MATH 3043, Numerical Analysis I
Fall 2020

Lab 1

This lab will have you implementing the Bisection method and fixed-point iteration to approximate solutions for several problems.

Solutions must be submitted on Canvas by **September 20** at **11:59 PM**. Please submit a single script file `Lab1Lastname.m` and the corresponding published file `Lab1Lastname.pdf` (for example, my submitted files would be `Lab1Zumbrum.m` and `Lab1Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

As part of your solution for each problem, output the error tolerance, the approximation, and the number of iterations required, formatted using the `fprintf` function as the sample output below:

```
Tolerance:  10e-8,  Approximation:  1.23456789,  Iterations:  23
```

For a solution accurate to within 10^{-k}, include at least k digits in the output.

1. Use the Bisection method to find a solution for $x - 2^{-x} = 0$ on the interval $[0, 1]$ accurate to within $\epsilon = 10^{-4}$. Set the maximum number of iterations to be 30, and use the stopping criteria
$$\frac{b_n - a_n}{2} < \epsilon.$$
2. Repeat Problem 1 using $\epsilon = 10^{-12}$.
3. Use the Bisection method to find an approximation of the solution of $x^3 + x - 4 = 0$ on $[1, 4]$ accurate to within $\epsilon = 10^{-4}$. Use the stopping criteria
$$\frac{b_n - a_n}{2} < \epsilon.$$
4. Use the Bisection method to find an approximation of $\sqrt{3}$ accurate to within $\epsilon = 10^{-8}$ using the initial interval $[1, 2]$. Use the stopping criteria
$$\left|\frac{p_n - p_{n-1}}{p_n}\right| < \epsilon.$$
5. The following four fixed-point iteration methods are proposed to compute $21^{1/3}$:

   a. $p_n = \dfrac{20p_{n-1} + 21/p_{n-1}^2}{21}$

   b. $p_n = p_{n-1} - \dfrac{p_{n-1}^3 - 21}{3p_{n-1}^2}$

c. $p_n = p_{n-1} - \dfrac{p_{n-1}^4 - 21p_{n-1}}{p_{n-1}^2 - 21}$

d. $p_n = \left(\dfrac{21}{p_{n-1}}\right)^{1/2}$

For each method, compute approximations for the value using $p_0 = 1$ and $\epsilon = 10^{-10}$ for the stopping criteria $|p_n - p_{n-1}| < \epsilon$. Rank the methods in order based on the apparent speed of convergence.

6. Use fixed-point iteration to find an approximation to the fixed point of $g(x) = 2^{-x}$ using $p_0 = 1$ accurate to within $\epsilon = 10^{-4}$. Use the stopping criteria

$$\left|\frac{p_n - p_{n-1}}{p_n}\right| < \epsilon.$$