. Approximate $f(x) = e^x$ by forming a natural cubic spline $S(x)$ using the nodes $x = 0, 1, 2, 3$. Plot $S(x)$ using a dashed blue curve and $f(x)$ using a solid black curve in the same figure.

   Note: The solution of this problem is included in Section 3.5 Example 2. Use the solution to check the output of your code, but write code general enough that it could easily be used for a different natural cubic spline problem (where only the input data would need to be updated).

2. Repeat Problem 1 for $f(x) = \sin 2x$ using the nodes $x = 0, 1, 2, \ldots, n$ where n is a random integer such that $3 \le n \le 10$.

3. The 2014 Kentucky Derby was won by a horse named California Chrome in a time of 2:03.66 (2 minutes and 3.66 seconds) for the $1\frac{1}{4}$ mile race. Times at the quarter-mile, half-mile, and mile poles were 0:23.04, 0:47.37, and 1:37.45.

   (a) Use these values together with the starting time to construct a natural cubic spline for California Chrome's race.

   (b) Use the spline to predict the time at the three-quarter-mile pole and compare this to the actual time of 1:11.80.

   (c) Use the spline to predict California Chrome's starting speed and speed at the finish.

4. Repeat Problem 1 using a clamped cubic spline $S(x)$ such that $f'(0) = 1$ and $f'(3) = e^3$. Plot $S(x)$ using a dashed blue curve and $f(x)$ using a solid black curve in the same figure.