MATH 3043, Numerical Analysis I
Fall 2020

Lab 4

This lab will have you implementing Neville's method to generate approximations based on Lagrange interpolating polynomials.

Solutions must be submitted on Canvas by **October 11** at **11:59 PM**. Please submit a single script file `Lab4Lastname.m` and the corresponding published file `Lab4Lastname.pdf` (for example, my submitted files would be `Lab4Zumbrum.m` and `Lab4Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

1. Use Neville's method to approximate $f(0.8)$, $f(1.2)$, and $f(1.7)$ given the data in the following table:

x	$f(x)$
0.5	1.772454
0.7	1.298055
1.0	1.000000
1.3	0.897471
1.5	0.886227
1.6	0.893515
2.0	1.000000

2. Generate function data for
$$f(x) = \frac{1}{1+x^2}$$
on N equally-spaced nodes on the interval [-5,5]. Use Neville's method to approximate $f(4.9)$ for $N = 11, 21, 41, 81, 121$. Output the value of N and the corresponding approximation in the following format:

```
N: 11   Approximation:  2.345678901
N: 21   Approximation:  1.234567890
N: 41   Approximation:  0.987654321
```

3. Repeat Problem 2 with function data at the Chebyshev nodes defined by
$$x = -5\cos\left(\frac{2k-1}{2N}\pi\right),\ k = 1, \ldots, N.$$

4. **Inverse Interpolation (another root-finding technique!)** Suppose $f \in C^1[a,b]$, $f'(x) \neq 0$ on $[a,b]$ and f has one zero p in $[a,b]$. Let $x_0, \ldots, x_n$ be $n+1$ distinct numbers in $[a,b]$ with $f(x_k) = y_k$, for each $k = 0, 1, \ldots, n$. To approximate p, construct the interpolating polynomial of degree n on the nodes $y_0, \ldots, y_n$ for f^{-1}. Since $y_k = f(x_k)$ and $0 = f(p)$, it follows that $f^{-1}(y_k) = x_k$ and $p = f^{-1}(0)$. Using iterated interpolation to approximate $f^{-1}(0)$ is called iterated inverse interpolation.

   Use iterated inverse interpolation to find an approximation to the solution of $x - e^{-x} = 0$, using the data

x	e^{-x}
0.3	0.740818
0.4	0.670320
0.5	0.606531
0.6	0.548812
0.7	0.496585