# Lines [Line (geometry) - Wikipedia]

In geometry, a point is an infinitely small dot that represents a specific place in space. Technically, a point has no size but it has location.
https://www.bing.com/videos/search?q=video+from+picnic+zoom+in+to+atoms+and+out+to+the+universe&docid=607997194206321339

Incredible zoom. View of atom to universe. – YouTube

A line is the set of all points that fall on a straight path and is infinitely long in both directions. $\leftrightarrow$
Lines have no size, but we write them whatever width our writing utensil shows.

Lines are defined by any two points. Two points define a line. There are an infinite number of points between any two points on a line.

Lines have two directions. The measurement between two points on a line is called the distance or magnitude. |x|

linear function

graph

function $f(x)=ax+b$

equation $y=mx+b$

two points

$$(x_1, y_1)\,(x_2, y_2)$$

$$(x_1, f(x_1))\,(x_2, f(x_2))$$

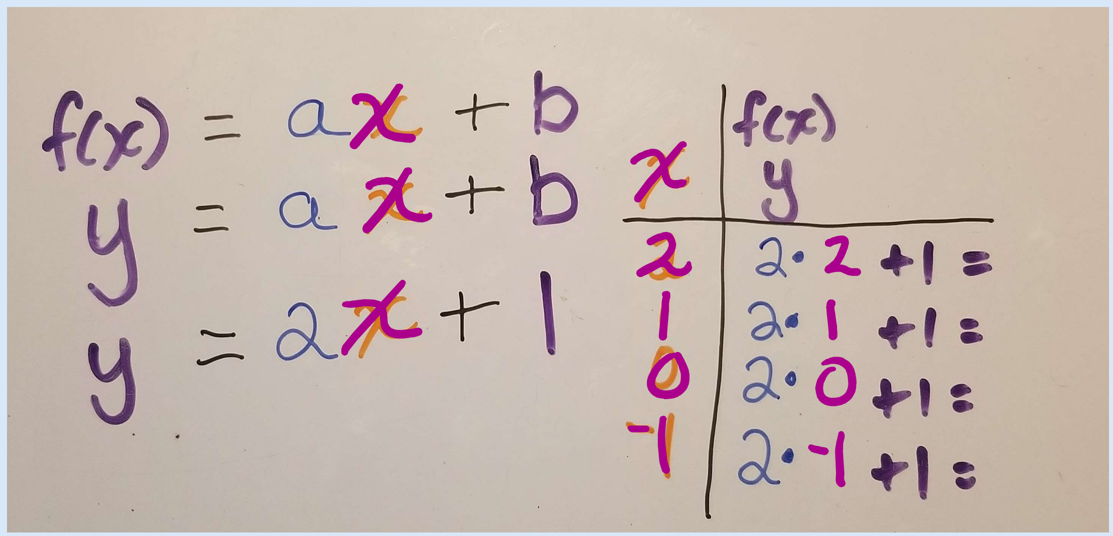
# y=ax+b or f(x)=ax+b is a linear function for ordered pairs (x, y) and (x, f(x))

y or f(x) is the output or total

a is the coefficient, the multiplier, the rate, or the slope

x is the linear term since it is x to the first power $x^1$ and it is the input.

b is the constant term, y-intercept or start



# The linear or first power term has an invisible 1 as the exponent. y=a* $x^1$+b

2x+3 is the same as 2*$x^1$+3

2 is the coefficient, rate, slope, or multiplier

3 is the constant, y-intercept on a graph, or start

When we write x, we don't bother writing the first power.

Every x is the same as $x^1$

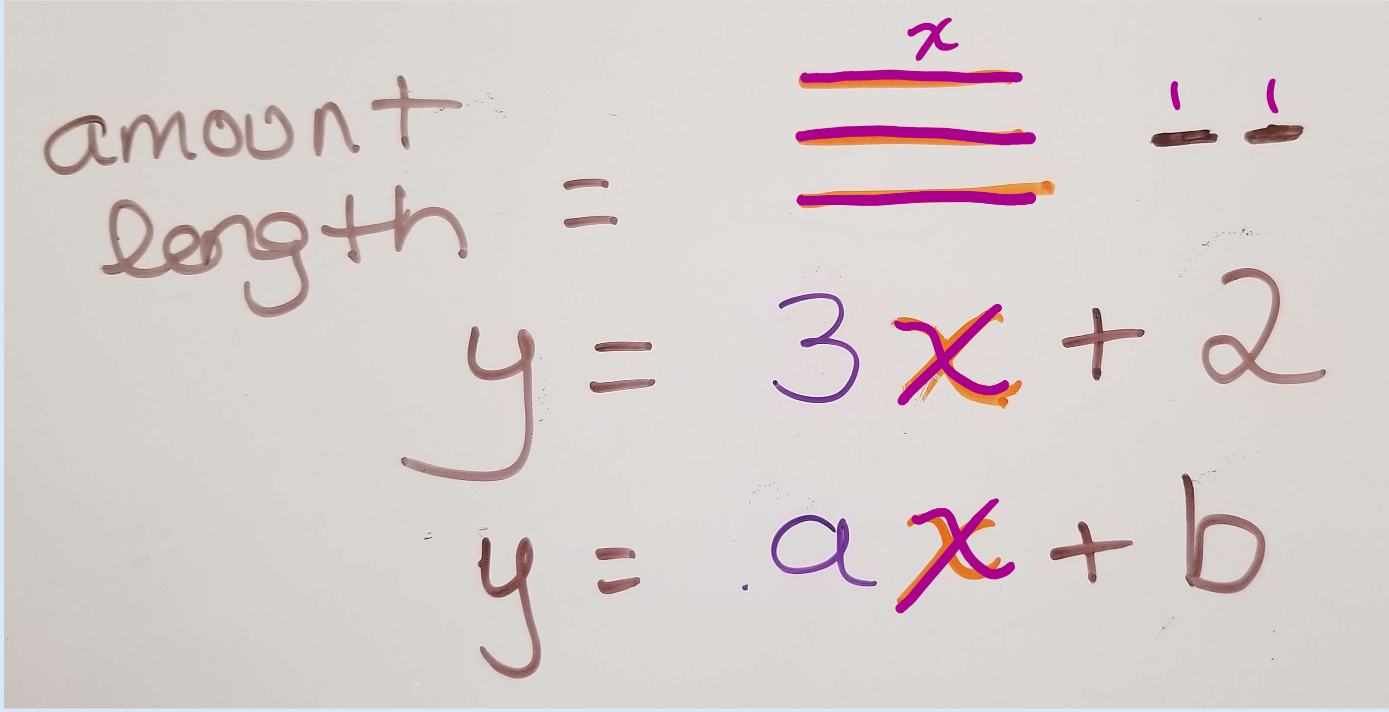
There is an invisible 1 as the power when we write a variable without another power. y=a* $x^1$+b

Thinking about linear, first-order, first-power, or first- degree functions and equations.

$f(x)=ax+b$ or $y=ax+b$

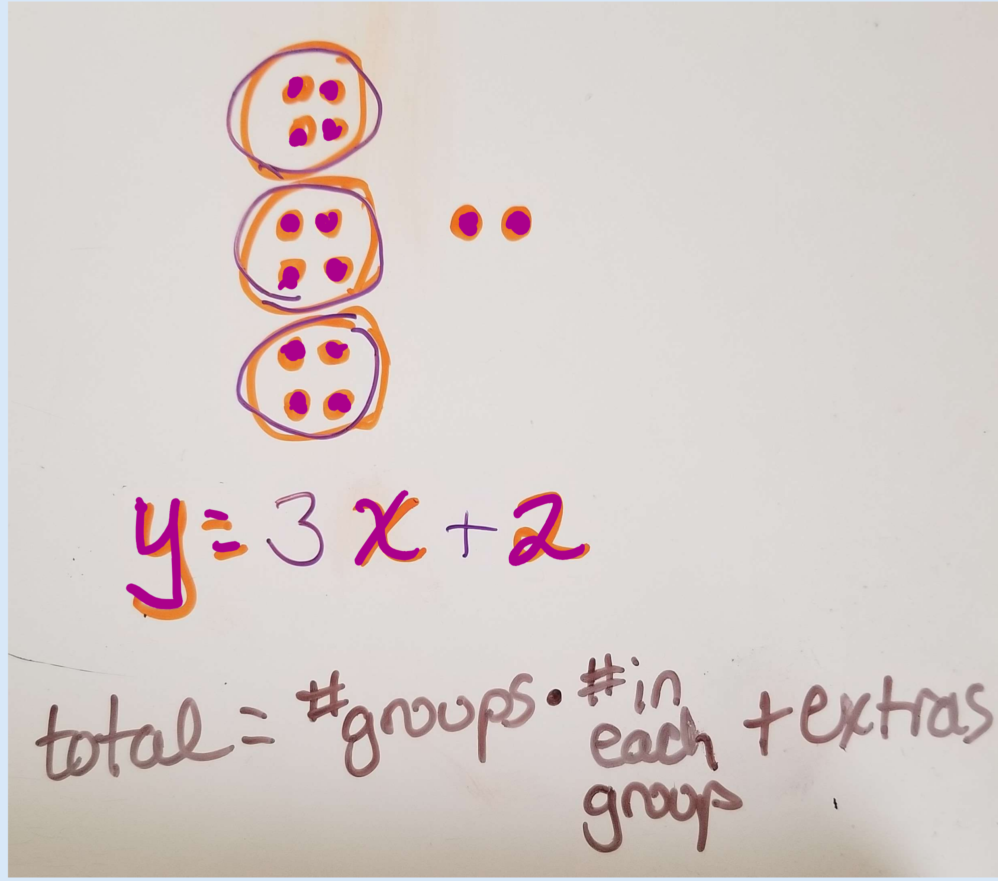
When the input and output have the same units, you can think of algebra tiles.

Output=multiplier*input + starting constant



If the output and the input have the same units, you can think of groups. If we change the input, which is the number in each group, then we change the output, or the total.

The output changes as a linear function of the input.



# For any equation of a line, you can plug in the x value to get the y value.

For y=2x+1 you can see that when x is zero y=1.

The point (0,1) is on this line

(1,3)

(0,1)

(-1, -1)

$y=2x+1$

| $x$ | $y$ |
|---|---|
| 1 | $2(1)+1=3$ |
| 0 | $2(0)+1=1$ |
| -1 | $2(-1)+1=-1$ |

Plugging in x to find y or putting the input in to get the output for any given input.

$f(x)$ $f(x)=2x+1$

$f(0)=2(0)+1=1$

$f(1)=2(1)+1=3$

$f(-1)=2(-1)+1= -1$

$$x \xrightarrow{\boxed{2x+1}} y$$

| $x$ | | $y$ |
|---|---|---|
| 0 | $\rightarrow$ | 1 |
| 1 | $\rightarrow$ | 3 |
| -1 | $\rightarrow$ | -1 |

$f(x+3)=2(x+3)+1= \ 2x+6+1=2x+7$

$f(x+n)=2(x+n)+1= 2x+2n+1$

$f(x)=2x+1$

$f(z)=2z+1$

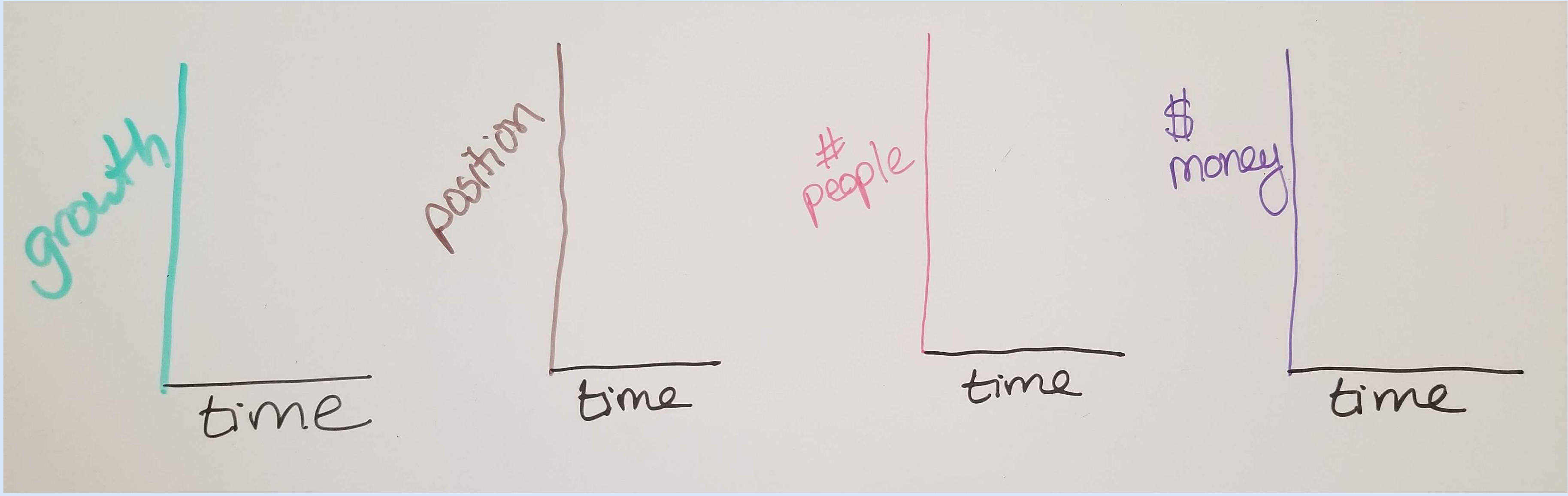
$f(x+3)=2(x+3)+1= \; 2x+6+1=2x+7$

$f(x+n)=2(x+n)+1= 2x+2n+1$

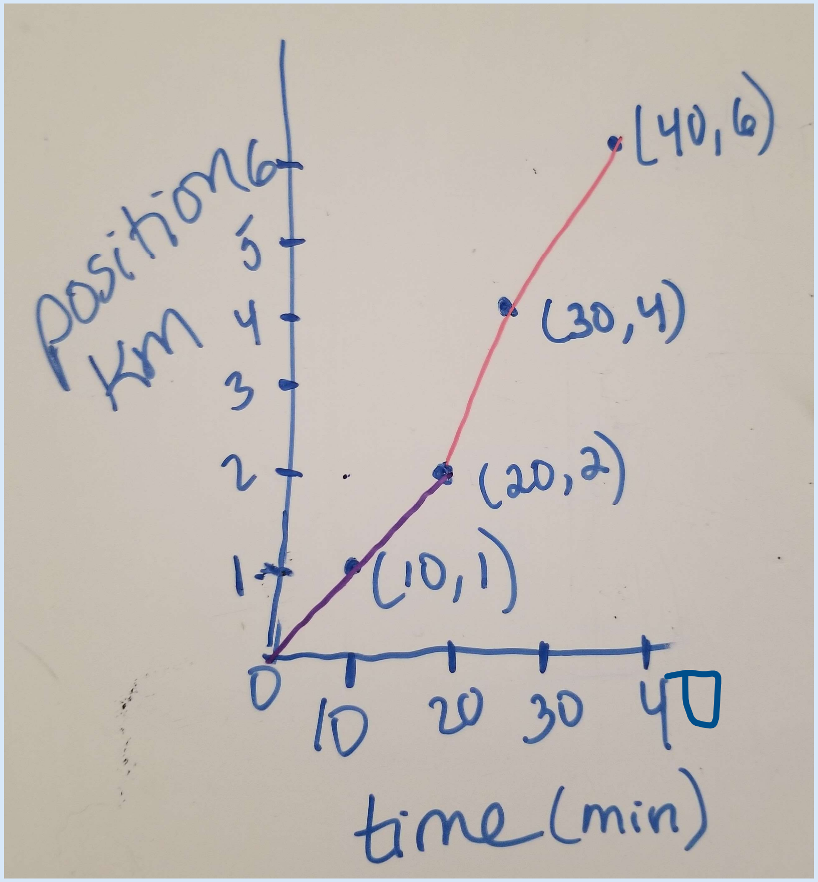
$$x \;\; \overset{2x+1}{\boxed{\qquad\qquad}} \;\; y$$

| | | |
|---|---|---|
| 0 | $\rightarrow$ | 1 |
| 1 | $\rightarrow$ | 3 |
| -1 | $\rightarrow$ | -1 |

Linear functions, as relations between the input and output, can be graphed on a Cartesian coordinate axis system. The input and output often have different units.



# Cartesian or coordinate axis graph




Lines or linear functions

$y=ax+b$

(x,y) are variables and points on the line, a and b represent numbers.

We want to be able to talk about what is going on and we can abstract with general algebraic formulas.

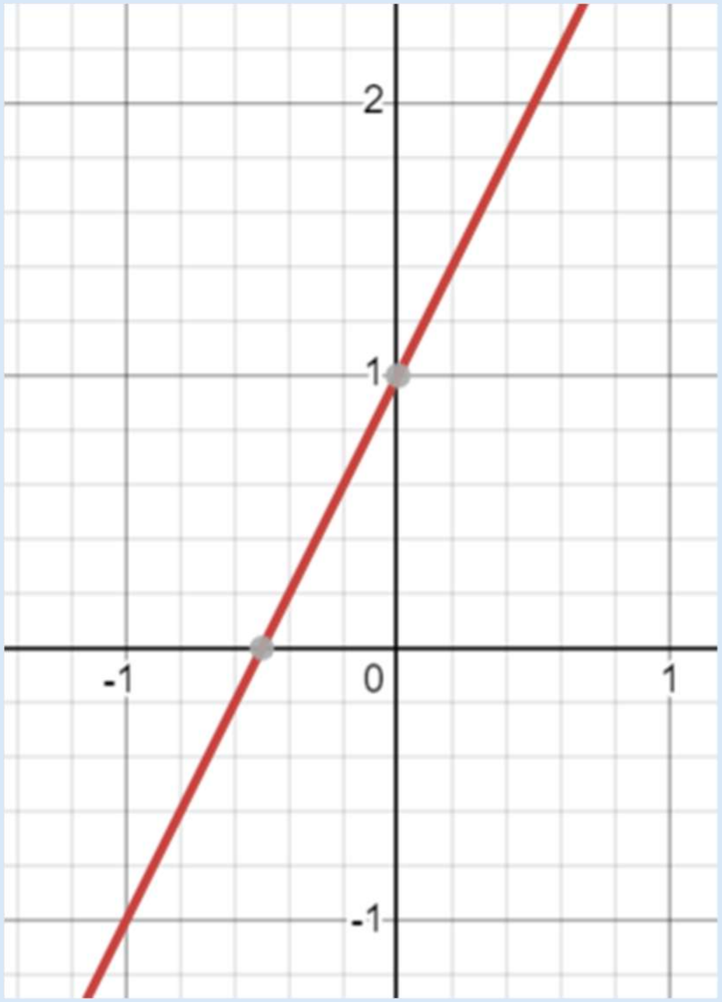
# $f(x)=ax+b=y$

Set of ordered pairs ($x$,$y$) that fall on the same line.

Have tilt or slope of **a** that is the multiplier or coefficient or rate of the input variable

Have a starting output of **b** that is the y coordinate where the line crosses the y axis. (0,**b**) called the y-intercept.

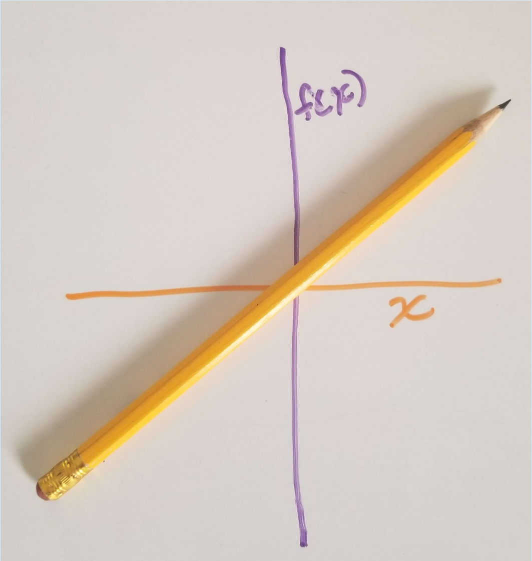
$ax+b=y$

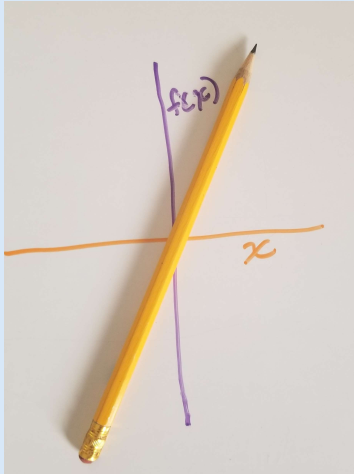


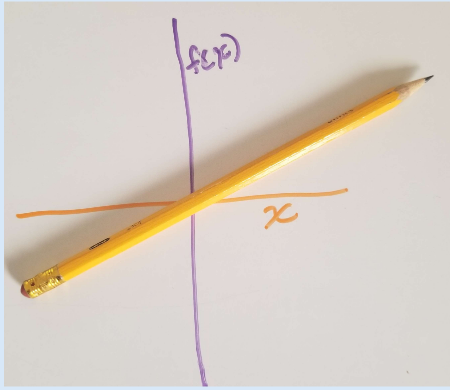
$f(x)=\color{red}{a}\color{black}x+b=y$

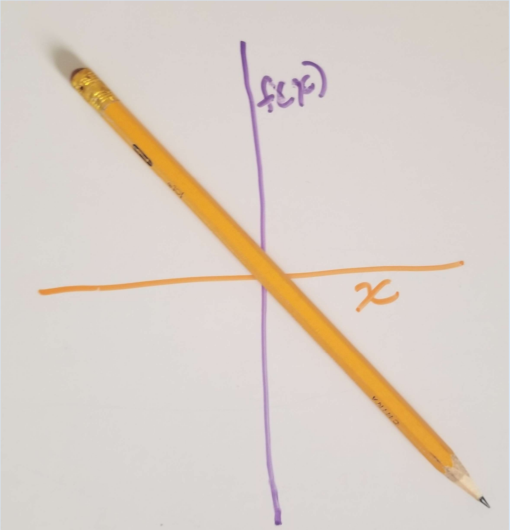
Can do two different actions to a line on a graph.

I can tilt it.



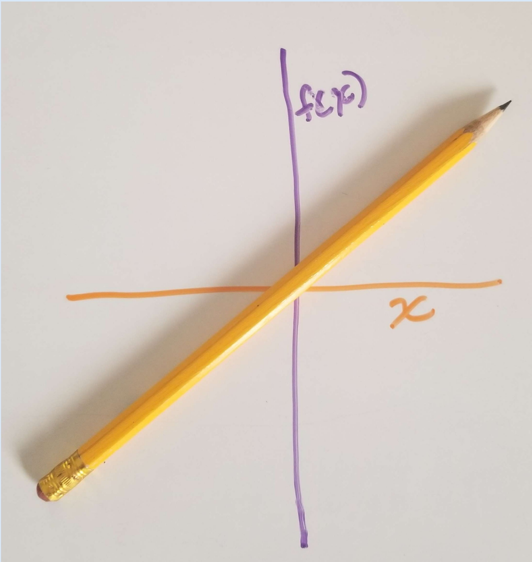



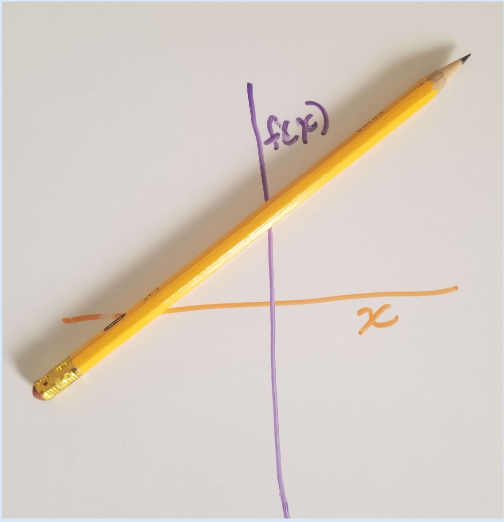



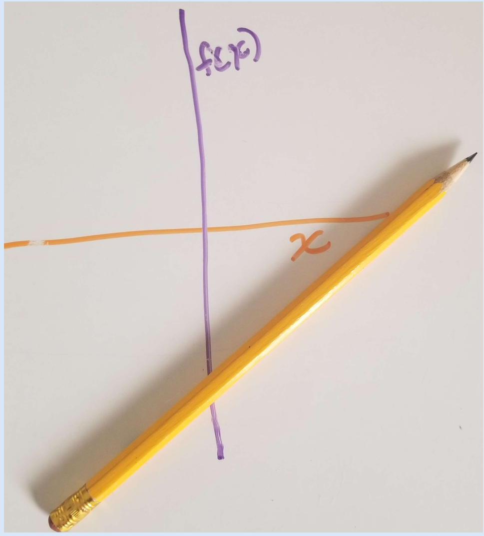
$f(x)=ax+b=y$

Can do two different actions to a line on a graph.

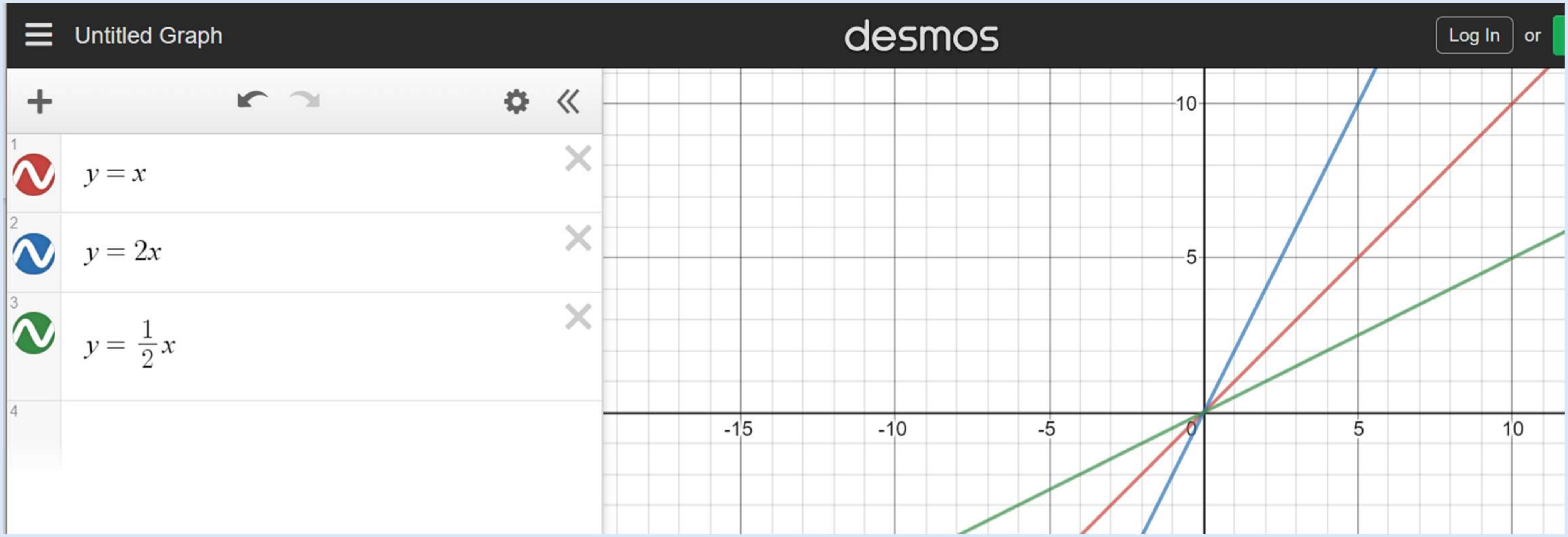
I can shift it up and down.



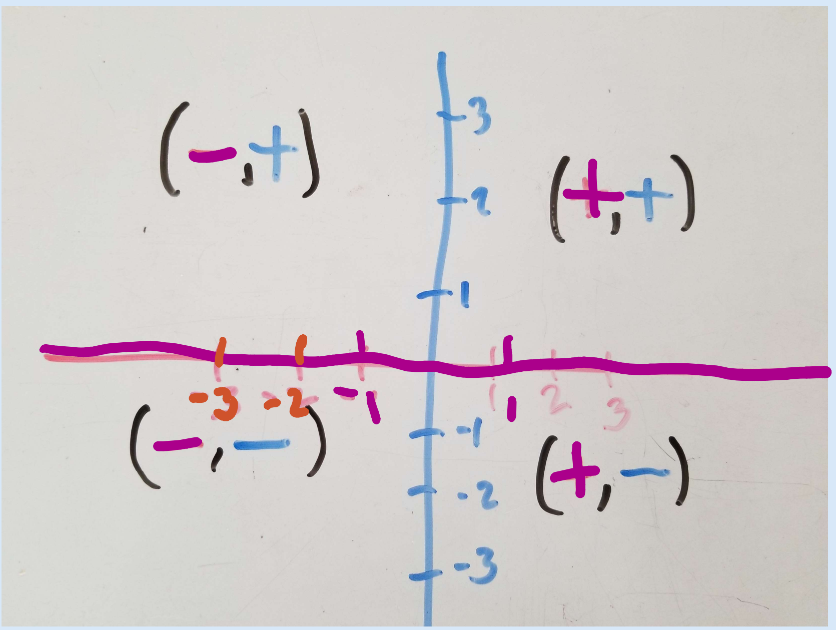



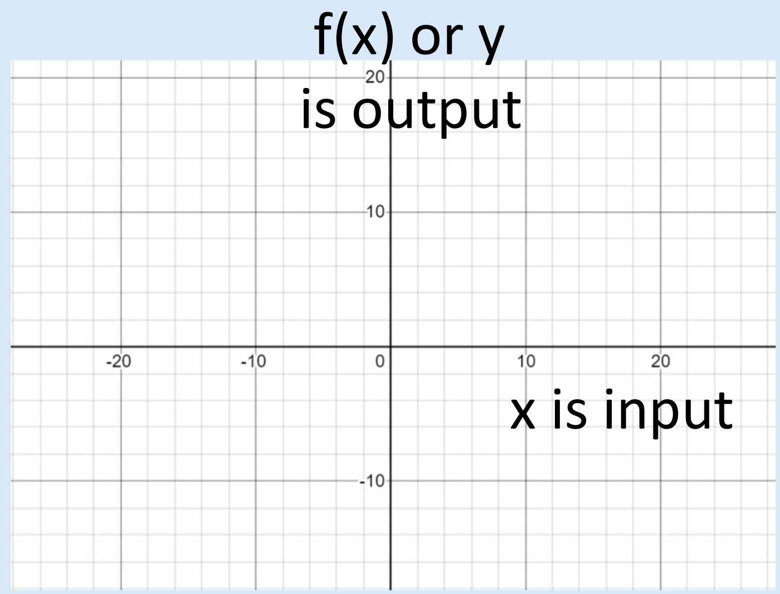



Desmos | Graphing Calculator is awesome and all the images that look like they are from Desmos are from Desmos.



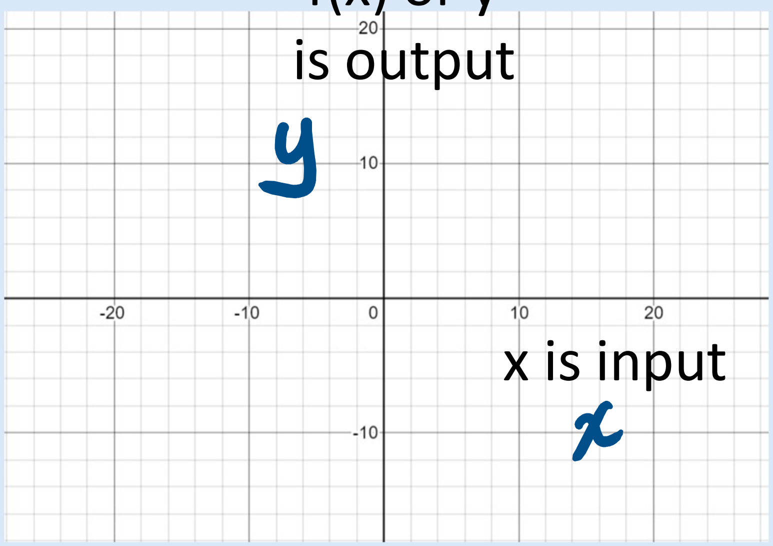
y or f(x) is the vertical, up/down axis, and is the output
x is the horizontal, right/left axis, and is the input.
Notice the signs of x and y in the different quadrants.
(x,y)



y or f(x) is the vertical, up/down axis, and is the output
x is the horizontal, right/left axis, and is the input.

input $x$ $\xrightarrow{\boxed{\text{function}}}$ output $f(x)$



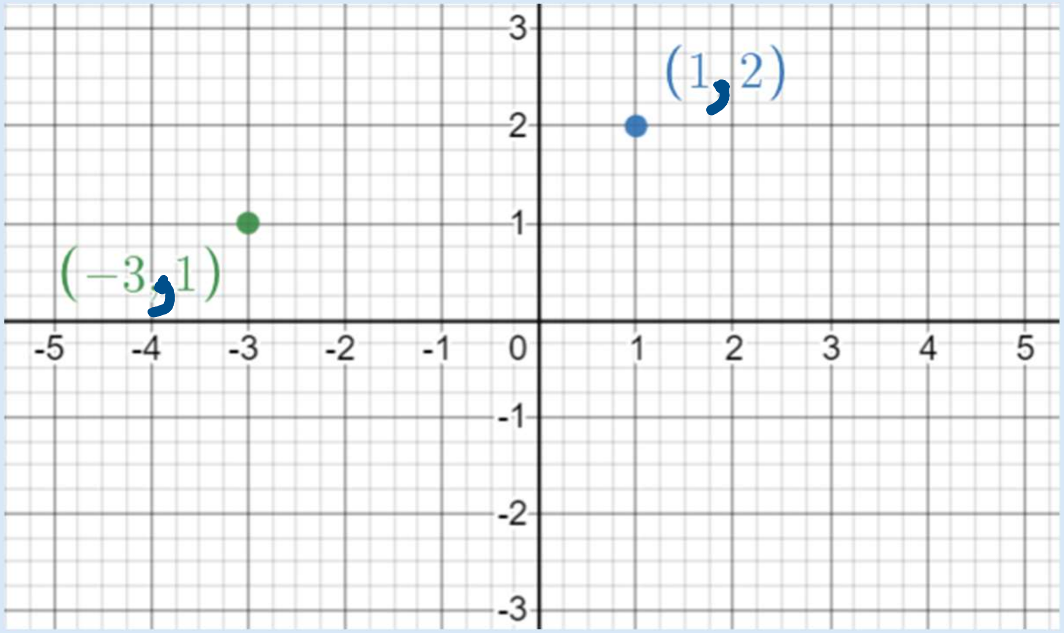
# (x,y) or (x, f(x)) is how we label a point
# It is an ordered pair (input, output)
# (1,2) and (-3,1) are shown here

thanks (AT)

vertical up and down Y gives Y

First term: x, input, domain, run, horizontal component

Second term: y, f(x), output, range, rise, vertical component



# Ordered pair is (x,y) with a comma to separate the first and second of the ordered pair.

Desmos has a very faint comma, so it looks like a period or a point or a dot on the slides, but it is a comma.

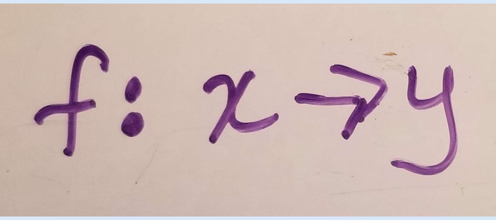
(1,2)

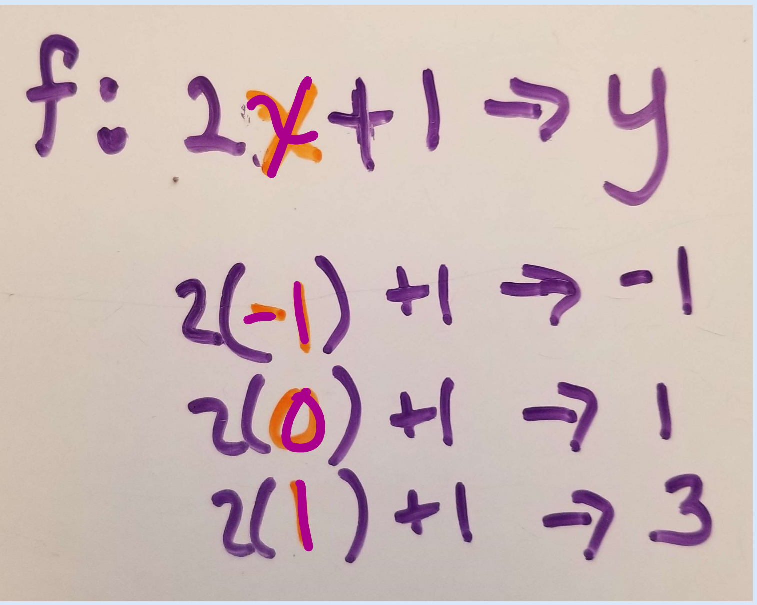
We say the point one     two with a slight pause.

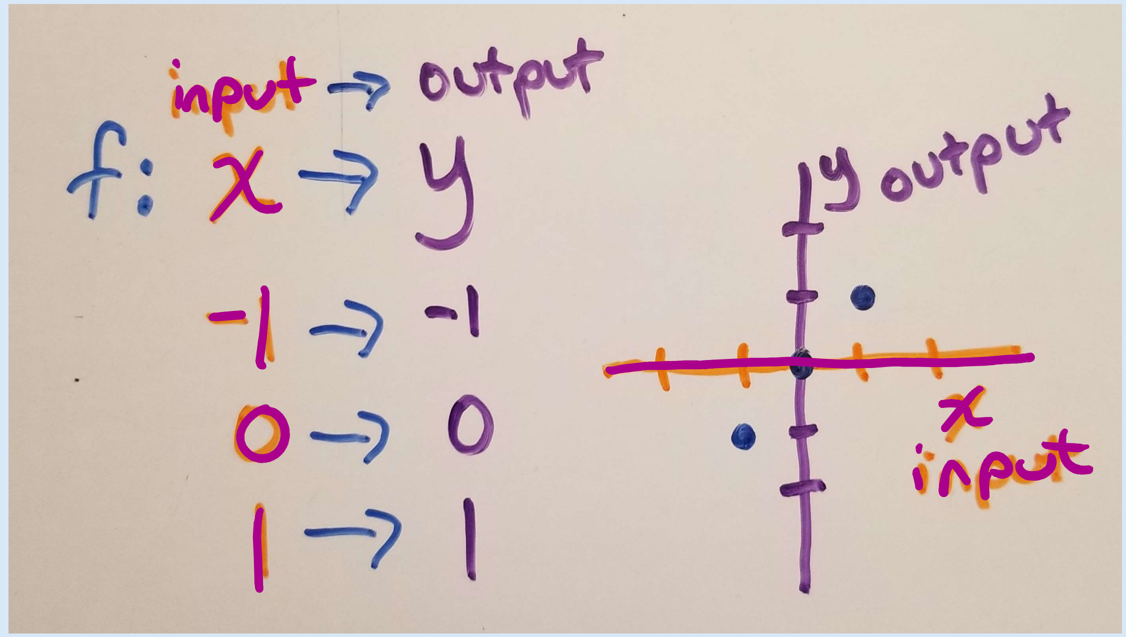
Or one comma two

Or x is 1 and y is 2

Function mapping may be an easier way to think of graphing a function.

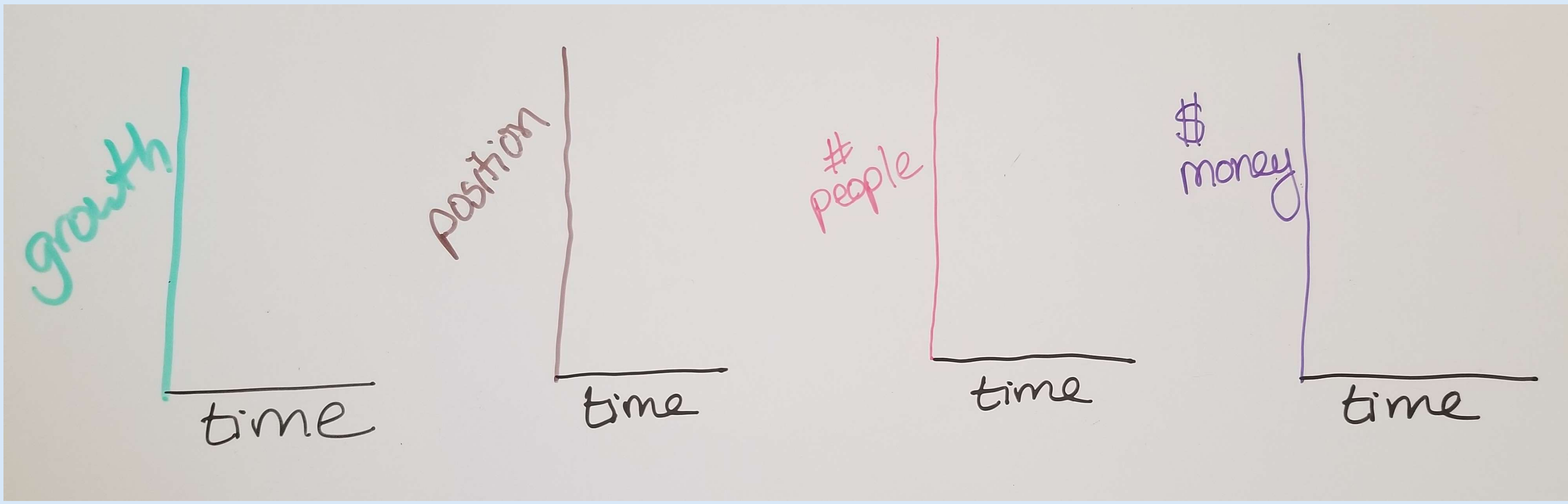


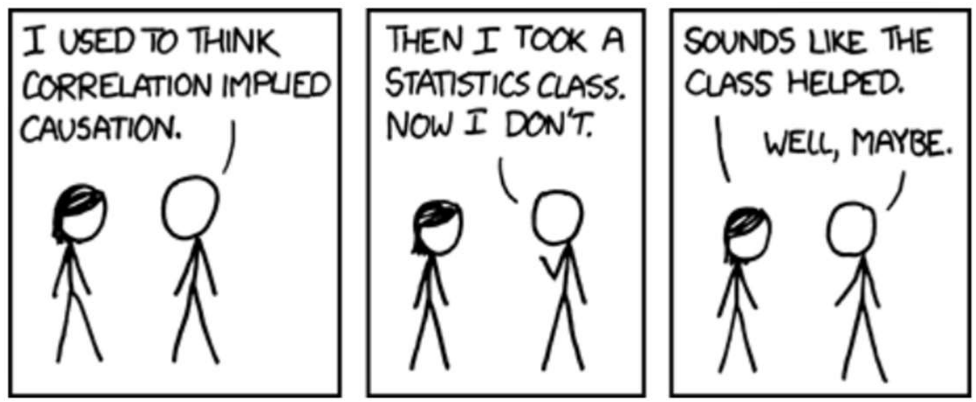



Time is very commonly the input or x coordinate.

$f(t) = at+b$



# It is easy in math to tell the input from the output but that can get really tricky in science, especially the biosciences!




**Correlation** (alt-text)

Correlation doesn't imply causation, but it does waggle its eyebrows suggestively and gesture furtively while mouthing 'look over there'.

[Correlation doesn’t imply causation! From XKCD comic]

# With set theory, a line is the infinite collection of all points that satisfy a linear formula.

$A=\{(x,y) : y=x\}$ (input, output) $\{(x,y) : x \to y\}$

$a=\{(0,0), (1,1), (5,5), (-1,-1)\}$ a is a subset of A as these are all points on A

$B=\{(x,y) : y=3x+1\}$ $b=\{(0, 1), (1, 4), (-1,-2)\}$

$3x+1$

$C=\{(x,f(x)) : f(x)=2x+3\}$ $c=\{(0, 3), (1, 5)\}$

Since I only need two points to define the line, I can also just give the subset that contains two points that are on the line to define the line.

# Remember that set theory deals with being a member or not a member of the set.

$A=\{(x,y) : y=x\}$   output = input

$a=\{(0,0), (1,1), (5,5), (-1,-1)\}$ a is a subset of A as these are all points on A

# Remember that set theory deals with being a member or not a member of the set.

$A=\{(x,y) : y=x\}$    output = input

$a=\{(0,0), (1,1), (5,5), (-1,-1)\}$ a is a subset of A as these are all points on A

$\{(0,1), (1,2), (5,4)\}$ are all points that are not in set A or on the line A.

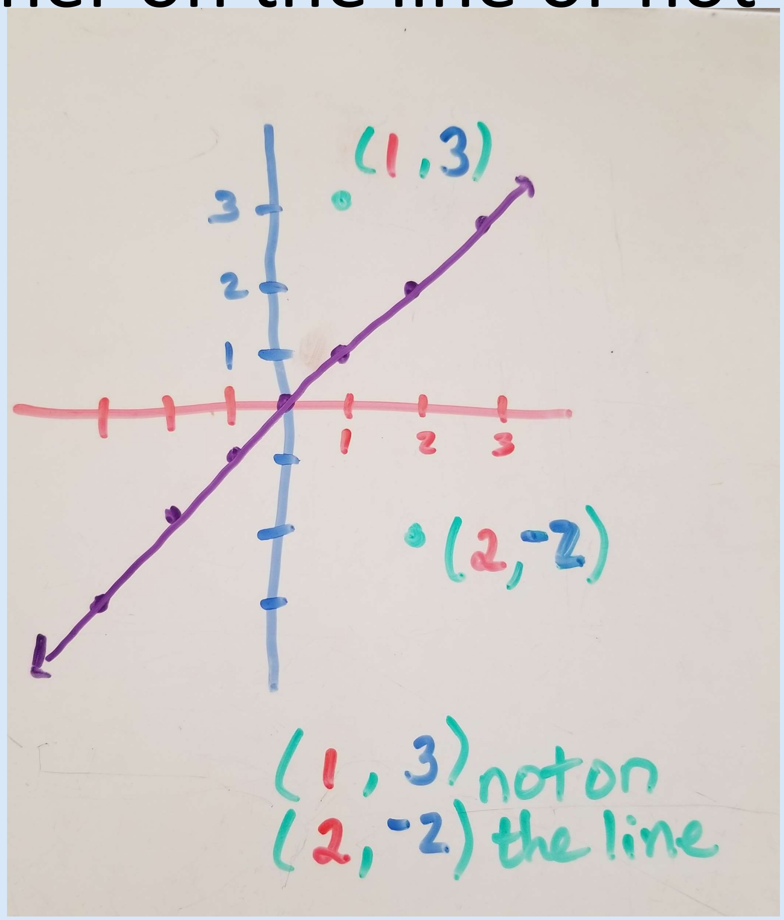
A $f(x)=x=y$

$(-5,-5)$ is on line A

Is $(-1, -1)$ on line A?

Is $(-1, 5)$ on line A?

# Points are either on the line or not on the line y=x




$x = y$
on

$x \neq y$
not on line

# Polynomial

Expression, equation or function with many terms with the x input variable to different whole number positive powers.

$y=5x^3+6x^2+7x+8$

No variables in the exponents, or in the denominator.

All positive integer powers of the variable.

$f(x)=b$ is a constant function as the output is always b

$g(x)=ax$ is the linear component of a polynomial function

$h(x)=ax+b$ is the equation for any line.

$y=ax+b$ variables (x,y) constants are a and b

$F(x)=ax+b$

Examples:

$G(x)=2x+1$

$f(x)=2.79x+345654432125325235$

$g(x)=-2x+1$

$H(x)=0x+4$  $H(x)=4$

$F(x)=0x+0$  $F(x)=0$

(x, y) = (input, output) There are two things that you can do to the input:

Multiply the input by a number.

Add a number to the that.

Output=a(input)+b

f(x)=ax+b or y=ax+b

x is the input and there are two things you can do to the input:

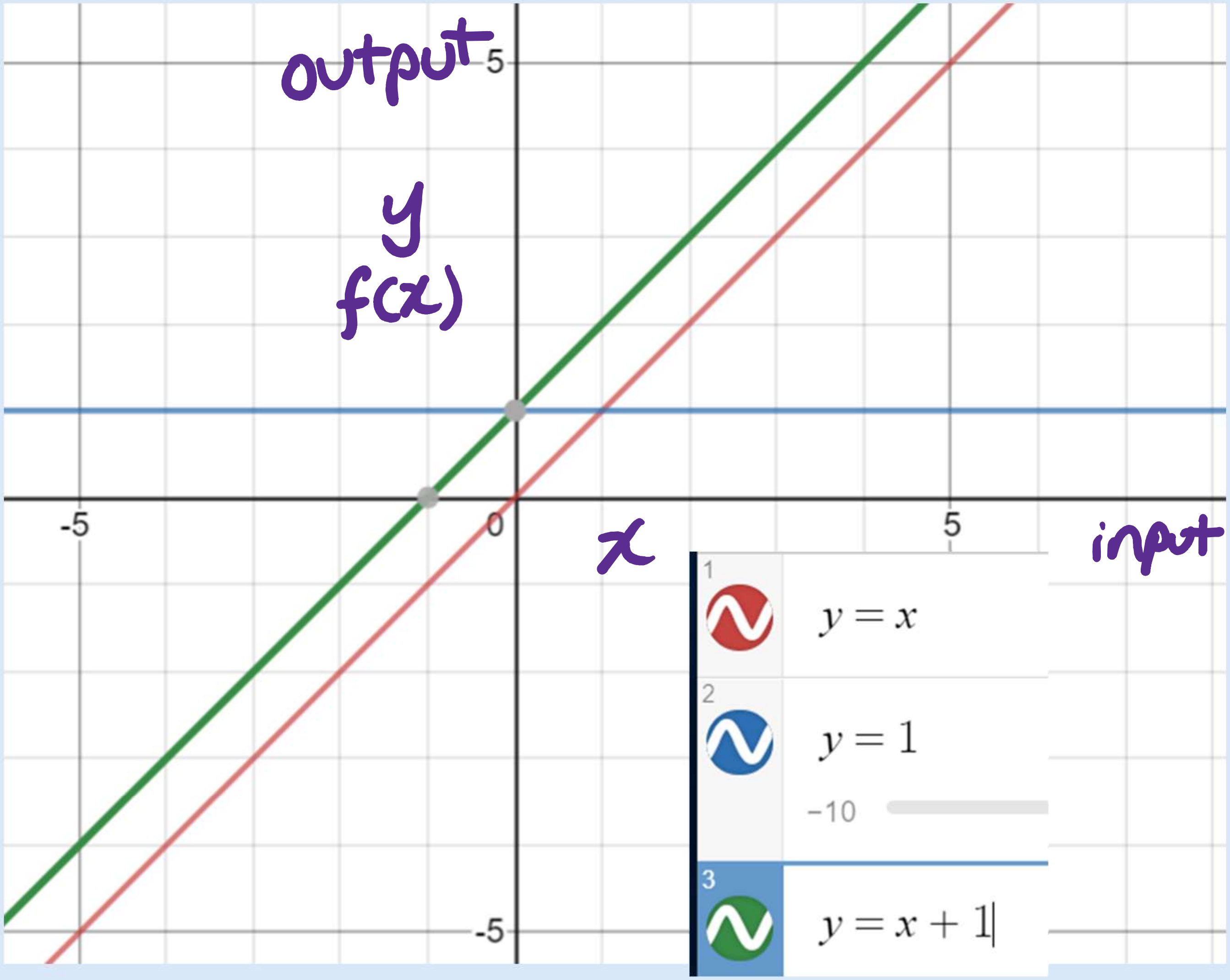
Multiply/divide the input by a number. (Since division is multiplication of the reciprocal, I just say multiplication)

Add/subtract a number to the that. (Since subtraction is addition of the negative number, I just say addition)

Output=a(input)+b

When we add the red line y=x to the blue line y=1, we move every point on the red line up by one to get the green line.

Green is y=x+1



# Adding a constant to a linear function

f(x)=2x

g(x)=1

h(x)=f(x) +g(x)=2x+1

if we use y for h(x)

y=2x+1

Can think of this as adding two different functions or just adding a constant to a linear function.

$x \ \overset{\cdot 2}{\boxed{f(x)}} \ 2x$ $\qquad x \ \overset{+1}{\boxed{g(x)}} \to 1$

$h(x) = \underset{\longrightarrow}{f(x)} \ 2x + \underset{1}{g(x)}$

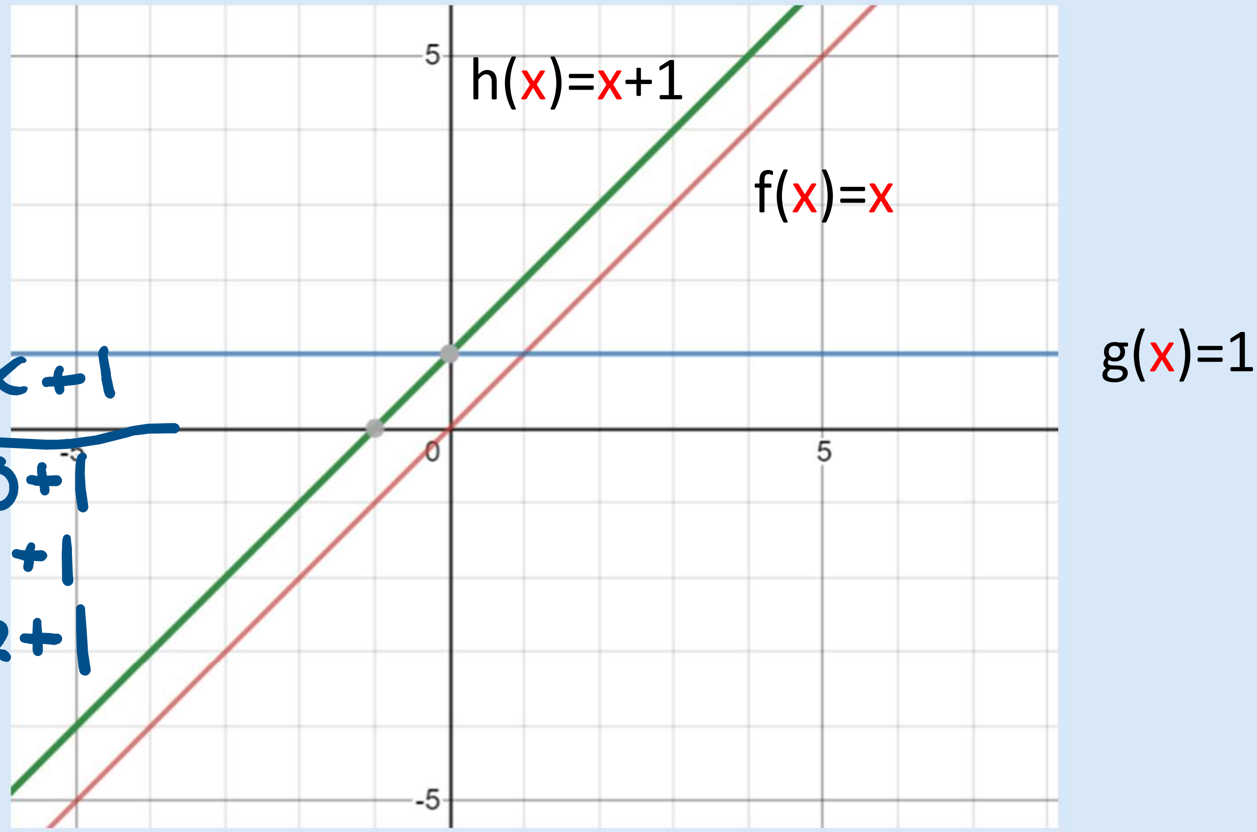
# Adding a constant to a linear function

$f(x)=x$

$g(x)=1$

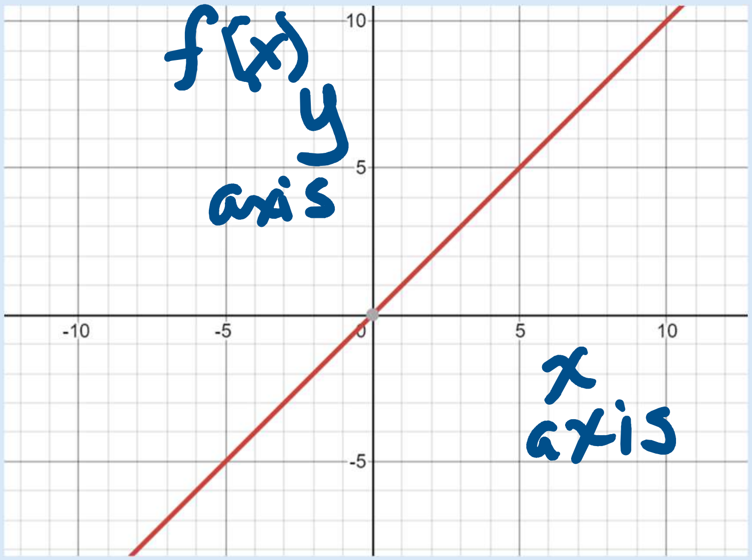
$h(x)=f(x)+g(x)=x+1$

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ | $x+1$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0+1 |
| 1 | 1 | 1 | 2 | 1+1 |
| 2 | 2 | 1 | 3 | 2+1 |



# Intercepts: where the line crosses the axis.

The y-intercept is where a line crosses the y axis. All linear functions cross the y axis.




The y-intercept is the output when x is zero.

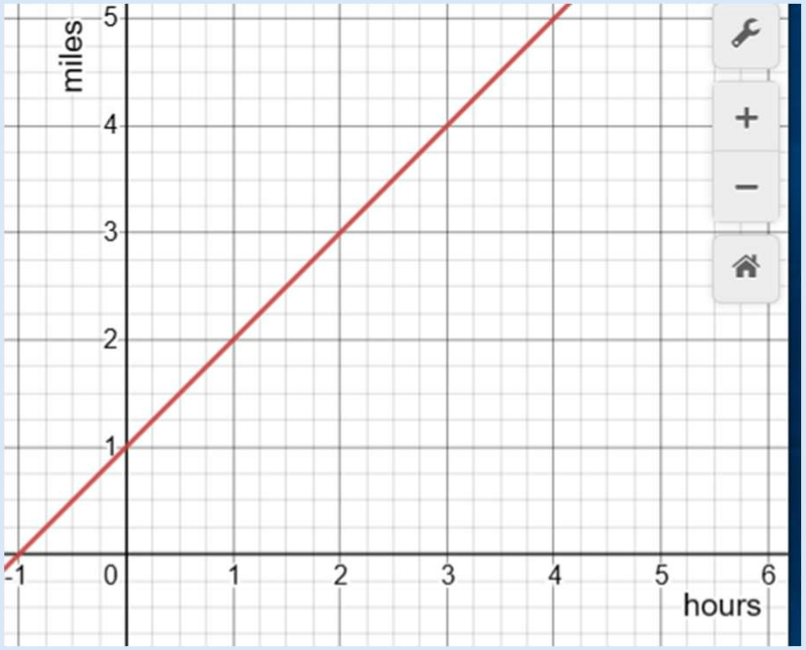
f(x)=ax+b

All y-intercept as points are (0,b)

The y-intercept has the same units as the y axis.

The y-intercept is where a line crosses the y axis. All linear functions cross the y axis.

y=x+1 miles=speed * time in hours +1 mile




All y-intercept as points are (0,b)
The y-intercept has the same units as the y axis.
The y-intercept is the starting point when the time is zero in this example.

For $f(x)=ax+b$ when $x$ is 0,

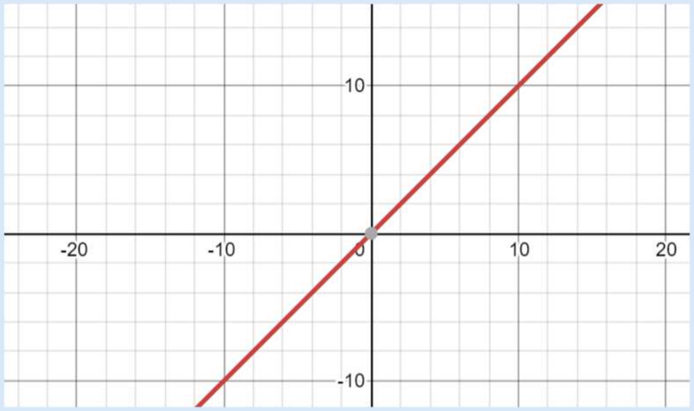
$f(0)=a(0)+b$

$f(0)=0+b=y$

$y=b$ when $x$ is 0

The point $(0,b)$ is on the line

# Linear Functions




A linear function is a function that defines a line.

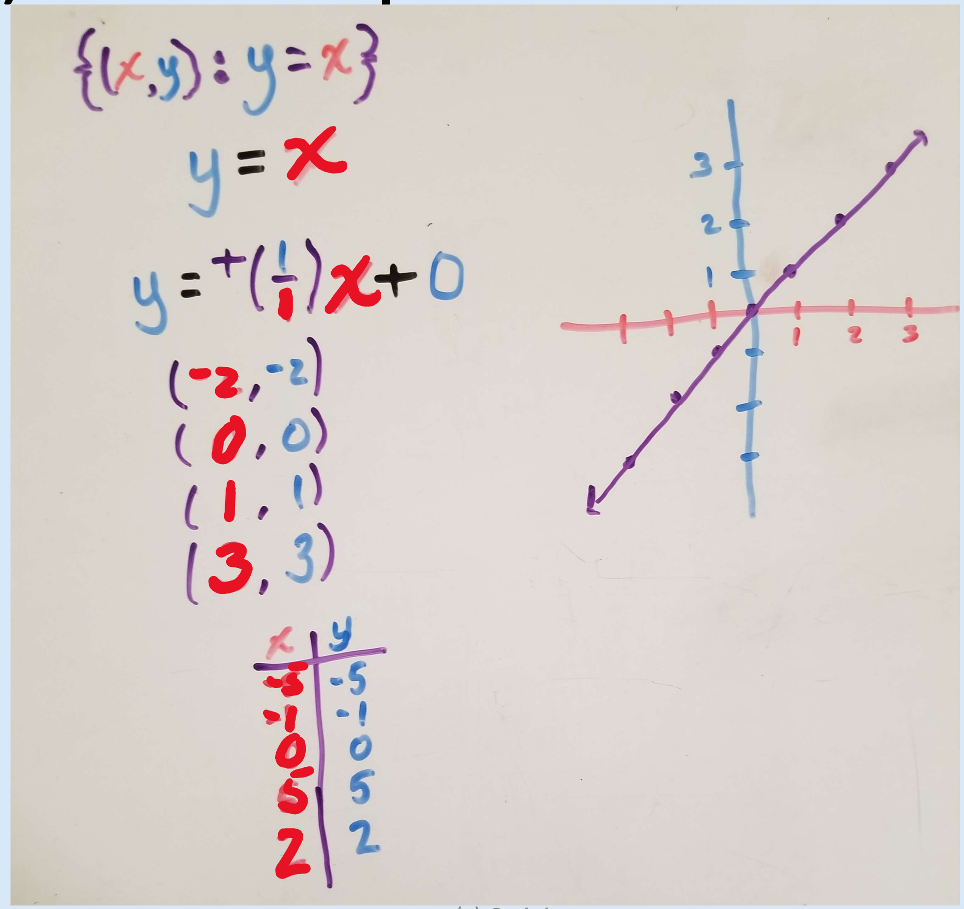
A graph of a linear function is a line.

$f(x)=x$ is the parent function for linear functions.

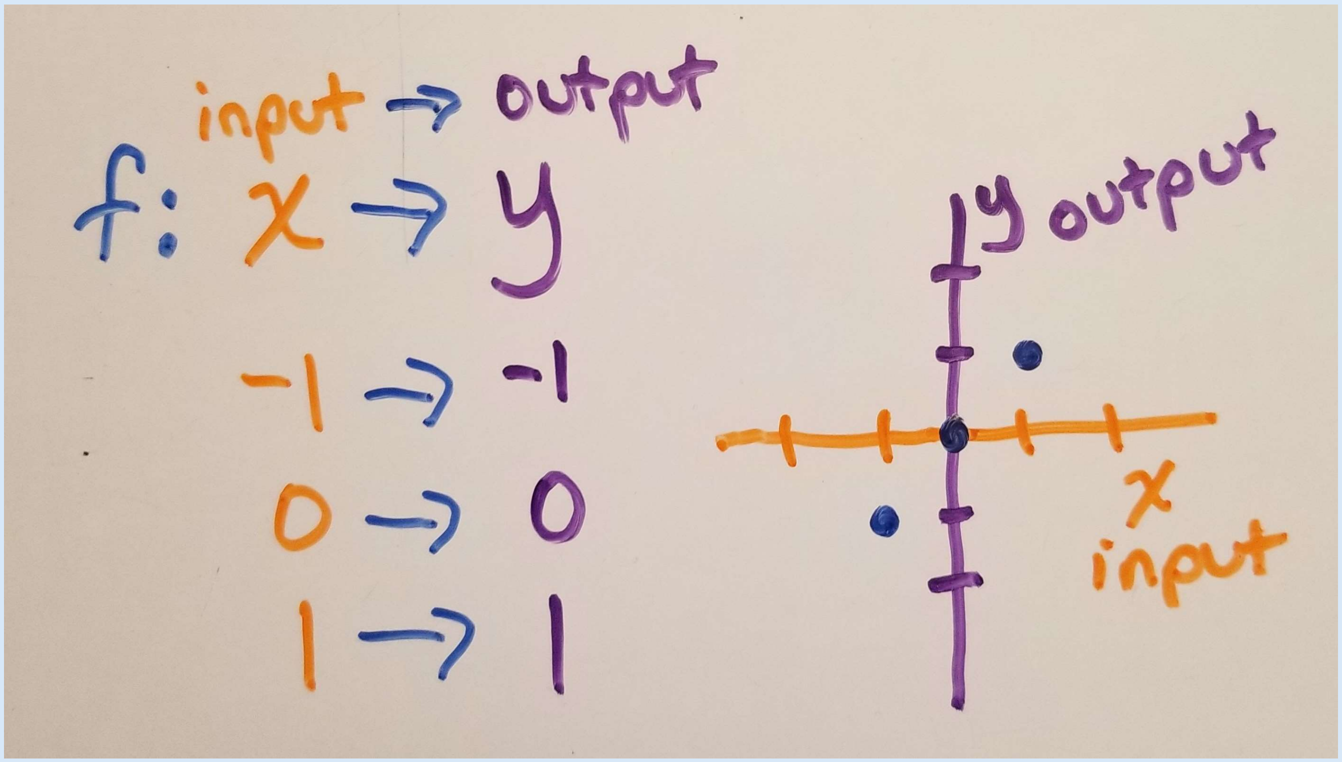
y is often used for $f(x)$ to mean the output

$f(x)=x$ is the identity for all functions because the output is identical to the input.

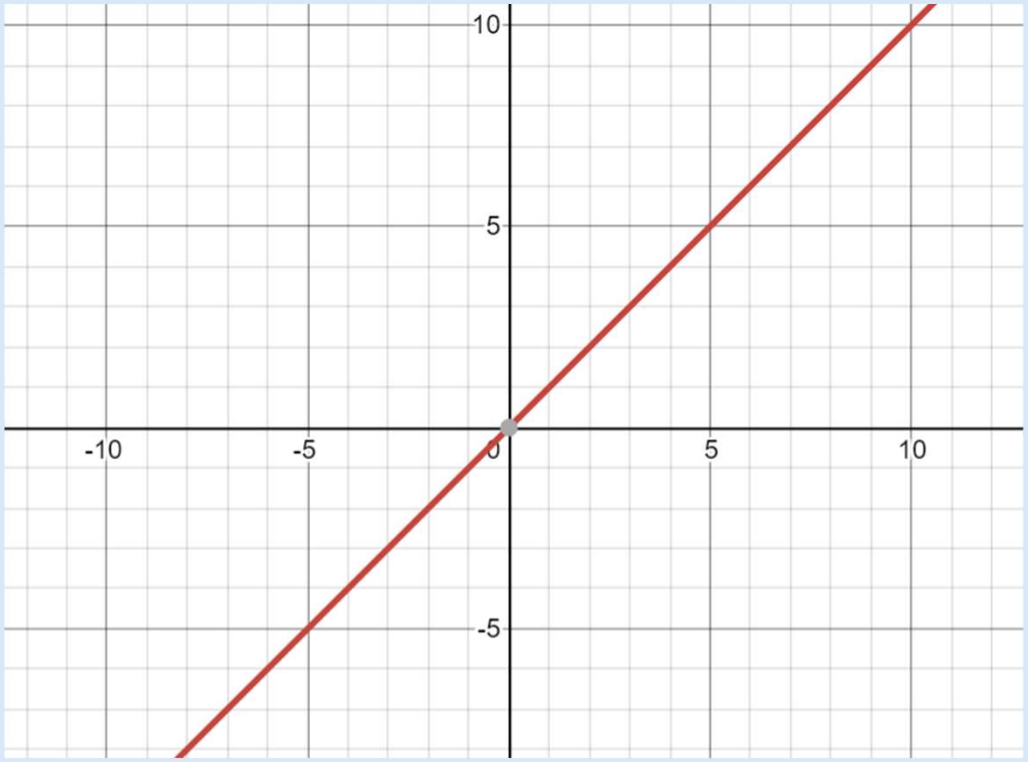
# y=x or f(x)=x is the parent function for lines

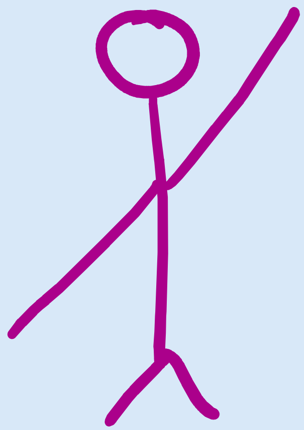


# y=x or f(x)=x is the parent function for lines



# $f(x)=x$ or $y=x$ is the parent linear function





$f(x) = x$

$y = 1x + 0$

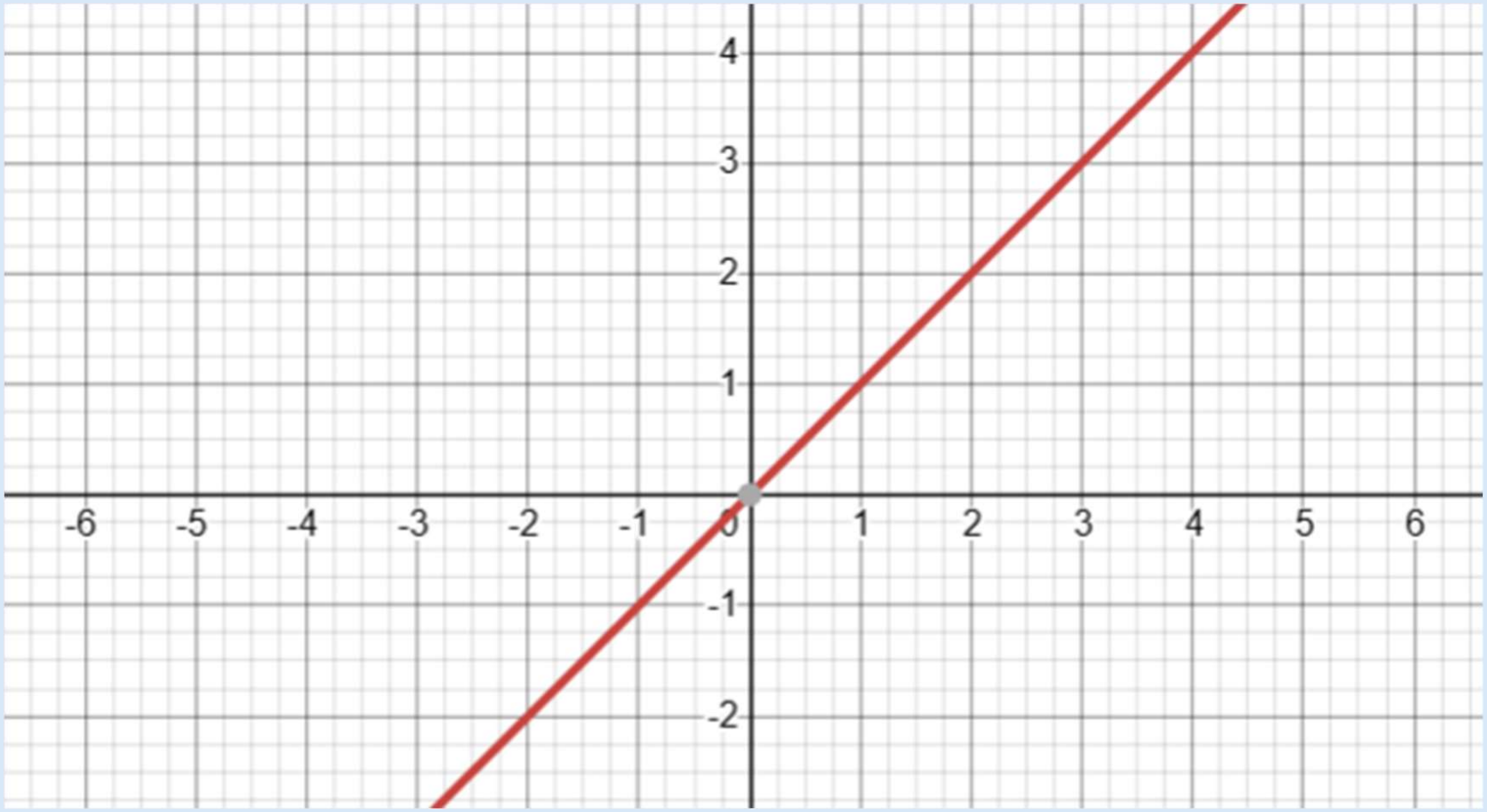
It has a y-intercept of zero and an x-intercept of zero because it crosses both axes at the origin (0,0)

f(x)=x=y

f(x)=ax+b=y

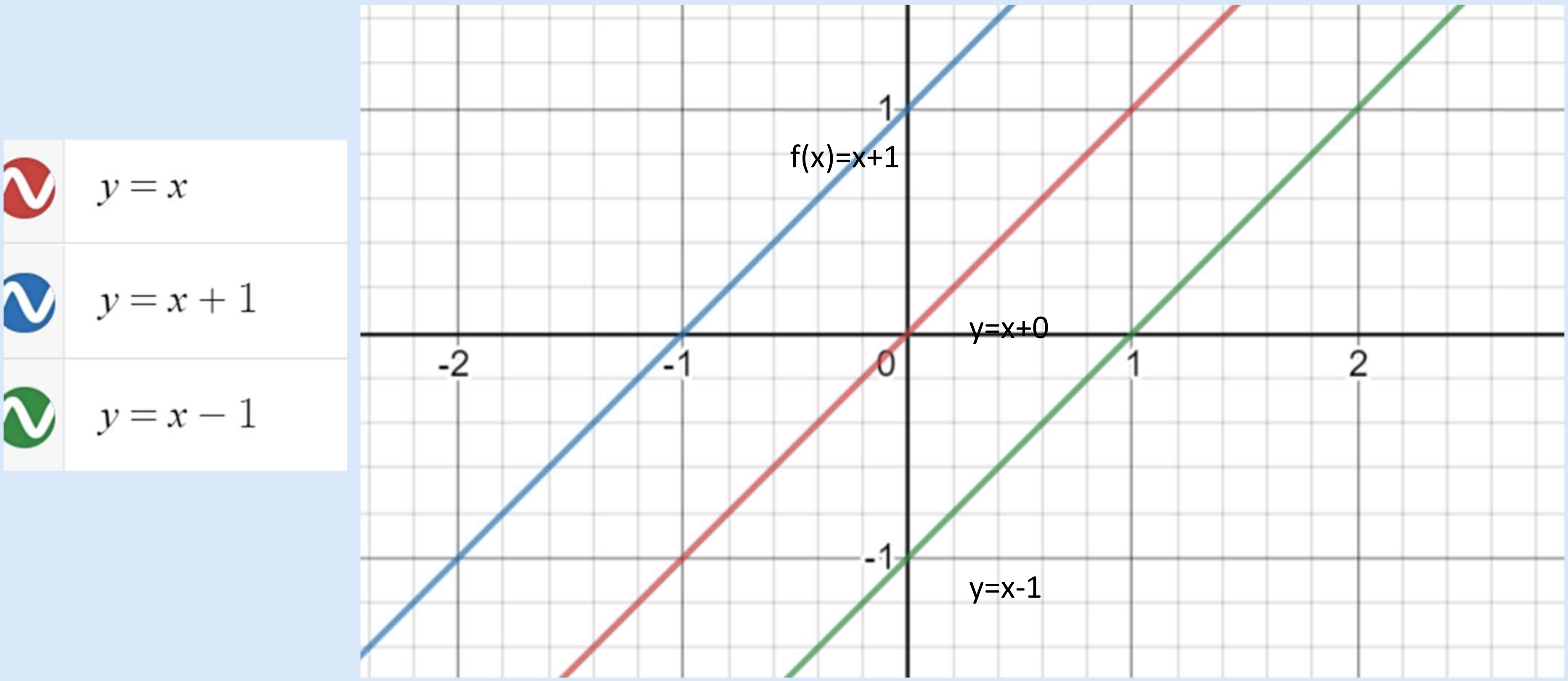
f(x)=1x+0

f(x)=x =y



**Transformation of functions**

The y-intercept is the output (up and down) when x (across) is zero. Notice what happens to the y-intercept when we add or subtract a constant to our parent function of f(x)=x.

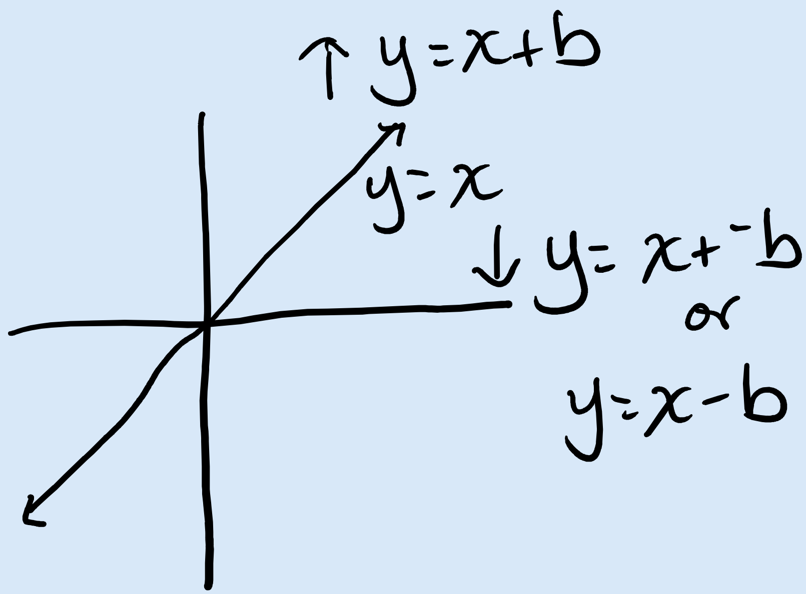


$f(x) = 1x + b = y$

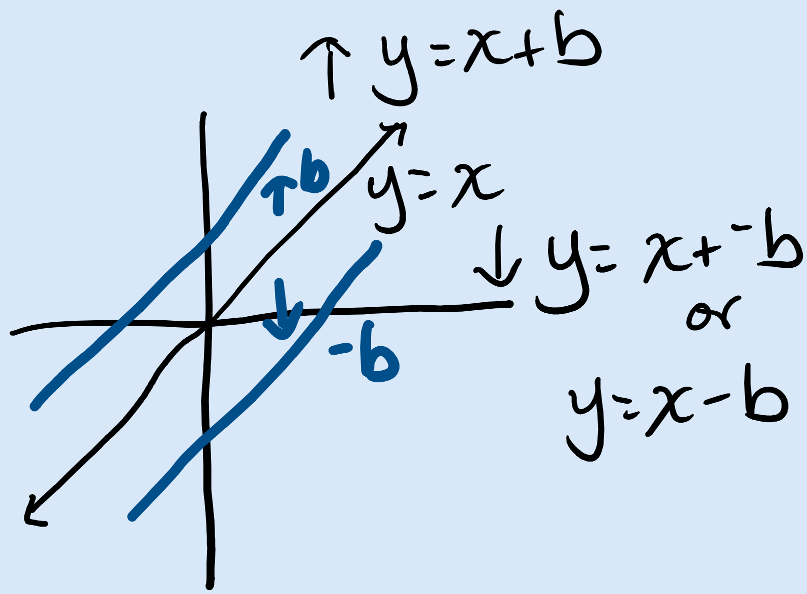
The parent function $y=x$ goes

up by b when it is +b

down by b when it is -b

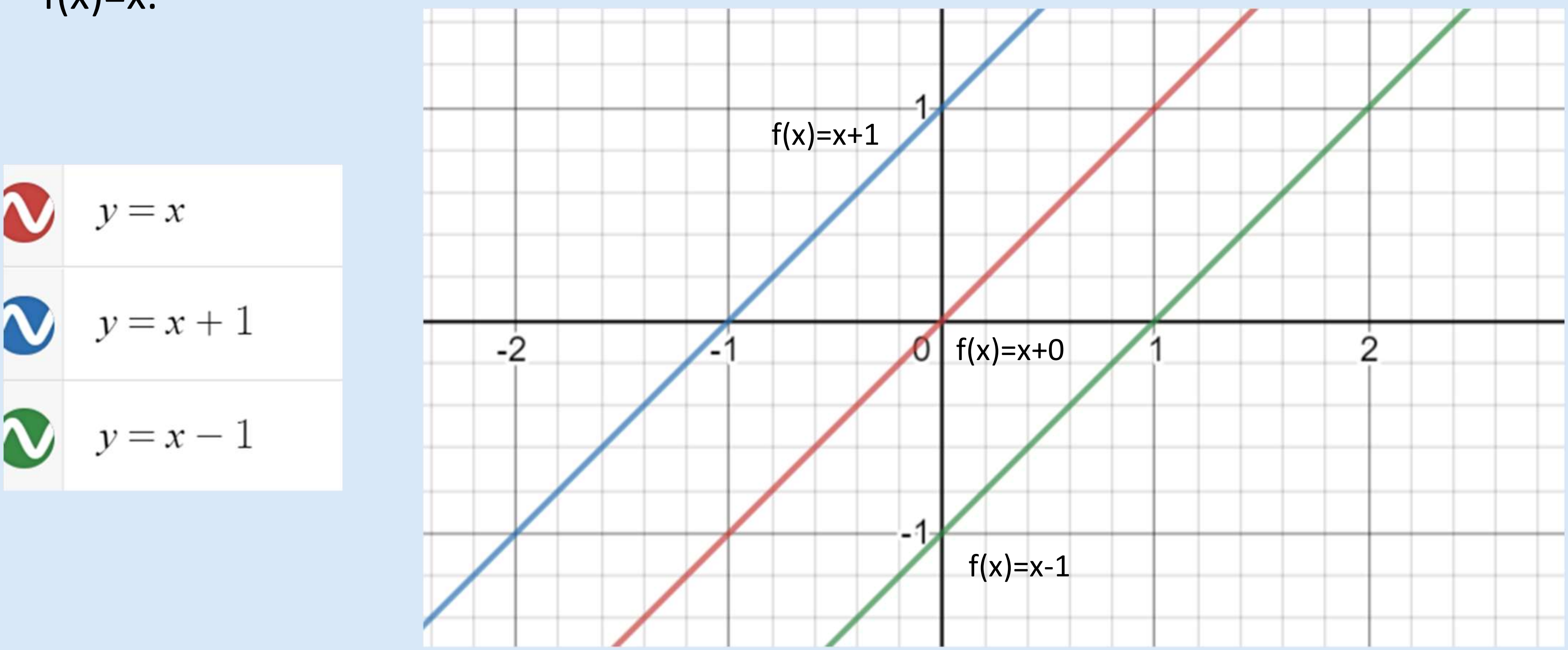


+b shifts the line up by b
-b shifts the line down by b



**Transformation of functions**

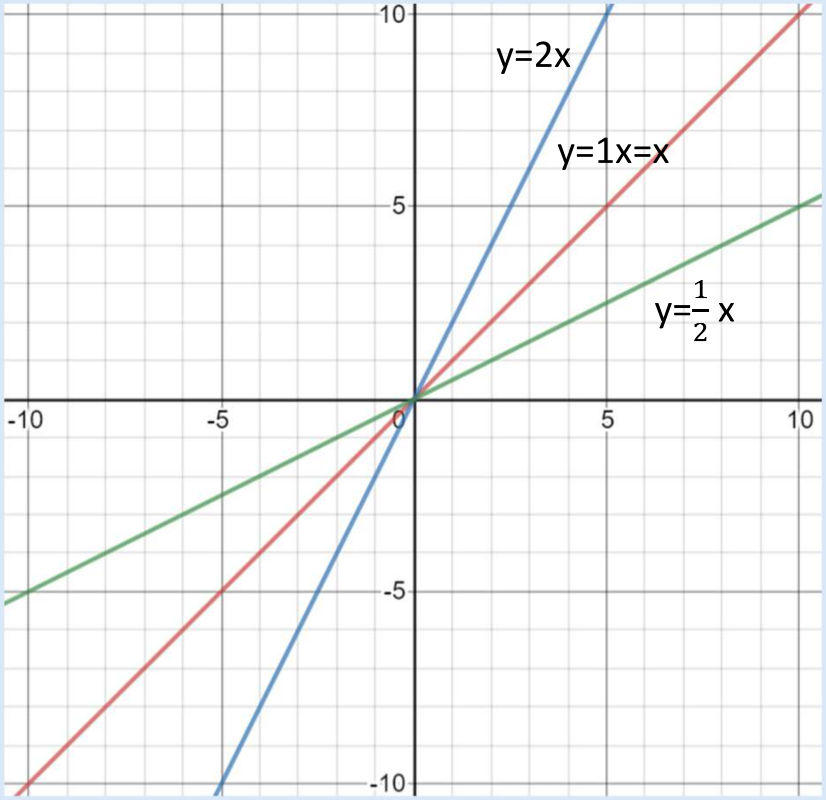
The y-intercept is the output (up and down) when x (across) is zero. Notice what happens to the y-intercept when we add a constant to our parent function of f(x)=x.



The **coefficient**, **rate**, or **multiplier** of x is the **slope** or tilt of the line.

**f(x)=mx+0    y=mx**

m is commonly used for the slope. It comes from the French word monter, which means to climb or go up. I will mainly use a but may use m.






(c) Crainix

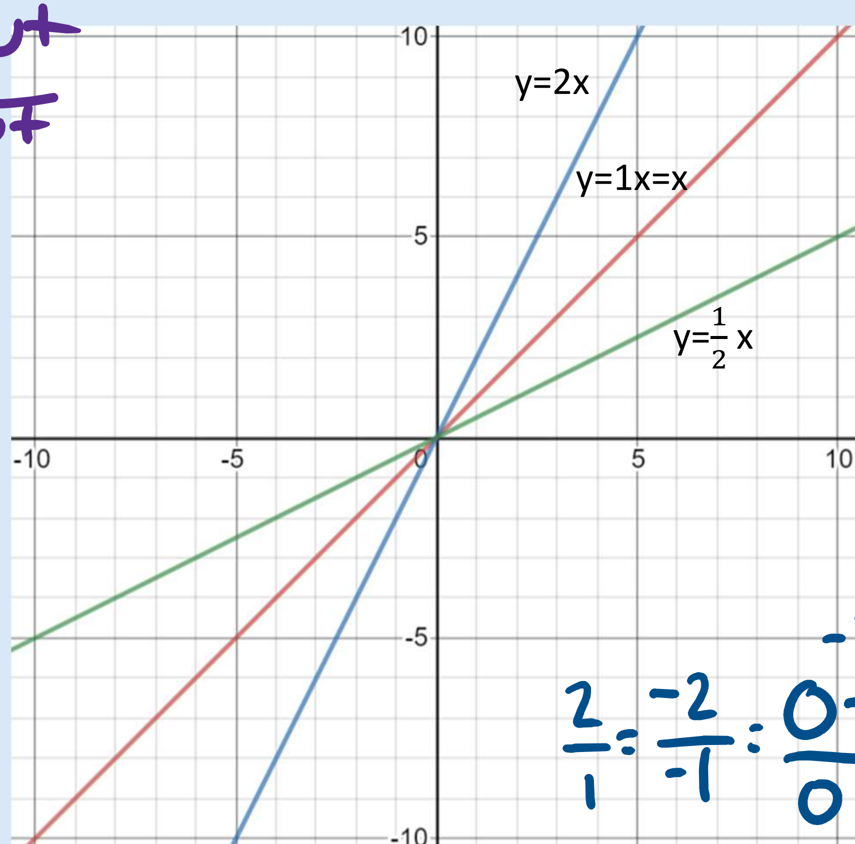
The **coefficient**, **rate**, or **multiplier** of x is the **slope** of the line.

**f(x)=ax+0 y=ax**

Here b, which is the y-intercept or constant term is zero.

slope $= \dfrac{\text{change of output}}{\text{change of input}}$

$\dfrac{\text{up}}{\text{over}}$




slope $= \dfrac{\Delta y}{\Delta x}$

point $(x, y)$

$(0,0)$ to $(1,2)$

slope $\dfrac{2}{1} = 2$

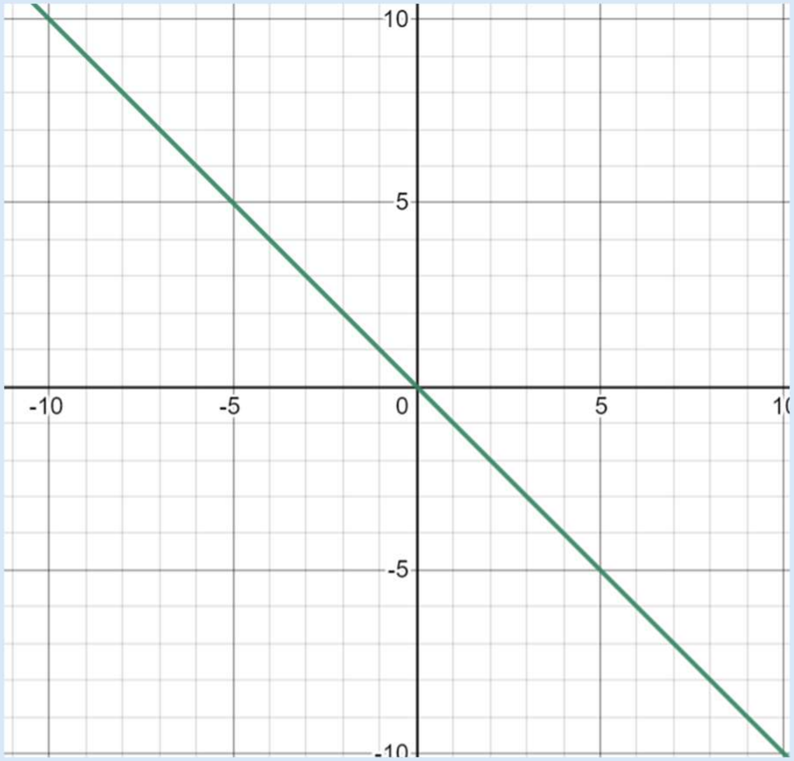
$\dfrac{2}{1} = \dfrac{-2}{-1} = \dfrac{0-2}{0-1} = \dfrac{2-0}{1-0} = \dfrac{2}{1}$

-2

-1

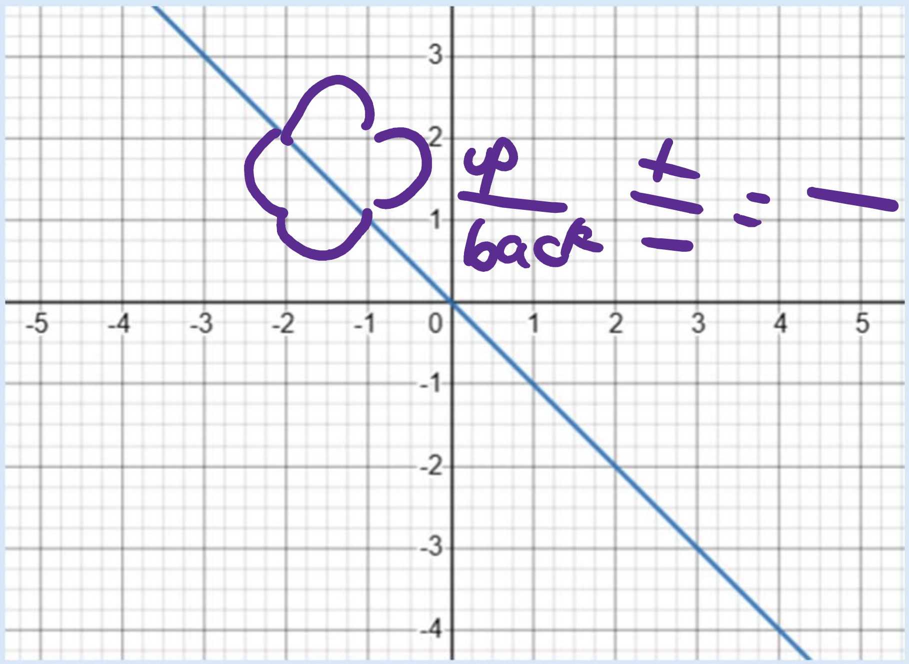
# f(x)=-1x or y=-1x or y=-x

y= - x



# Do you see how the output has the opposite sign from the input for f(x)= -x ?

$- = \frac{\text{down}}{\text{over}} \frac{-}{+}$



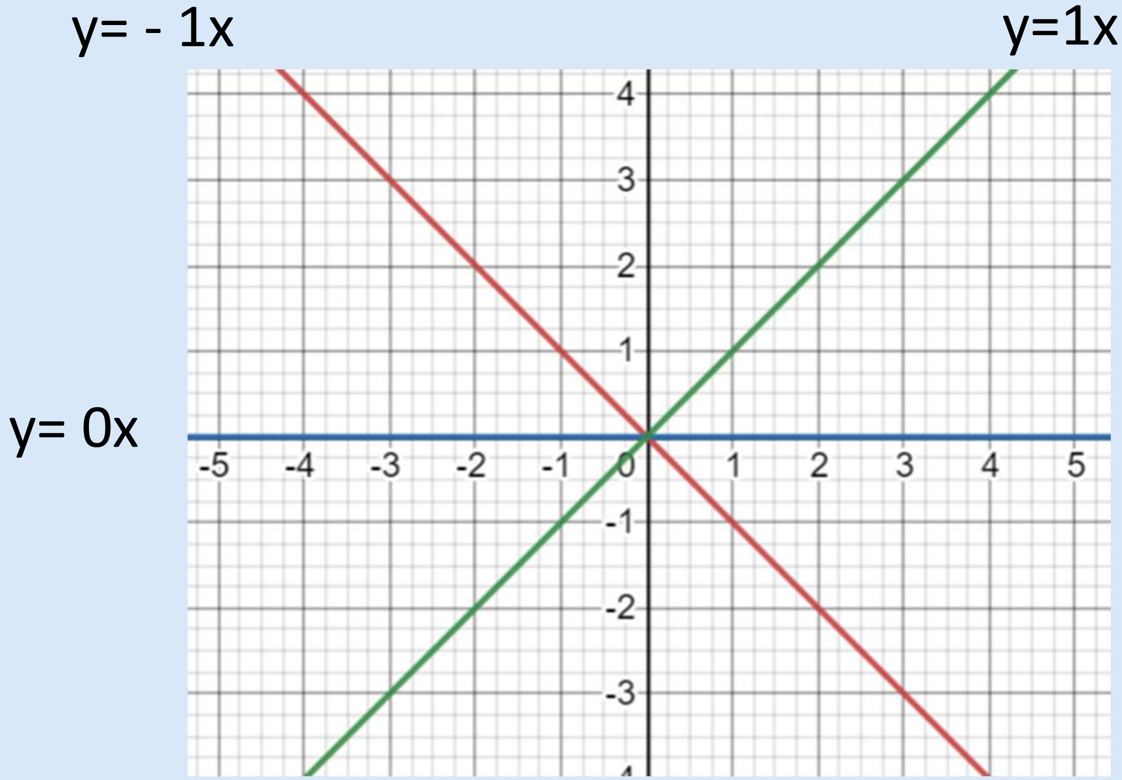

$\frac{\text{up}}{\text{back}} \frac{+}{-} = -$ slope

# Slope as a decimal or a fraction.

2 miles in 5 hours is $\frac{2}{5}$ mph and we used to talk about slopes as fractions a lot. Now we have computers do the computations for us and it is much easier to think of slopes as a unit rate, which is a decimal.

With a decimal, the rate or slope tells how much the output changes for each unit of input. 2 miles in 5 hours is 0.4 miles per hour. It is much easier to use the decimal than typing the fraction and it is much easier to think in terms of **how much the output increases or decreases when the input increases by one**.

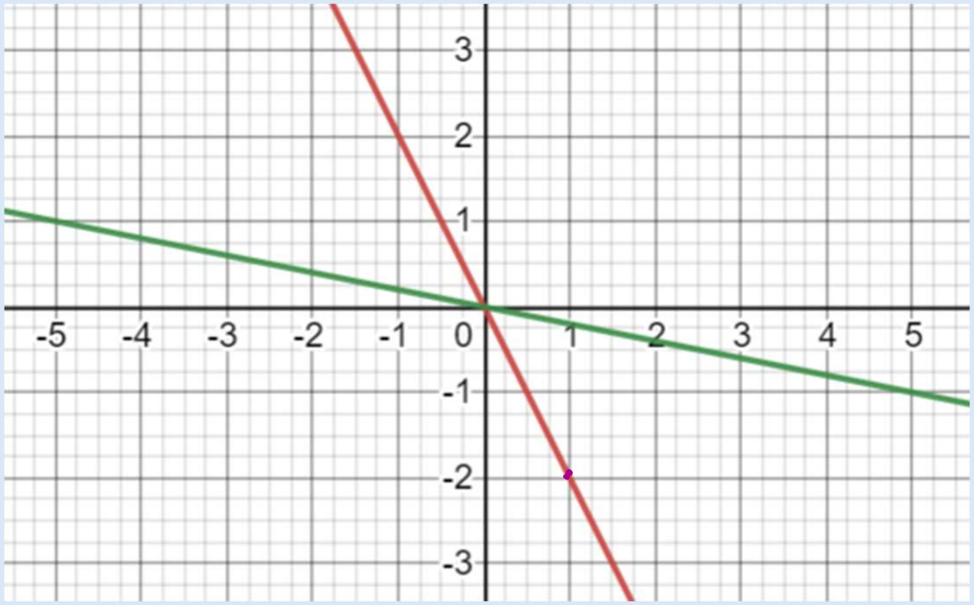
With a negative rate or slope, the output goes down as x increases.
A zero rate or slope stays flat.
With a positive rate or slope, the output goes up as x increases.



With a negative rate or slope, the output goes down as x increases.
The higher the rate, the steeper the slope, which is the tilt of the line.
The lower the rate, the less steep the tilt, which is the slope of the line.

y= - 2x

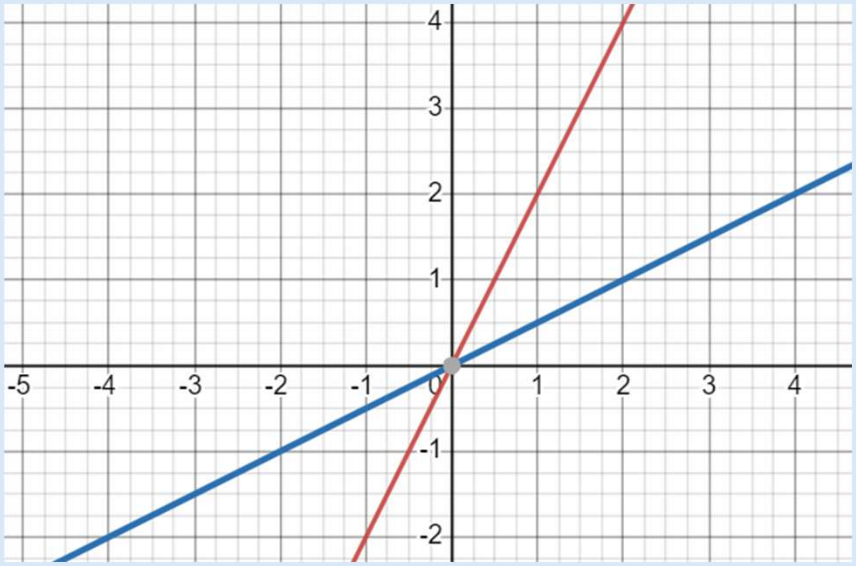
y= -0.2x



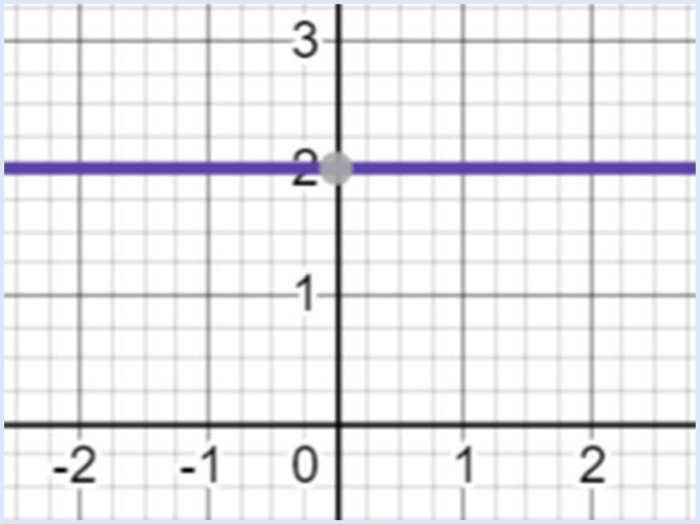
A positive rate, the output goes up as x increases. The slope goes uphill left to right.

The higher the rate or slope, the steeper the tilt of the line.

The lower the rate or slope , the less steep the tilt of the line.



A flat line has a rate of zero and a flat slope.
$y=0x+2$

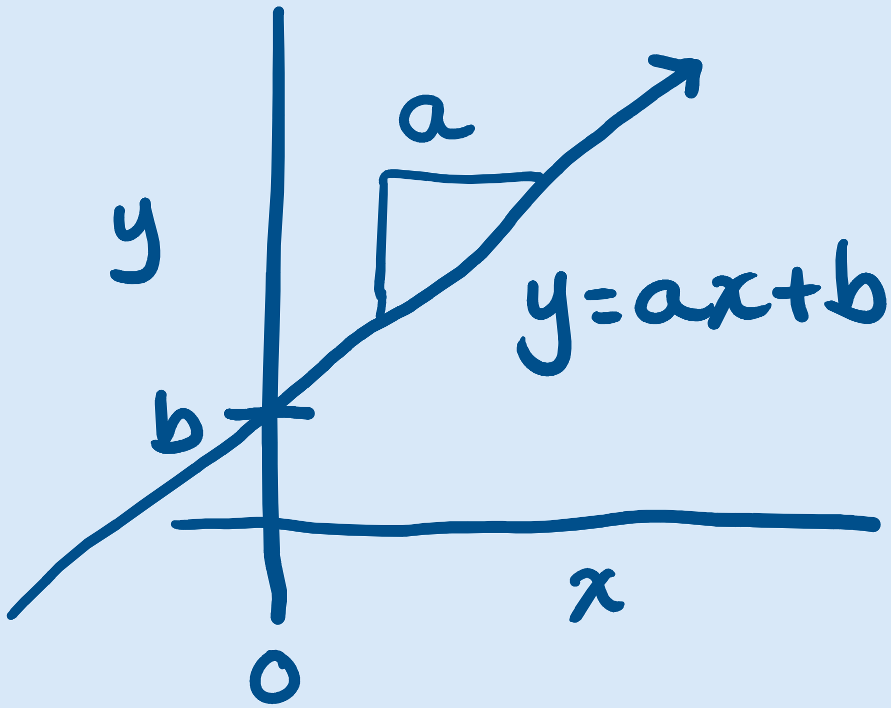


The slope of the line is the rate, coefficient or multiplier of x

y=ax+b

a is the slope, the rate, the coefficient or the multiplier of x

b is how much the a*x line is shifted up or down

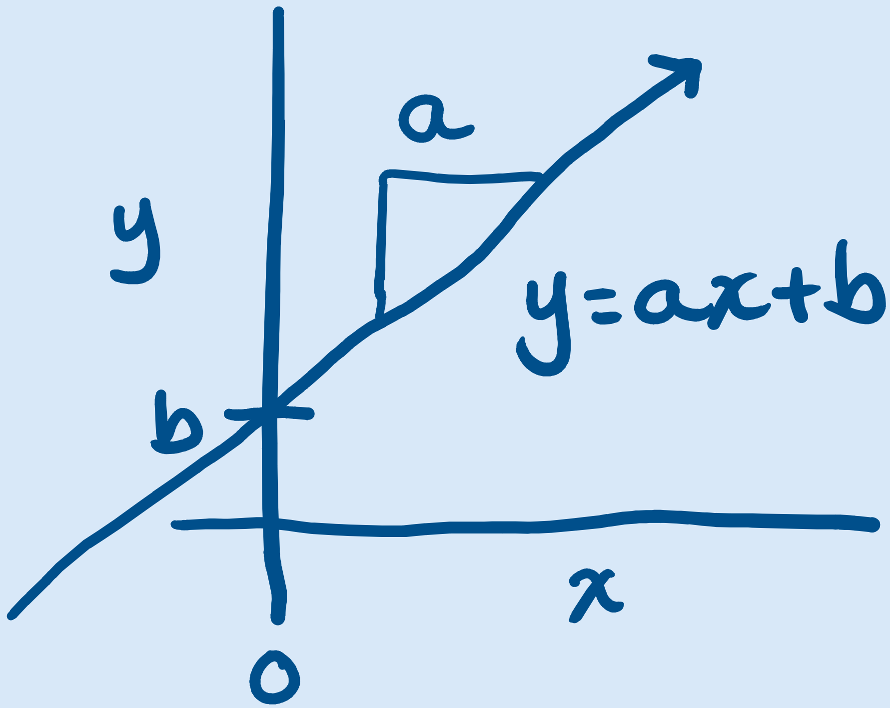


y=ax+b

a is the slope, the rate, the coefficient or the multiplier of x

a tilts the line and tells the steepness. It says how much you go up as you go over.

b is how much the a*x line is shifted up or down and where it crosses the y axis.



output = coefficient (input) + starting output

output = multiplier (input) + starting output

$y =$ tilt $\cdot x +$ y-intercept

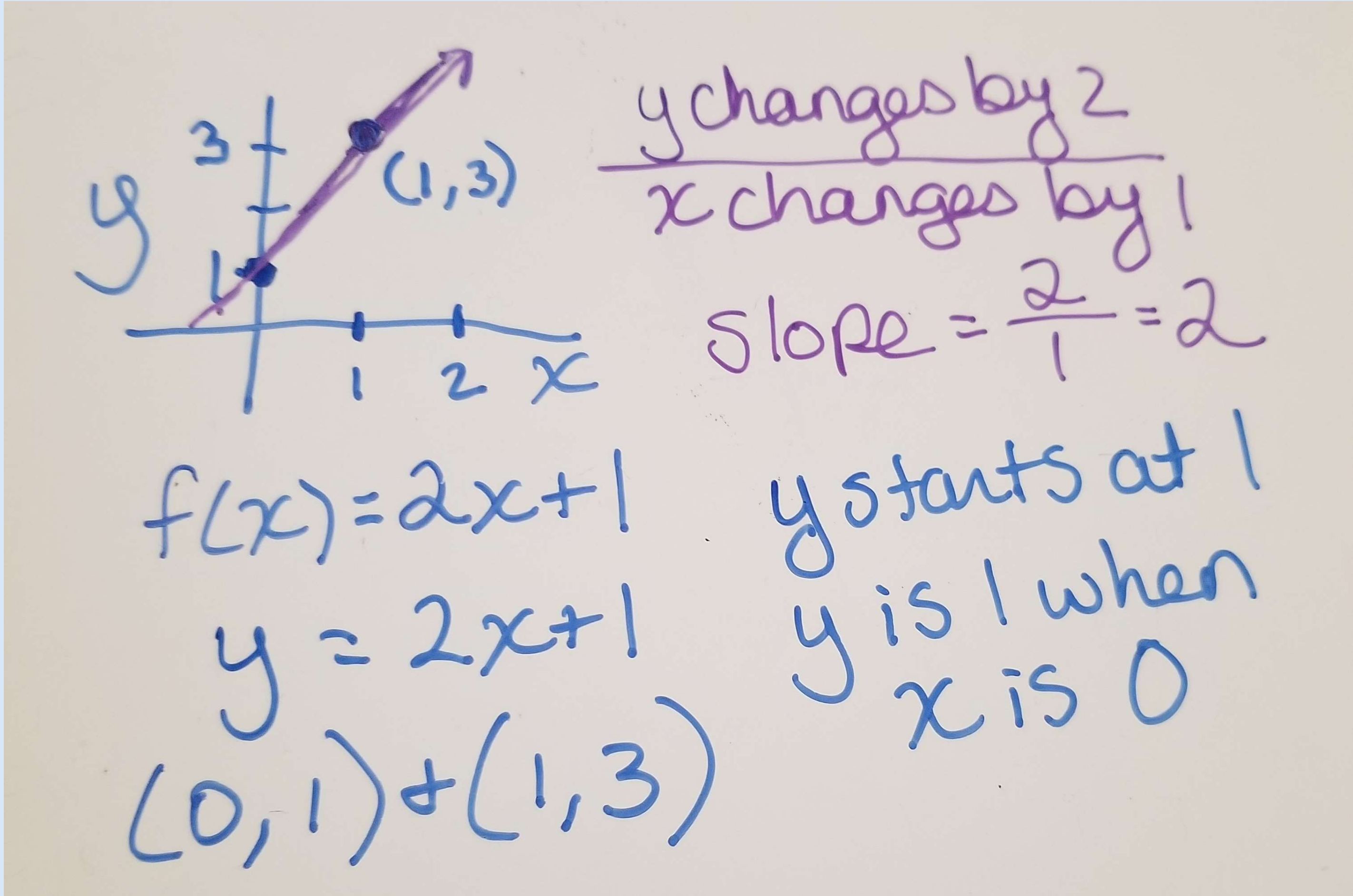
$y =$ rate $\cdot x +$ y-intercept

$y =$ slope $\cdot x +$ y-intercept

$$y = ax + b$$

$$f(x) = ax + b$$

y=2x+1



3
(1,3)
y
1
1 2 x
y changes by 2
x changes by 1
slope = 2/1 = 2
f(x)=2x+1
y = 2x+1
(0,1) + (1,3)
y starts at 1
y is 1 when
x is 0

The slope tells how much you go up or down as you go over.

It is the change of y over the change of x.

It is how much the output changes as the input changes.

It is easy when the input is changing by one and then you get a decimal rate or slope.

$\Delta$ is the symbol for change and I will use it a lot when we do calculus. The derivative in calculus is the rate of change of a function at any given point. For a line the rate of change is the same for any input and it is a or m.

slope a or m $= \dfrac{\Delta y}{\Delta x}$ $\dfrac{\text{change of output}}{\text{change of input}}$

The slope tells how much you go up or down as you go over.

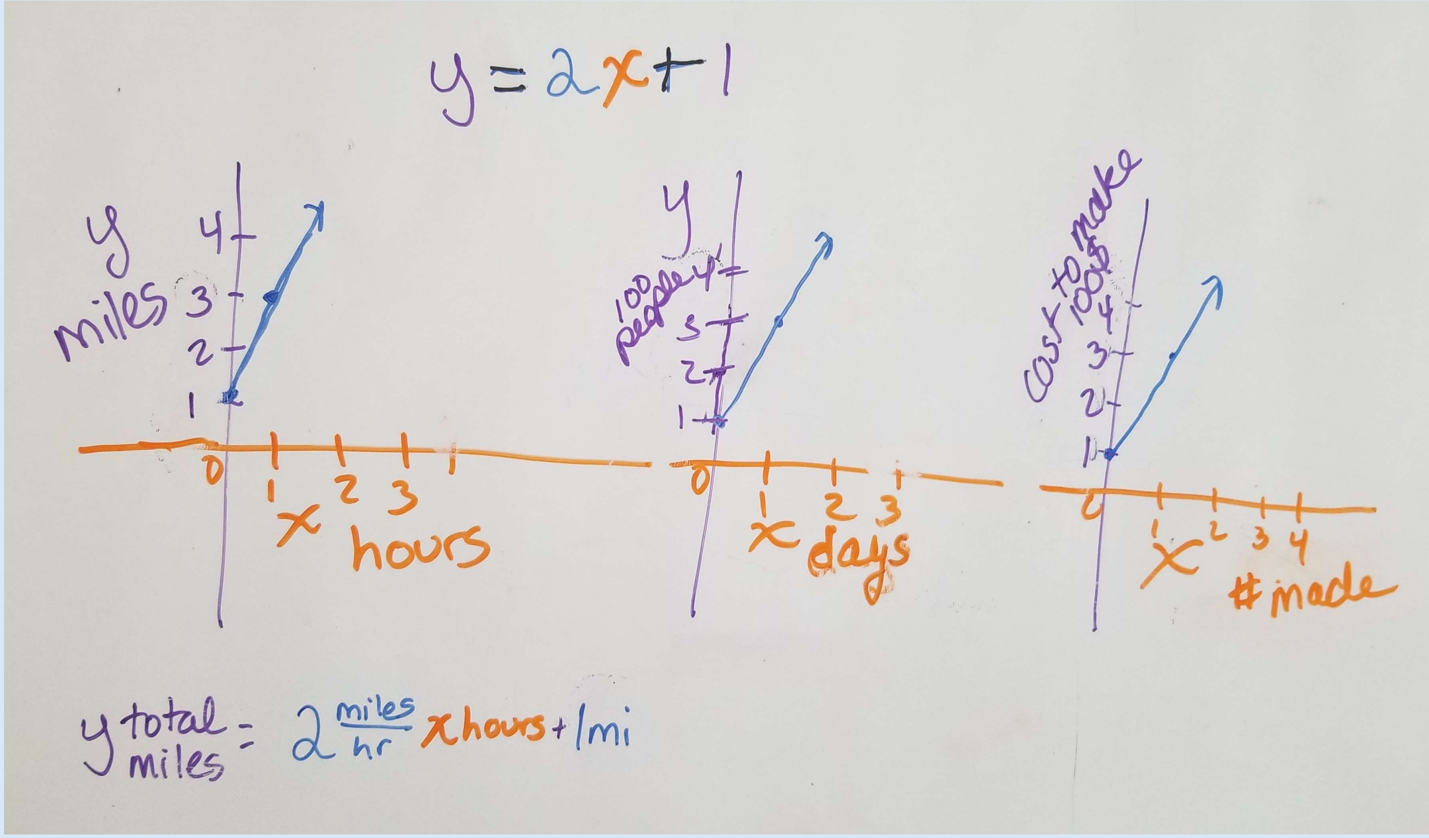
It is the change of y over the change of x.

Warning! In an ordered pair, x comes first and for the slope, y is on top and x is on the bottom.

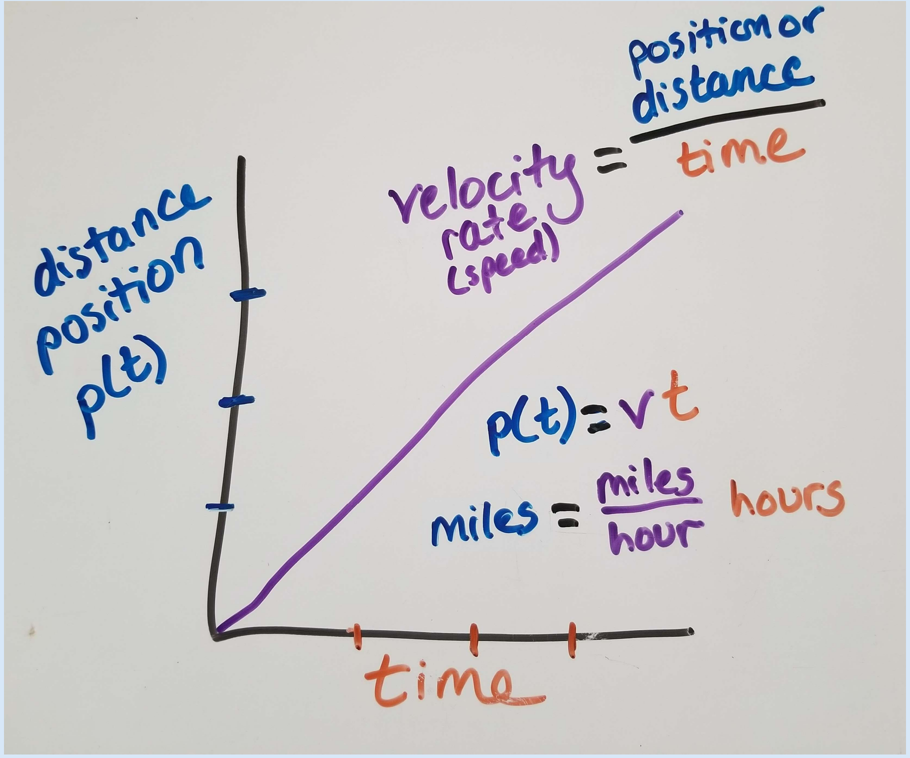
$(x, y)$

slope $a$ or $m$ $= \dfrac{\Delta y}{\Delta x}$ $\dfrac{\text{change of output}}{\text{change of input}}$

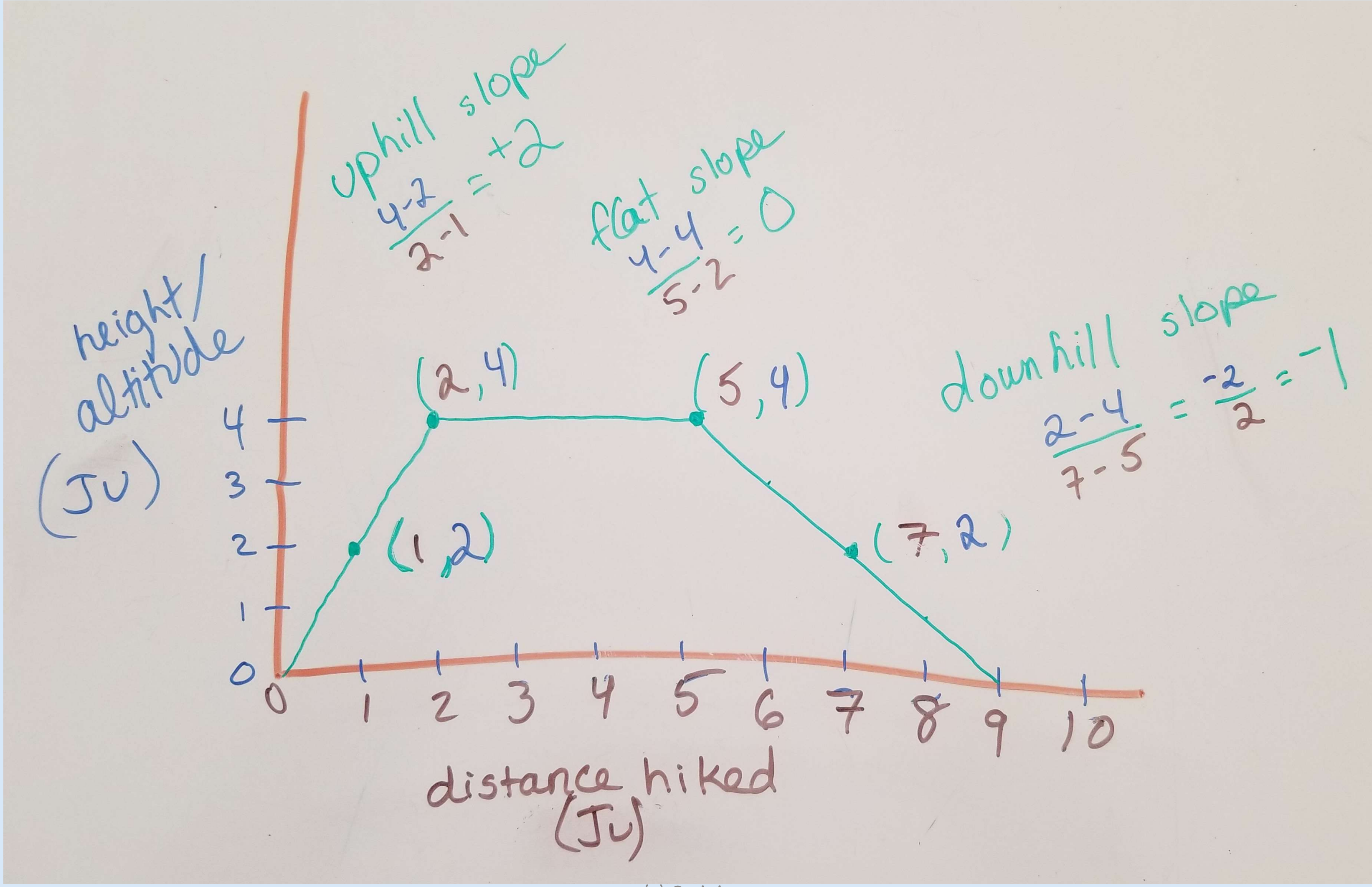
The y and x are variables that change in relation to each other. The slope or multiplier and starting point or y-intercept are fixed properties of a particular line.



# Going a constant speed is a linear function.



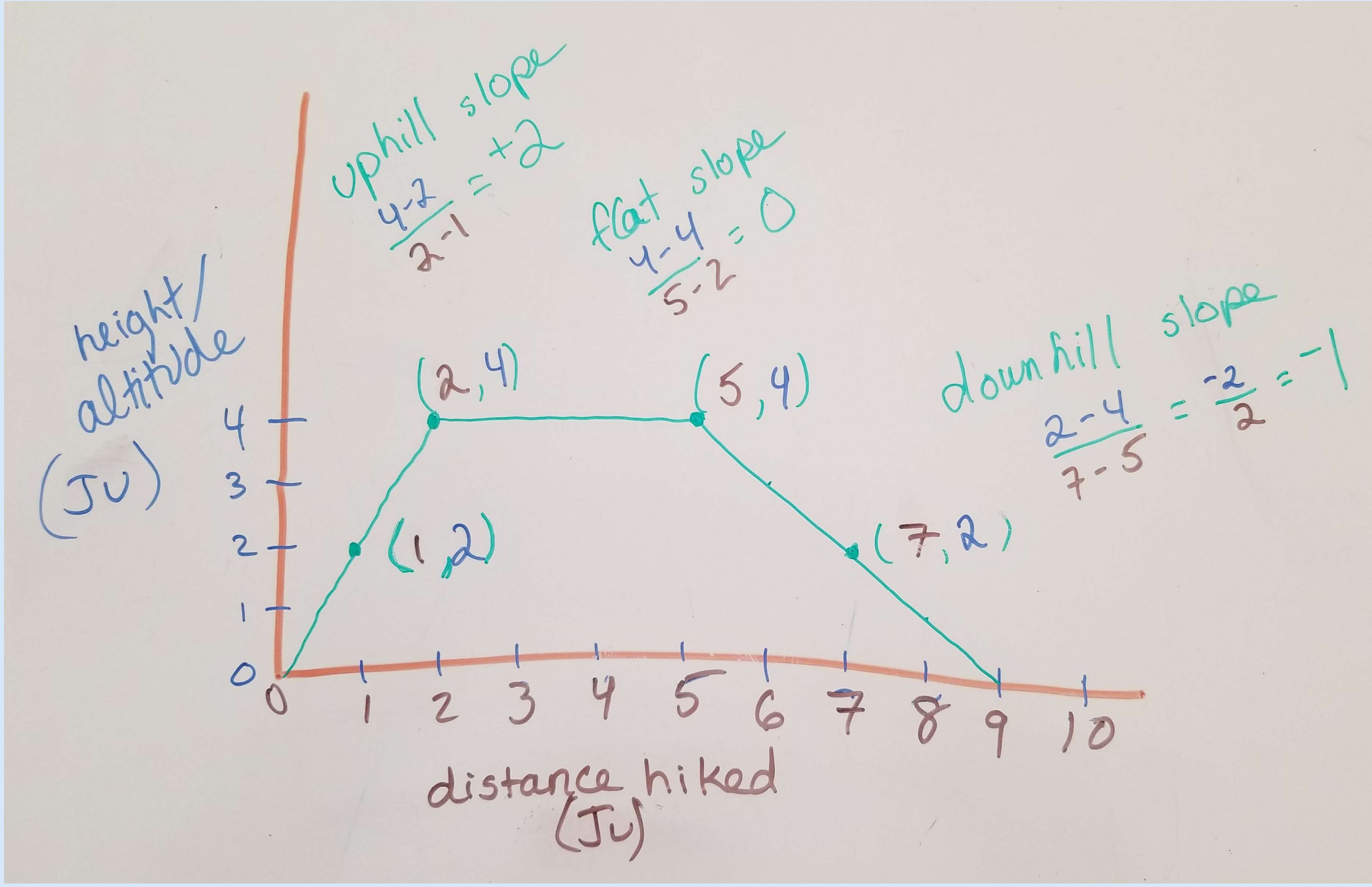
Hiking with height or altitude vs distance hiked in Jae units

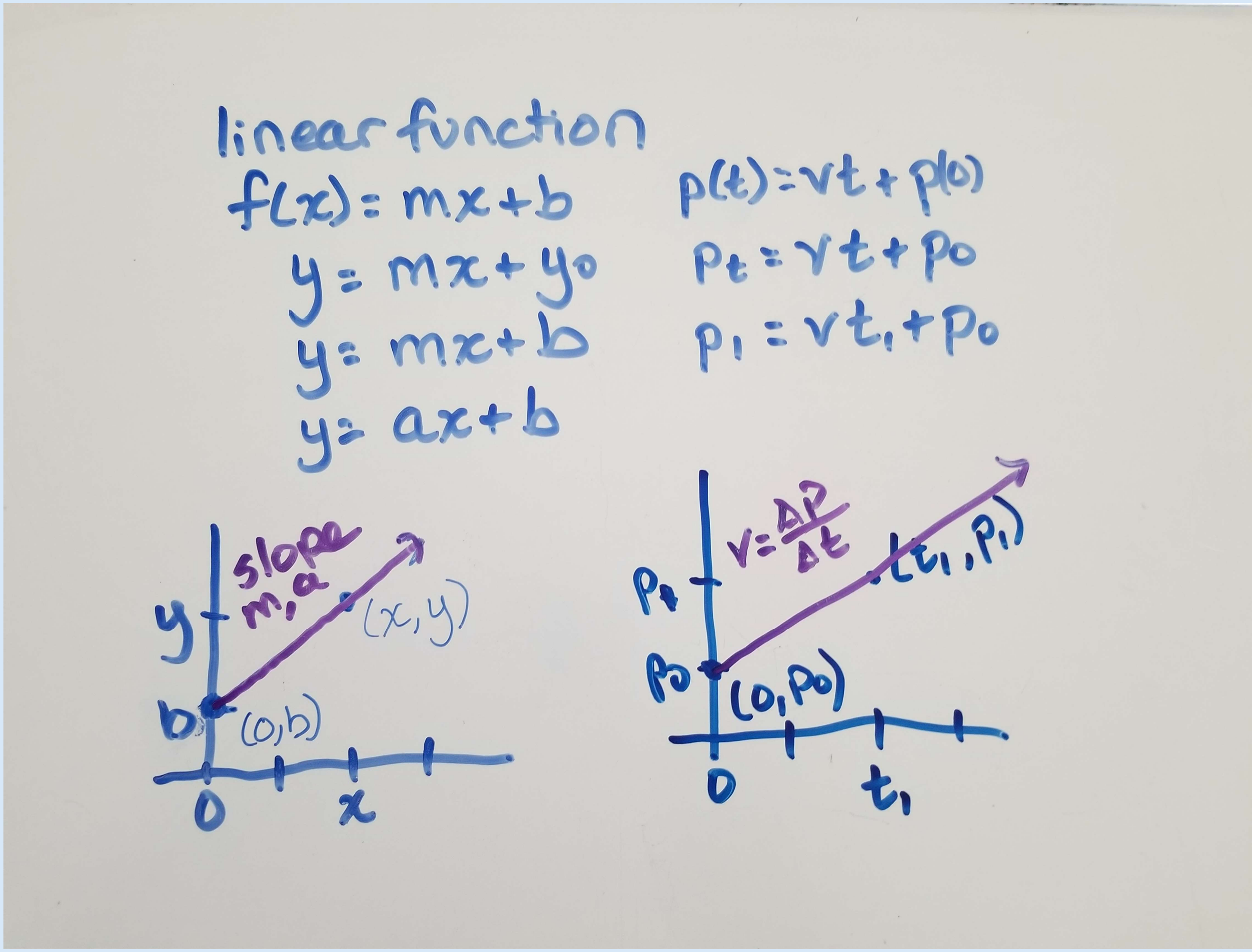


Start with a slope of positive 2 so going up twice as much as hiking.

Then going with a slope of 0 along a flat ridge so still hiking but not going up or down.

Then coming down a shallower grade with a slope of -1




linear function
$f(x) = mx + b$
$y = mx + y_0$
$y = mx + b$
$y = ax + b$
$p(t) = vt + p(0)$
$p_t = vt + p_0$
$p_1 = vt_1 + p_0$
slope
m, a
$(x, y)$
$y$
$b$
$(0, b)$
$0$
$x$
$v = \frac{\Delta P}{\Delta t}$
$(t_1, p_1)$
$P_t$
$P_0$
$(0, P_0)$
$0$
$t_1$

$$p = vt + p_0$$

$$\text{position at time } t = \text{velocity} \cdot \text{time} + \text{initial position}$$

$$\text{velocity} = \frac{\Delta\, \text{position}}{\Delta\, \text{time}}$$

$$v = \frac{p_2 - p_1}{t_2 - t_1}$$

$(t_1, p_1)$

$(t_2, p_2)$

# Ways of representing linear functions