Z Example Taken from Some Homework

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Introduction

The Class Manager's Assistant module must provide an interface for dealing with students enrolling in a class. Students enroll in a class. The result of the enrollment is specified by *EnrollResponse*. At some point in time, a student may be tested; it is assumed that a student will pass the test. When a student is tested, the Class Manager's Assistant must provide an appropriate *TestResponse*.

```
Given Set [Student]  | size : \mathbb{N}   EnrollResponse ::= success | alreadyenrolled   TestResponse ::= success | alreadytested | notenrolled
```

Abstract State Schema

For a *Class* in the Class Manager's Assistant, a student must enroll in a class to be considered enrolled. Once a student is enrolled, the student may be tested.

```
Class enrolled: \mathbb{P}Student tested: \mathbb{P}Student tested \subseteq enrolled \#enrolled \le size
```

Initialization of Abstract State

It is initially assumed that a class is empty, and therefore has no students enrolled.

```
[Class' \mid enrolled' = 0]
```

Operation - Enroll

When a student requests to be enrolled, that student is added to the Class.

```
\triangle Class
S?: Student
R!: EnrollResponse
S? \notin enrolled
enrolled' = enrolled \cup \{S?\}
R! = success
```

If a student has already enrolled, then that student can not enroll in the Class again. Please note that this would be regardless of the student's test status since $tested \subseteq enrolled$.

```
egin{align*} AlreadyEnrolled & & & & \\ & & & & \\ & & & & \\ S?:Student & & & \\ R!:EnrollResponse & & & \\ & & & \\ S? \in enrolled & & \\ S? = alreadyenrolled & & \\ \end{aligned}
```

Therefore, the enroll operation must be the disjunction of EnrollOK and AlreadyEnrolled.

 $Enroll \triangleq EnrollOK \lor AlreadyEnrolled$

Operation - Test

When an enrolled student is tested, the student passes and is now part of the student population in the class marked as tested.

```
TestOK \triangle Class
S?: Student
R!: TestResponse

S? \in enrolled
S? \notin tested
tested' = tested \cup \{S?\}
enrolled' = enrolled
R! = success
```

If a student has not enrolled, then the attempt to test the student must necessarily fail.

NotEnrolled \subseteq $\Xi Class$ S?: Student R!: TestResponse $S? \not \in enrolled$ S? = notenrolled

In addition, a student can not be tested twice.

AlreadyTested $\Xi Class$ S?: Student R!: TestResponse S? = tested R! = alreadytested

Therefore, the test operation must be the disjunction of TestOK, NotEnrolled, and AlreadyTested.

 $Test \triangleq TestOK \lor NotEnrolled \lor AlreadyTested$