"Application of FEA Solutions and Analytical Techniques in Resolving a Fixed Beam Problem"

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1 Problem Statement

A construction company is debating the spacing of steel H-beams for a construction project. They're debating between spacing of 2.5 m and 4 m beams. The spacing of 2.5m will result in the structure supporting stronger loads while the 4m spacing will result in using fewer beams and thus save the cost of the project. It is estimated that the beam will need to support up to 10 kN of a uniform force. Which spacing should be most appropriately used?

Before we begin, it is important to know what the maximum deflection limit and shear stress of a steel beam are. According to the International Building Code (IBC), the maximum deflection limit, δ_{max}, of a beam should be the span of the beam, L, divided by the 360. We can mathematically define this by,

$$\delta_{max} = \frac{L}{360}. \tag{1}$$

The maximum shear strength for steel varies between 345 and 525 MPa. For the sake of safety, we will consider 345 MPa to be the limit. [Chi20]

2 Method to solve

In this paper, we will use a finite element analysis (FEA) solution and analytical techniques to solve the problem. The beams used are W100mm x 100 mm x 19.3 H-beams.

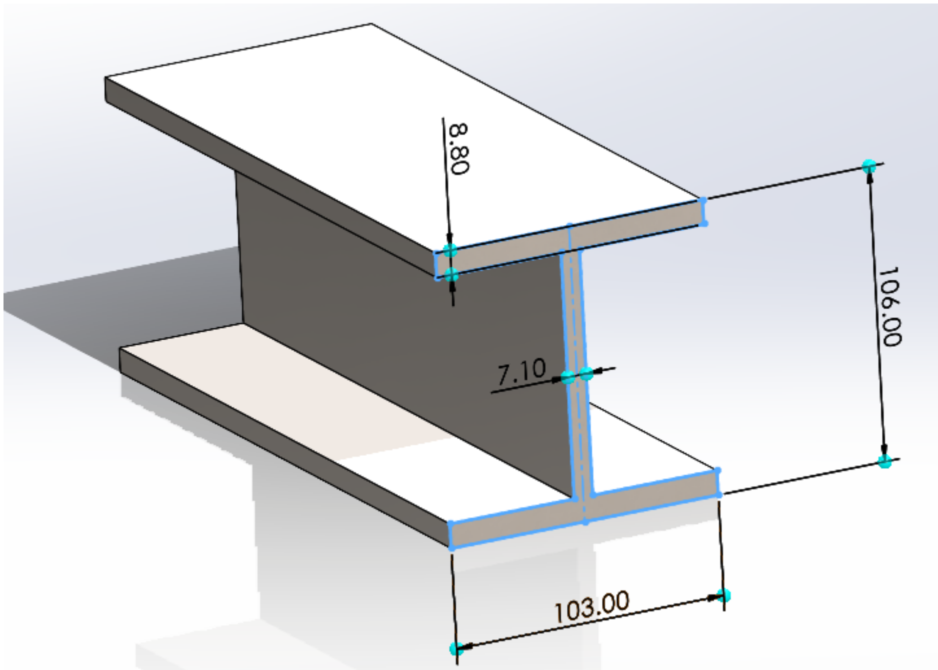

Figure 1: This is representation the W100mm x 100 mm x 19.3 H-beam. The depth and width of the beam are 106mm and 103mm respectively. The web and flange thicknesses are 7.1mm and 8.8mm respectively. The span (length) is not defined in this image.

2.1 Finite Element Method

For this analysis, we will use Ansys Student R2 as our FEA solver software. The geometry was created in SOLIDWORKS and then imported into Ansys. For meshing, the element size was set to .05m. The element's shapes were quadrilateral and also 2nd order. The end of the support beams were set to "fixed" and a 10kN force was top surface of the beam.

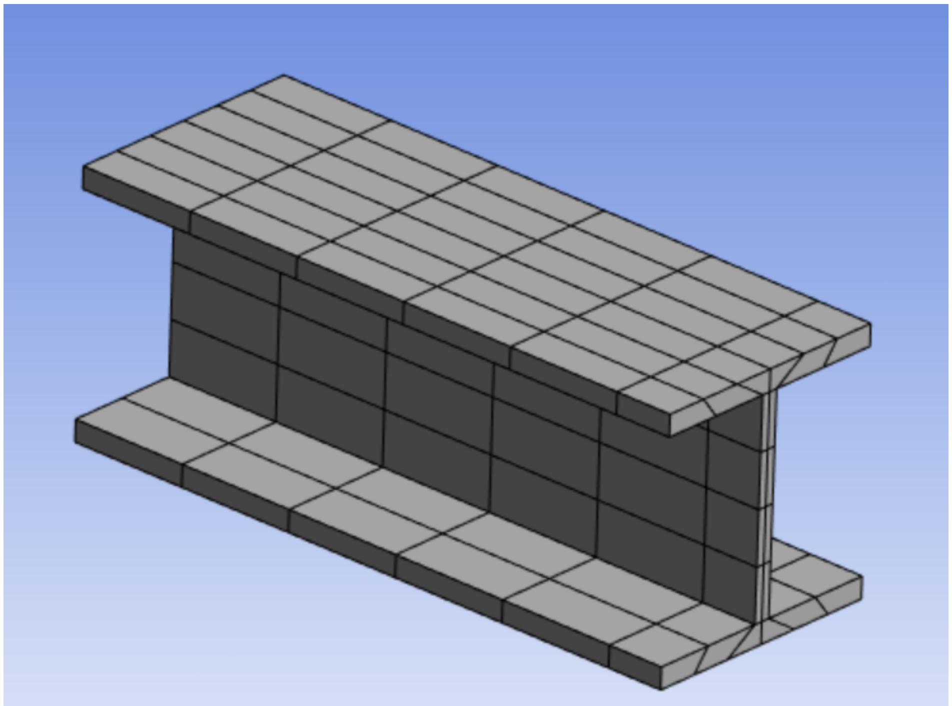

Figure 2: This represents the mesh generated for a section steel beam with element size set to .05”.

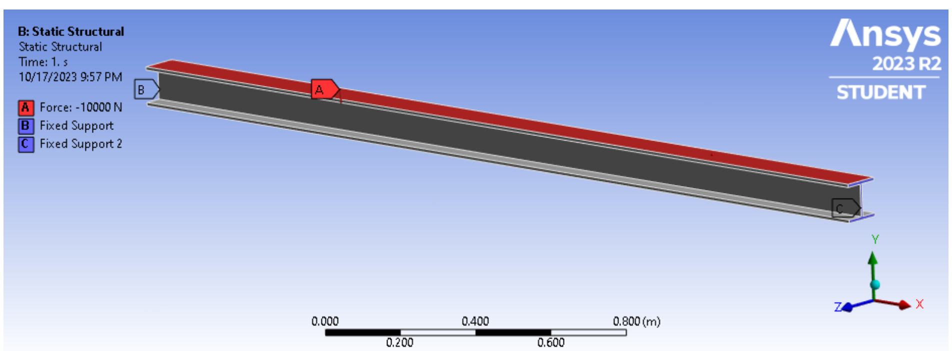

Figure 3: Here is an image of the steel beam with the constraints and load applied to the system. Note that the two faces (Faces B and C) on the end are fixed and a uniform force of 10 kN is applied in the downward y direction (Face A).

2.2 Analytical Method

Although the system is composed of many parts with many complex calculations, there are still ways to use analytical techniques. Consider the system to be completely rigid across its width and depth and only the length will deform. This will reduce our system down to a 1-D analysis where the deformation, y, acts as the dependent variable. Although this simplifies our system, it gives a general framework to validate the FEA solution.

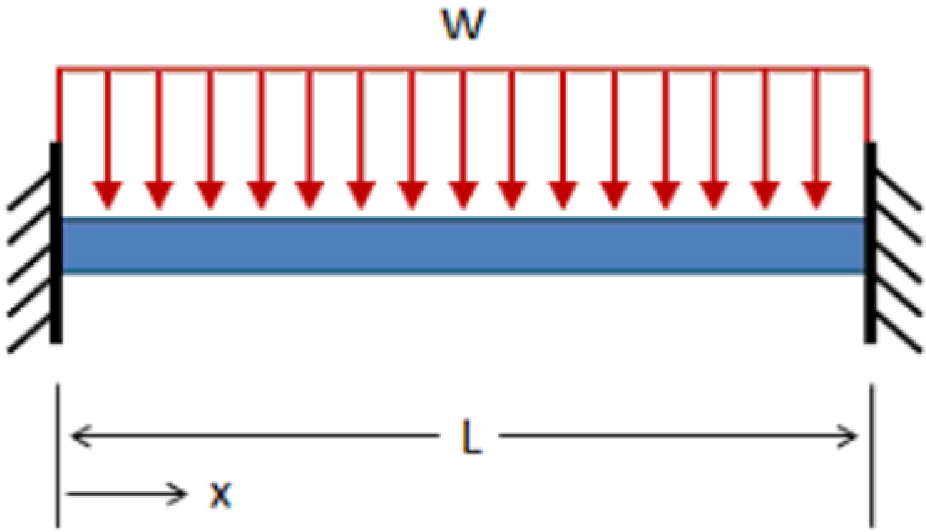

Figure 4: The free body diagram (FBD) of our 1-D system. Note that the two ends of the beam are fixed and a uniform force, w, is applied to the system. The variable L represents that span of the system.

2.2.1 Elastic Curve

The elastic curve defines the deflection, y, of the beam based on the distance from the origin, x, as modeled in Figure 4. The elastic curve for a beam with 2 fixed ends and a uniform force, w, is mathematically defined as,

$$y(x) = \frac{wx^2}{24EI}\left(L - x\right)^2 \tag{2}$$

2.2.2 Maximum Deflection

The maximum deflection occurs at the point where the derivative of the elastic curve is equal to 0. This can defined as,

$$\frac{dy}{dx} = 0. \tag{3}$$

Note that,

$$\frac{dy}{dx} = \frac{w}{24EI}\left(2L^2x - 6Lx^2 + 4x^3\right). \tag{4}$$

By combining Equations 3 and 4,

$$x = L/2. \tag{5}$$

Thus, by combing equation 2 and 5, our max deflection, y_{max} is,

$$y_{max} = \frac{wL^4}{384EI}. \tag{6}$$

[Hib23]

Using equations 1 and 6, it should be noted that

$$\begin{aligned} \delta_{max} &\geq y_{max} \\ \implies \frac{L}{360} &\geq \frac{wL^4}{384EI}. \end{aligned} \tag{7}$$

If Equation 7 is satisfied, then the beam should be safe to use, assuming complete rigidity among the width and depth of the entire beam.

3 Results

3.1 FEA Solution

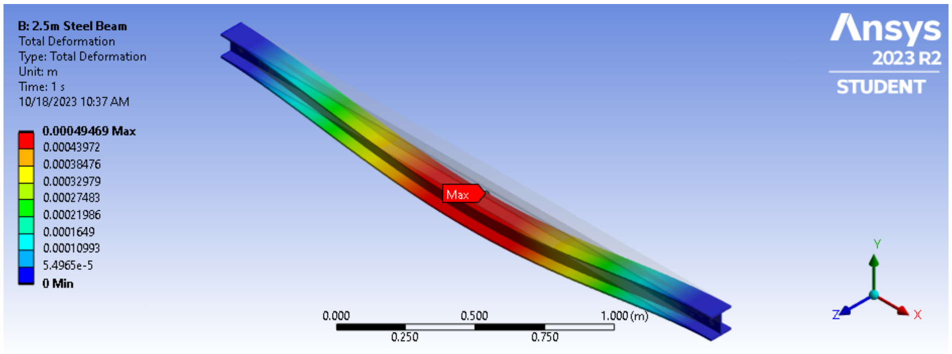

Figure 5: The image provided depicts the FEA solution for the total deflection of a 2.5m steel beam with fixed ends, subjected to a uniform 10kN distributed force applied along the top surface of the beam. It should be noted the deflection is grossly exaggerated to show a clear distinction of bending.

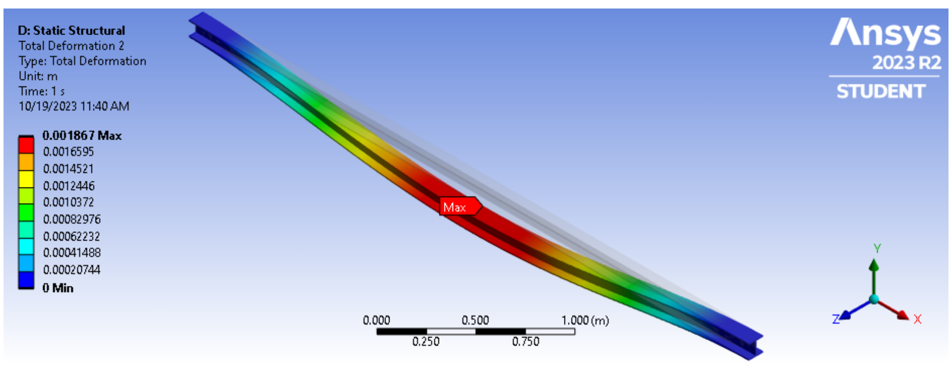

Figure 6: The image provided depicts the FEA solution for the total deflection of a 4m steel beam with fixed ends, subjected to a uniform 10kN distributed force applied along the top surface of the beam. It should be noted the deflection is grossly exaggerated to show a clear distinction of bending.

	2.5m Beam	4m Beam
Average Deflection [mm]	.26728	.99964
Max Deflection [mm]	.49469	1.867
Max Shear Stress, XY [MPa]	7.9288	8.2131
Max Shear Stress, YZ [MPa]	.88298	1.2297
Max Shear Stress, XZ [MPa]	6.2168	9.5802

Table 1: Results from the FEA simulation of 2.5m steel beam and 4m steel that experienced a 10 kN uniformly distributed force along the top face of the beam with fixed ends.

3.2 Validating the FEA solution with 1-D Analytical Solution

3.2.1 2.5m Deflection Path

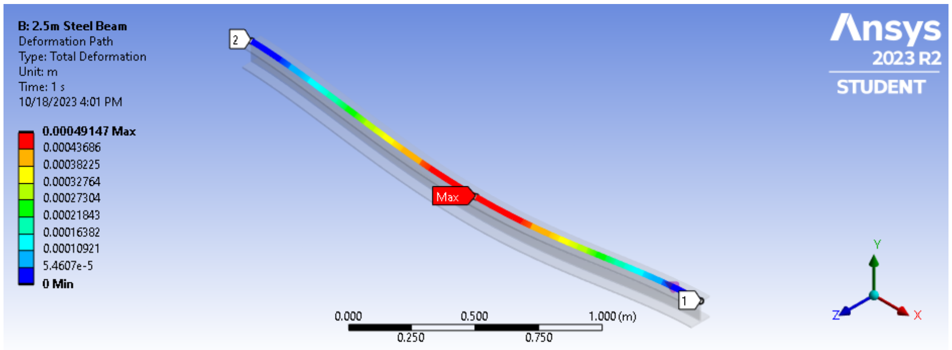

Figure 7: Deflection of the 2.5m steel beam along the center of the top surface.

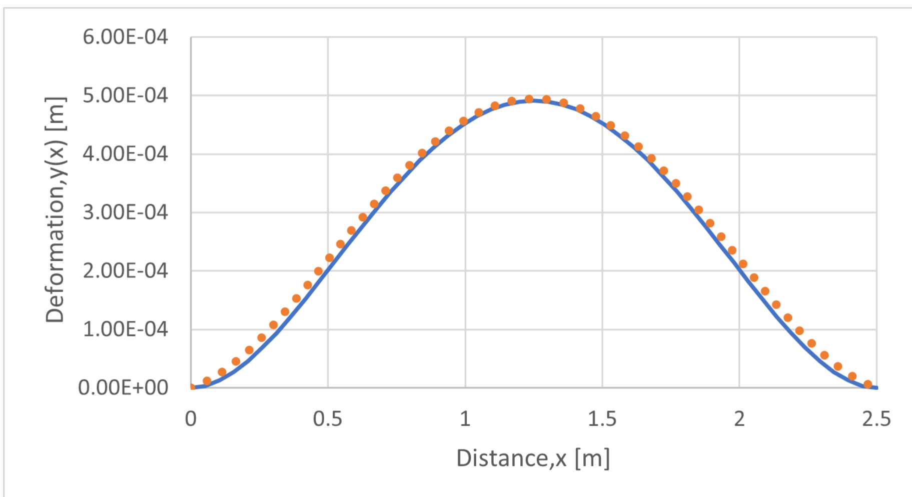

Figure 8: The blue line represents the elastic curve to the analytical solution from Equation 2 for a 2.5m beam. The orange dots represent the FEA solution pictured in Figure 7. Notice how the FEA solution slightly overestimates the amount of deflection but still converges to the analytical solution.

3.2.2 4m Deflection Path

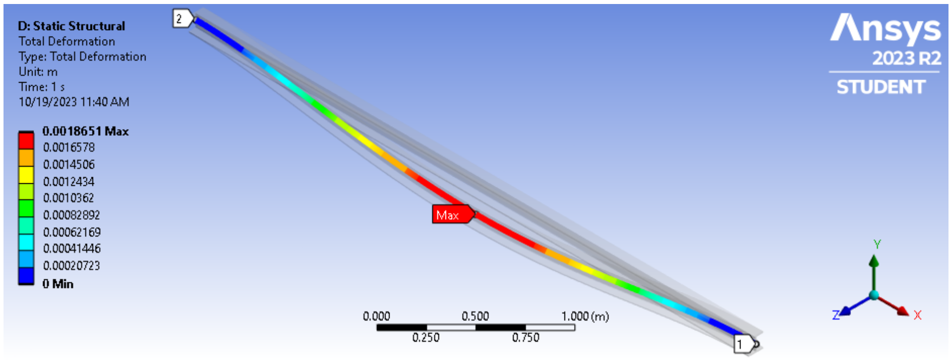

Figure 9: Deflection of the 4m steel beam along the center of the top surface.

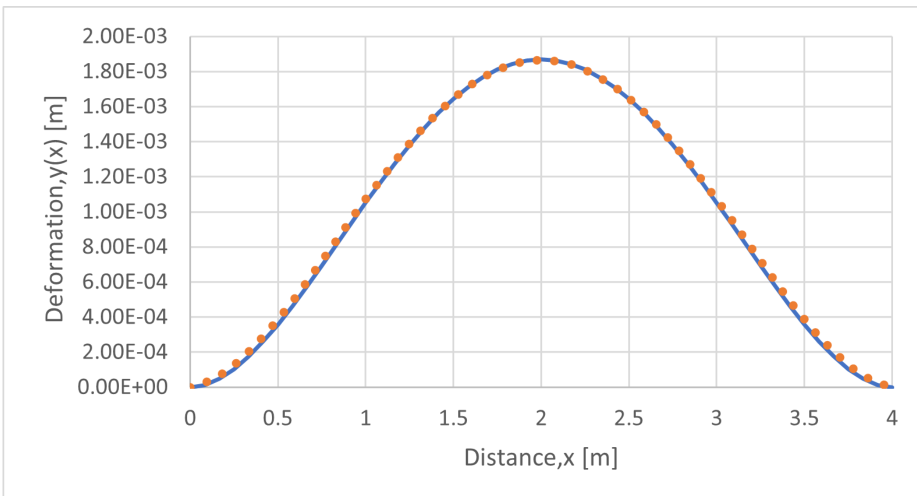

Figure 10: The blue line represents the elastic curve to the analytical solution from Equation 2 for a 4m beam. The orange dots represent the FEA solution pictured in Figure 9. Notice how the FEA solution slightly overestimates the amount of deflection but still converges to the analytical solution.

4 Discussion

Using a 2.5m steel beam, the maximum limit was calculated as $\delta_{max} = 6.9$ mm. According to Table 1, the maximum deflection was found to be 0.49469 mm, significantly below the threshold of δ_{max}. This outcome was further substantiated by the application of Equation 6, yielding a value of $y_{max} = 0.4915$ mm, which also remains well under δ_{max}. Similarly, the maximum shear stress, recorded as 7.9288 MPa in Table 1, falls comfortably below the established limit of 345 MPa. Consequently, it can be concluded that this beam is entirely safe for use, considering the imposed loads on the system.

In the case of the 4m steel beam, the maximum limit, δ_{max}, is 11.1 mm. As indicated in Table 1, the maximum deflection was calculated to be 1.867 mm, significantly below the limit of δ_{max}. Further validation through the utilization of Equation 6 demonstrated a corresponding y_{max} value of 1.87003 mm. Additionally, the maximum shear stress of 7.9288 MPa, as outlined in Table 1, remains comfortably below the 345 MPa threshold. Consequently, it is deemed safe to utilize this particular beam, given the loads exerted on the structure.

Based on the comprehensive structural analysis conducted, it has been determined that the maximum deflection and shear stress for both beams are notably below their respective maximum limits, indicating the structural integrity and safety of the beams. In light of the construction company's objective to optimize cost efficiency, the application of a 4m beam spacing should be endorsed as a prudent choice, as it not only meets safety requirements but also utilizes less material, thereby effectively reducing overall costs. However, it is advised to consider the feasibility of employing a higher beam spacing, such as 8 or 12 meters, and subject it to rigorous analysis to ensure that the structural stability is maintained while potentially yielding further cost savings. This approach would allow for a comprehensive evaluation of the structural design, enabling the company to strike an optimal balance between safety, functionality, and cost-effectiveness in the construction project.

The core focus of the project was to address and analyze a tangible real-life issue, thereby emphasizing the practical application of theoretical concepts. Employing the Finite Element Analysis (FEA) method played a pivotal role in facilitating the precise numerical computation of results, contributing to a comprehensive understanding of the problem's intricacies. Furthermore, the integration of analytical techniques served as a crucial step in validating the FEA outcomes, ensuring their convergence with the expected solution. This amalgamation of computational and analytical approaches not only enhanced the accuracy of the results but also solidified the project's credibility, underscoring the importance of a robust and verifiable methodology in addressing complex real-world challenges.

References

[Chi20] Francis; D.K. Ching. *IBC 2021: International building code*. ICC Publications, 2020.

[Hib23] R. C. Hibbeler. *Mechanics of Materials*. Pearson, 2023.