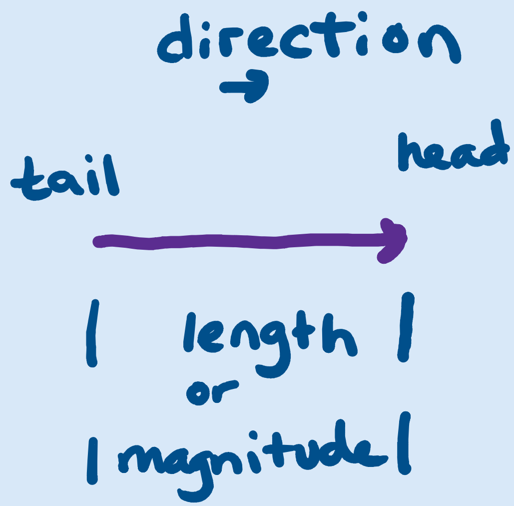
Vectors have magnitude (meaning length) and direction. The absolute value sign can be used, or double bars are used for vectors to make it clear that it is a vector length. $|v|$ or $\|v\|$

Vectors are written on a Cartesian axis system in [Euclidean space - Wikipedia] with a direction going from the tail to the head.



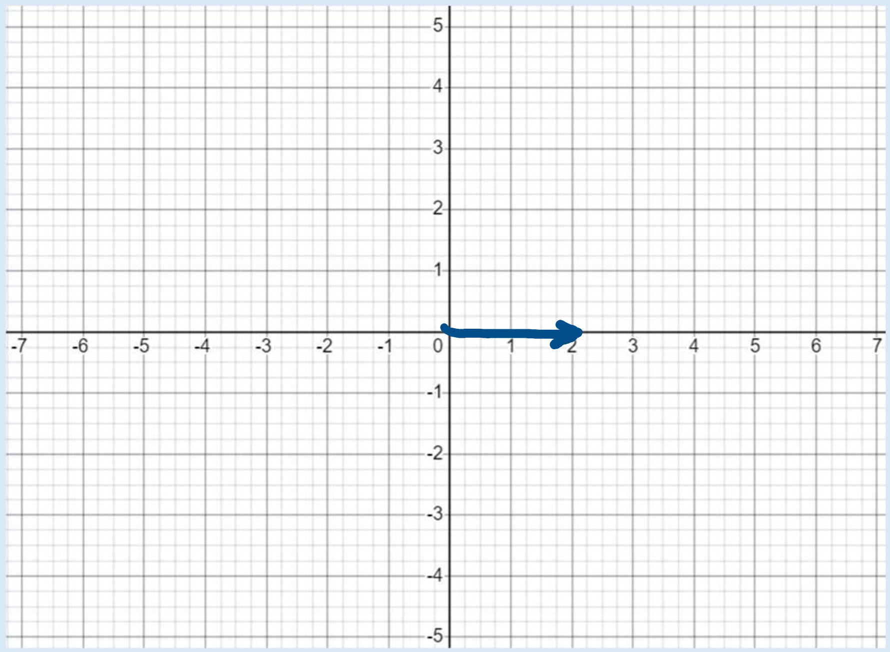
Vectors have magnitude or length and direction. In one variable, we can have a vector in the positive or negative direction on the x or y axis. The direction is from tail to head, with the arrow as the head. Here is a vector of length 2 in the positive x direction. ***x*** is +|2|

When the tail starts at the origin (0,0), then we write the vector as the endpoint or the head of the vector.

$\langle 2, 0 \rangle$

(0,0) to (2,0)

$2\hat{x}$



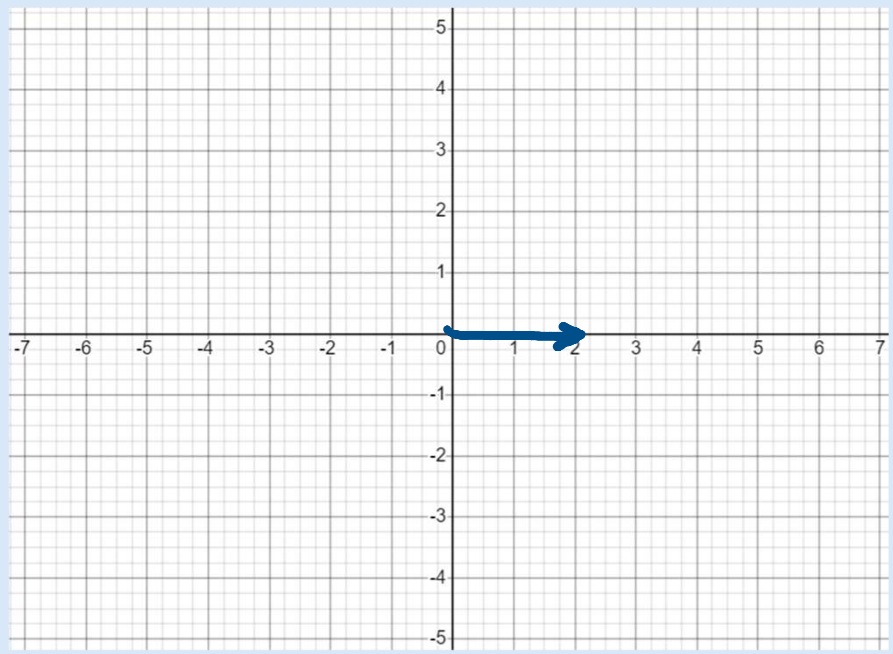
When the tail starts at the origin (0,0), then we write the vector as the endpoint or the head of the vector. Parentheses can be used for the vector, but I am going to use angle brackets or square brackets to distinguish the vectors from the points to make it clear.

$\langle 2, 0 \rangle$

$2\hat{x}$

scaler

vector



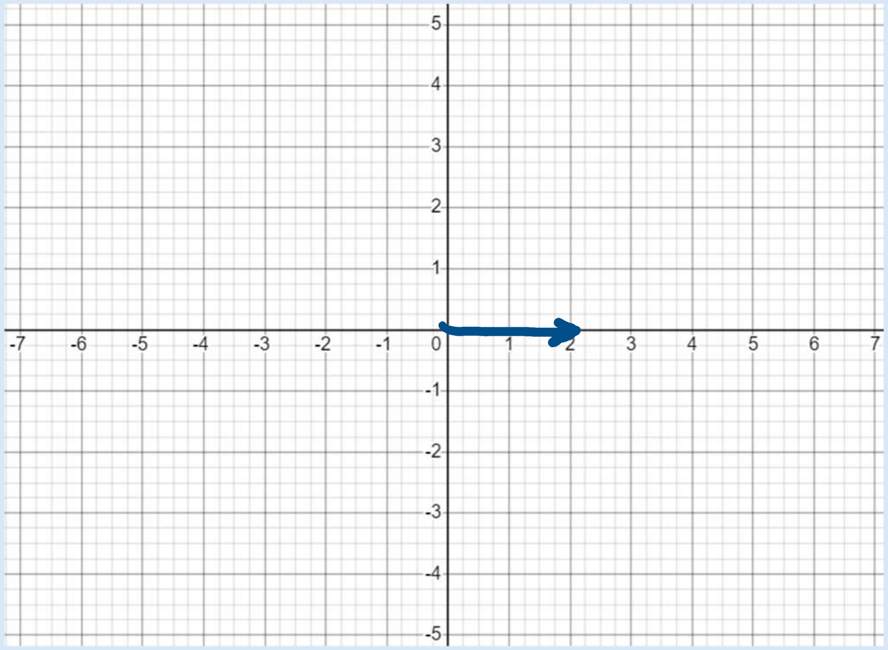
A scaler scales or multiplies and changes the length of the vector.

$\langle 2, 0 \rangle$

$2\hat{x}$

scaler

vector

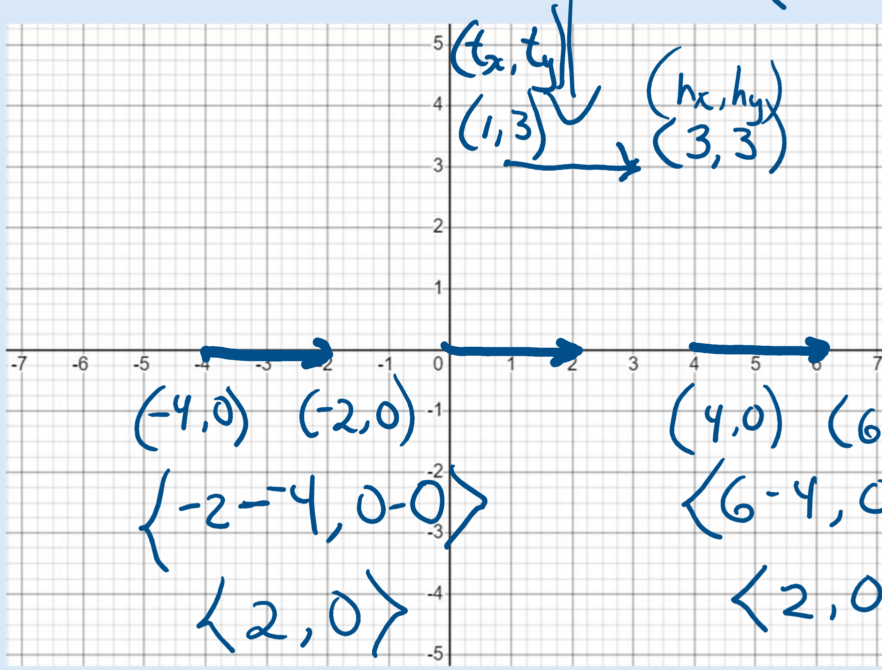


Vectors don't have location, but they can be written on the coordinate axis system as if they started at the origin. If they start elsewhere, then the starting point gets subtracted from the endpoint. These are all +2 in the x direction and could be written as the vector <2,0>

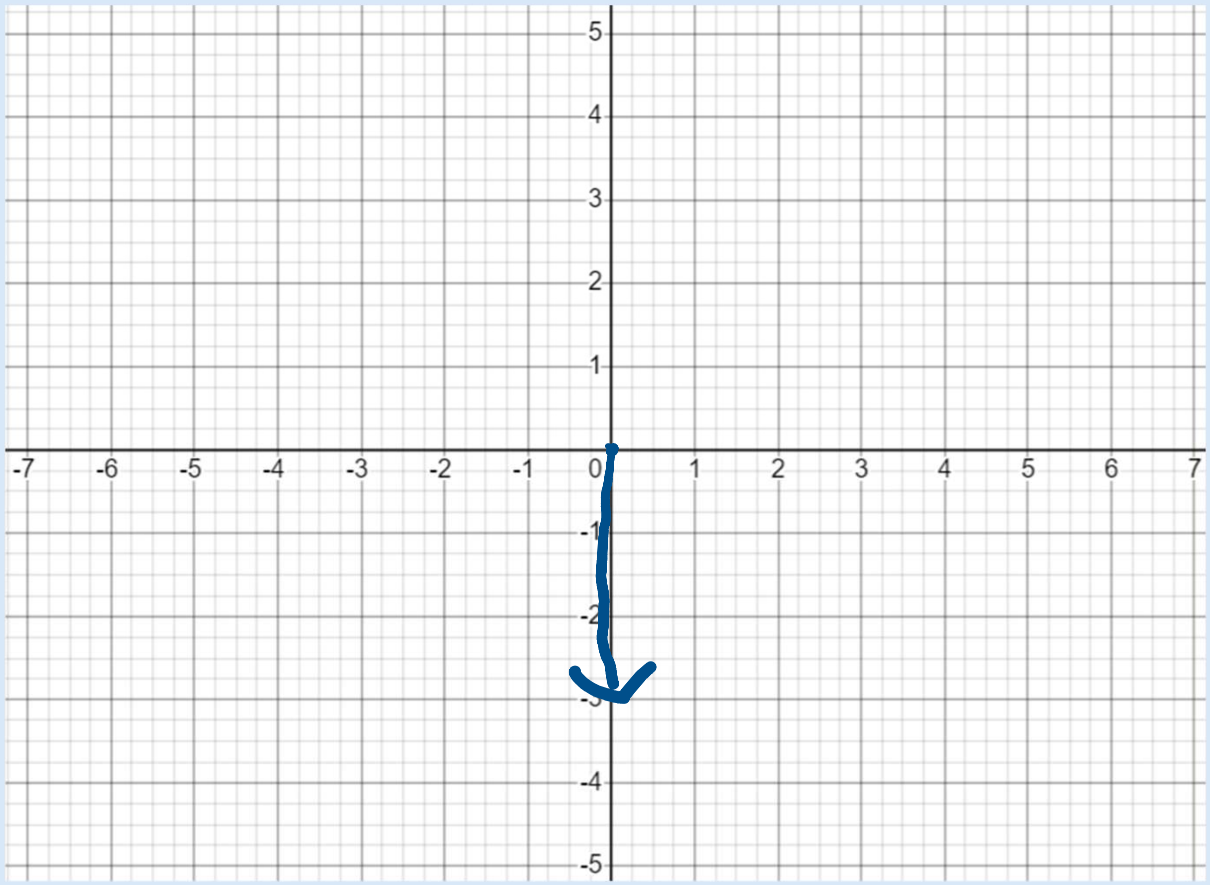
$\langle h_x - t_x, h_y - t_y \rangle$

$\langle 3-1, 3-3 \rangle$

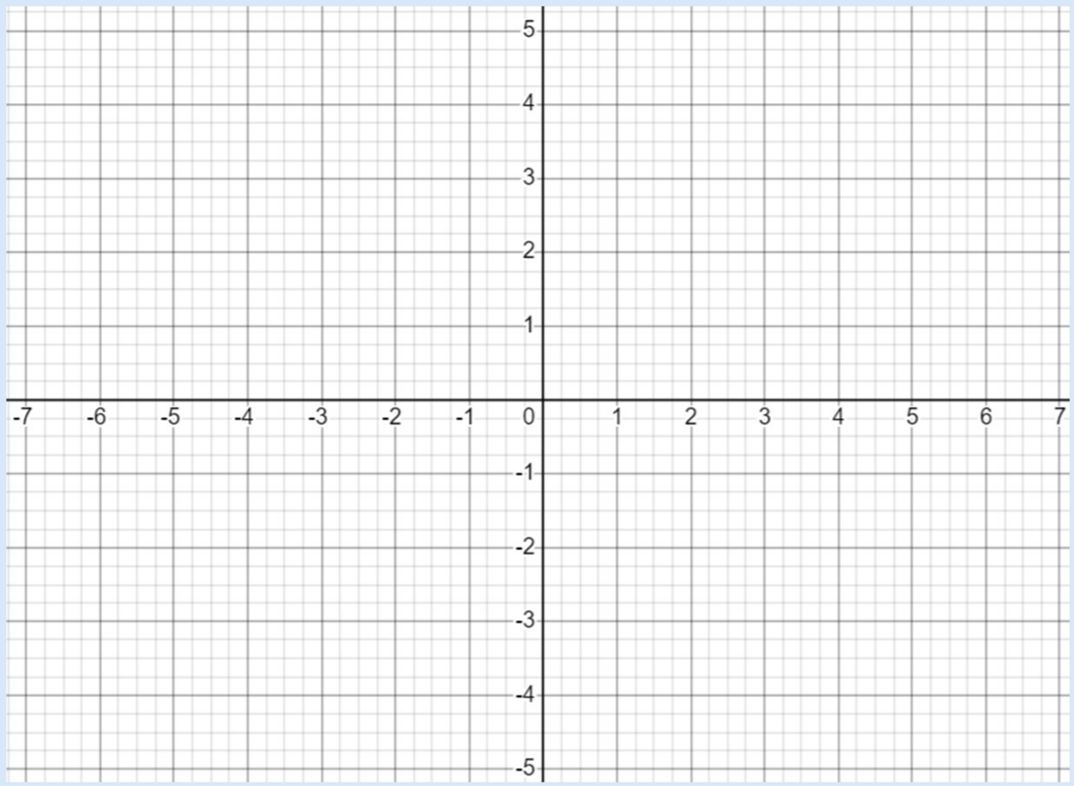
$\langle 2, 0 \rangle$



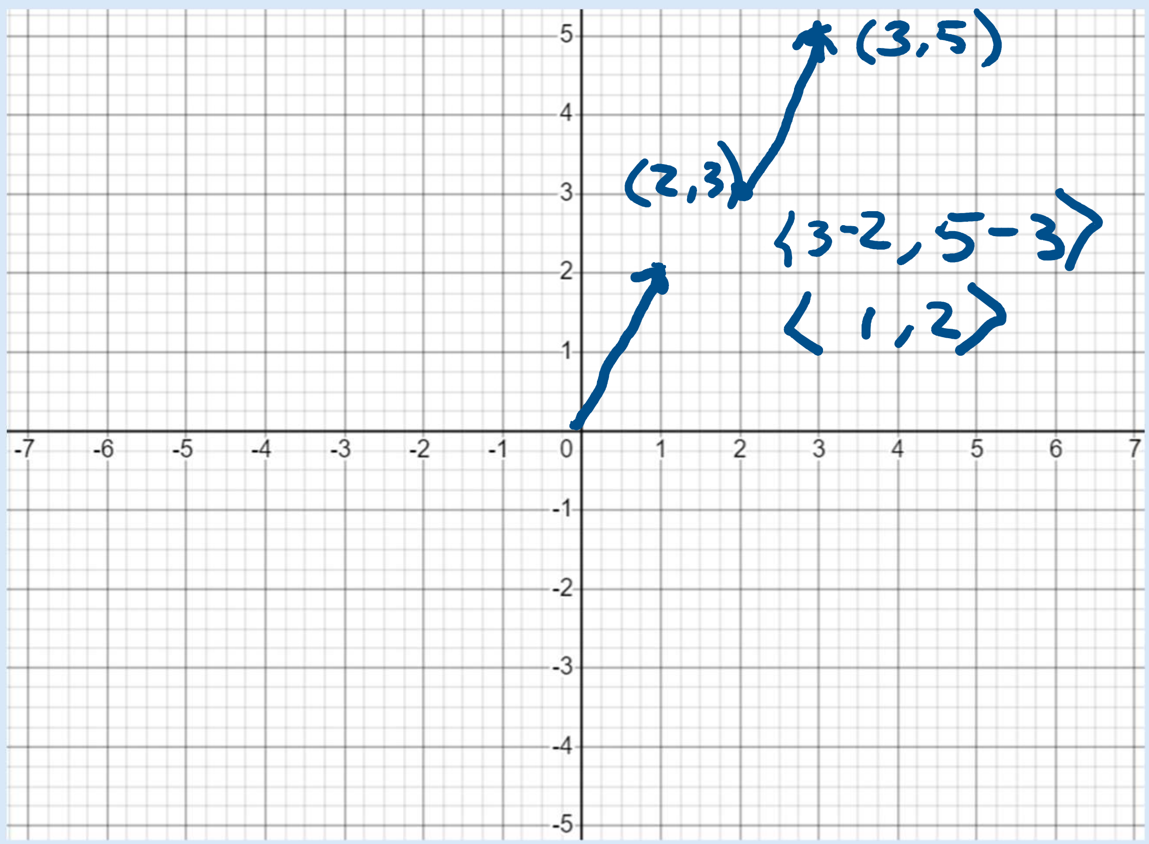
Here the vector is negative 3 in the y direction. We can write ***y*** is -|3| in the y direction. <0,-3> $= -3\hat{y}$



Are you good with the difference between a point and a vector? How would you explain the difference? You can use words, drawing, and/or movement to explain.



A vector can have position by naming the beginning and ending points. It is the same vector as the vector with the same length and direction starting from the origin. Points have parentheses and vector have the vector symbols.



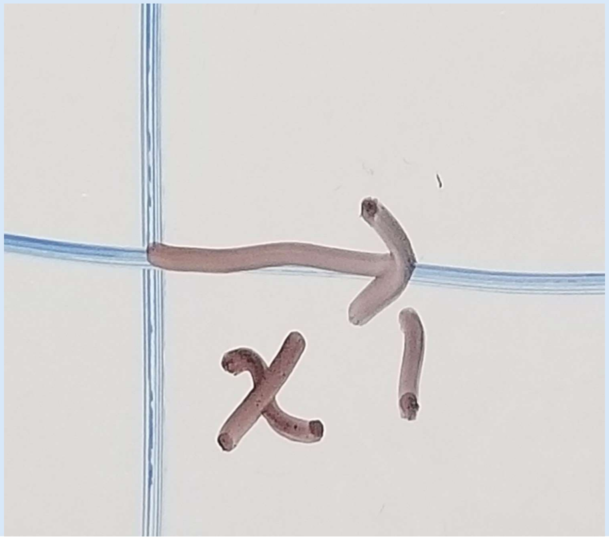
Vectors have magnitude (meaning length) and direction.

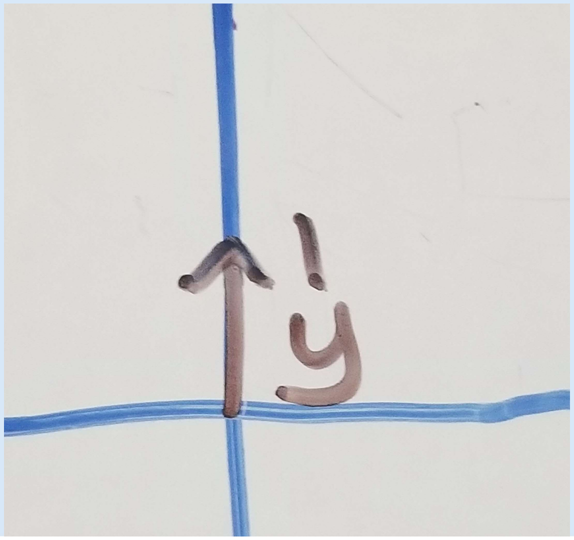
Vectors are written on a Cartesian axis system with a direction going from the tail to the head.

Along the x and y axis, positive and negative are used for direction.

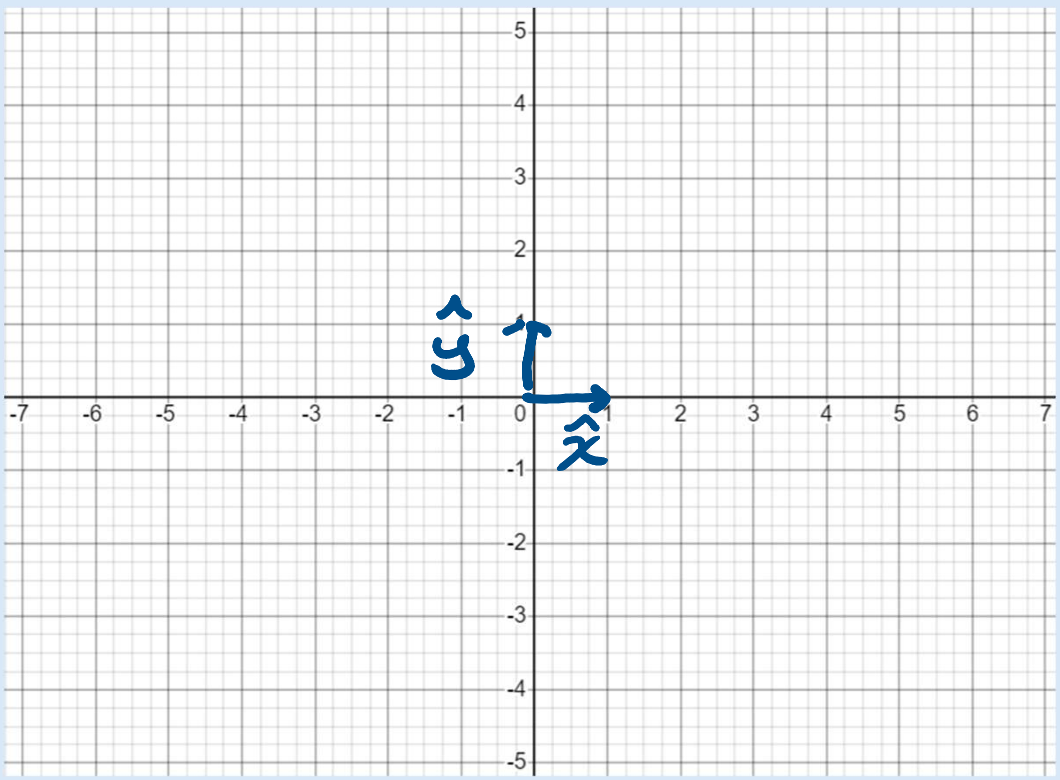
A unit vector has a length of 1.

$\hat{x}$ $\hat{y}$





$\hat{\mathbf{v}}$ This symbol, called v hat, is used for a unit vector. A unit vector in the x direction would be $\hat{x}$ and a unit vector in the y direction would be $\hat{y}$

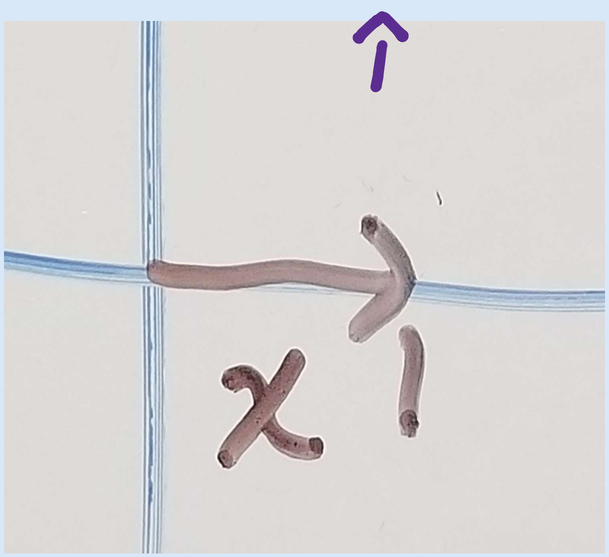


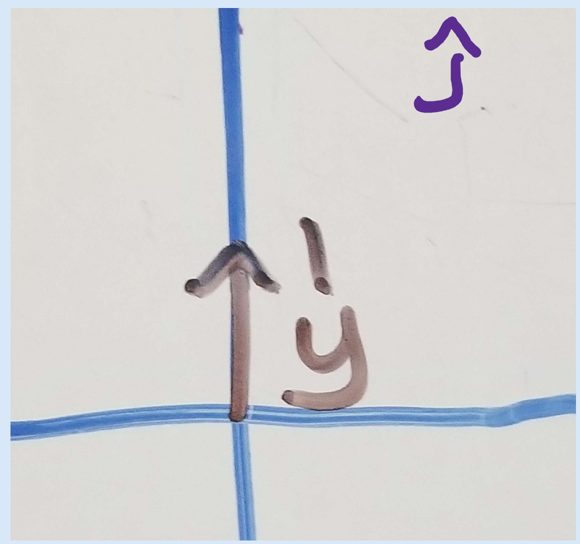
Unit vectors have a length or magnitude of one unit.

Bold i is sometimes used for a unit vector in the x direction (1,0) or <1,0>

j is sometimes used for a unit vector in the y direction (0,1) or <0,1>

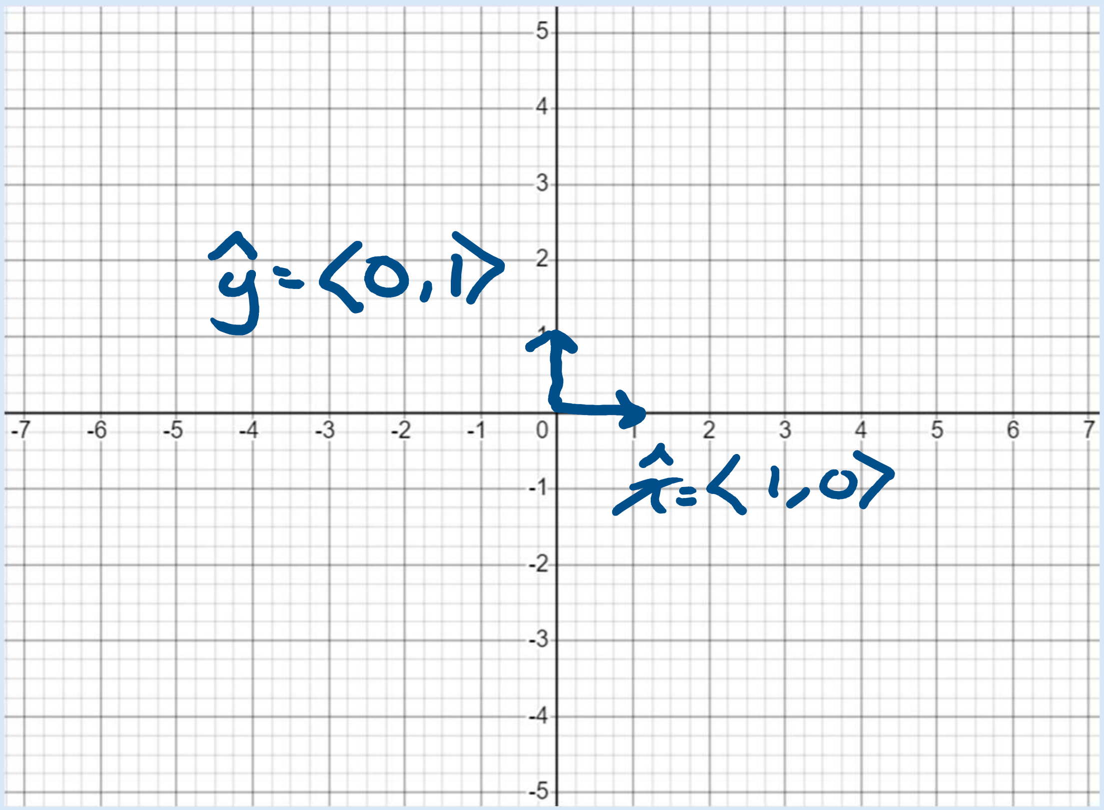
Parentheses or < > or square brackets can be used to name the vectors with the endpoints that start at the origin. I use parentheses for points and the other for vectors so there is not confusion.





Unit vector – Wikipedia

Unit vectors can be written as coordinate pairs <1,0> and <0,1> for **coordinate vectors**.



Vectors occur in vector spaces, and here are some common ones:

[Vector (mathematics and physics) - Wikipedia]

Vectors in specific vector spaces [edit]

- Column vector, a matrix with only one column. The column vectors with a fixed number of rows form a vector space.
- Row vector, a matrix with only one row. The row vectors with a fixed number of columns form a vector space.
- Coordinate vector, the n-tuple of the coordinates of a vector on a basis of n elements. For a vector space over a field F, these n-tuples form the vector space F^n (where the operation are pointwise addition and scalar multiplication).
- Displacement vector, a vector that specifies the change in position of a point relative to a previous position. Displacement vectors belong to the vector space of translations.
- Position vector of a point, the displacement vector from a reference point (called the *origin*) to the point. A position vector represents the position of a point in a Euclidean space or an affine space.
- Velocity vector, the derivative, with respect to time, of the position vector. It does not depend of the choice of the origin, and, thus belongs to the vector space of translations.

Caution! In data science and computer science, some things are referred to as vectors that aren't really mathematical vectors.

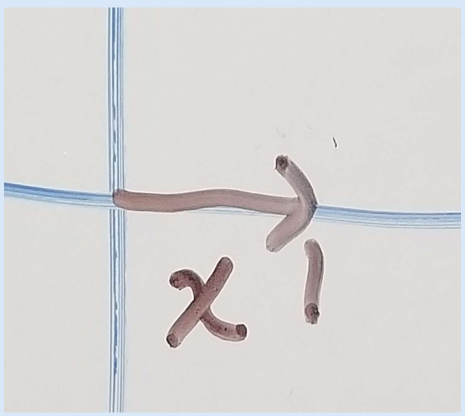
[Vector (mathematics and physics) - Wikipedia]

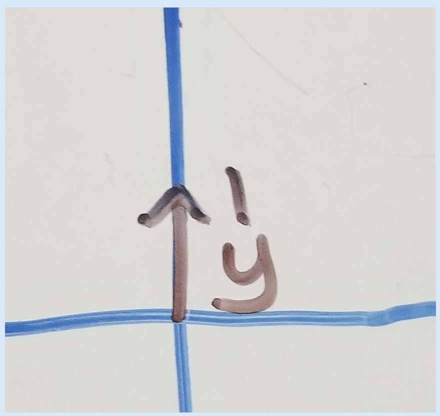
Tuples that are not really vectors [edit]

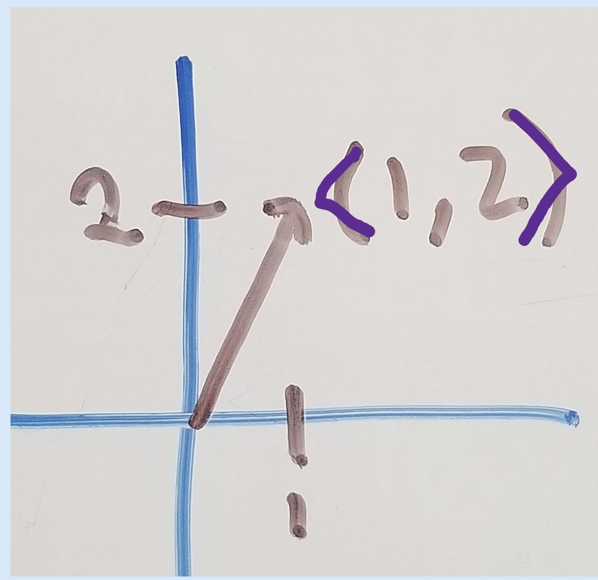
The set $\mathbb{R}^n$ of tuples of n real numbers has a natural structure of vector space defined by component-wise addition and scalar multiplication. When such tuples are used for representing some data, it is common to call them *vectors*, even if the vector addition does not mean anything for these data, which may make the terminology confusing. Similarly, some physical phenomena involve a direction and a magnitude. They are often represented by vectors, even if operations of vector spaces do not apply to them.

- Rotation vector, a Euclidean vector whose direction is that of the axis of a rotation and magnitude is the angle of the rotation.
- Burgers vector, a vector that represents the magnitude and direction of the lattice distortion of dislocation in a crystal lattice
- Interval vector, in musical set theory, an array that expresses the intervallic content of a pitch-class set
- Probability vector, in statistics, a vector with non-negative entries that sum to one.
- Random vector or multivariate random variable, in statistics, a set of real-valued random variables that may be correlated. However, a *random vector* may also refer to a random variable that takes its values in a vector space.
- Vector relation, a binary relation determined by a logical vector.

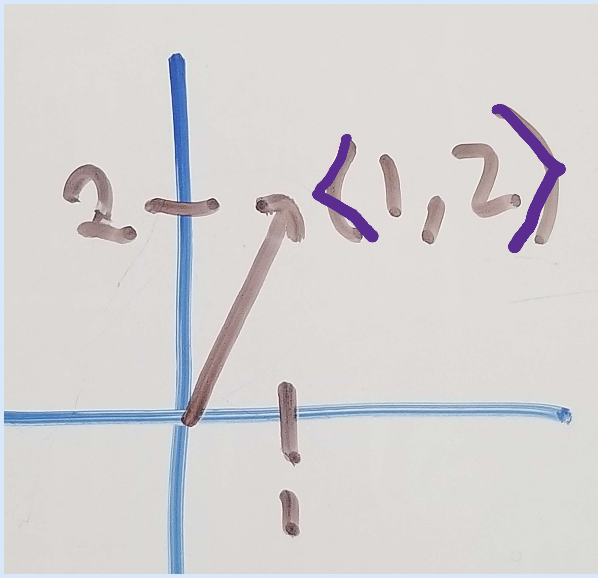
v=<1,2> is a way of writing a vector v from the origin (0,0) to the point or coordinate (1,2)

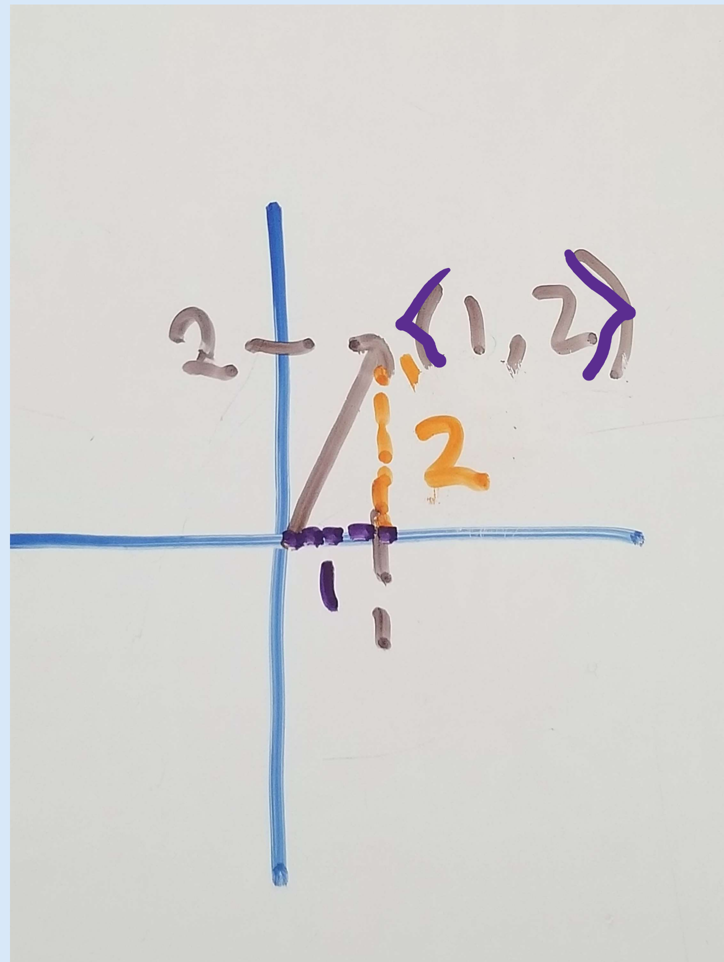




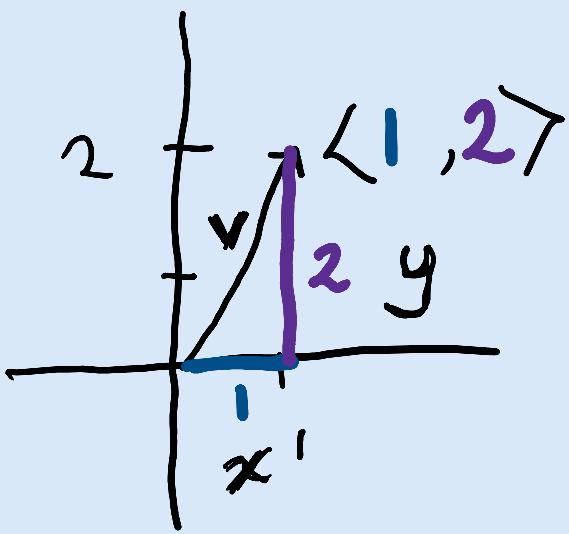


Vectors can be broken into x or horizontal, and y or vertical components.





Notice that vectors drawn in this way make right triangles so we can use the Pythagorean theorem to calculate the length of the diagonal if we know the lengths of the x and y sides of the triangle.



$$|v| = \sqrt{|x|^2 + |y|^2}$$

$$x^2 + y^2 = v^2$$

The triangle can be put on a coordinate axis system for what we call analytic geometry. Analytic geometry - Wikipedia

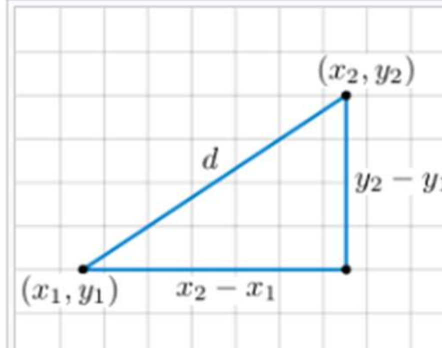
Distance and angle [edit source]

Main articles: Distance and Angle

In analytic geometry, geometric notions such as distance and angle measure are defined using formulas. These definitions are designed to be consistent with the underlying Euclidean geometry. For example, using Cartesian coordinates on the plane, the distance between two points (x_1, y_1) and (x_2, y_2) is defined by the formula

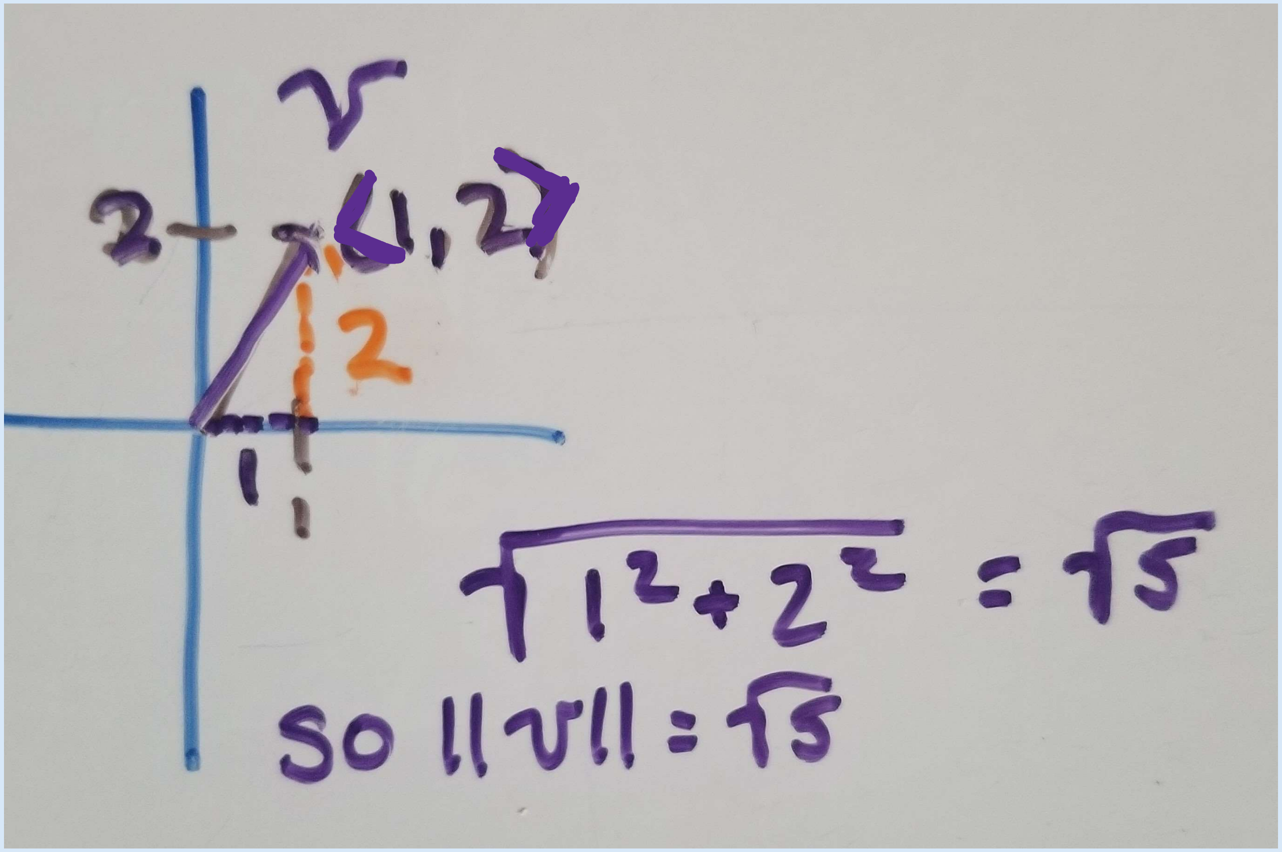
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

which can be viewed as a version of the Pythagorean theorem. Similarly, the angle that a line makes with the horizontal can be defined by the formula



The distance formula on the plane follows from the Pythagorean theorem

The magnitude or length can be calculated with the distance equation or Pythagorean formula.



Desmos has a special vector graph, and it can do the magnitude:

Vectors (desmos.com) I think it is hard to use.

Vector Calculator (mathsisfun.com) has a vector calculator.

Vector Add, Subtract Calculator – Symbolab only seems to do vectors in 3D space, which isn't that useful for us now.

Graphing Calculator – GeoGebra seems hard to get used to using.

2D Vector Calculator | Physics Calculators (icalculator.info)

Here is a good summary Scalar, Vector, Matrix (mathsisfun.com)

Vectors (mathsisfun.com)

This seems the most useful: [Vector Calculator - with detailed explanation (mathportal.org)]

Result:

You have entered the following vector:

$v_1 = (1,\ 0)$

The Magnitude of vector v_1 is: 1

Explanation:

To find magnitude of vector $v = (a, b)$ we use formula $||v|| = \sqrt{a^2 + b^2}$

In this example $a = 1$ and $b = 0$, so:

$$||v|| = \sqrt{1^2 + 0^2} = \sqrt{1 + 0} = \sqrt{1} = 1$$

There are three ways of writing the symbol for vectors.

Parentheses.

Angle brackets that I have been using.

Square brackets which is the matrix form. Because I took these slides from another unit, I am going to use square brackets now.

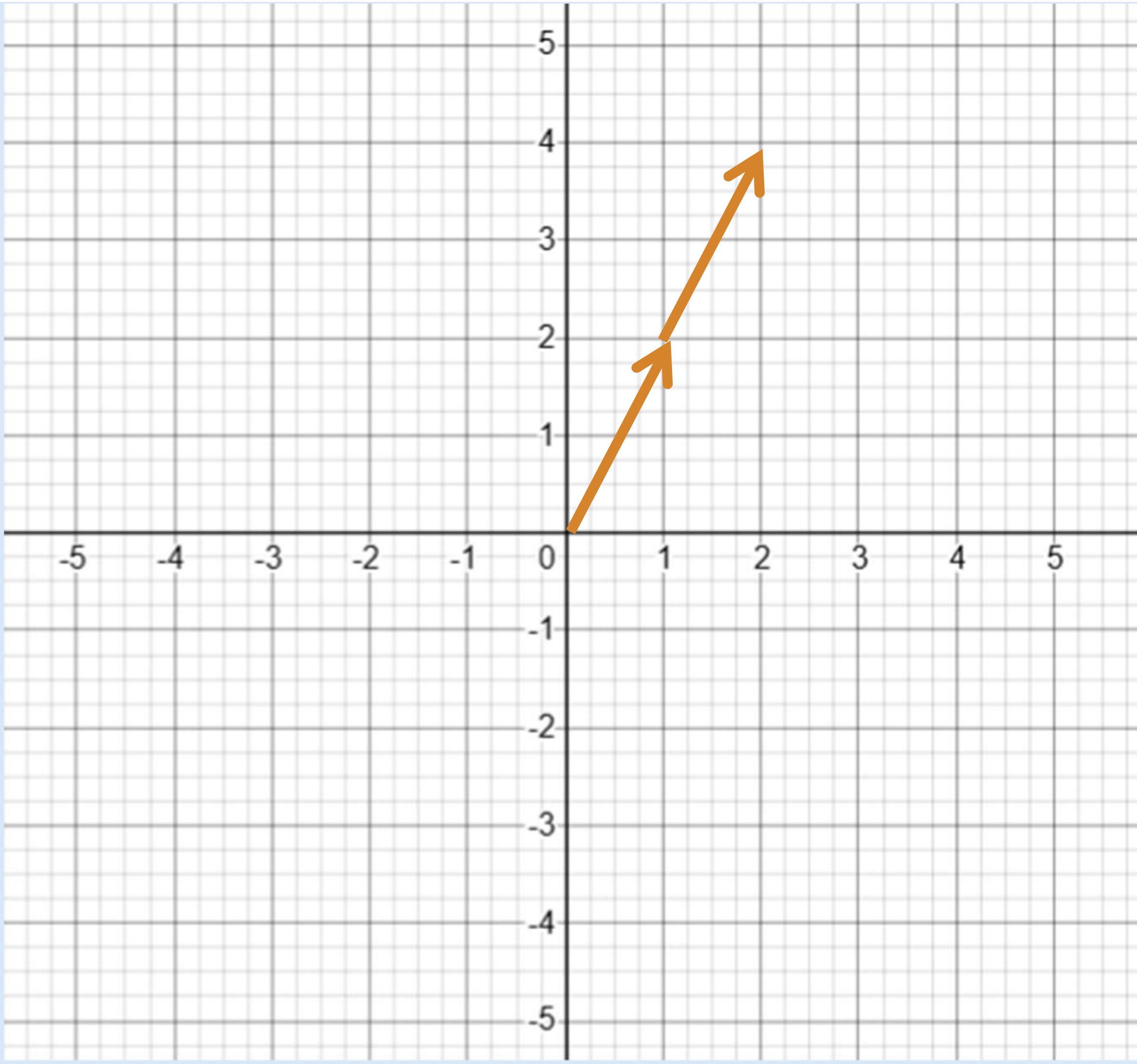
We will start with some operations on vectors, and we can multiply (or divide) by a scaler or scaling factor that is a number and not a vector.

Multiplying a vector by a scalar increases the length of the vector by the scalar.

[Scalar - Simple English Wikipedia, the free encyclopedia]

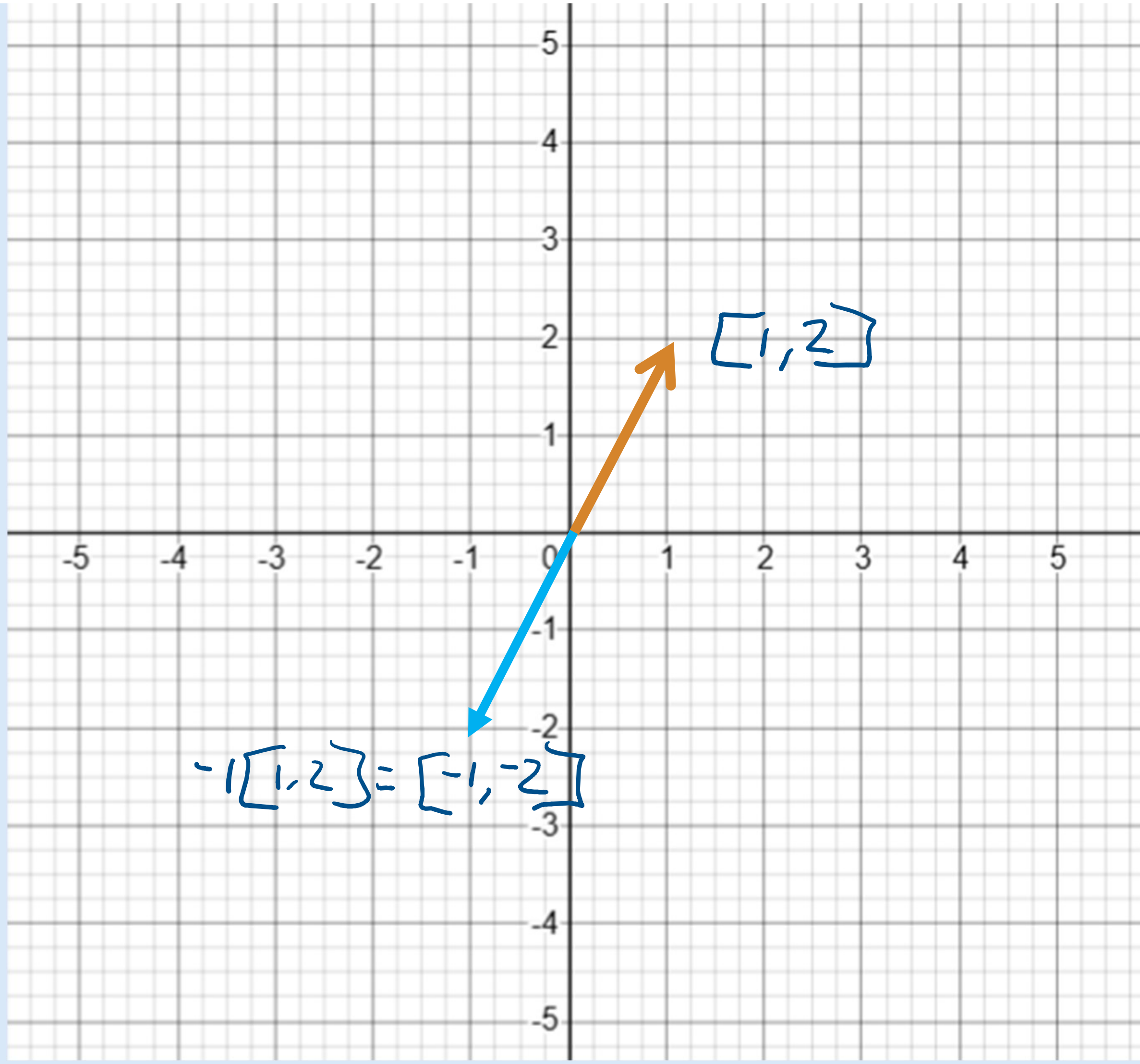
The scalar multiplies or scales the magnitude or length but doesn't change the direction. (Unless it is negative, in which case it reverses the direction.)

2·[1,2] = [2,4]



Multiplying a vector by -1 changes the direction of the vector by pi radians or 180 degrees. It changes the sign of both the x and y components of the vector.

-1*[1,2]=[-1,-2]

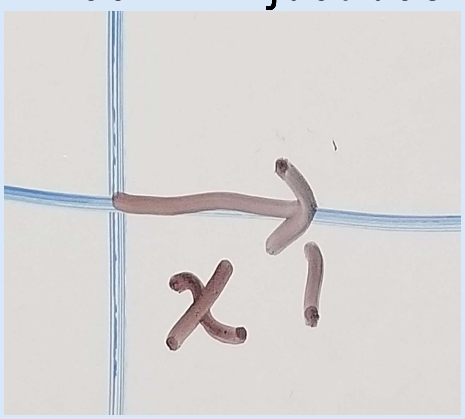


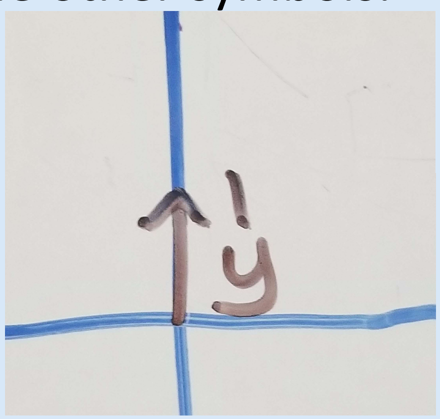
v=<1,2> is a way of writing a vector v from the origin (0,0) to the point or coordinate (1,2)

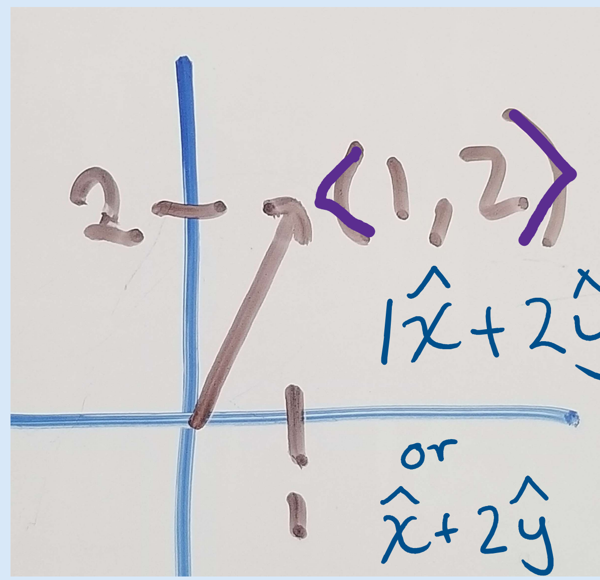
Another way is to write the vector in terms of the x and y unit vectors multiplied by how long each vector is

<1,2> could be written as $\hat{x} + 2\hat{y}$

My spellcheck is being a pain about using bold small I and I need spellcheck so I will just use the other symbols.

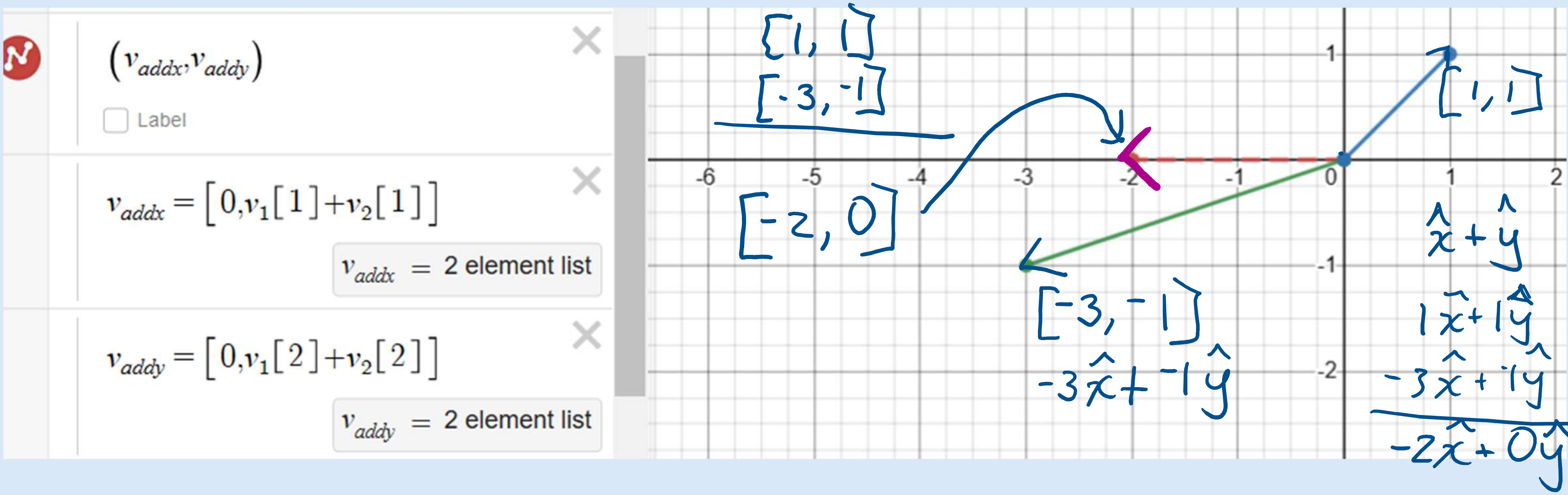






The basic vector operations are addition and subtraction, scalar multiplication, and the dot product or inner product. You can also move vectors around by doing transformations.

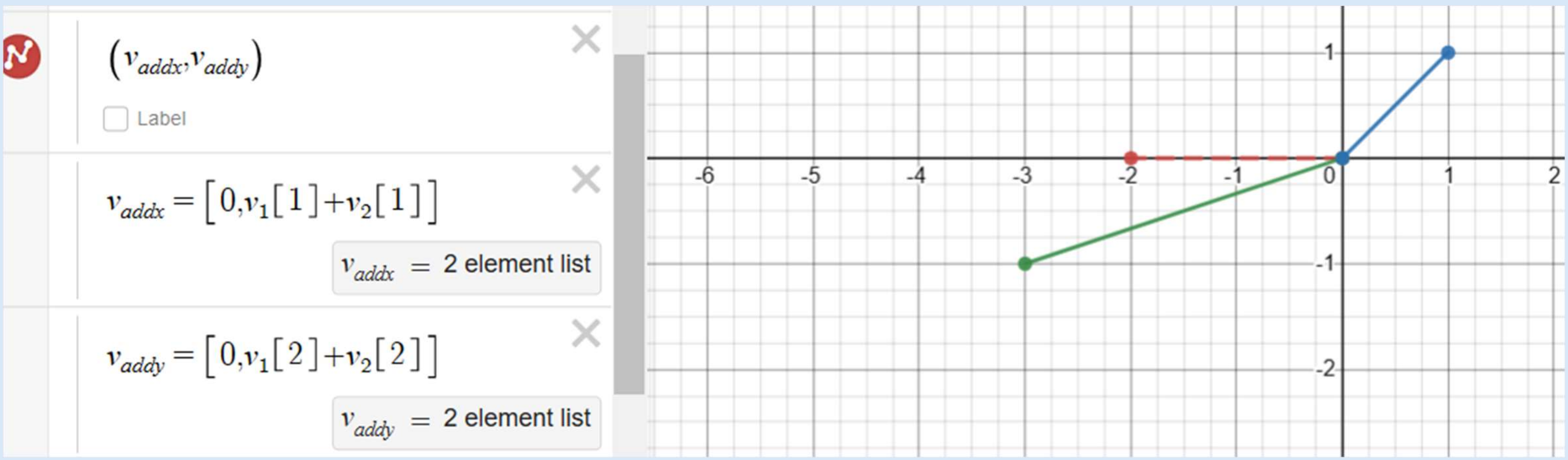
[Vectors (desmos.com)](https://www.desmos.com) Here Desmos is showing vector addition:



[Vectors (desmos.com)]() has a vector package that I find a bit cumbersome. A ***v*** is often used for a vector and ***u*** for a second vector.

V_x can be used for the x component and V_y for the y component

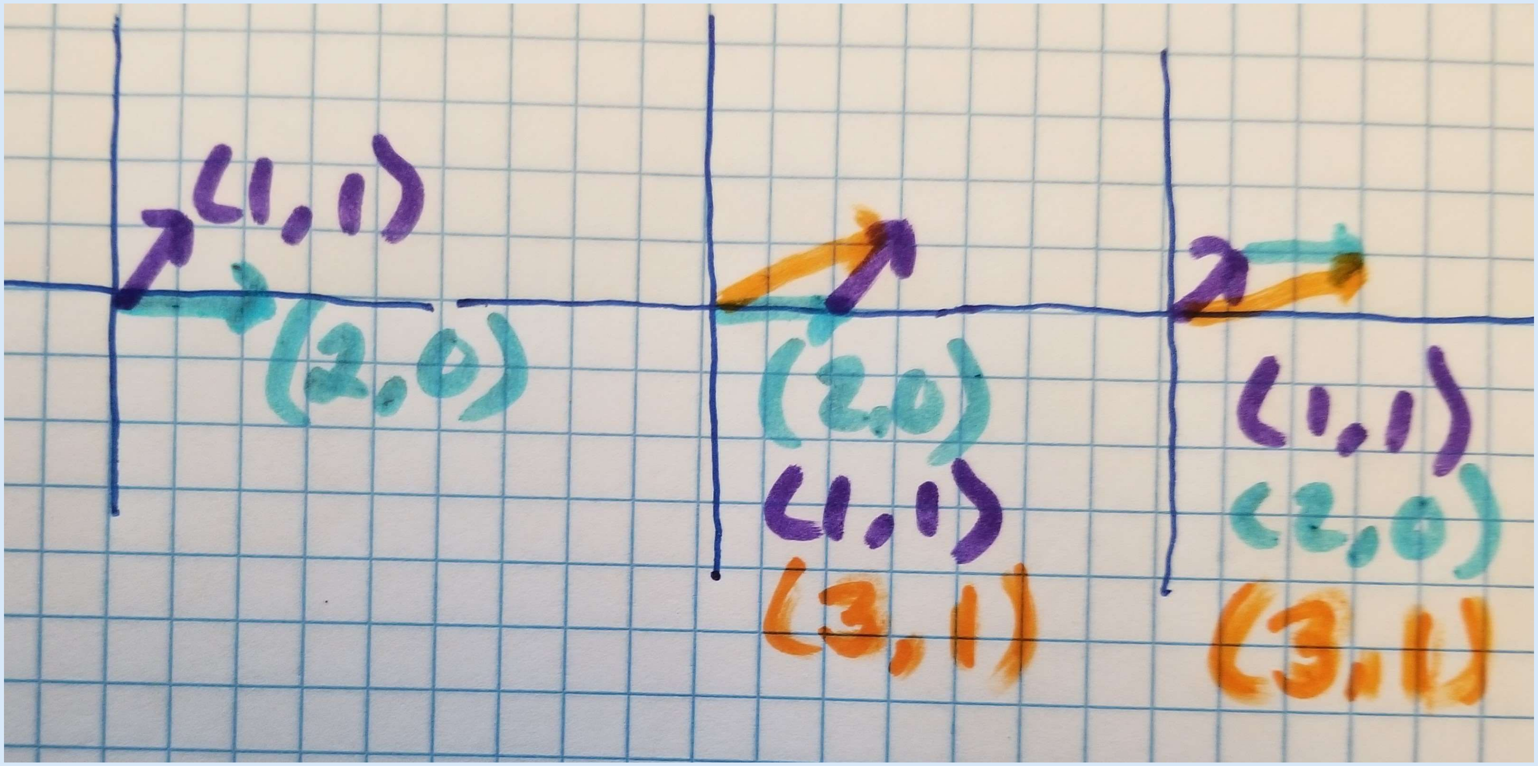
$$\langle V_x, V_y \rangle$$



For vector addition, you add the x components and the y components.

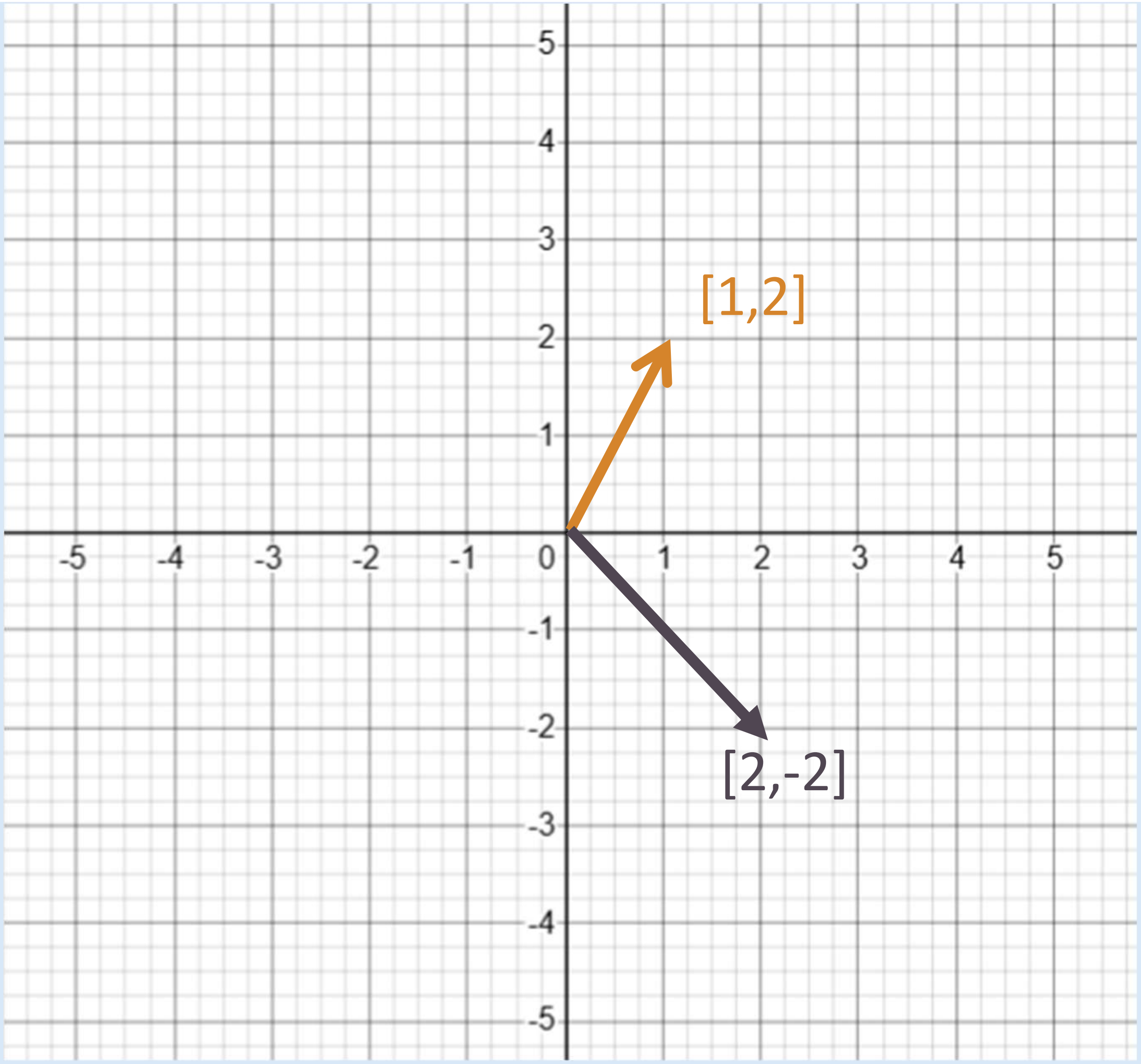
On a graph, this is like moving them head to tail next to each other.

[1,1] + [2,0] = [1+2,1+0] = [3,1] Often the term **resultant** vector is used for the answer.



Adding vectors on a graph.

[1,2] + [2,-2]

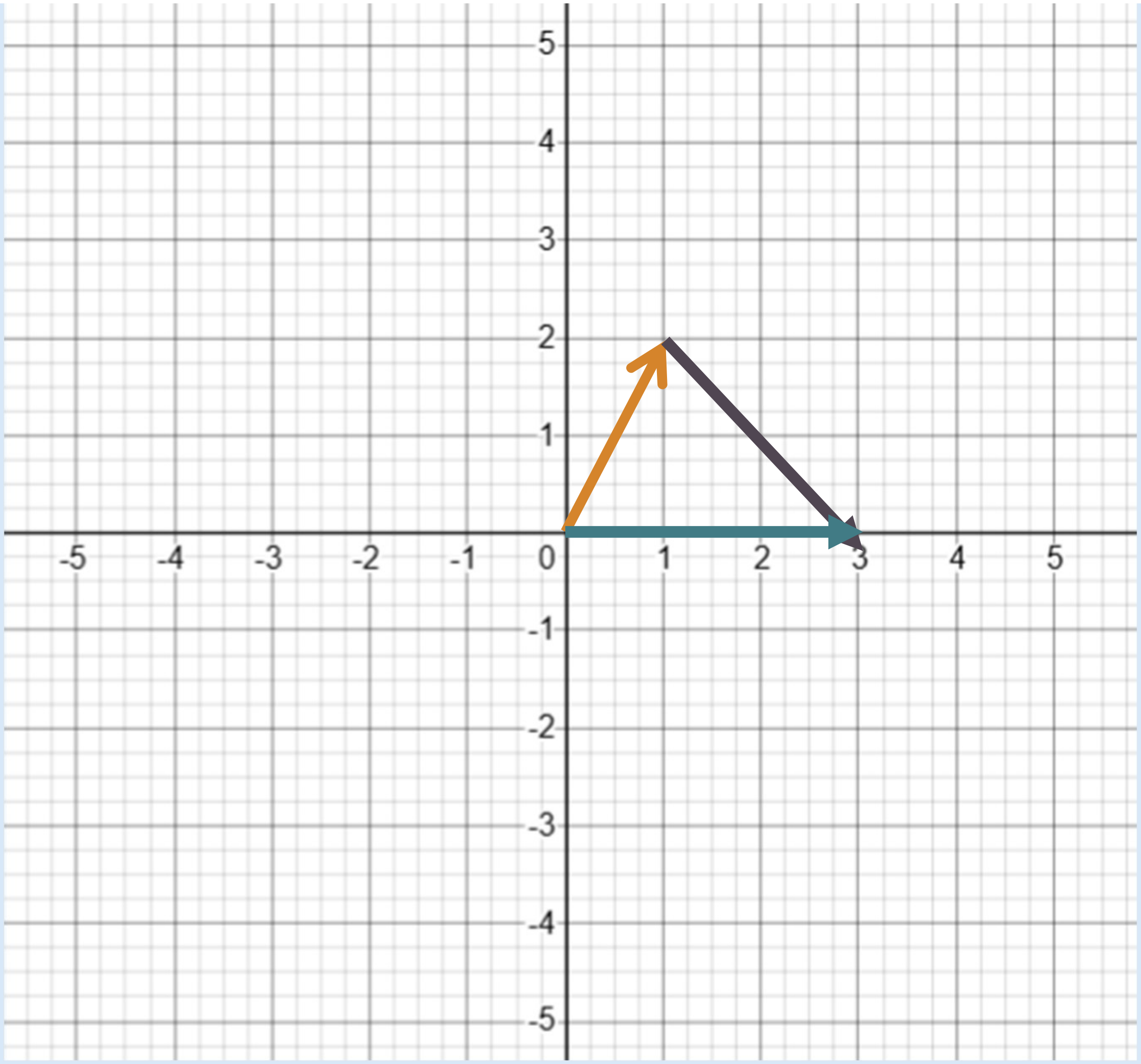


Adding vectors on a graph.

Move them head to tail.

[1,2] + [2,-2]

=[1+2, 2-2] = [3,0]



Adding vectors on a graph.

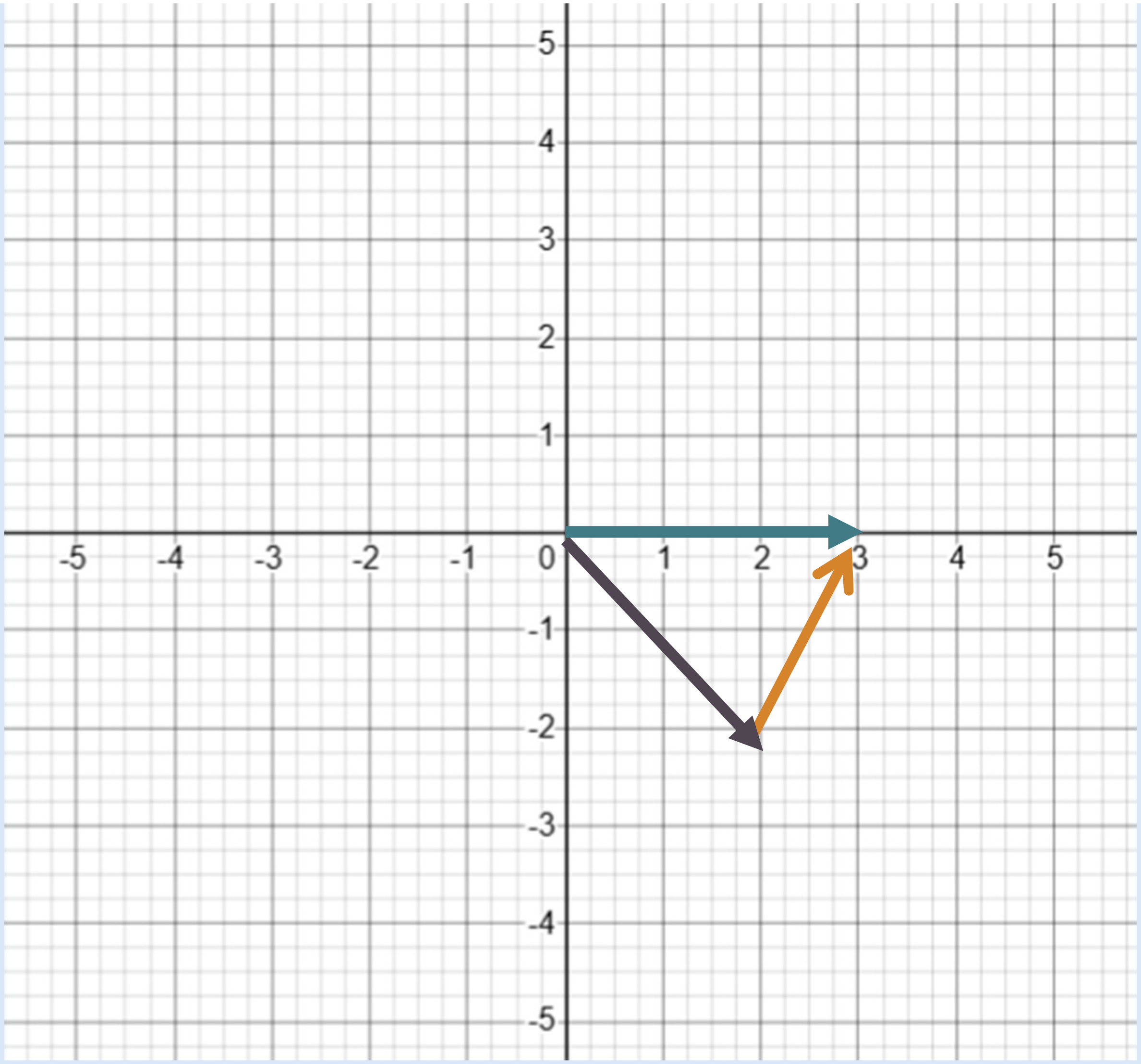
[1,2] + [2,-2]

=[1+2, 2-2] = [3,0]

Order doesn't matter

for adding

[2,-2] + [1,2] = [3,0]

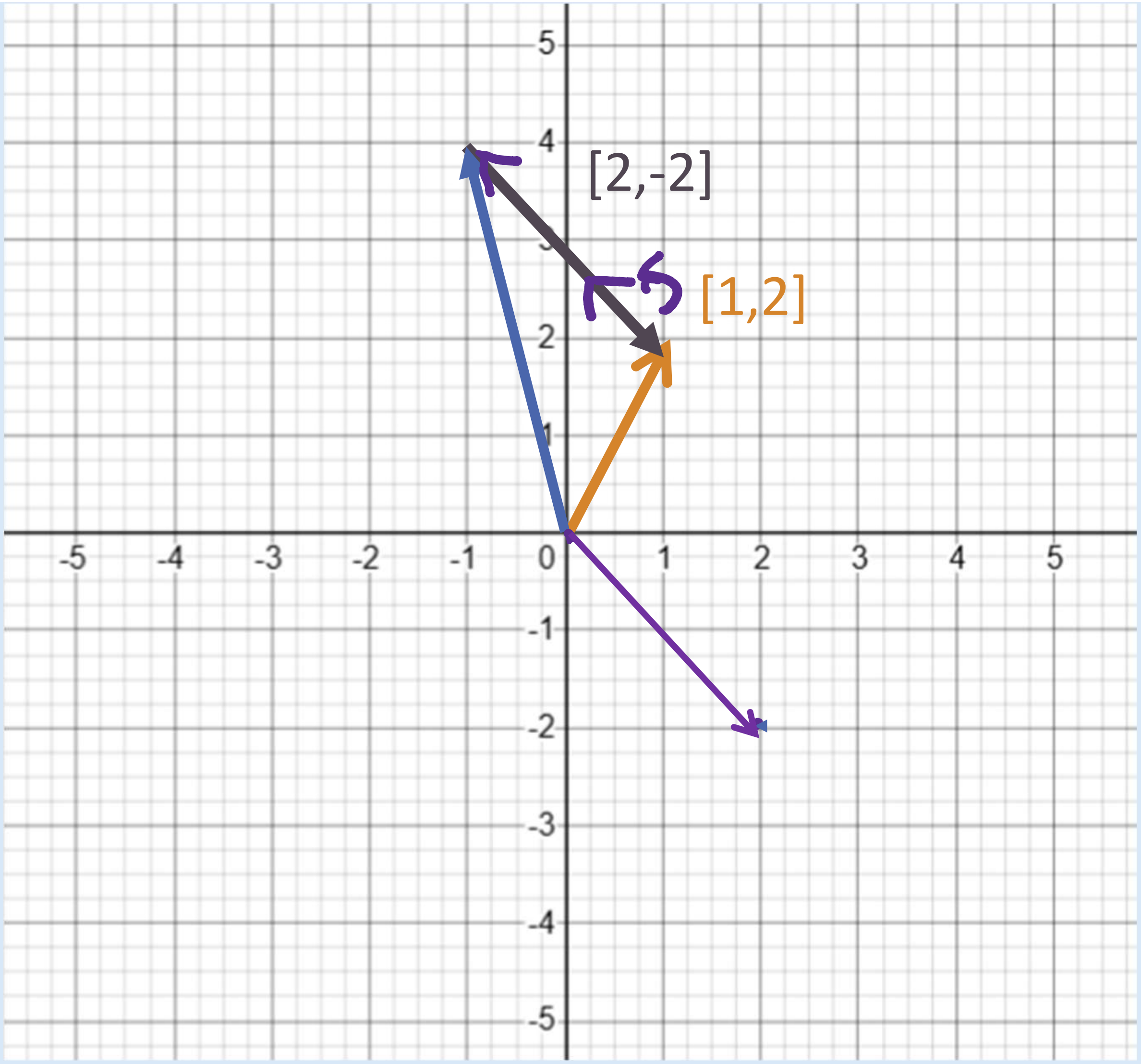


To subtract vectors, change the direction of the vector that you are subtracting.

Here the order matters

[1,2] - [2,-2] =

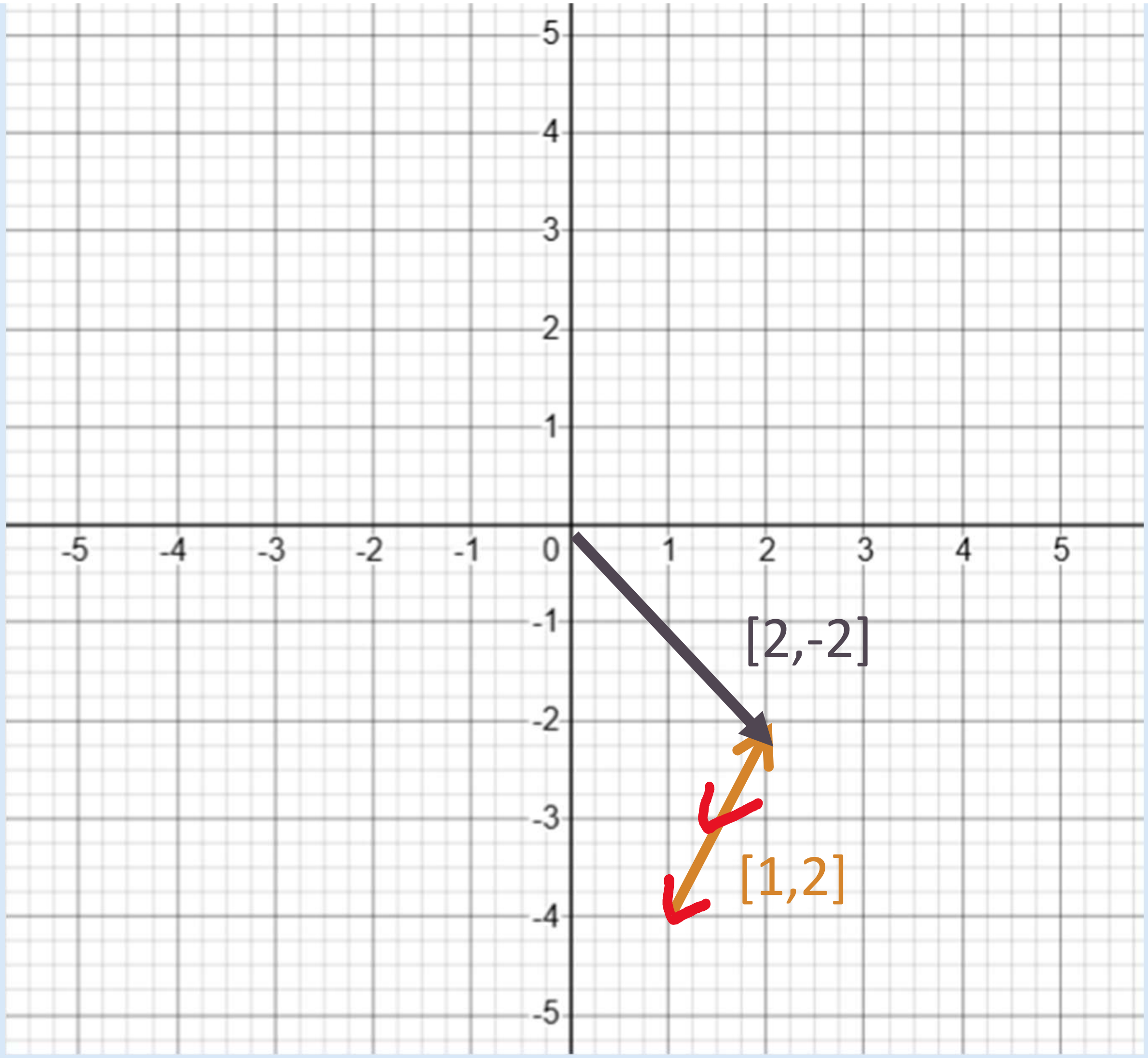
[1-2, 2- -2] = [-1,4]



Here the order matters

[2,-2] - [1,2] =

I find that lining them up head-to-head is too confusing, so I change direction and add.



Add or Subtract vectors

Multiply or divide a vector by a scalar (real number)

You can't multiply or divide a vector by a vector but there are multiplication like operations that you can do that some people call vector multiplication. Don't be fooled as this isn't really multiplication.

Vector Multiplication – The Physics Hypertextbook

Dot product

Cross product

Vector multiplication – Wikipedia includes matrix operations

Outer Product Outer product - Wikipedia

For matrices we can do the Hadamard product (matrices) – Wikipedia

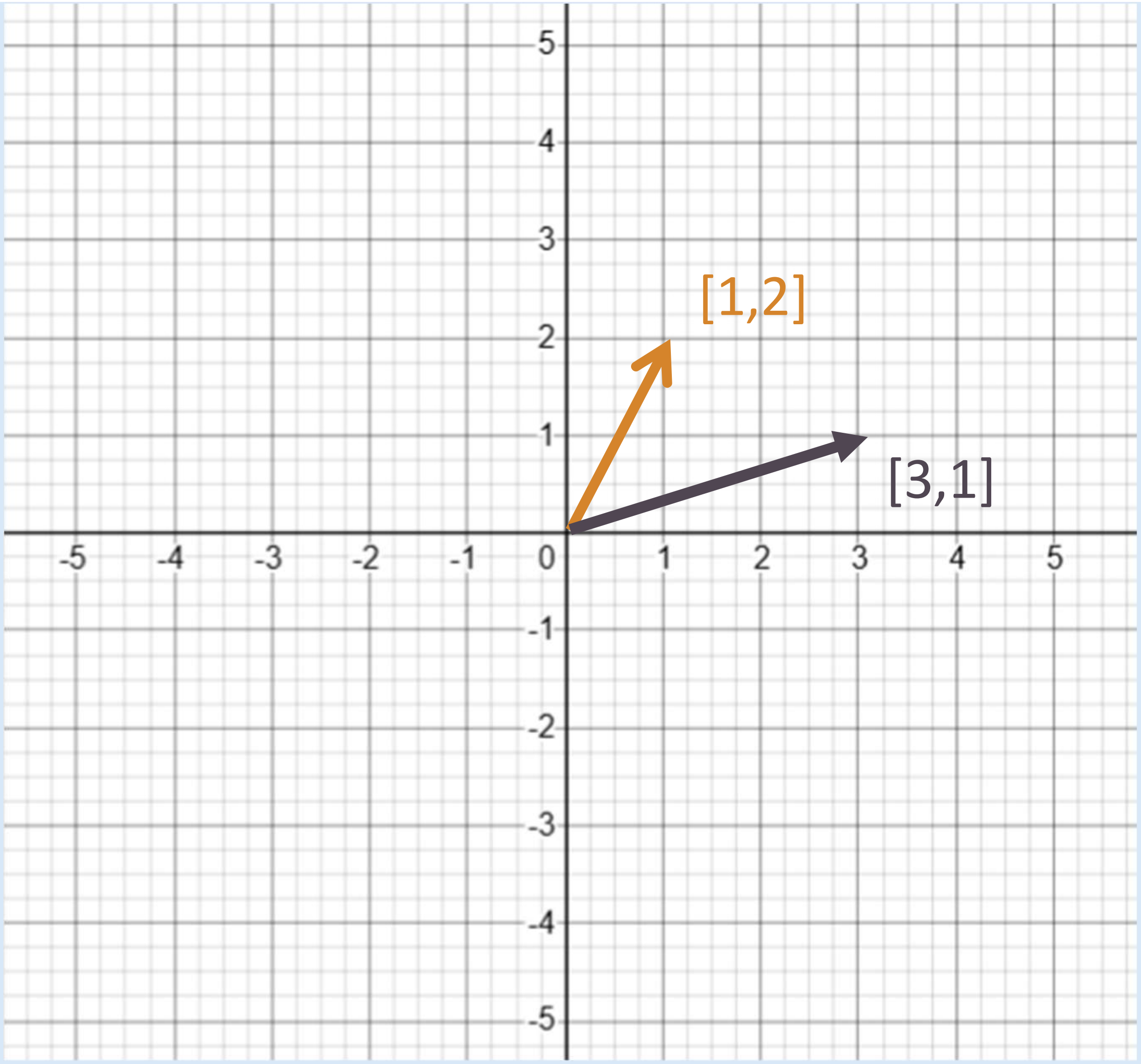
We will just look at the dot product.

The dot product gives a scalar that is the amount that the two vectors are reinforcing or opposing each other.

[1,2] · [3,1] is

1*3+ 2*1 = 5

I also think of it as how much the two vectors are similar.



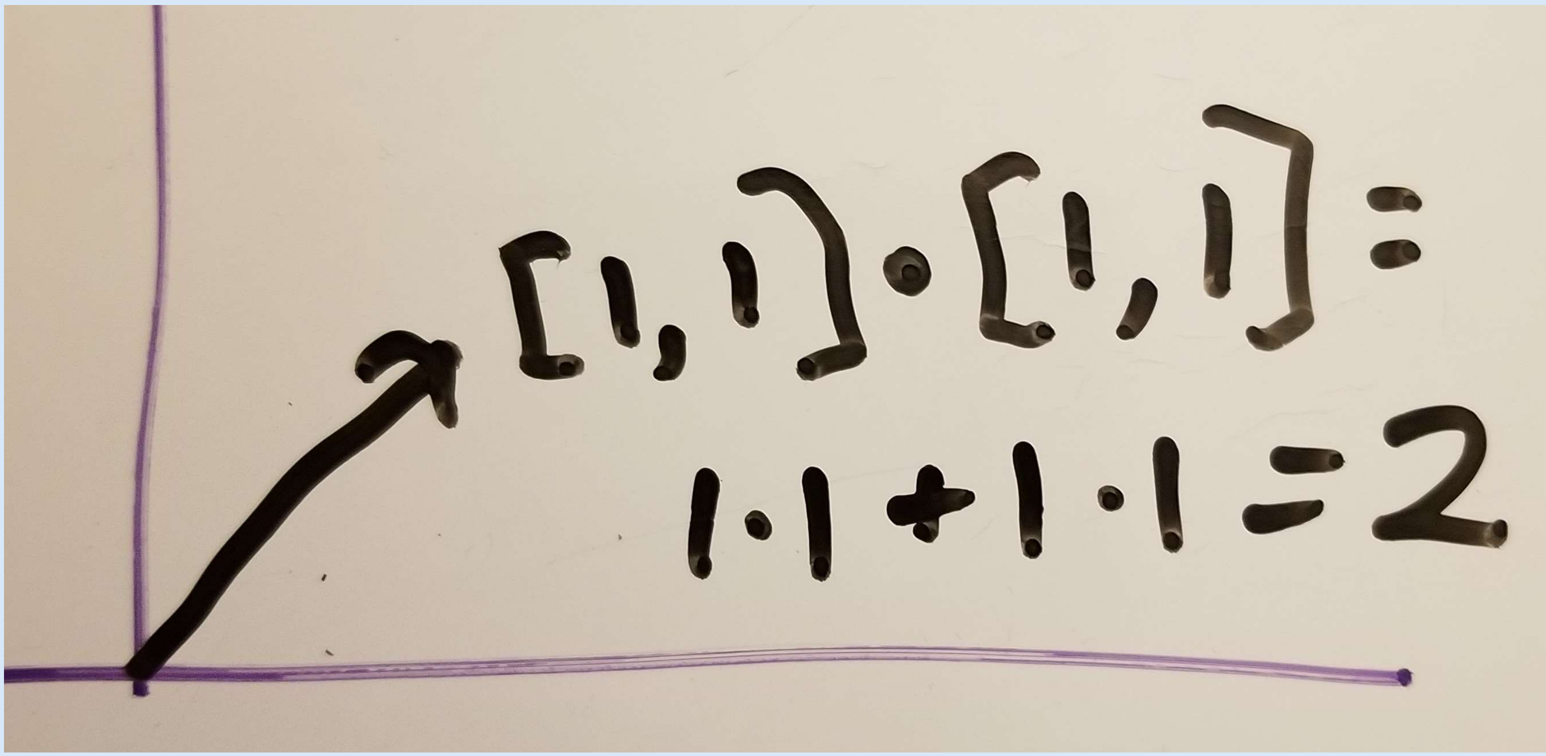
Here is a good summary Scalar, Vector, Matrix (mathsisfun.com)

Vectors (mathsisfun.com)

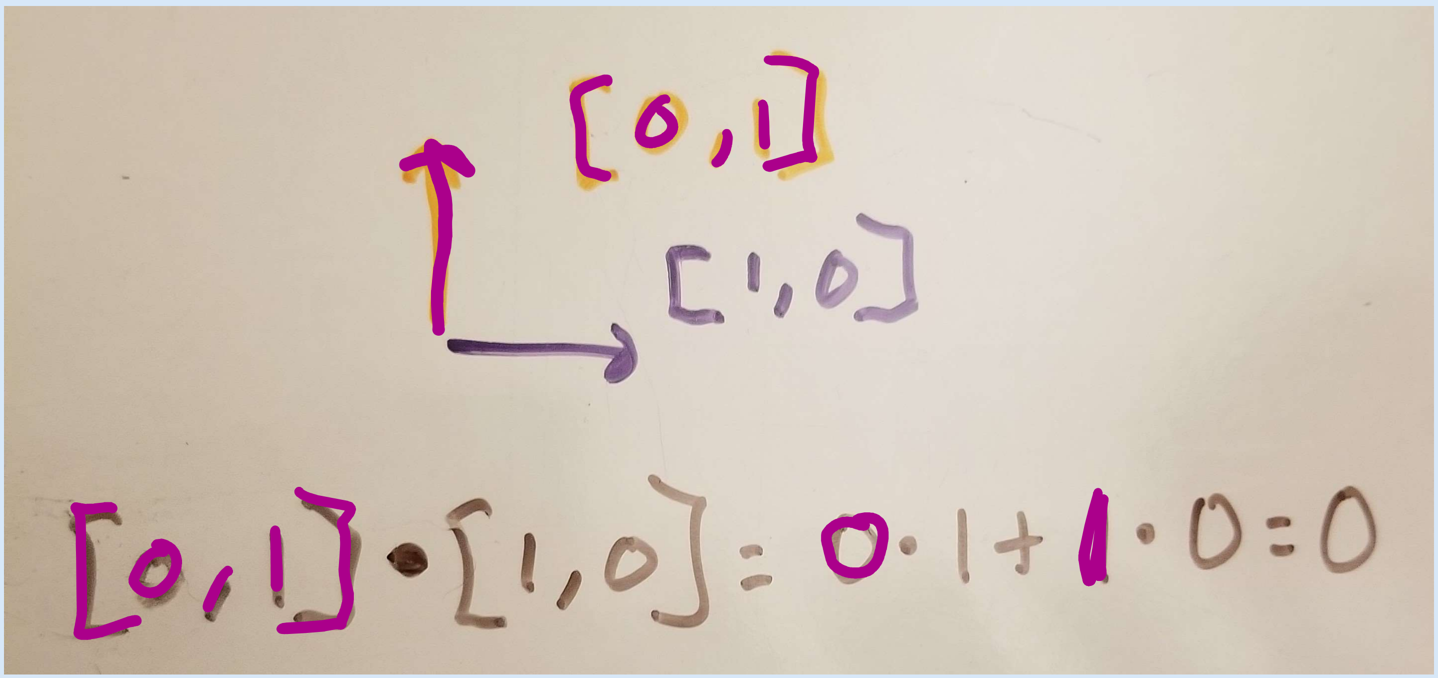
The dot product gives a scalar that is the amount that the two vectors are reinforcing or opposing each other.

Vector Calculus: Understanding the Dot Product – BetterExplained has a good explanation.

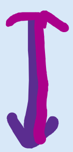
The dot product gives a scalar that is the amount that the two vectors are reinforcing or opposing each other. Two vectors going in the same direction double their impact. Two people pulling in the same directions will have twice the pull.

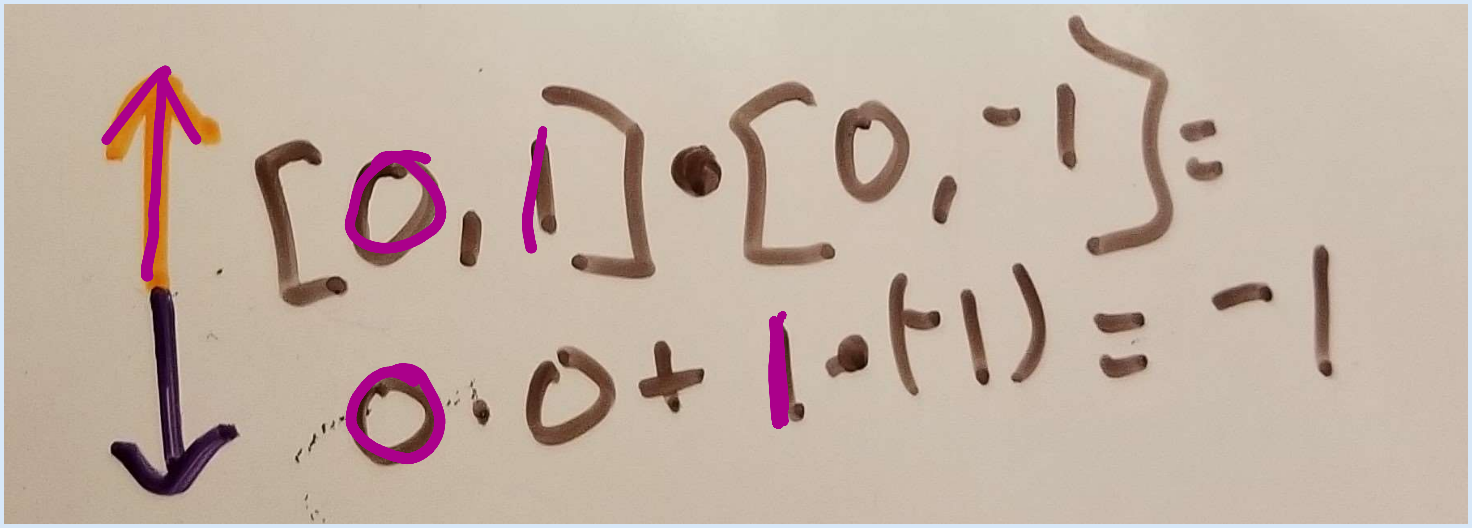


The dot product gives a scalar that is the amount that the two vectors are reinforcing or opposing each other. Two vectors going perpendicular have no net impact.



The dot product gives a scalar that is the amount that the two vectors are reinforcing or opposing each other. Two vectors going in opposite directions have a negative impact. Here the second person is pulling against the first one.



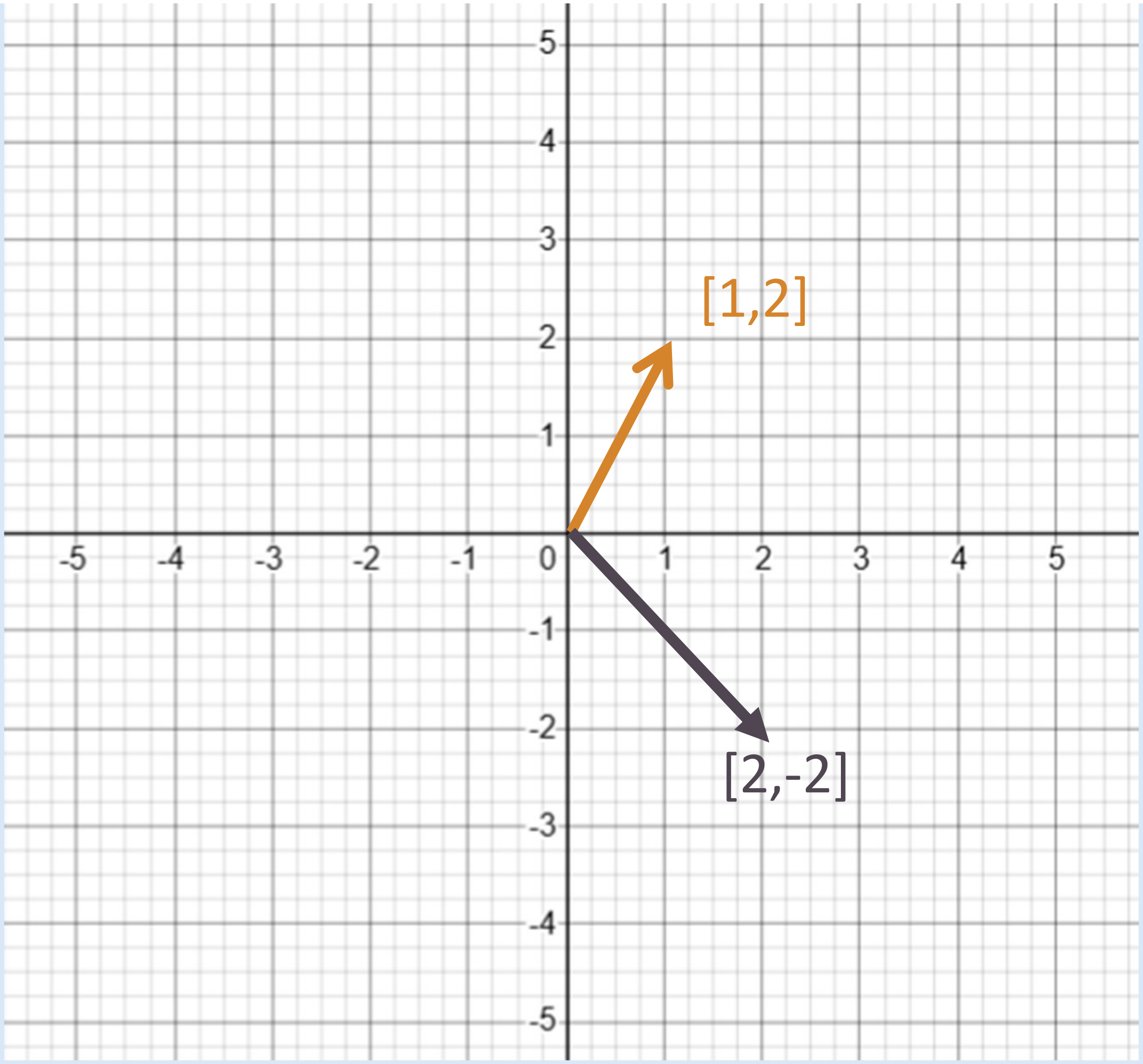


The dot product gives a scalar that is the amount that the two vectors are reinforcing or opposing each other.

It can be easier to do the calculation than to understand what it means.

[1,2] · [2,-2] is

1*2 + 2* -2 = -2



[1,2] · [2,-2] is

1*2 + 2* -2 = -2

The dot product can be used in physics and data science.

In data science, we can take information and put it into vectors. Take a rating scale that has two different questions. Then we can use the dot product to see how alike the two people respond to the questions.

Power is the dot product of Force and Velocity.

https://en.wikipedia.org/wiki/Dot_product