

# Set Notation Help

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## Some set notation

Let  $Z$  be the set of integer  $\geq 0$ . Show  $Z \times Z \times Z$  is countable by constructing the bijection  $f : Z \times Z \times Z \rightarrow \mathbb{N}$   
I am assuming that  $0 \notin \mathbb{N}$ , so  $\mathbb{N} = (1, 2, 3, \dots)$

*Proof.* First we show that there is a bijection  $g : Z \times Z \rightarrow \mathbb{N}$ .

blah, blah, blah

$g : Z \times Z \rightarrow \mathbb{N}$  is a bijection and therefore  $Z \times Z$  is countable.

We may now construct  $f : Z \times Z \times Z \rightarrow \mathbb{N}$  via composition:

$$f(Z_1, Z_2, Z_3) = h(g(Z_1, Z_2), Z_3)$$

□

*Proof.*  $L1 = \{ M \mid M \text{ accepts } w \text{ if } w \text{ contains the substring } 10 \}$  is undecidable

Blah, blah, blah

If  $M(w)$  accepts, then  $L(HWAMw) = \{ w \mid w \text{ contains the substring } 10 \}$

If  $M(w)$  rejects or fails to halt, then  $L(HWAMw) = \emptyset$

Thus, we know  $(M, w) \in A_{TM} \Leftrightarrow HWAMw \in L1$

□

*Proof.*  $L_2 = \{ M \mid M \text{ accepts an odd number of strings} \}$  is undecidable

$P(M)$  if  $|L(M)|$  is odd.

$x \in \{ 000, 111, 101 \}$

Blah, blah, blah

If  $M(w)$  accepts, then  $L(HWBMw) = \{ 000, 111, 101 \}$ . Note that  $|L(HWBMw)| = 3$ , which is odd.

If  $M(w)$  rejects or fails to halt, then  $L(HWBMw) = \emptyset$ , so  $|L(HWBMw)| = 0$

Thus, we know  $(M, w) \in A_{TM} \Leftrightarrow HWBMw \in L_2$

□