

Type Theory Study Group

Meeting 1

13.11.2015

Notes

by Ian Godik

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A judgement is a statement about the world.

A proposition is an object we can study.

→ Jon: Rather a statement about the mental state of the creating subject.

Danny: A judgement is meaningless until we define it, provide the rules.

Ast: Defines the form.

* Don't need sorts initially.

Sorts example:

- Expr
- Statement

Leaves vs Inner nodes:

$\text{num}[x]$ defines an infinite amount of nullary operator, the operators don't take a parameter AST-wise.

$\text{num}[x]$ is just the name of the operator.

* Leaves in the book are defined as variables only.

Inner nodes: take a list of parameters, may be empty.

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ABT's:

Invented before, under a different name.
(70's)

Formalizes variable bindings.

* Many different impl.s for variable binding.

Aliasing problem:

Binding:

$\lambda x. \{\text{term that may contain } x\}$

'x' is meaningless, for documentation purposes.

But when implementing or formalizing,
need to worry about collisions.

ABT's encode binding up to α -equivalence.

- De Bruijn indices (read de-brown),
Solves the issue easily,

get rid of the names entirely,
but uncomfortable for humans.

* ABT's frequently implemented using
de-Bruijn indices.

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Operators that bind variables:

$\text{lam } (x. \underline{x})$
can use x here.

$\lambda x. \underline{x}$
can use x here.

"Sorts can be thought as the types of ABTs"

Jon: The reason sorts and types feel similar is that ABTs themselves constitute a kind of type theory.

Induction:

Most of the proofs will go by induction on rules.
Judgement- thinking about something in our brain,
then claiming some rules represent it,
then go by a mechanical process on the rules,
and arriving to a result which
respect our mental model.

* "All things that this judgement defines
provide the means for finitely analyzing them."

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Jon:

Distinguish between infinite proofs in general
- which are fine,
and non-well-founded proofs,
for example, proofs of infinite depth.

Example of proof of infinite breadth:
the "omega rule"