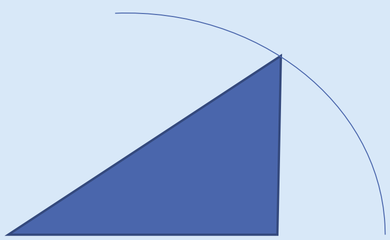
Geometry

Geometry is the study of the way shapes work.
https://en.wikipedia.org/wiki/Geometry

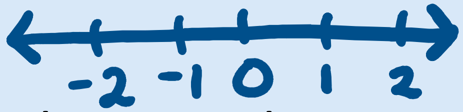
There are some mind-blowing concepts that come up because shapes don't all conform to the same measuring systems.

This is called incommensurability, meaning "no common measure".



And there is a lot of infinity. ∞

We measure in units that are continuous variables. This means that in between each integer, there is an infinite number of rational numbers.



Rational numbers can be written as a ratio of two integers.

$\frac{a}{b}$ $b \neq 0$ $\frac{2}{3}$ $\frac{3}{11}$ $\frac{1}{3}$

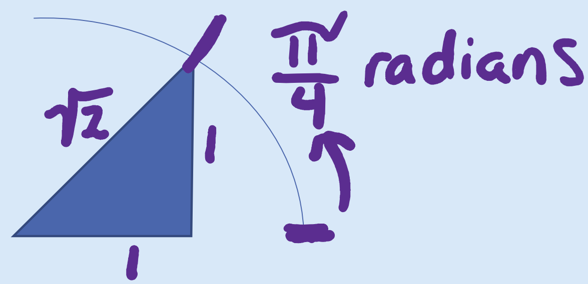
In between all of the rational numbers, there are an infinite number of irrational numbers that can't be written as a ratio of integers.

$\sqrt{2} \approx 1.414\ldots$ $\pi \approx 3.1415\ldots$

We will only be using a few irrational number categories so we will give them names and symbols so we can talk about them as exact numbers. For geometry, those are the square roots of numbers that aren't perfect squares and pi $\sqrt{2}$ $\sqrt{3}$ $\sqrt{5}$ $\sqrt{6}$ π

Measure systems for sides of rectangles, diagonals of rectangles, and going around in a circle are incommensurable, meaning “no common measure”.

If the sides of the rectangle are rational numbers, then the diagonal can be a square root, and the measure around the circle is in terms of an irrational number π that we call pi. We will come back to all of this after we start with the basics.



First, there is the tricky concept of infinity.

A point **has location but no size**. Although we draw them as a dot, since they have no size there can be infinitely many points between any two locations.

Zooming in and out to get a feel for large infinity and small infinity.

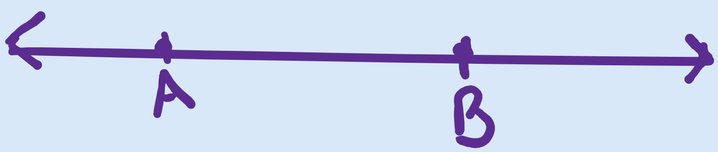
Eye to Universe, Eye to Outer Space, Eye Zooms into Universe – YouTube

Zooming into Australia from outer space to a cell Zooming in to Cell level – YouTube

A point has location but no size. We draw them as a dot.

In geometry, a straight line is defined as the set of all points on the shortest distance between two points and it extends on to infinity in both directions. There are infinitely many locations on the line.

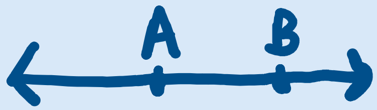
There are infinitely many points or locations between any two points.



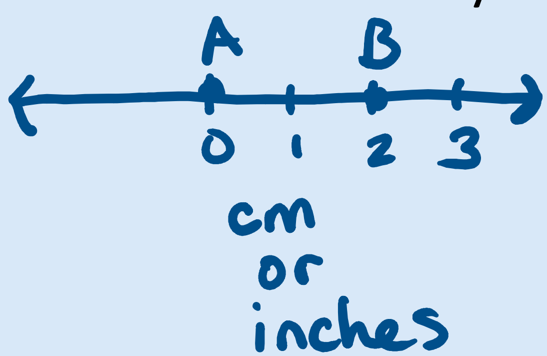
Lines can get their names from any two points on the line. For example, if a line contains two distinct points, named A and B, then the line can be called either $\overleftrightarrow{AB}$, or $\overleftrightarrow{BA}$.

From: Line - Simple English Wikipedia, the free encyclopedia

Since a line is made up of points, lines have location but no size. They don't have a length or a width.

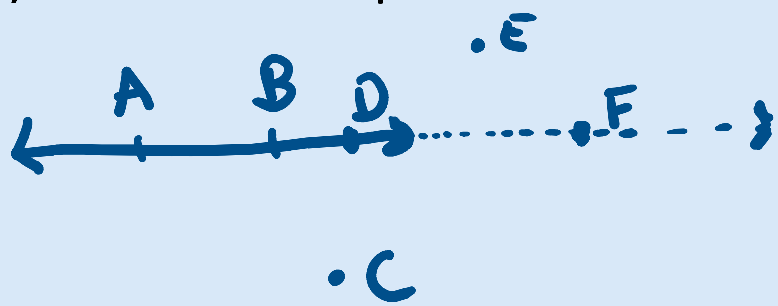


We can have a measurement system that we use to name the location of the points. Then we can talk about length in terms of the distance along the measurement system. We can talk about the distance, or length in that measurement system between two points.



Two points define a line. This means that a line is created from two points.

Other points can be on the line (colinear) or not on the line (noncolinear) with the two points that defined the line.



In geometry, a line segment is a line between two different end points.

There are infinitely many points on the line segment and there are infinitely many points between any two points on the line segment.

We can name the line segment by the endpoints. We write it as the endpoints with a line over. $\overline{AB}$ $\overline{BA}$

We can join line segments at their endpoints to make shapes.

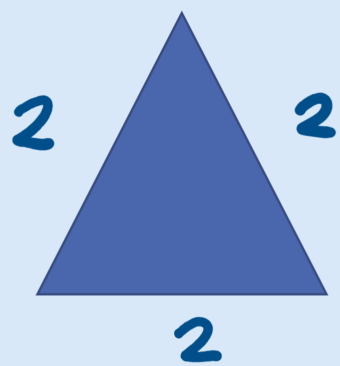
A polygon is the name for a many-sided figure. There are special names for some of the number of sides.

A triangle has three sides.

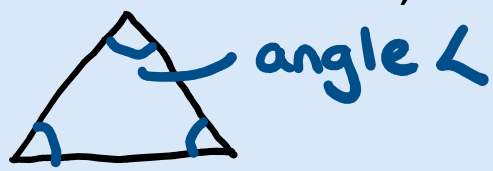
A rectangle has four sides.

We are going to start by focusing on triangles.

Triangles have three sides. The sides are line segments that have a length that we can measure with our linear system.



Triangles have three sides. They also have three angles. In order to understand angle measurements, we are going to switch over to talking about circles.

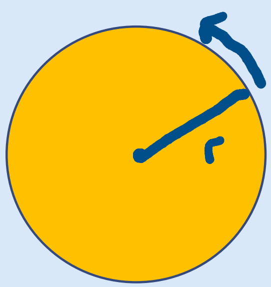


Some or many of you may be used to angle measurements in degrees but in math, we measure angles in radians, which are the angular distance around a circle.

There is a direct ratio between the measurement across and around a circle so if we use radians, we are using a direct ratio. Degrees add in a whole other measurement system that is not a direct ratio, so it gets complicated to use degrees in math.

A circle is a nonlinear or curved shape that is created by taking a line segment, fixing one endpoint, and then spinning the second endpoint around.

The length of the line segment that gets spun around is called the **radius**. We will talk about unit circles that have a radius of one unit. (It can be meters, inches, miles… in math we just call it a length of one.)



You can try making a circle with a writing utensil or a ruler or a piece of string that you pull tight.

It turns out that the measure around the circle (circumference) compared to the measure of the radius of the circle does not give a rational number.

They are incommensurable. The distance around a circle divided by twice the radius gives a constant. The Greeks apparently used the diameter, which is twice the radius.

$$\frac{\text{distance around (circumference)}}{2 \cdot \text{radius (diagonal)}} = \pi$$

$$C = 2\pi r$$

The distance around a circle (called the circumference) divided by twice the radius gives a constant. The Greeks apparently used the diameter, which is twice the radius and much easier to measure from the outside of a column or a tree trunk or anything solid.

This constant was named after the Greek letter pi with the symbol π and it is an irrational number.

$$\frac{\text{distance around (circumference)}}{2 \cdot \text{radius (diagonal)}} = \pi$$

$$C = 2\pi r$$

Since we are talking about a unit circle, with a radius of one, we call the measurement around “radians”.

For a unit circle (with a radius of one), C=2π radians. The circular measurement is in the same units as the radius but the distance to go all the way around is an irrational number.

$$\frac{\text{distance around (circumference)}}{2 \cdot \text{radius (diagonal)}} = \pi$$

$$C = 2\pi r$$

C=2π radians for a unit circle.

The circular measurement is in the same units as the radius but the distance to go all the way around is twice the irrational number pi.

If the radius is one centimeter, then the circumference would be 2π cm or about

$$\frac{\text{distance around (circumference)}}{2 \cdot \text{radius (diagonal)}} = \pi$$

$$C = 2\pi r$$

C=2π radians for a unit circle.

The circular measurement is in the same units as the radius but the distance to go all the way around is twice the irrational number pi.

If the radius is one centimeter, then the circumference would be 2π cm. This is the exact number of the distance or length around the circle.

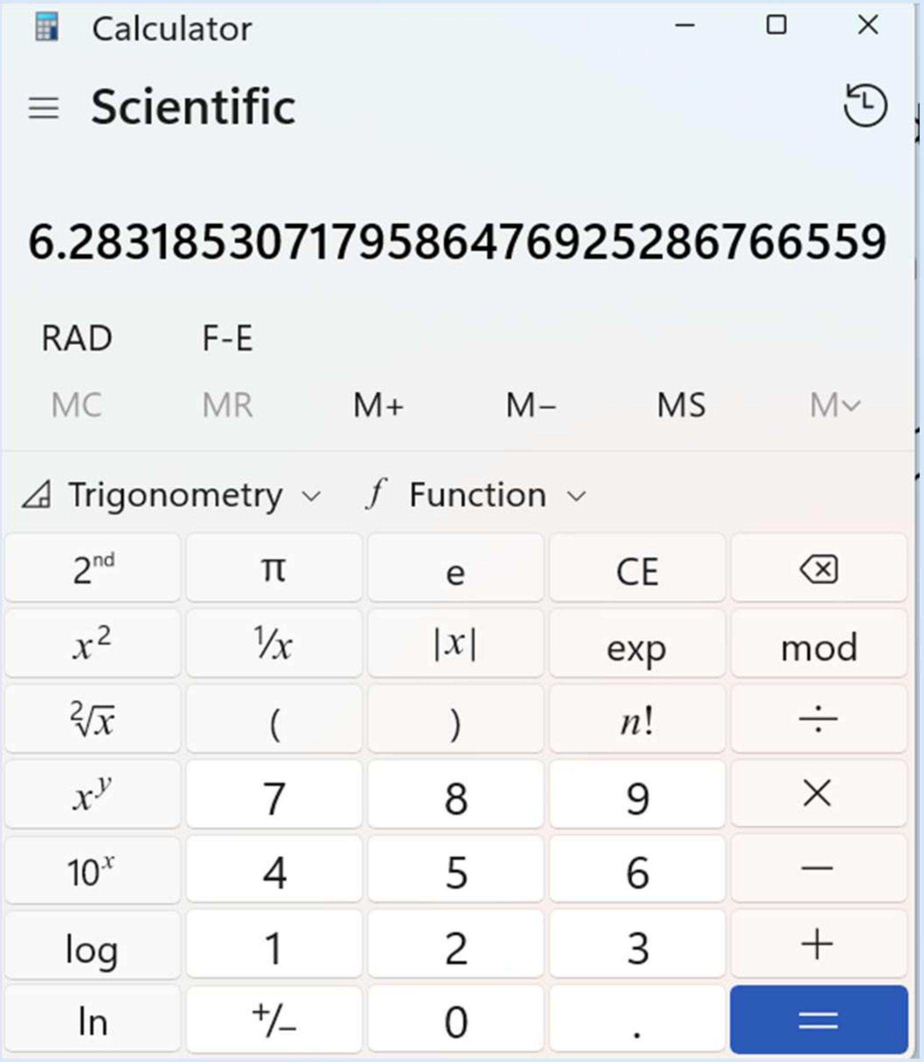
$$\frac{\text{distance around (circumference)}}{2 \cdot \text{radius (diagonal)}} = \pi$$

$$C = 2\pi r$$

If the radius is one centimeter, then the circumference would be 2π cm. This is the exact number of the distance or length around the circle.

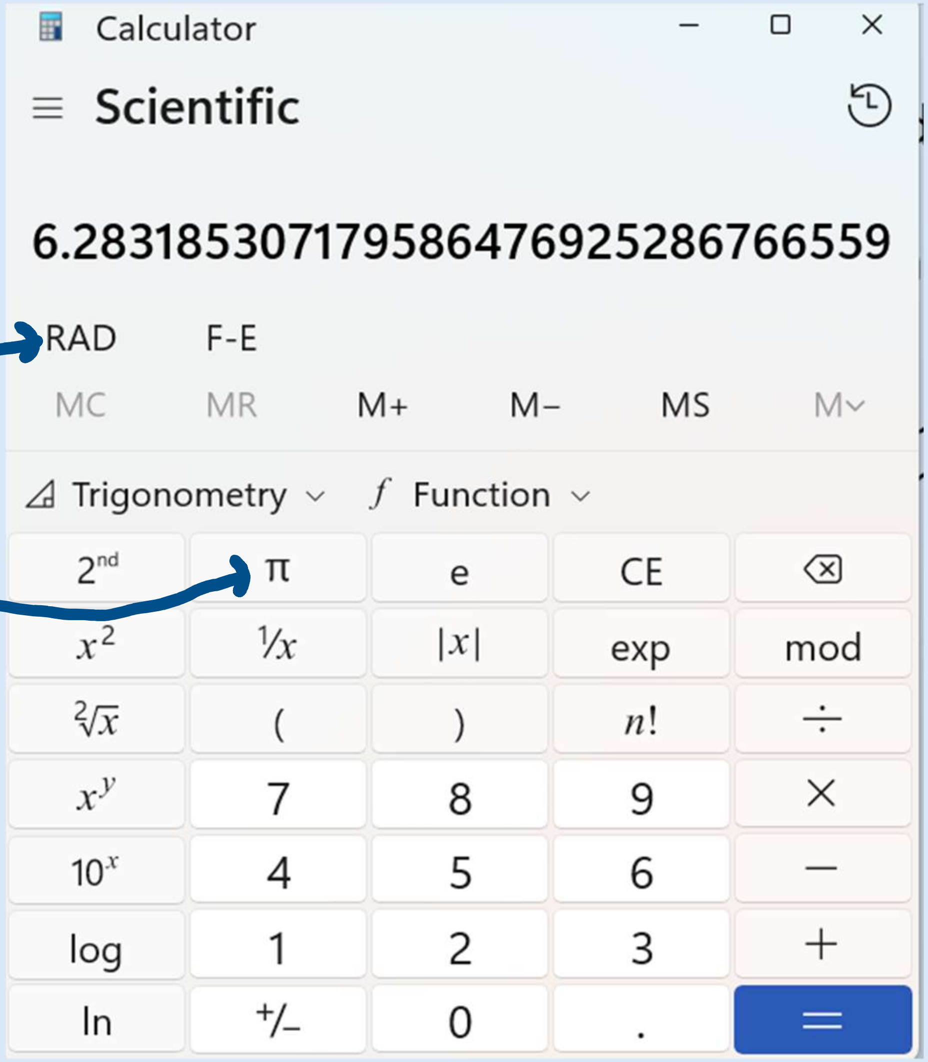
We can use a calculator to get an approximation.

$C = 2\pi r$

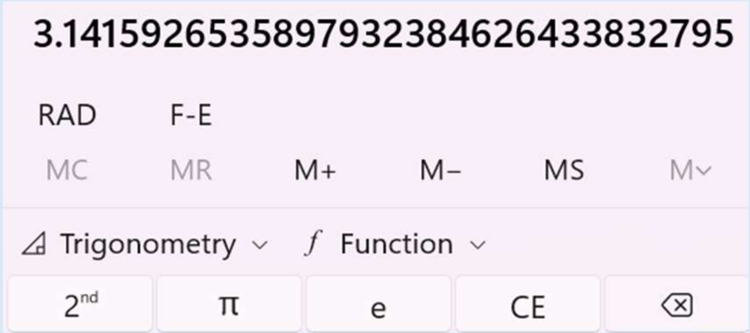


Notice that I set it to radians instead of degrees.

Notice the π button.



Pi is the irrational number that is the conversion factor for the linear distance of the diameter (twice the radius) to the circular distance around the circle. https://simple.wikipedia.org/wiki/Pi has a nice animation.



Here is pi on my calculator. Computers do calculations with series to calculate pi.

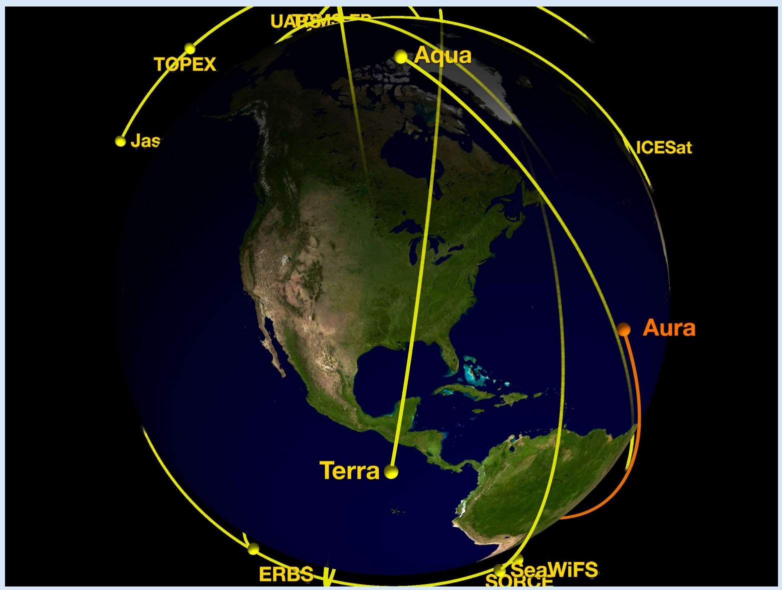
Calculating Pi (π) - Maths Careers

In math, we always use the symbol pi in equations, since pi represents the exact number, no matter how many digits we go out to.

If I am measuring ribbon for around a round cat basket, then I can use 3.15 or 3.2 to multiply by twice the radius and cut off any excess.

In math, we always use the symbol pi in equations, since pi represents the exact number, no matter how many digits we go out to.

If we are measuring the orbit of a satellite around the earth, then I need to go way further out on my digits to be more accurate. [SVS: NASA's Orbiting Earth Observing Fleet (includes Aura in orange)]

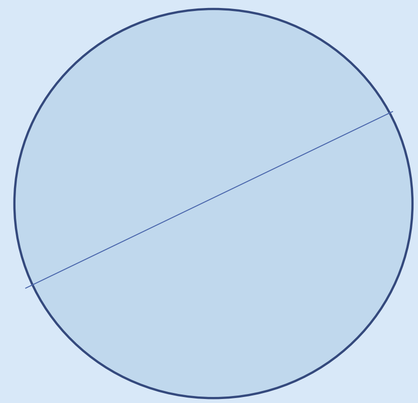


We use radians in terms of pi to measure angles, since it is a direct ratio and not a different measuring system, like degrees.

Degrees are a different measuring system, but pi is a Real number that is the ratio as long as the circumference and radius are in the same units.

$\pi = 180^\circ$

$\frac{\pi}{180^\circ}$ or $\frac{180^\circ}{\pi}$



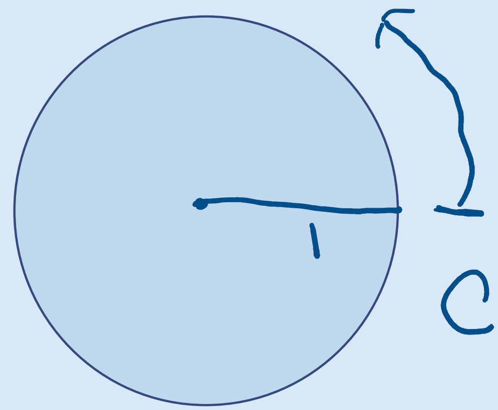
$\frac{C \text{ ft}}{2r \text{ ft}} = \pi$ direct ratio

For math, we use pi radians and then convert to degrees if we need to later.

Halfway around a unit circle is π or 180°

$\pi = 180^\circ$

$\frac{\pi}{180^\circ}$ or $\frac{180^\circ}{\pi}$



$C = 2\pi \approx 6.25$

For math, we use pi radians and then convert to degrees if we need to later. Pi radians equals 180 degrees.

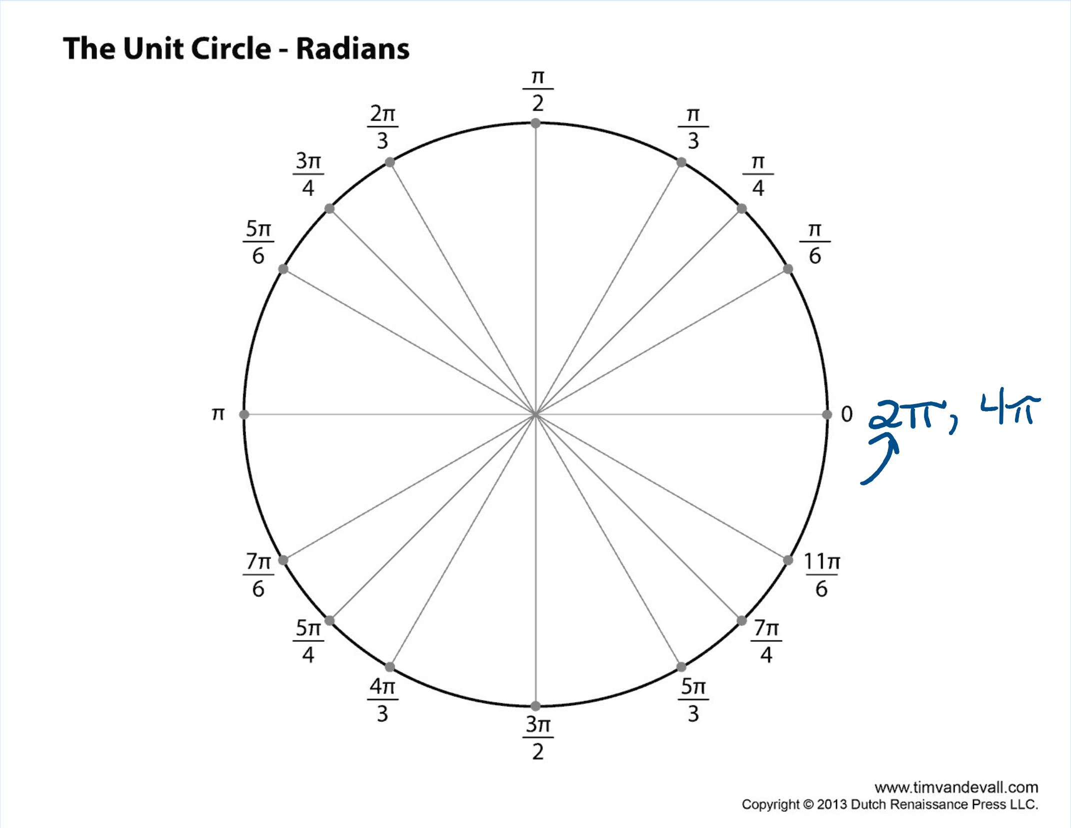
Halfway around a unit circle is π or 180°

All the way around is 2π or 360°

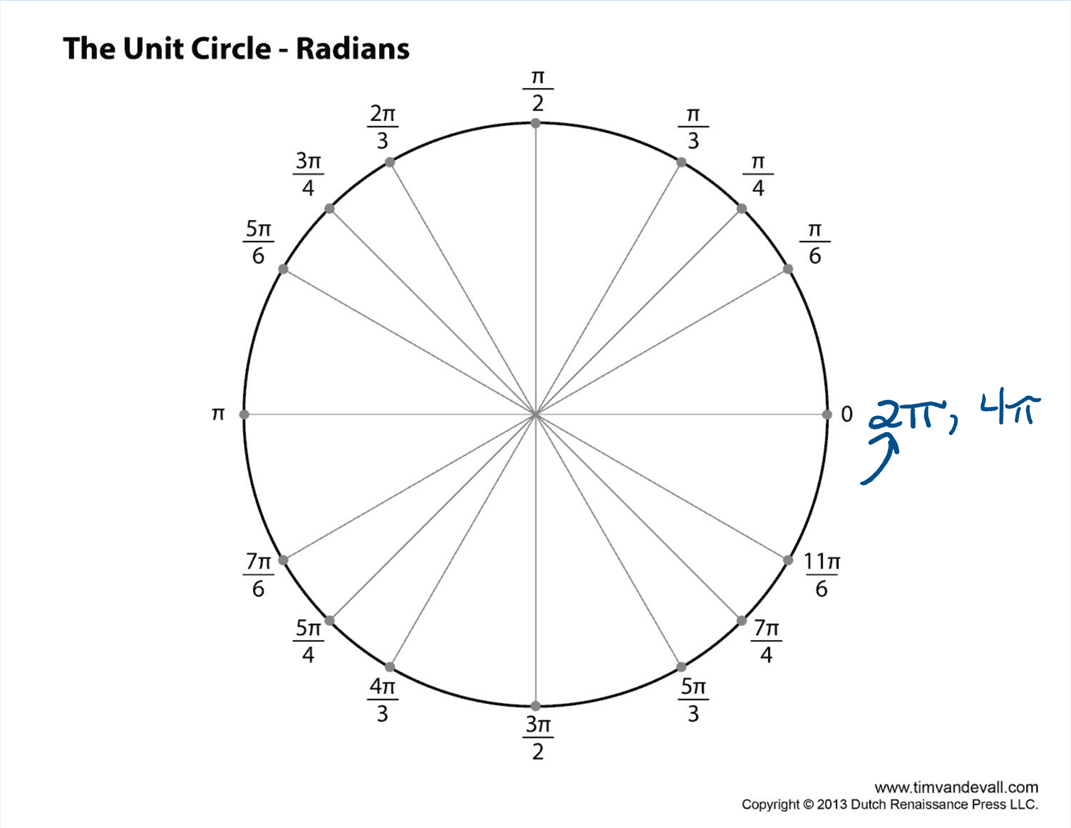
One pie is 2π

Pies and photo from
Dr Susan Marcolina
AKA Dr Pie or Doctor 2π

The unit circle is defined as having a radius of one, so the angle measurements are defined for a unit circle. We go around in a counterclockwise direction, which can get confusing for some people.

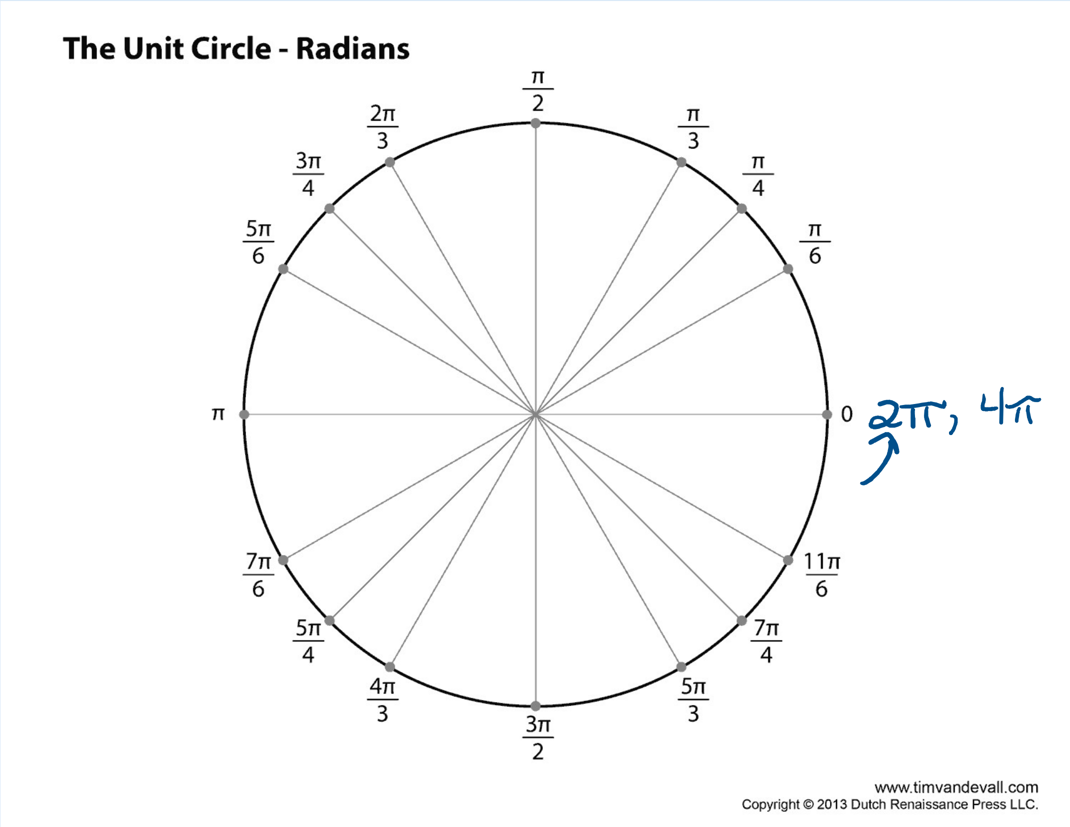


Because the distance loops back, we can scale the number of times we go around by multiplying the distance around times the number of times we go around. So going around once is 2π and going around twice is 4 π.

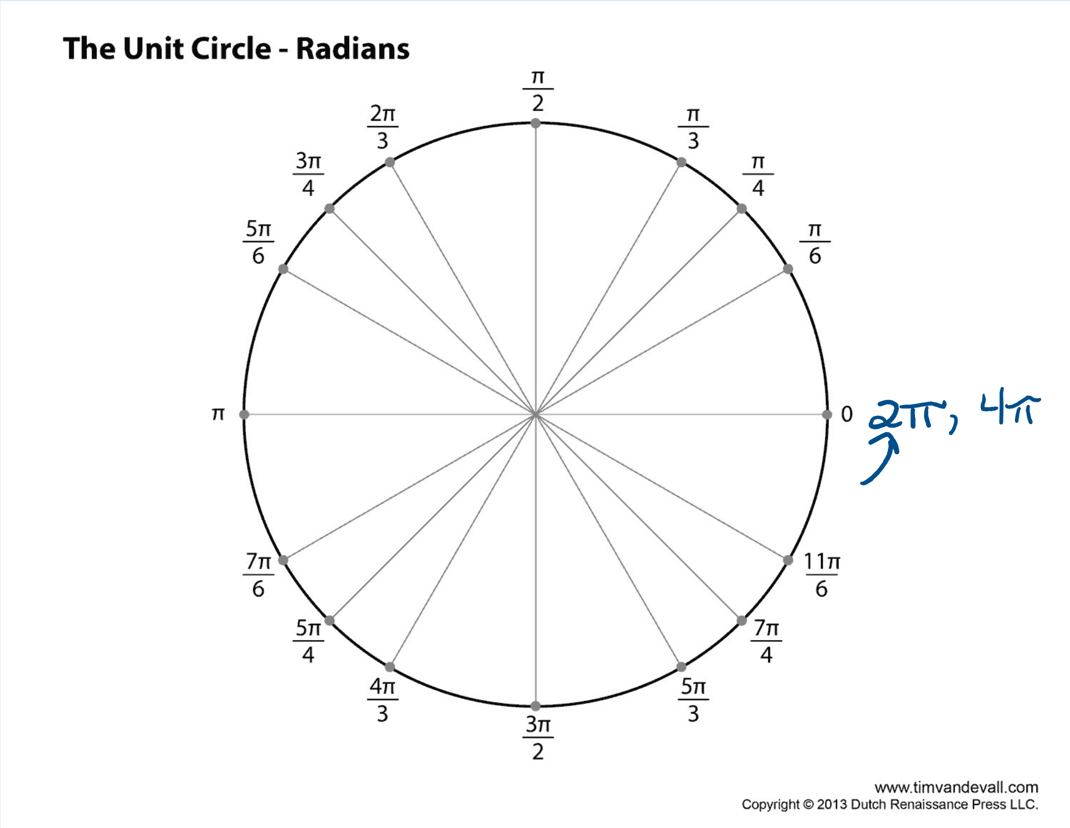


What if we went around 3, 4, or 5 times?

Do you notice the pattern?

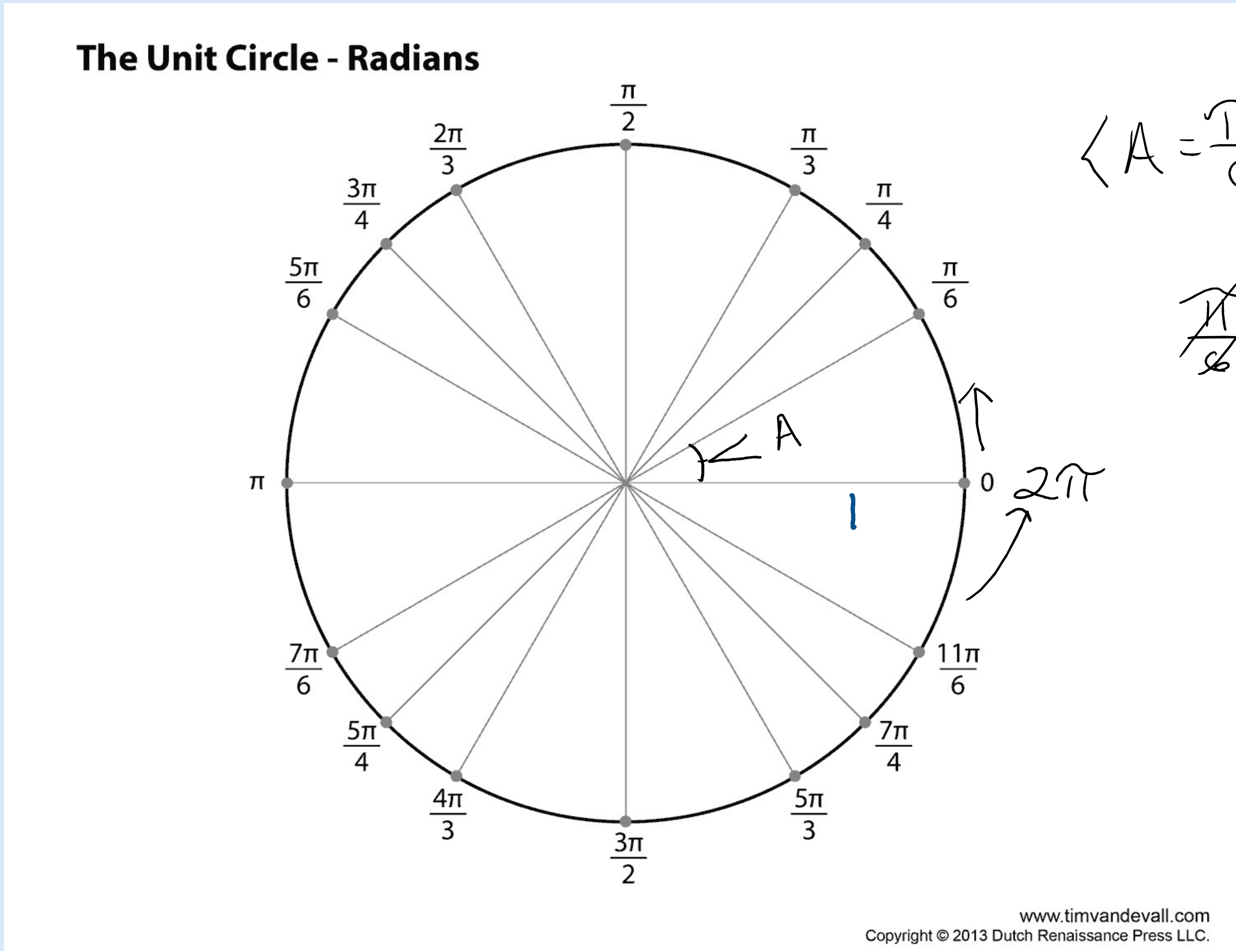


Since we are talking about angles of triangles right now, we will focus on the lengths between 0 and π because all the angles in a triangle add up to between 0 and π radians.



$2\pi, 4\pi$

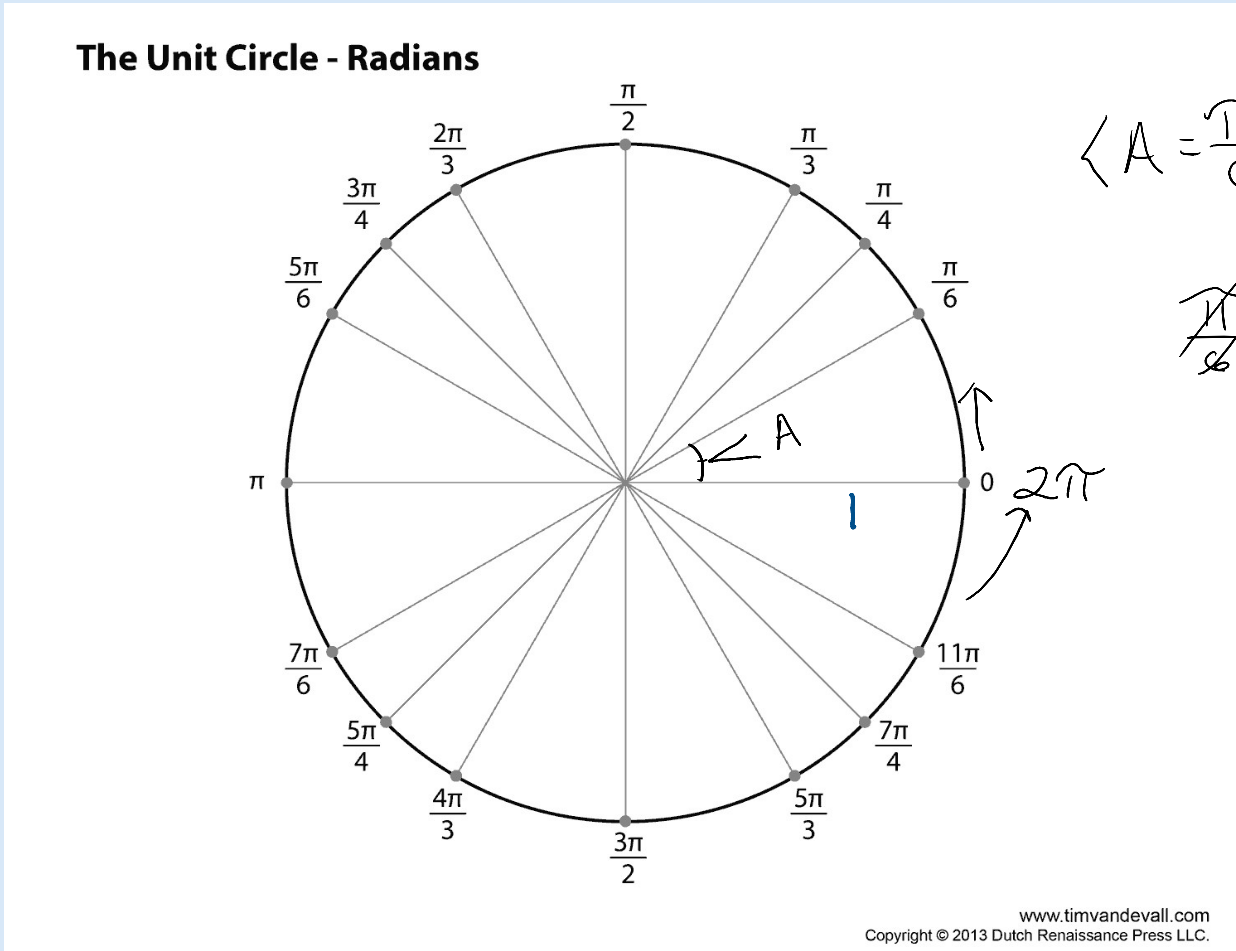
We measure the angles in terms of the distance around a unit circle.



$\angle A = \frac{\pi}{6}$ radians

$$\frac{\cancel{\pi}}{\cancel{6}} \cdot \frac{\cancel{180^\circ}^{\,30^\circ}}{\cancel{\pi}} = 30^\circ$$

We measure the angles in terms of the distance around a unit circle.

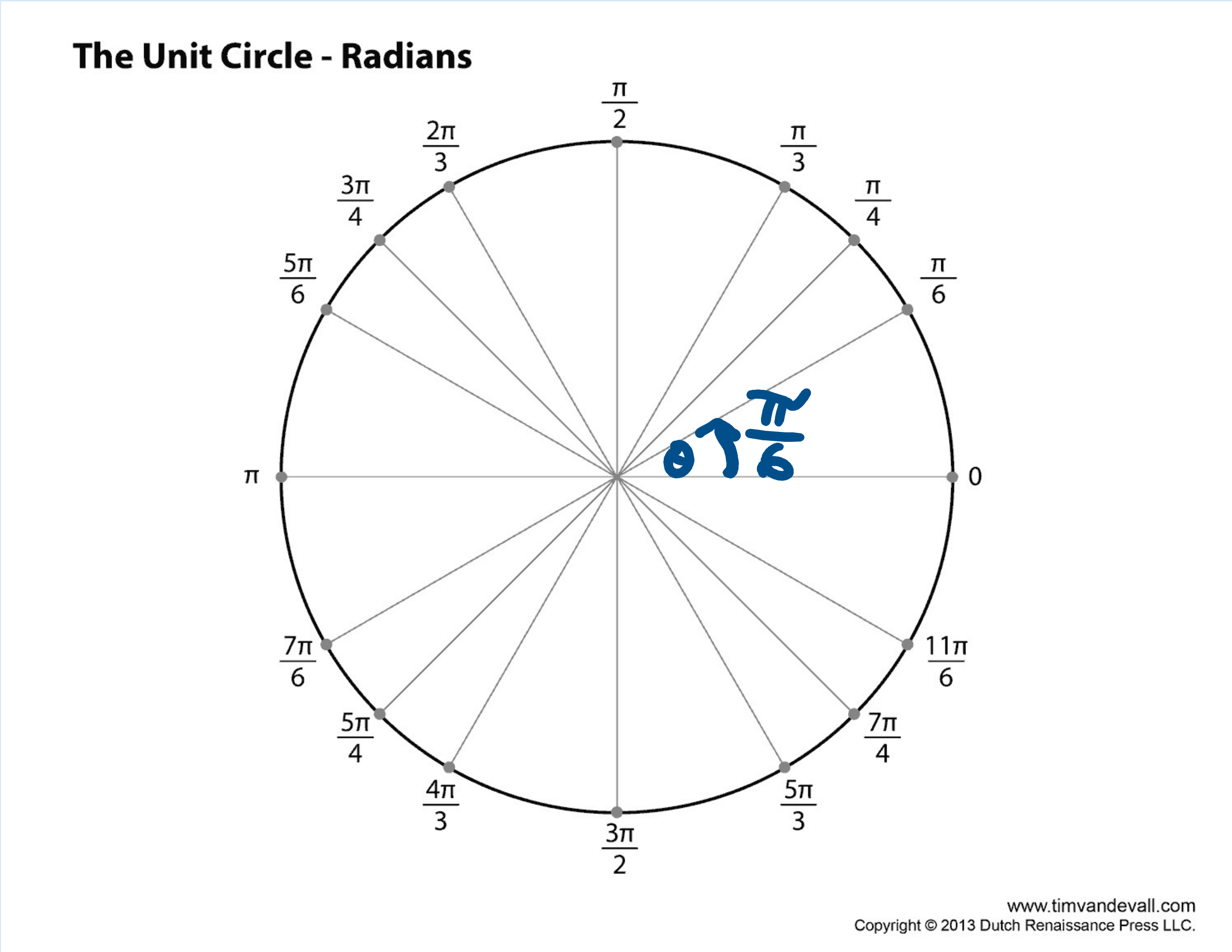


$\angle A = \frac{\pi}{6}$ radians

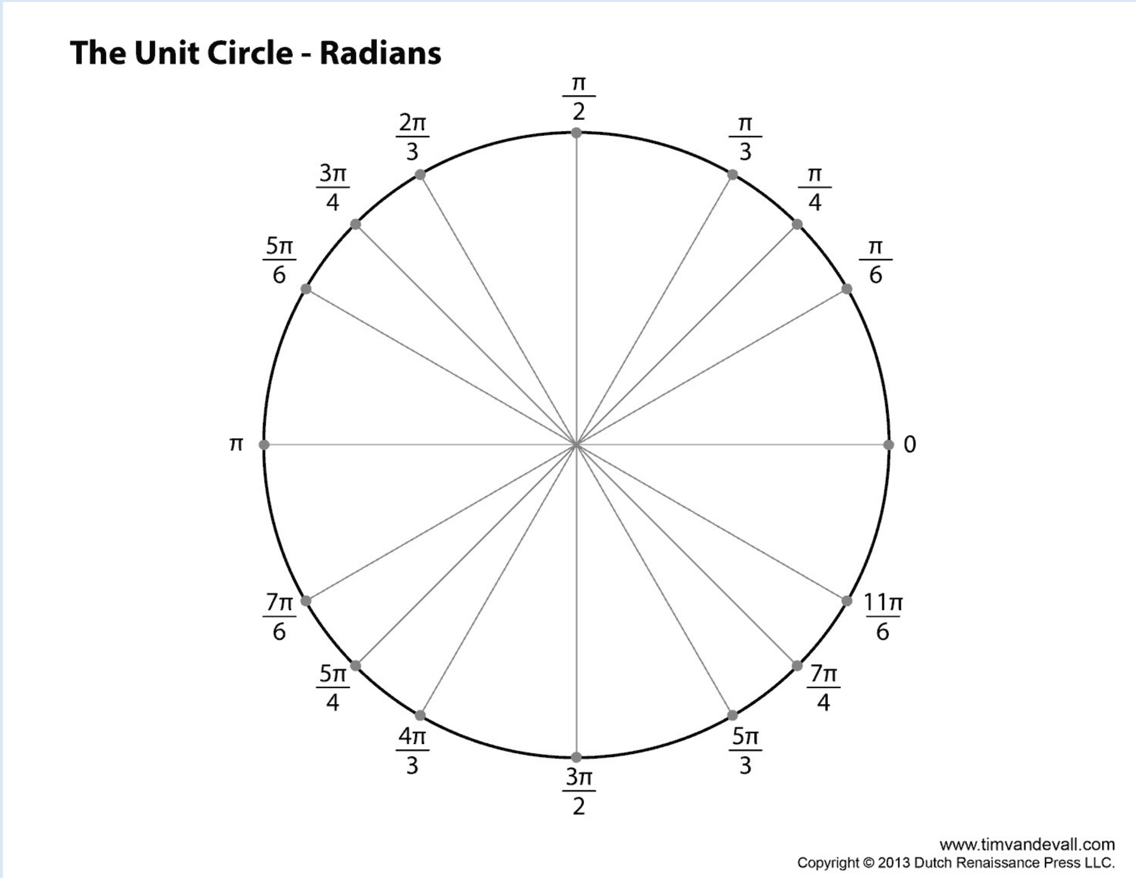
$$\frac{\cancel{\pi}}{\cancel{6}} \cdot \frac{\overset{30^\circ}{\cancel{180^\circ}}}{\cancel{\pi}} = 30^\circ$$

We use the Greek letter theta θ to represent the angle measurement.

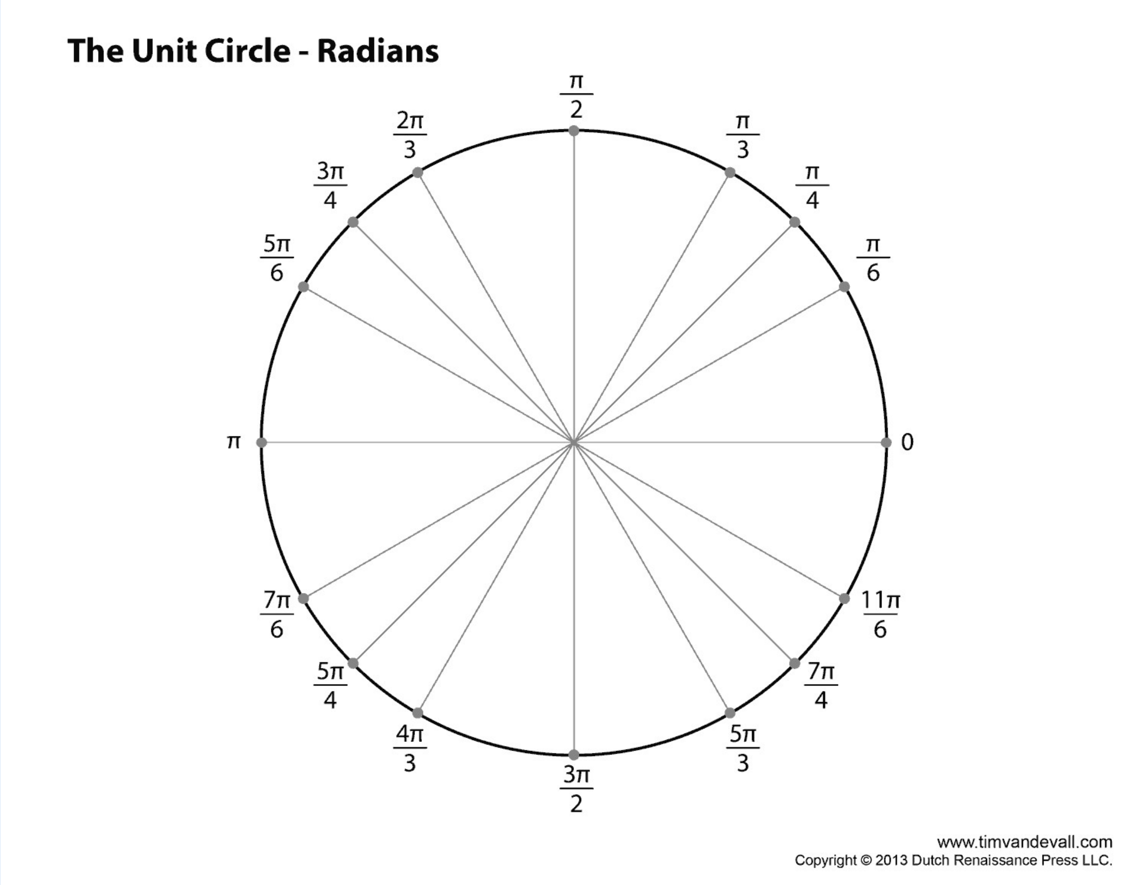
$\angle\theta = \frac{\pi}{6}$



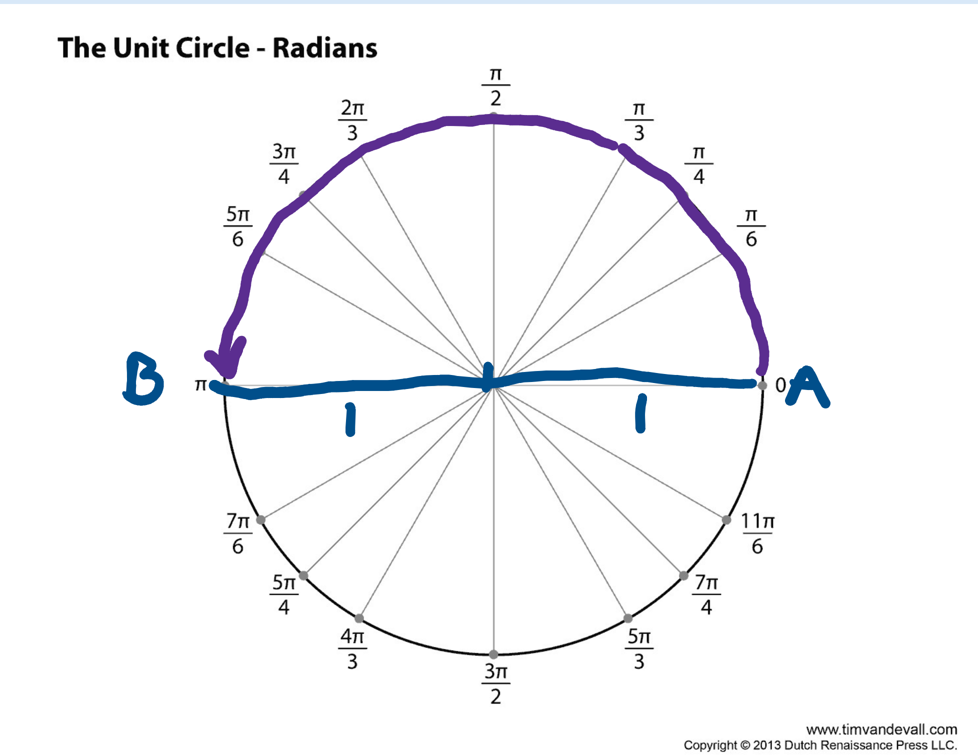
We can find all the pi over 6 angles that are on this graph.



We can find all the pi over 4 angles that are on this graph.



The distance around a unit circle, which has a radius of 1, is the linear measurement times the angle measurement in radians. Going from A to B across the diameter is two units and going around is π units. In math, we leave it as the exact number, π.

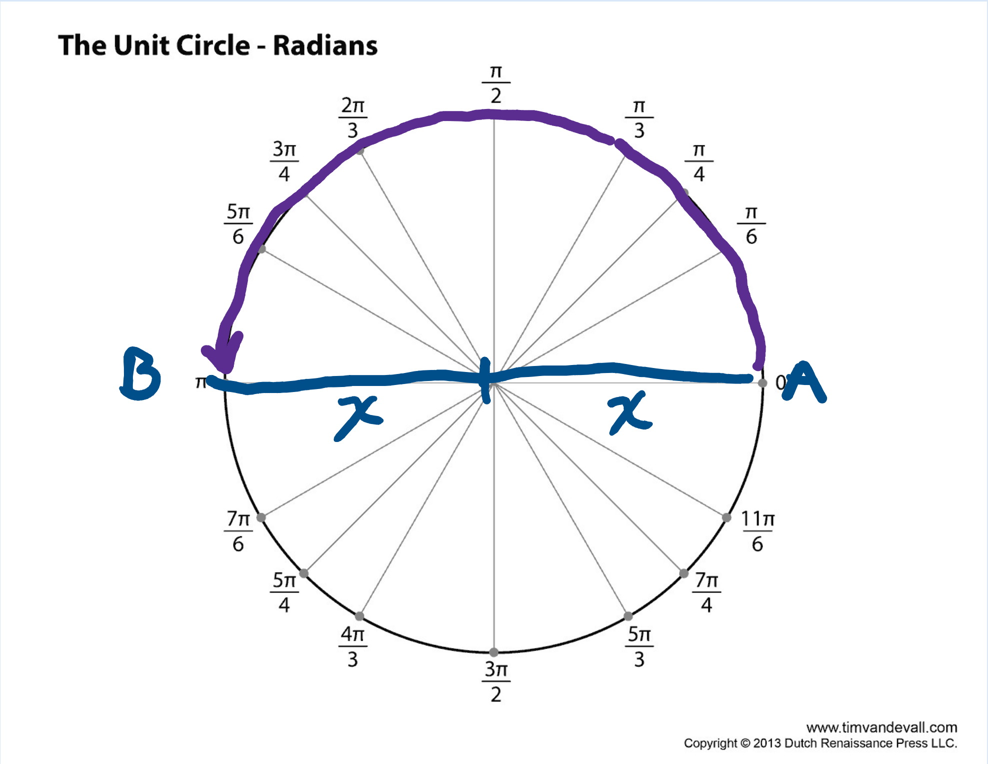


A to B linear is 2

A around to B is π in length or about 3.14

≈ 3.14

If the radius was another number x, going from A to B across the diameter is two times x units and going around is π times x units. In math, we leave it as the exact number, π.

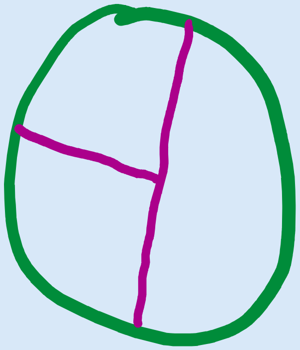


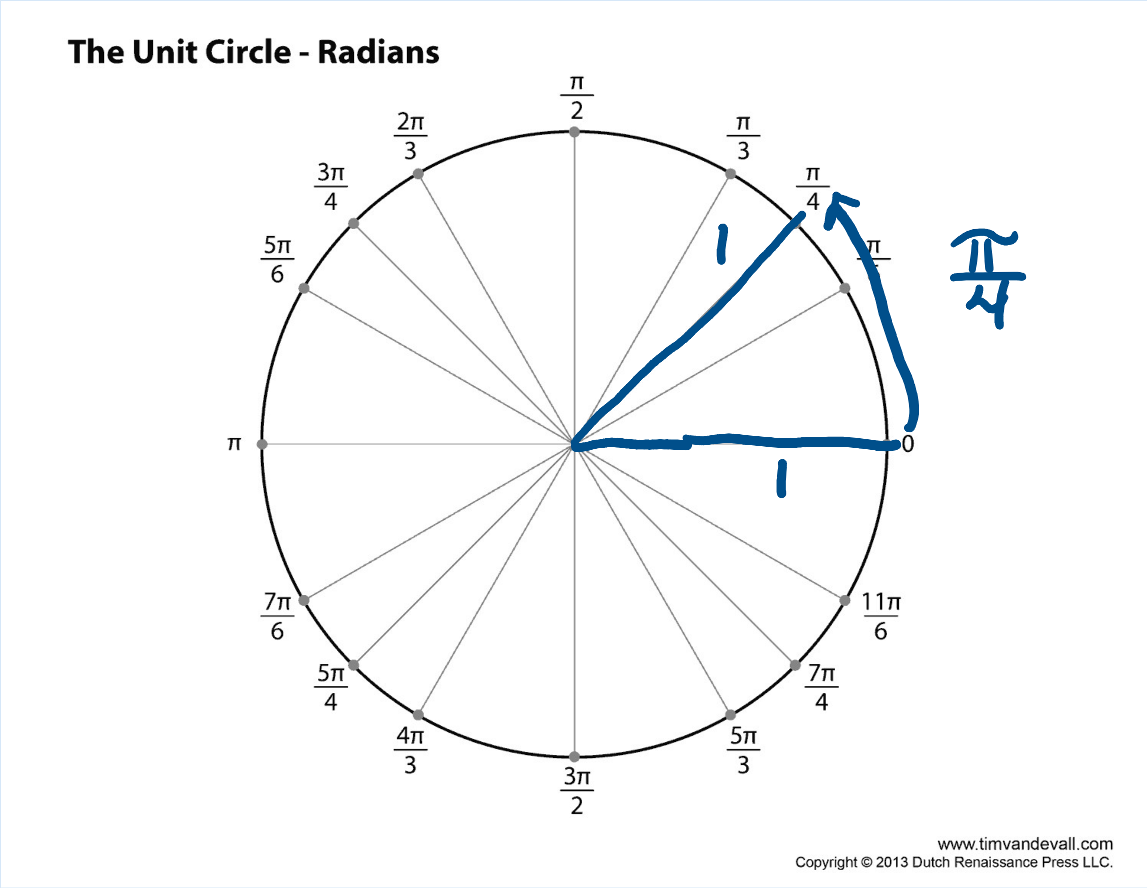
A to B linear is $2x$

A around to B is πx in length or distance.

Can you find a circle with a diagonal through it? Or create a story with drawings?

I am doing restoration on a tiny native forest that is in a circle in our neighborhood. If I wanted to know the distance around vs going across a trail in the middle, I would use π to do the calculations.




The Unit Circle - Radians
π/2
2π/3
π/3
3π/4
π/4
5π/6
π/6
π
0
7π/6
11π/6
5π/4
7π/4
4π/3
5π/3
3π/2
1
1
π/4
www.timvandevall.com
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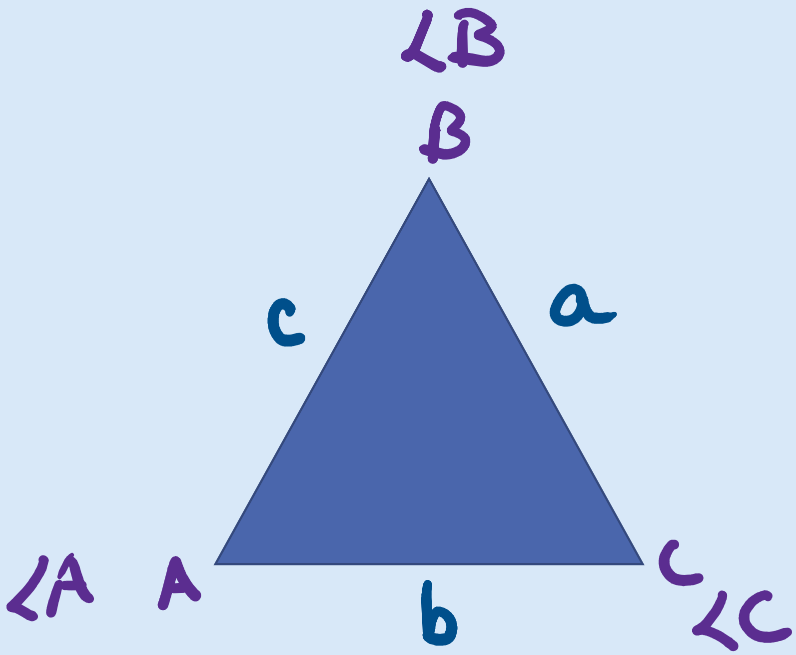
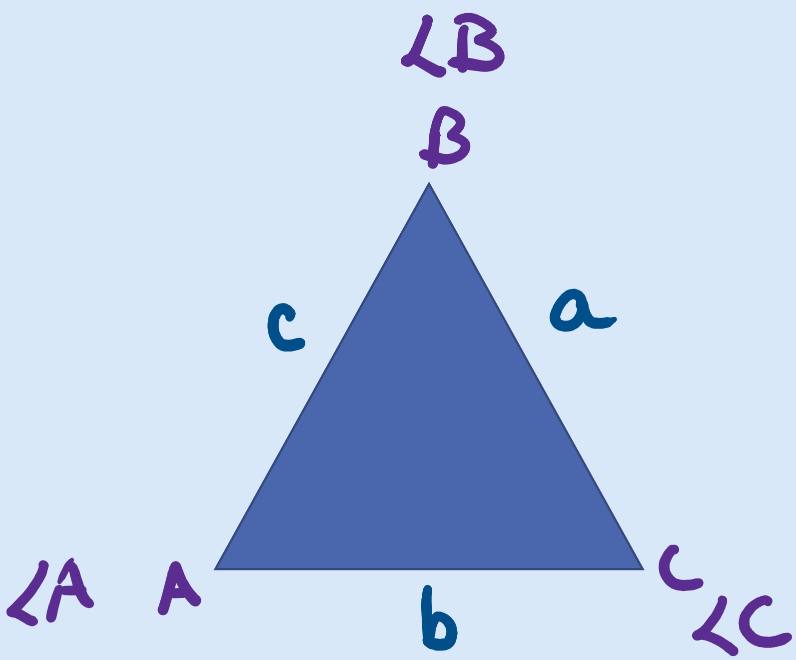
We left triangles to learn about angles and measuring angles in radians. Now we can come back to triangles.

We use Capital letters for the endpoints, the angle symbol with the endpoint for the angles, small letters for the side lengths opposite the endpoints.

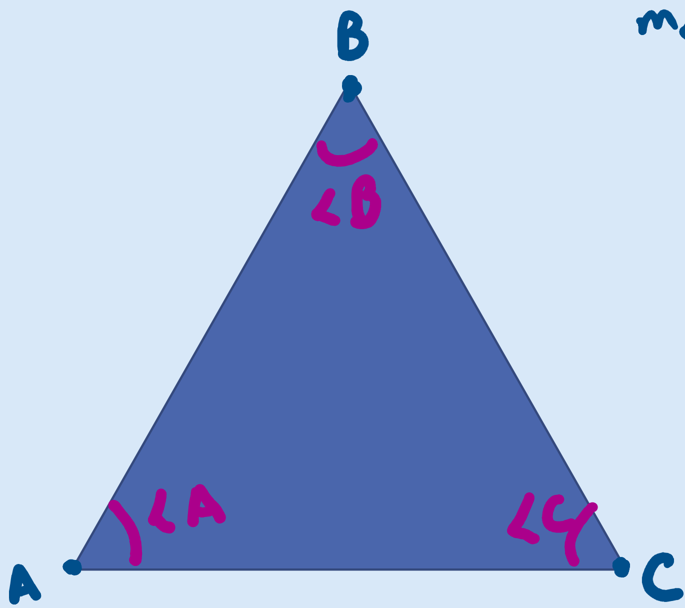


Notice that side a is across from point and angle A, side b is across from point and angle B, and side c is across from point and angle C.



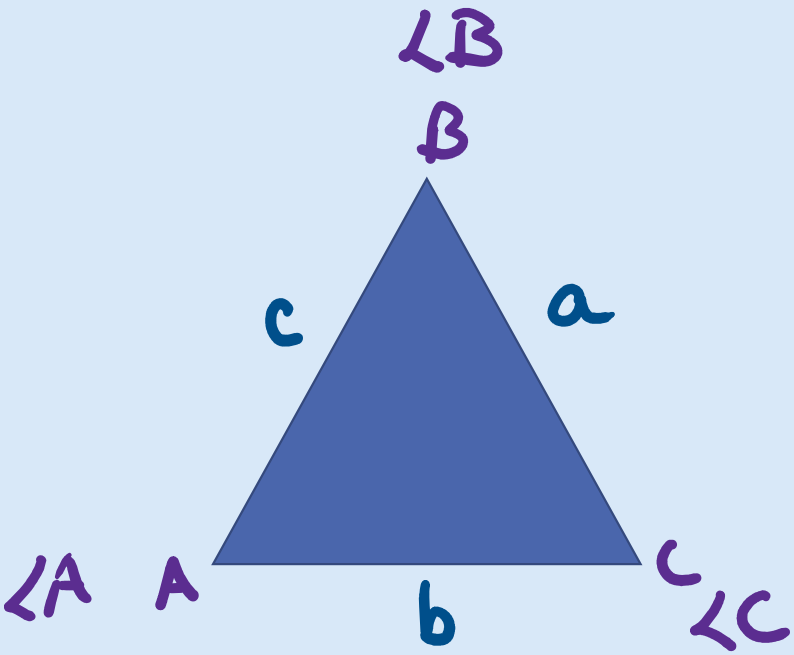
The angles of a triangle add up to 180 degrees or π radians. Since we are talking about the measurement of the angle, I use the m to mean measurement of the angle. The angle itself has location but no measurement so I add the m to show that we are measuring the angle and here it is obviously in radians.

$$m\angle A + m\angle B + m\angle C = \pi$$

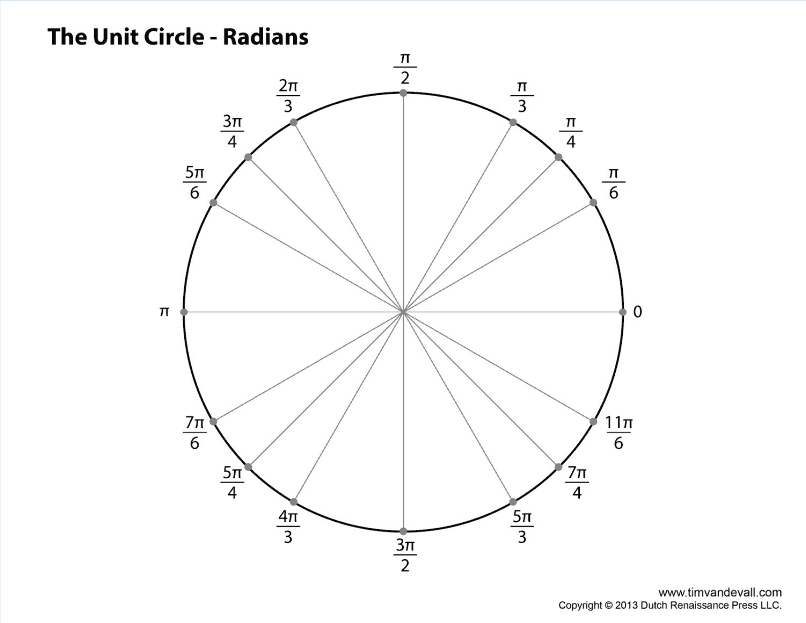


The angles are proportional to the length of their opposite sides.

$$\frac{\angle A}{a} = \frac{\angle B}{b} = \frac{\angle C}{c} \quad \text{or} \quad \frac{a}{\angle A} = \frac{b}{\angle B} = \frac{c}{\angle C}$$



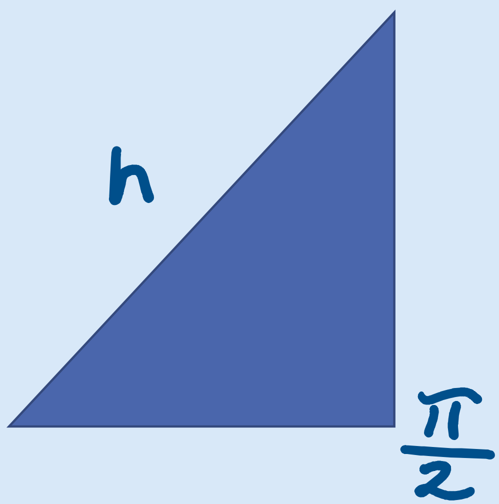
The angles in a triangle all add up to π radians.



We call an angle of $\frac{\pi}{2}$ radians (90 degrees) a right angle.

When there is an angle that is $\frac{\pi}{2}$ radians, in a triangle, we call it a right triangle.

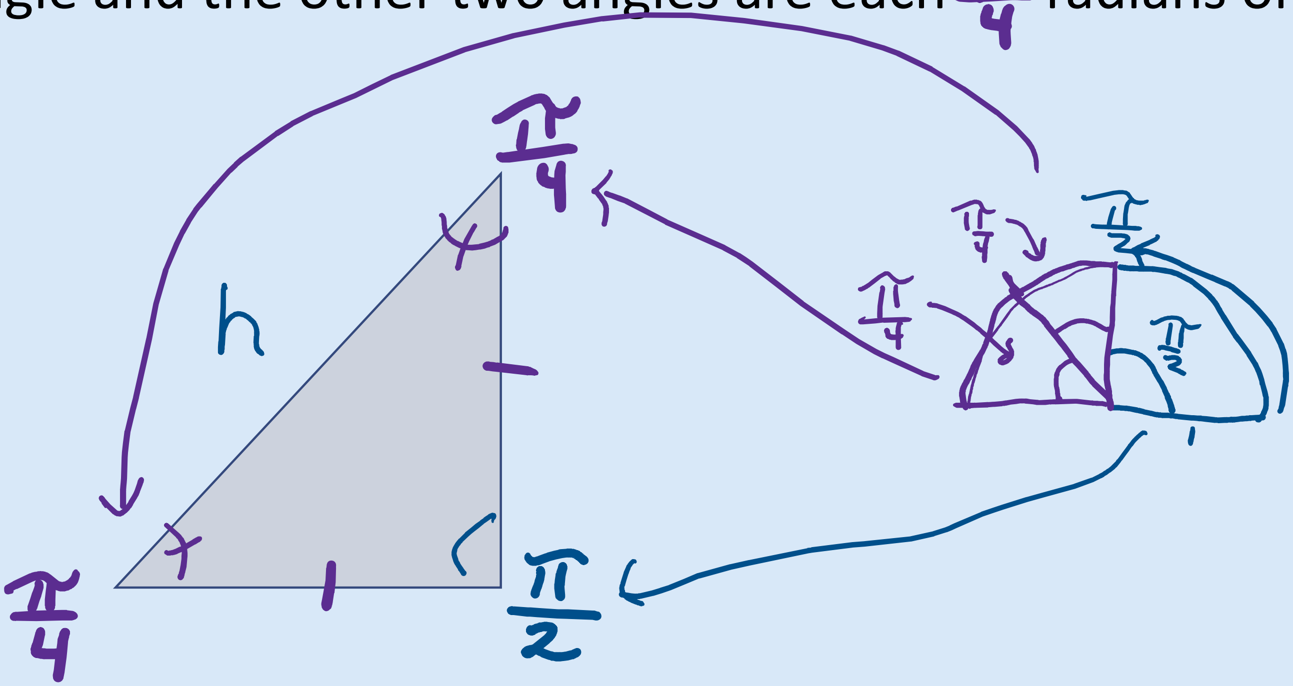
The side opposite the right angle has a special name called the hypotenuse.



We call an angle of $\frac{\pi}{2}$ radians a right angle.

When there is an angle that is $\frac{\pi}{2}$ radians in a triangle, we call it a right triangle.

If the other two sides have the same length, then it is an isosceles right triangle and the other two angles are each $\frac{\pi}{4}$ radians or 45 degrees.



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If the other two sides have the same length, then it is an isosceles right-trianglem and the other two angles are each $\frac{\pi}{4}$ radians.