Systemtheorie der Sinnesorgane

1 Übung

1.1

For the underlying problem, we constructed a sinusoidal wave with an amplitude of $u_{max} = 1\,\mathrm{V}$ around the steady component of $u_{DC} = 0.5\,\mathrm{V}$. Thereby, the frequency of the oscillation was set to $f = 1\,\mathrm{Hz}$ in the time interval of $t \in [0,\ 2]$ s. For further analyses, a sampling frequency of $f_s = 20\,\mathrm{Hz}$ was chosen. The sampling points are depicted as squares (Fig. 1).

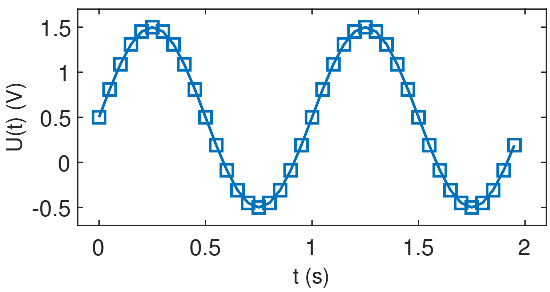

FIGURE 1 – Sinusoidal base signal for further analysis. The wave was constructed after $u(t) = u_{max}\sin(\omega t) + u_{DC}$ with $\omega = 2\pi f$.

1.2

In this exercise, the signal should be decomposed into its underlying frequencies by a Discrete Fourier Transformation (DFT), in this case Fast Fourier Transformation (FFT), to a single-sided amplitude spectrum. We are allowed to reduce the complete spectrum, because the transformed signal $u(t)$ is real and hence the relation $U(f) = U(-f)^*$ is valid. In this context, a^* expresses the complex conjugation of a.

As the amplitude spectrum depicts the amplitudes of the composing waves, we can expect the amplitude of 1 at $1\,\mathrm{Hz}$ created by our generating parameters u_{max} and f. Further, the steady component u_{DC} can be expressed as a wave with no frequency, but 0.5 amplitude. The composition of these two then forms $u(t)$.

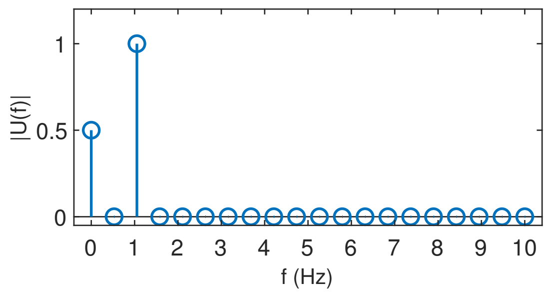

FIGURE 2 – Single-sided FFT amplitude spectrum of the input signal $u(t)$.

1.3

Subsequently, the voltage level of input signal $u(t)$ was computed using equation 1 and is shown in figure 3. 1 V was selected as reference voltage level U_0. A Fourier transformation to the voltage spectrum (shown in Fig. 4) shows many more different frequencies overlapping to the signal.

$$L_U = 20 \log_{10}\left(\frac{U}{U_0}\right) \text{ dBV} \tag{1}$$

$$RMS(L_U) = L_{U,eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} L_U^2 dt} \tag{2}$$

$$\overset{discretized}{\underset{equidist.\ grid}{=}} \sqrt{\frac{1}{N}\sum_{n=1}^{N} |L_{Ui}|^2} \tag{3}$$

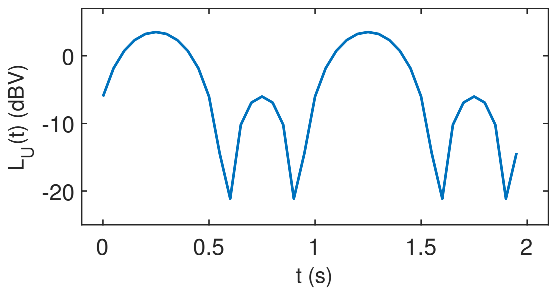

Figure 3 – Voltage level computation after eq.1 from $u(t)$ to the reference voltage $U_0 = 1\,\mathrm{V}$

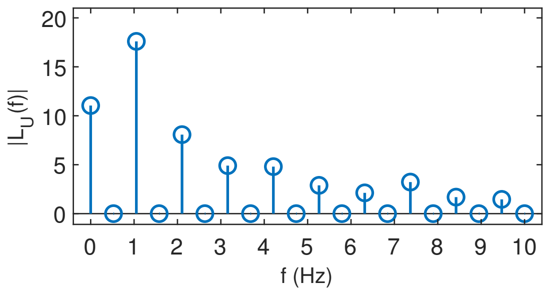

Figure 4 – Single-sided FFT-spectrum of L_U

Additionally, the Root-Mean-Square (RMS) was calculated according to equation 3. To achieve this, we discretized the continuous integral in eq. 2 with $T = n \cdot \Delta t$ under the assumption of constant time steps. Plugging the values of L_U into the equation above gives a RMS of 18.69.