Set Notation Help

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Some set notation

Let Z be the set of integer ≥ 0. Show $Z \times Z \times Z$ is countable by constructing the bijection $f : Z \times Z \times Z \rightarrow \mathbb{N}$

I am assuming that $0 \notin \mathbb{N}$, so $\mathbb{N} = (1, 2, 3, \cdots)$

Proof. First we show that there is a bijection $g : Z \times Z \rightarrow \mathbb{N}$.

blah, blah, blah

$g : Z \times Z \rightarrow \mathbb{N}$ is a bijection and therefore $Z \times Z$ is countable.

We may now construct $f : Z \times Z \times Z \rightarrow \mathbb{N}$ via composition:

$f(Z_1, Z_2, Z_3) = h(g(Z_1, Z_2), Z_3)$

$\square$

Proof. $L1 = \{\, M \mid$ M accepts w if w contains the substring 10$\,\}$ is undecidable

Blah, blah, blah

If M(w) accepts, then L(HWAMw) = $\{\, w \mid$ w contains the substring 10$\,\}$

If M(w) rejects or fails to halt, then L(HWAMw) = $\emptyset$

Thus, we know $(M, w) \in A_{TM} \Leftrightarrow HWAMw \in L1$

$\square$

Proof. $L2 = \{ M \mid \text{M accepts an odd number of strings} \}$ is undecidable

$P(M)$ if $|L(M)|$ is odd.

$x \in \{ 000, 111, 101 \}$

Blah, blah, blah

If M(w) accepts, then L(HWBMw) = $\{ 000, 111, 101 \}$. Note that $|L(HWBMw)| = 3$, which is odd.

If M(w) rejects or fails to halt, then L(HWBMw) = $\emptyset$, so $|L(HWBMw)| = 0$

Thus, we know $(M, w) \in A_{TM} \Leftrightarrow HWBMw \in L2$

□