Algebra

Algebra uses variables, which are letters or symbols that represent numbers.

In the US, we use English and Greek letters most commonly for variables.

Constants are quantities that don't change, and letters or symbols can represent constants in any given example. It can be confusing because we can have variables represent constants.

Some constants have set names or symbols like pi π or e or $\sqrt{2}$

We will start with one variable x and we will be talking about doing operations on numbers or data that represent numbers.

Algebra with Real numbers

We will start with one variable and will use x for that variable. We will be talking about doing operations on numbers or data that represent Real numbers.

Real numbers are the set of numbers that include whole numbers, integers, rational numbers, and irrational numbers.

They don't include imaginary numbers, which are part of the Complex number system (quadratic units) or infinitesimals, which are part of the Surreal number system (calculus units).

While addition works on any of those systems, we will talk about them when we meet them in later units.

Algebra with Real numbers

In English, I could say that one pile of chips plus one pile of woodchips equals one big pile of chips.

But we will explicitly use the language of algebra, where we are talking about operations on Real numbers that can be modeled on a number line. So we can say: one plus one equals two.

Operations

Something that you do to numbers, constants, and variables that gives a predictable outcome. The basic operations are:

Addition

  Subtraction (inverse of addition)

Multiplication

  Division and factoring (inverse of multiplication)

Exponentiation

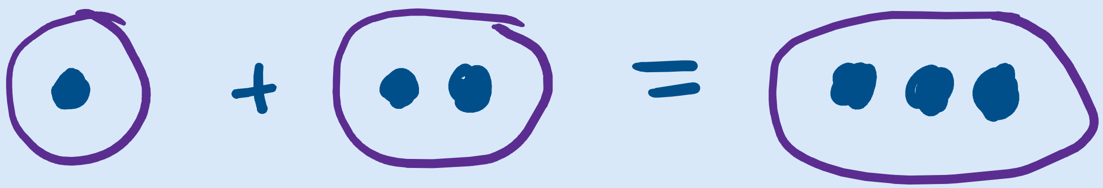
Doing multiple operations

Addition

5 ways to think about addition:
Combining groups or sets
With algebra tiles
On a number line
With vectors
As a function

You can only add like **variables** /You can only add like **terms**
Use the plus sign +

Combining groups or sets



This model works well for **discrete** variables or data

Discrete vs Continuous [Continuous or discrete variable - Wikipedia]

Discrete colors

Continuous color scale

Digital (discrete) vs analog (continuous) time scales.

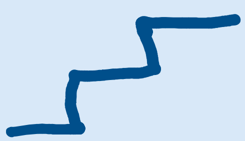
Piano (discrete) vs slide whistle or trombone (continuous).

[Trombone - Wikipedia]

Plucking a guitar string vs tuning a guitar string.

Steps are discrete measurements and a ramp is a continuous measurement.

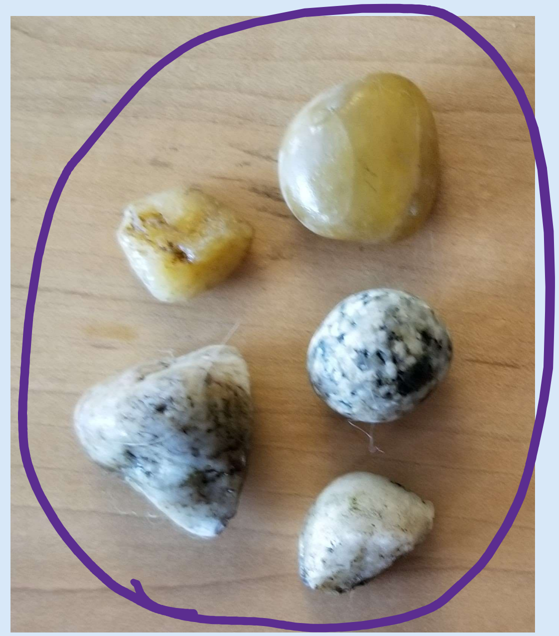
steps



ramp

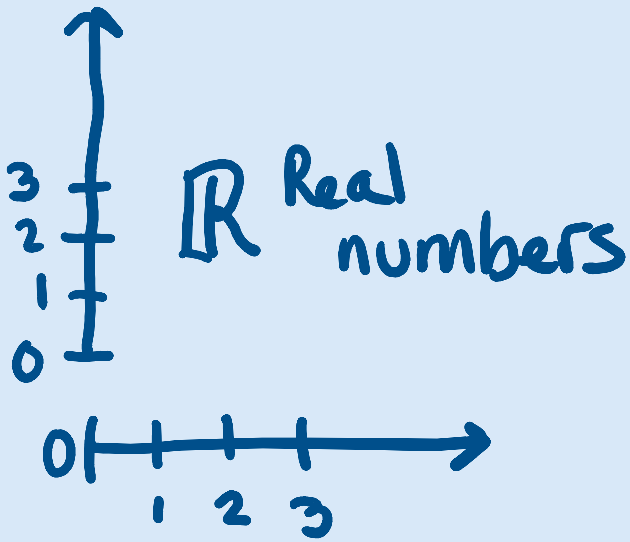
My favorite discrete variables or examples

Can add groups of discrete variables or data and count the total.



$$2 + 3 = 5$$

We can use a number line to model continuous variables or measurement like length or height.



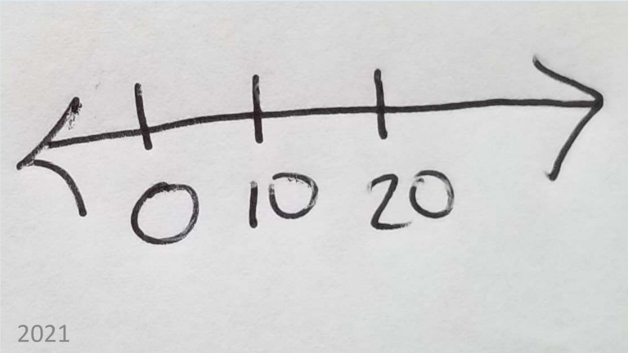
ramp

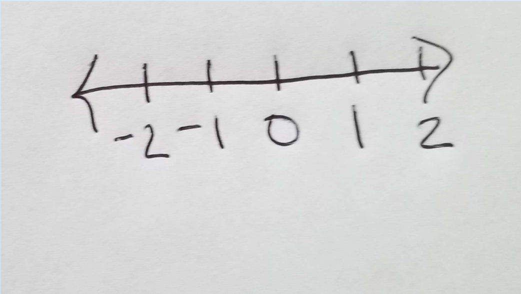
Remember: horizontal number lines get greater to the right.

In our decimal system, bigger numbers are to the left, and we write from greatest or biggest to least or smallest.

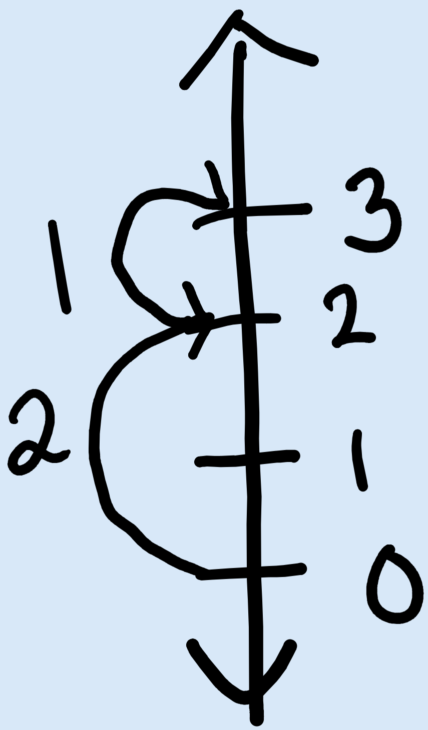
2139.04 the 2 represents two thousand which is much bigger than the 4 which represent 4 hundredths or $\frac{4}{100}$.

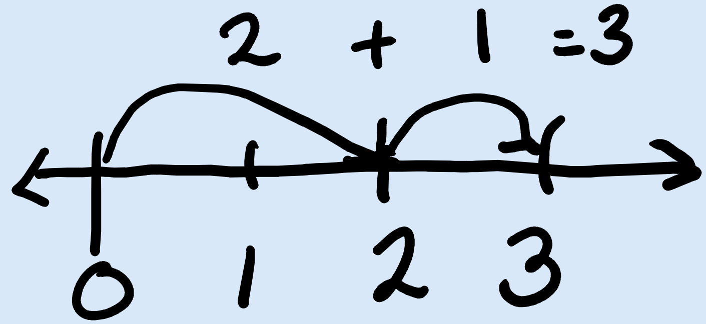
Caution! We are trained to think of the bigger numbers to the left and then the number line has the greater numbers to the right!





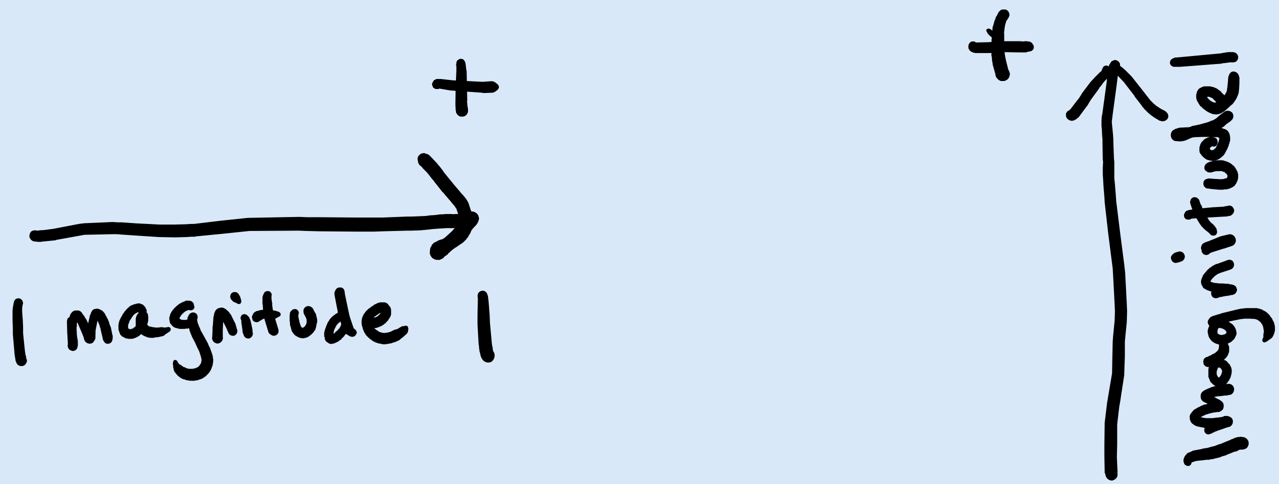
Adding is going up or right on a number line



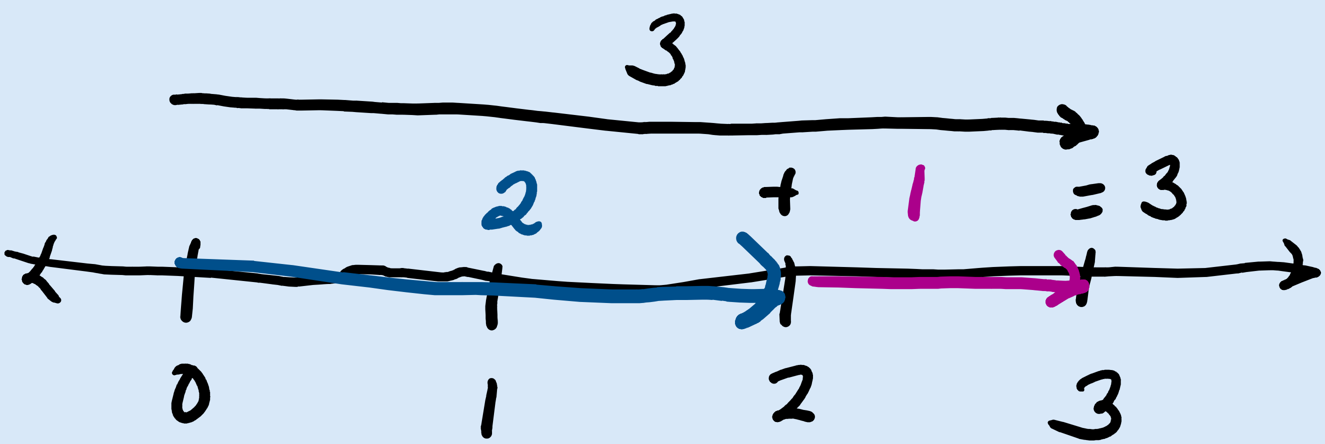


Number lines work well for continuous variables or data.

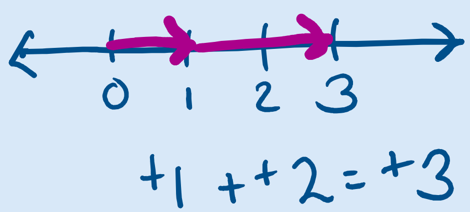
A vector has magnitude which is |length| and direction.
We use the bars to mean length without direction.

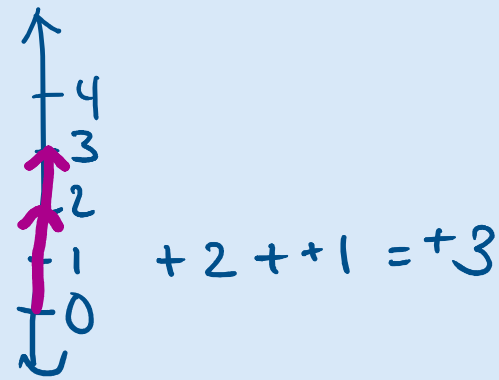


Adding with vectors: the vectors line up on a Real number line. Positive numbers are to the right of zero and adding lines them up head to tail to the right.



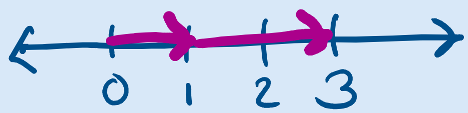
To add vectors, place them head to tail on a number line.



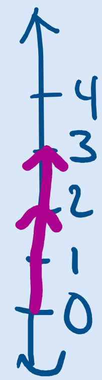


If we are just talking about one variable, then right/left or up/down don't matter as there are only two directions for each of those variables and we use + and – for the two directions.

To add vectors, place them head to tail on a number line. Notice that the order that we add the terms or vectors doesn’t matter.



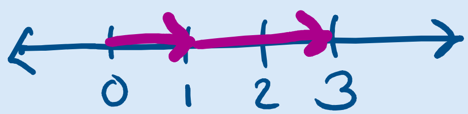
$^{+}1 + {}^{+}2 = {}^{+}3$



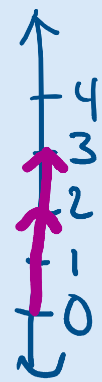
$+2 + {}^{+}1 = {}^{+}3$

The Commutative Property of Addition says that the order that you add the terms doesn't matter. 1+2=2+1

You don't need to remember the name, but I think that the commute or how you get there doesn't matter.

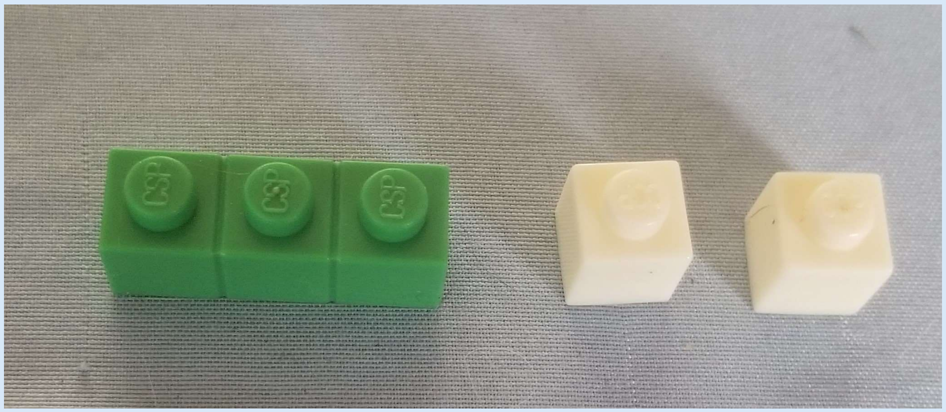


$^{+}1 + ^{+}2 = ^{+}3$



$+2 + ^{+}1 = ^{+}3$

With algebra tiles



x+2 here x has the value of 3

We can also draw algebra tiles.

$x + 2$

$x + 1$

$2x + 3$

A **function** is an operation, procedure, or process that has a predictable outcome. We will soon do a unit on functions but for now, you can think of it as a machine or process that has a predictable output for any given input.

$+1$

2 input x $\boxed{\text{function}}$ 3 output $x+1$

Can only add like terms, units or like variables

The groups have to be made of the same types of things.

It has to be on the same number line or a number line with the same scale.

2 cats + 1 cat = 3 cats

2 cats + 1 bunny can't be added together,

but we can convert:

2 pets + 1 pet = 3 pets

Can only add like terms, units or like variables

2 inches + 1 foot

2 in + 12 in = 14 inches

$\frac{1}{6}$ ft + 1 foot = $1\frac{1}{6}$ ft

2x+1x=3x

x+y can't be added together

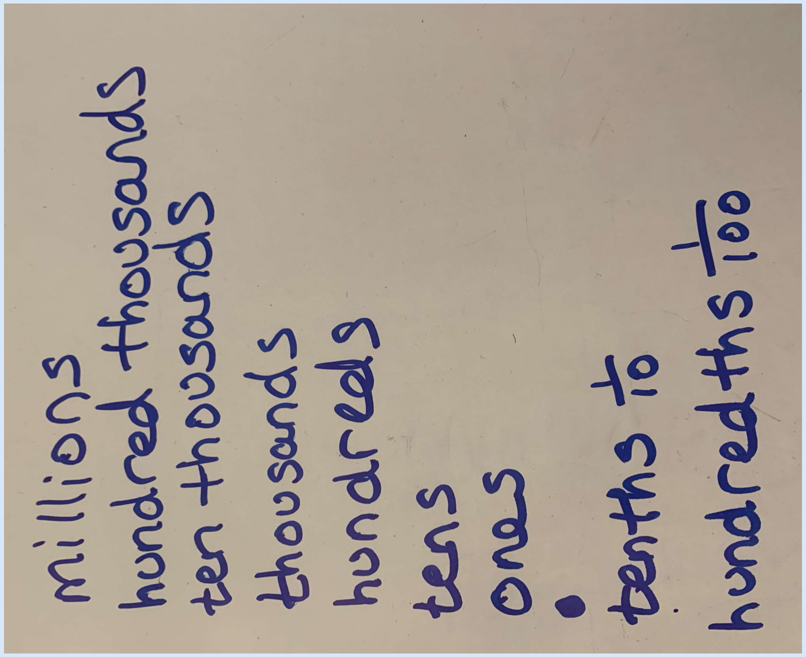
Can you think of some examples of adding like terms or converting to like terms?

I like to add up the like terms to keep them straight. Our number system has place value to tell the like terms.

$$\begin{array}{r} x + 2 + y \\ + \ \underline{x + 1 + 2y} \\ 2x + 3 + 3y \end{array}$$

$$\begin{array}{r} 235 \\ + \ 102 \\ \hline 337 \end{array} \qquad \begin{array}{r} 235 \\ + \ 12 \\ \hline 247 \end{array}$$

You have to add numbers with the same place value.



$$
\begin{array}{r}
431. \\
+\ 25. \\
\hline
456.
\end{array}
$$

Different variables are not like terms. Different units are not like terms.
You can only combine like terms.

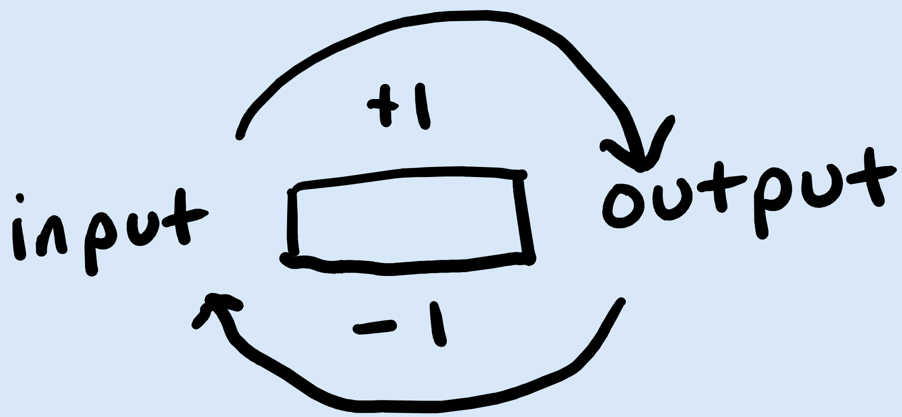
$2x + 2y$

2 inches + 2 feet

Subtraction is the inverse of addition
Subtraction undoes addition of the same number

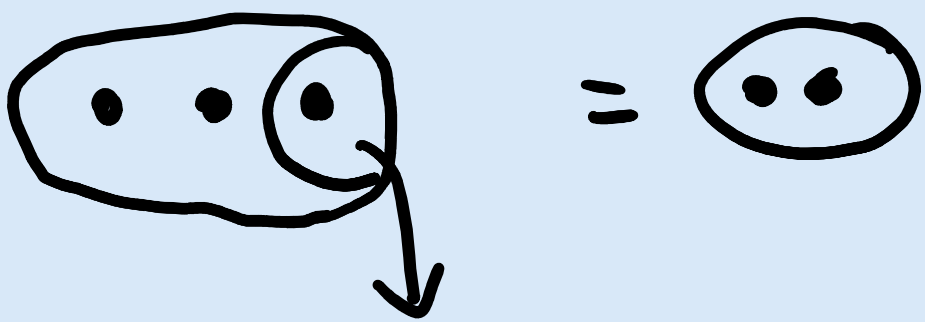
Must subtract like terms or variables

Use the minus sign -



For grouping or sets we can think of taking away.

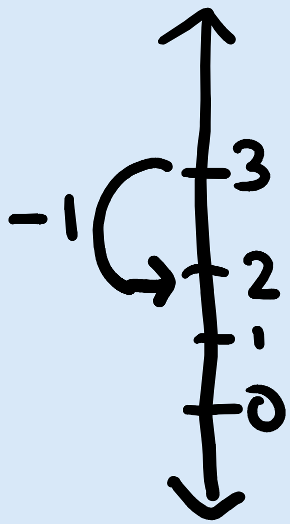
$$3-1=2$$



Subtraction on a number line goes down or to the left. It is the opposite direction of addition.

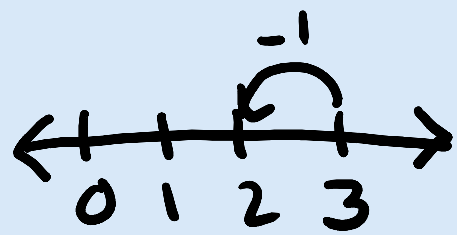
start at 3 down 1 → ends up at 2

$3-1=2$

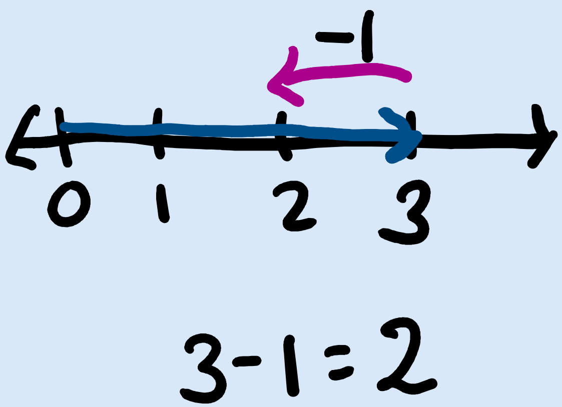


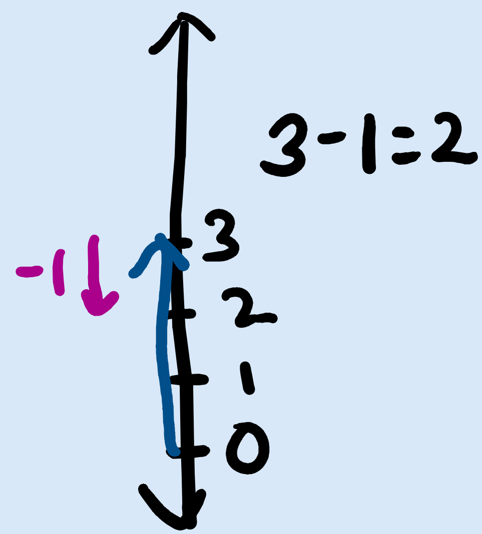
go left 1 end up at 2

$3-1=2$



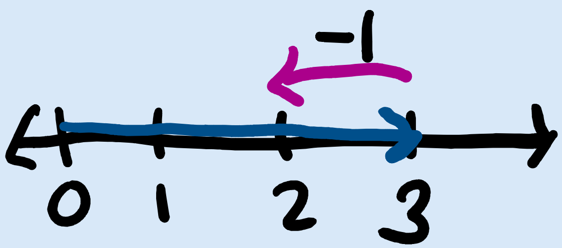
Subtraction with vectors is the same as adding a vector in the opposite direction.
For vectors, negative or minus is the opposite direction of adding or positive.





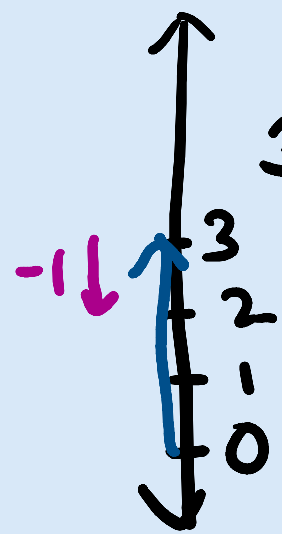
Vectors are helpful for understanding integers, which are the positive and negative counting numbers. Minus a positive number is the same as adding the negative number.   3-1=3+(-1)

$$3-1=3+{}^{-}1$$



$3-1=2$

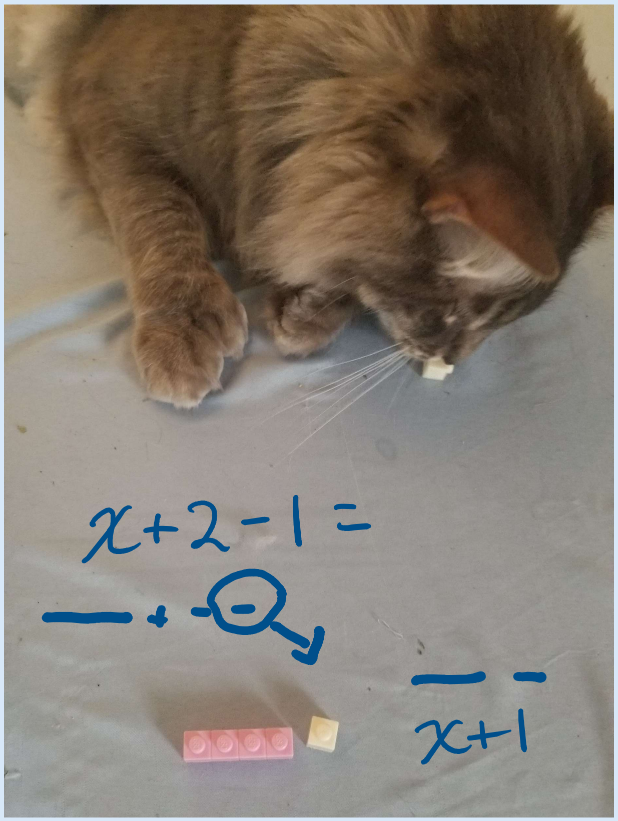
$3+{}^{-}1=2$

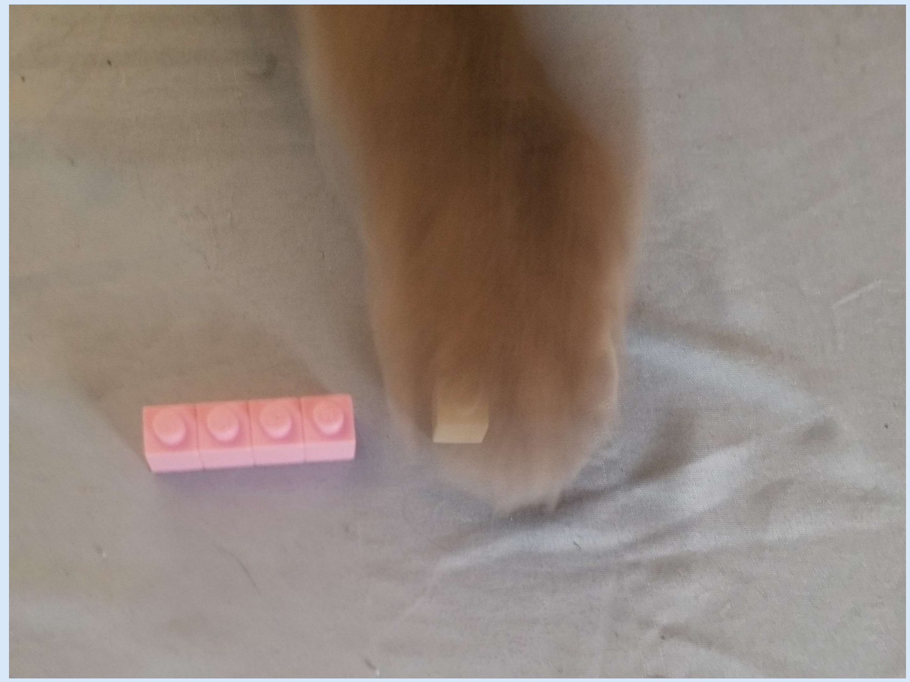


$3-1=2$

$3+{}^{-}1=2$

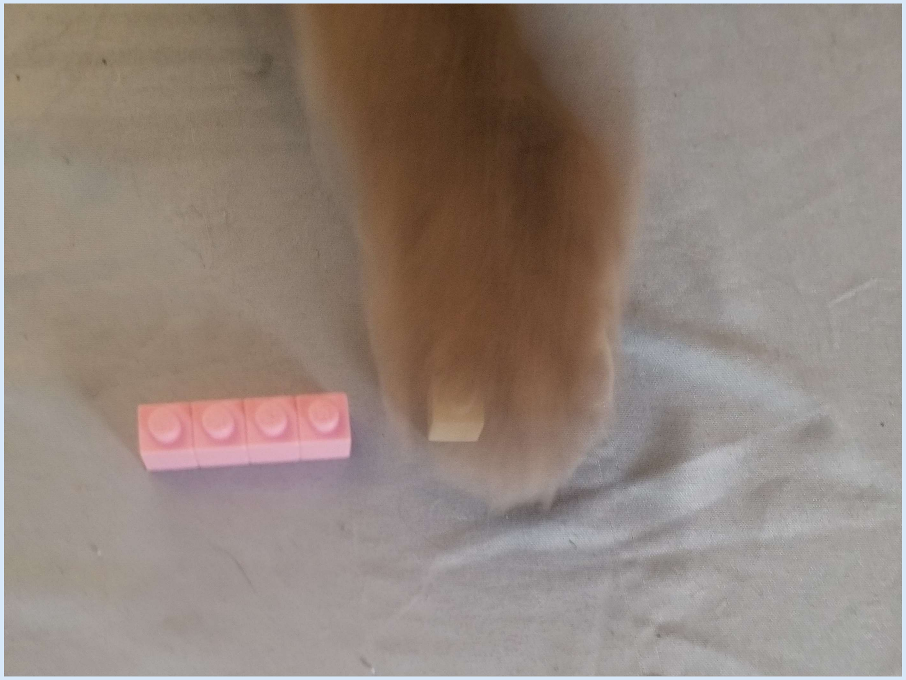
Subtraction with algebra tiles and Doc





$x+1-1=x$

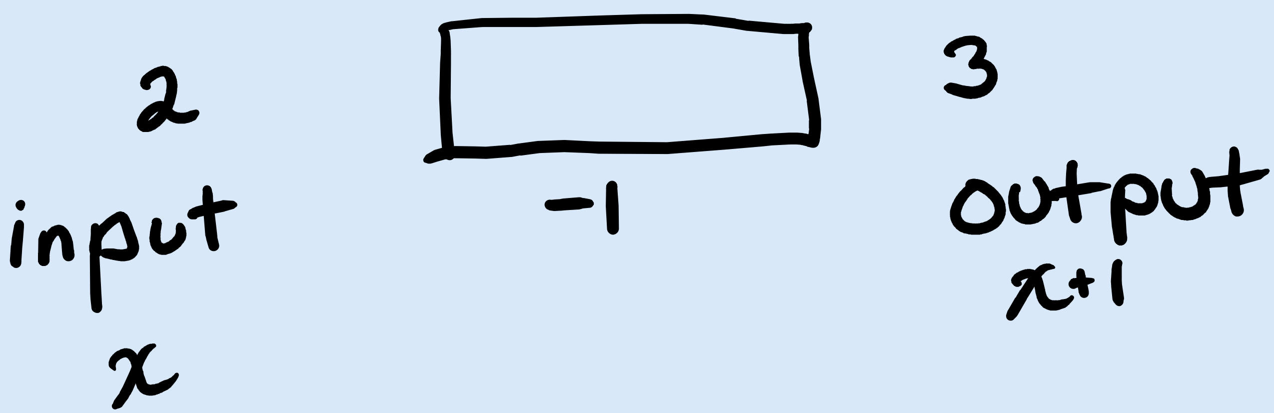
Doc is pointing out that anytime you add a number to a variable and then subtract that same number, you get just the variable.



$$x+1-1=x$$

Subtraction as a function

Subtraction is the **inverse** of addition and undoes the addition function.



The order that we write the terms matters for subtraction.

As we saw with vectors and will see more of when we look at adding and subtracting with integers, subtraction can be converted into adding the opposite sign of the number or variable.

$1 - 2 \neq 2 - 1$

$1 + {}^{-}2 = {}^{-}2 + 1$

Zero is the identity of addition and subtraction.

If we add or subtract zero, the value stays identical.

$2+0=2$ $2-0=2$

$x+0=x$ $x-0=x$

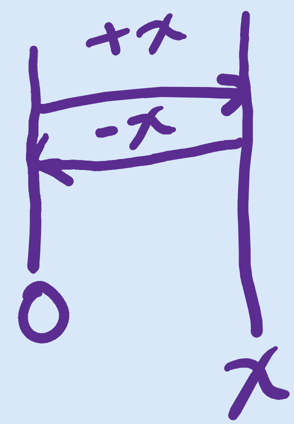
$n+0=n$ $n-0=n$

Any number plus the opposite sign of that number is zero.

Adding and then subtracting the same number gets you back to zero.

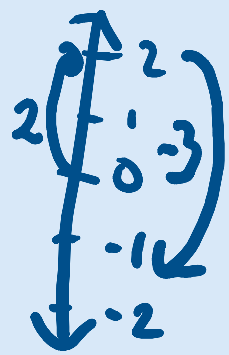
$+x-x=0$

$-x+x=0$



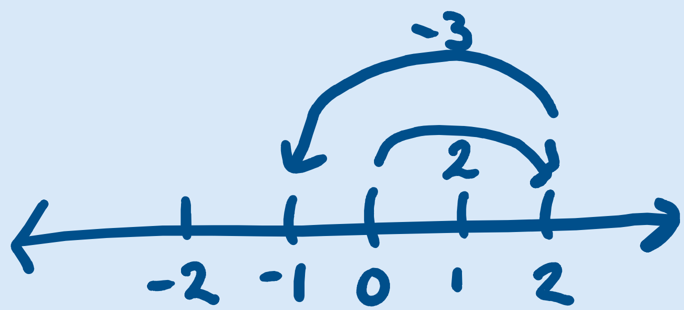
We have been working with whole number integers. Now that we have subtraction as an operation, we can get negative numbers. Next, we will talk about addition and subtraction with Integers, that are the set of positive and negative whole numbers.

$2 - 3 =$



$2 - 3 =$

$2 + {}^{-}3 =$



$2 - 3 =$

$2 + {}^{-}3 =$