MATH 3043, Numerical Analysis I

Fall 2020

Lab 3

This lab will have you implementing Aitken's and Steffenson's methods and constructing Lagrange interpolating polynomials.

Solutions must be submitted on Canvas by **October 4** at **11:59 PM**. Please submit a single script file `Lab3Lastname.m` and the corresponding published file `Lab3Lastname.pdf` (for example, my submitted files would be `Lab3Zumbrum.m` and `Lab3Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

1. Use fixed-point iteration to approximate the solution of $x = 3^{-x}$ accurate to within 10^{-8} using $p_0 = 0.5$. Repeat the problem using Aitken's method and Steffenson's method. Use the stopping criteria
$$\left|\frac{p_n - p_{n-1}}{p_n}\right| < \epsilon,$$
and output the error tolerance, the approximation, and the number of iterations required, formatted using the `fprintf` function as the sample output below:
```
Tolerance:  10e-8,  Approximation:  1.23456789,  Iterations:  23
```

2. For $f(x) = \sin \pi x$, let $x_0 = 1$, $x_1 = 1.3$, and $x_2 = 1.6$. Construct the Lagrange interpolating polynomial of degree at most two (using all three nodes), and plot the underlying function $f(x)$ (as a solid black line), the interpolating polynomial (as a dashed black line), and data points (as red circles) in a single figure window. Approximate $f(0.85)$, $f(1.15)$, $f(1.45)$, and $f(1.75)$, and find the absolute error of each approximation.

3. For $f(x) = \cos(\ln x)$, let $x_0 = 0.5$, $x_1 = 1$, $x_2 = 1.5$, and $x_3 = 2$. Construct a Lagrange interpolating polynomial of degree at most one, at most two, and at most three (using appropriate nodes for each). Plot the underlying function (as a solid black line), the interpolating polynomials (as dashed black lines), and data points (as red circles) in a single figure window. Approximate $f(1.75)$ using each interpolating polynomial and find the absolute error of each approximation. Which interpolating polynomial gives the best approximation for $f(1.75)$?