Z Example Taken from Some Homework

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Introduction

The Class Manager's Assistant module must provide an interface for dealing with students enrolling in a class. Students enroll in a class. The result of the enrollment is specified by *EnrollResponse*. At some point in time, a student may be tested; it is assumed that a student will pass the test. When a student is tested, the Class Manager's Assistant must provide an appropriate *TestResponse*.

Given Set [Student]
$| \; size : \mathbb{N}$

$EnrollResponse ::= success \mid alreadyenrolled$
$TestResponse ::= success \mid alreadytested \mid notenrolled$

Abstract State Schema

For a *Class* in the Class Manager's Assistant, a student must enroll in a class to be considered enrolled. Once a student is enrolled, the student may be tested.

Class

$enrolled : \mathbb{P} Student$
$tested : \mathbb{P} Student$

$tested \subseteq enrolled$
$\#enrolled \leq size$

Initialization of Abstract State

It is initially assumed that a class is empty, and therefore has no students enrolled.

$[Class' \mid enrolled' = 0]$

Operation - Enroll

When a student requests to be enrolled, that student is added to the Class.

EnrollOK

$\Delta Class$
$S? : Student$
$R! : EnrollResponse$

$S? \notin enrolled$
$enrolled' = enrolled \cup \{S?\}$
$R! = success$

If a student has already enrolled, then that student can not enroll in the Class again. Please note that this would be regardless of the student's test status since $tested \subseteq enrolled$.

AlreadyEnrolled

$\Xi Class$
$S? : Student$
$R! : EnrollResponse$

$S? \in enrolled$
$S? = alreadyenrolled$

Therefore, the enroll operation must be the disjunction of EnrollOK and AlreadyEnrolled.

$Enroll \triangleq EnrollOK \lor AlreadyEnrolled$

Operation - Test

When an enrolled student is tested, the student passes and is now part of the student population in the class marked as tested.

TestOK

$\Delta Class$
$S? : Student$
$R! : TestResponse$

$S? \in enrolled$
$S? \notin tested$
$tested' = tested \cup \{S?\}$
$enrolled' = enrolled$
$R! = success$

If a student has not enrolled, then the attempt to test the student must necessarily fail.

NotEnrolled

$\Xi Class$
$S? : Student$
$R! : TestResponse$

$S? \notin enrolled$
$S? = notenrolled$

In addition, a student can not be tested twice.

AlreadyTested

$\Xi Class$
$S? : Student$
$R! : TestResponse$

$S? = tested$
$R! = alreadytested$

Therefore, the test operation must be the disjunction of TestOK, NotEnrolled, and AlreadyTested.

$$Test \triangleq TestOK \lor NotEnrolled \lor AlreadyTested$$