Solving multistep linear equations in one variable

When we are solving equations, we are undoing what was done to the variable.

You do the inverse to the output that you do to the input.

+1

input   function   output

-1

Review of solving one-step equations

$x+n=a$

$x=a-n$

```
>>> a=3
>>> n=1
>>> x=a-n
>>> print(x)
2
```

$x-n=a$

$x=a+n$

```
>>> a=3
>>> n=1
>>> x=a+n
>>> print(x)
4
```

$n*x=a$ $nx=a$

$x=\frac{a}{n}$

```
>>> a=3
>>> n=5
>>> x=a/n
>>> print(x)
0.6
```

```
>>> print(str(a)+'/'+str(n))
3/5
```

$\frac{x}{n}=a$

$x=a*n$ $x=an$

```
>>> a=2
>>> n=3
>>> x=a*n
>>> print(x)
6
```

Review of solving one-step equations

$x+1=3$
$x=3-1$

$x+n=a$
$x=a-n$

$x-1=3$
$x=3+1$

$x-n=a$
$x=a+n$

$2x=6$
$x=\frac{6}{2}$

$nx=a$
$x=\frac{a}{n}$

$\frac{x}{2}=5$
$x=5\cdot 2$

$\frac{x}{n}=a$
$x=a\cdot n$

$n\neq 0$

We haven’t run into problems that don’t work or equations that aren’t true.

$x = x + 1$ not true for any numbers

$0 \neq 1$

$\emptyset$ empty solution set

$x = x$ true for any value of x

$1 = 1$

$\pi = \pi$

$10 = 10$

$\mathbb{R}$ true for all real numbers

Can be just addition and multiplication

subtraction is addition with the negative of the number

$x - n = x + {}^{-}n$

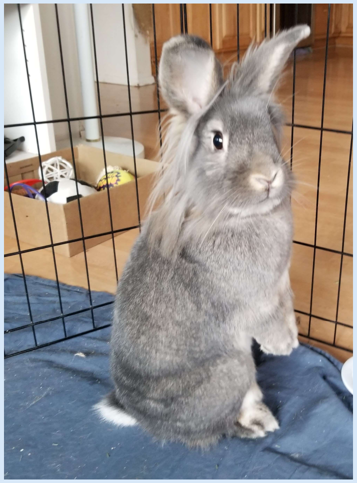
division is multiplication with the reciprocal of the number

reciprocal $a \neq 0$ $b \neq 0$

$\frac{3}{5} \sim \frac{5}{3}$ $\frac{a}{b} \sim \frac{b}{a}$ $\frac{x}{n} = \frac{1}{n} \cdot x$

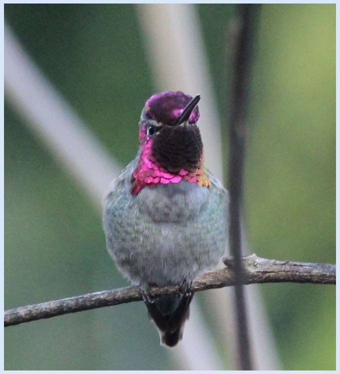
$\frac{3}{5} \cdot \frac{5}{3} = 1$ $\frac{a}{b} \cdot \frac{b}{a} = 1$

Undoing more than one step. On the left is Simba the bunny, in a crate, behind Doc the cat. Simba had to go around Doc and out of the crate to get free. Silly Doc stayed in for a nap.



While we are focusing on numbers, this also works for objects, vectors, data points etc.

I do photography and the photo editor lets me crop, rotate, and make adjustments that are all digitalized as numbers and the actions are operations on the pixels.



Equation has an equal sign so one side equals the other side. Isolate the variable by undoing the output by what was done to the variable. Or doing the same thing to both sides.

$$3(2x+1)=9$$

$$2x+1=3$$

$$2x=2$$

$$x=1$$

Warning! For these problems, we don't ever divide by the variable. If you want to move the variable, you add or subtract it to move it because you don't want to end up dividing by zero.

$3x=2x$ you need to use addition and subtraction to move the pieces.

$3x-2x=0$

$x=0$

You can't divide by zero and this is not a multiplication problem.

What kind of number the variable is doesn't matter

x+3=2

x=2-3= -1

x+1= 3.5

x=3.5-1 = 2.5

2x=5

x= $\frac{5}{2}$

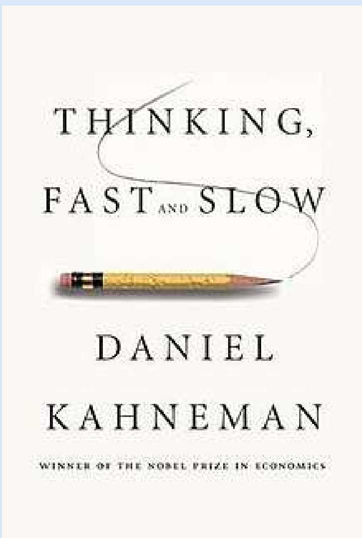
$\frac{x}{2} = \frac{1}{4}$

x= $\frac{2}{4} = \frac{1}{2}$

Want to use the right tool for the right job.

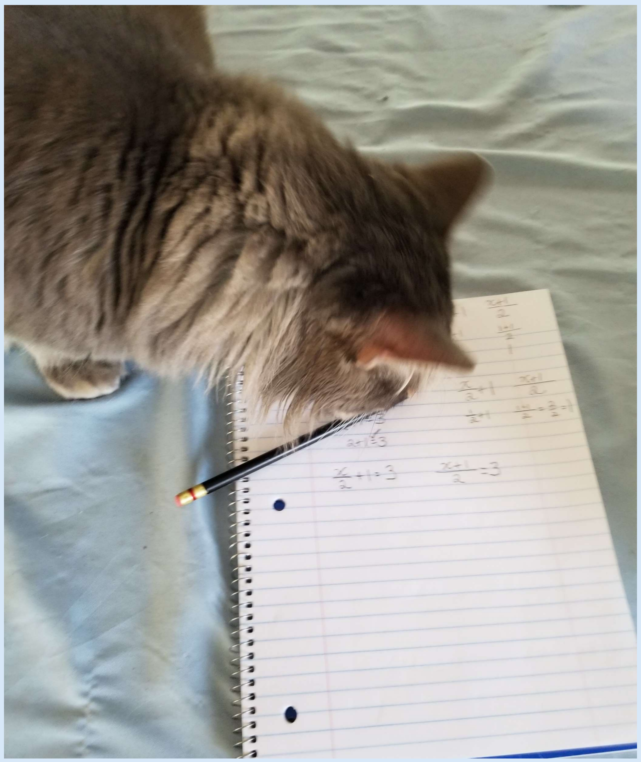
Can be hard to do in working memory!

Remember system 1 and system 2 or the fast brain processing and the slow brain processing.



Want the individual steps to be in system 1 or fast recognition. Want to have ways of doing the steps systematically since they are slow processing.

Develop your own system for writing out problems and keeping them for future reference.



You can just use technology

Dsolved slowly as there aren't many of them. You can always use technology to help with the problem solving. Microsoft math solver Microsoft Math Solver - Math Problem Solver & Calculator

Wolfram|Alpha Widgets: "Solving Literal Equations" - Free Education Widget

Literal Equations Calculator (mathcelebrity.com)

Literal Equation Calculator | Solve for Variable Calculator (easycalculation.com)

Literal Equation Calculator (mathsite.org)

Undo in the reverse order of what was done.

Getting dressed to go out in the cold: first I put on a mask (in Covid times), then a scarf, and then gloves.

When I come back in, first I take off the gloves, then the scarf, then the mask comes off last.

2x+1=3 Solving equations with multiple steps: first we undo the last thing that was done, which is adding one. Then we undo the next which is multiplying by 2.

$2x+1=3 \; -1$

$2x=\frac{3-1}{2}$

$x=1$

When solving an equation, can undo or distribute

2(x+1)=4

Can start by either dividing both sides by 2 or by distributing the two over the (x+1). Can't start by subtracting one from both sides.

2(x+1)=4

I will divide by 2 first since I get a whole number   x+1=2   x=1

2(x+1)=5 Dividing 5 by 2 gets me a fraction and I don't want to get into that yet so I will distribute first.

2x+2=5

2x=3

$$x=\frac{3}{2}$$

Distribute or not?

$2(x+1)=6$ first undo the multiplication by 2,

$x+1=3$ then undo the adding 1.

That is quicker, more efficient, and I am less apt to make a mistake.

But if it were: $2(x+1)=7$

$2(x+1)=7$ then I would distribute first

$2x+2=7$ then I would undo the adding 2

$2x=5$ then I would undo the multiplying by 2

Do you see why I distributed first?

If I tried to do it the other way, I would have:

2(x+1)=7

x+1= ugh a fraction that I will then have to subtract a whole number from, yuck!

You are welcome to finish the problem this way, but I choose not to, haha. If I distribute first, then I do the division at the end, and I don't get a fraction until the end.

2(x+1)=7 then I would distribute first

2x+2=7 then I would undo the adding 2

2x=5 then I would undo the multiplying by 2

$$2(x+1)+3= 7$$

$$2(x+1)=4$$

$$x+1=2$$

Solving an equation

Solving an equation for a variable means that we are unwrapping to get the variable by itself on one side.

$$10(2x+1)+5=35$$

$$10(2x+1)=30$$

$$2x+1=3$$

$$2x=3-1$$

$$x=1$$

In computer programming there is a FILO order:

First in, last out

What if the variable is negative?

$2(x-3)=-8$

$2x-6=-8$

$2x=-2$

$x=-1$

$2(x-3)=-8$

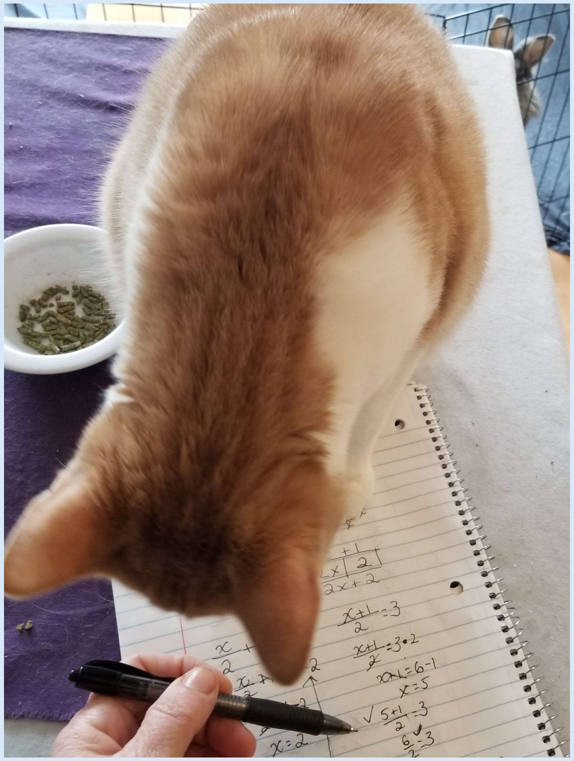
$x-3=-4$

$x=-1$

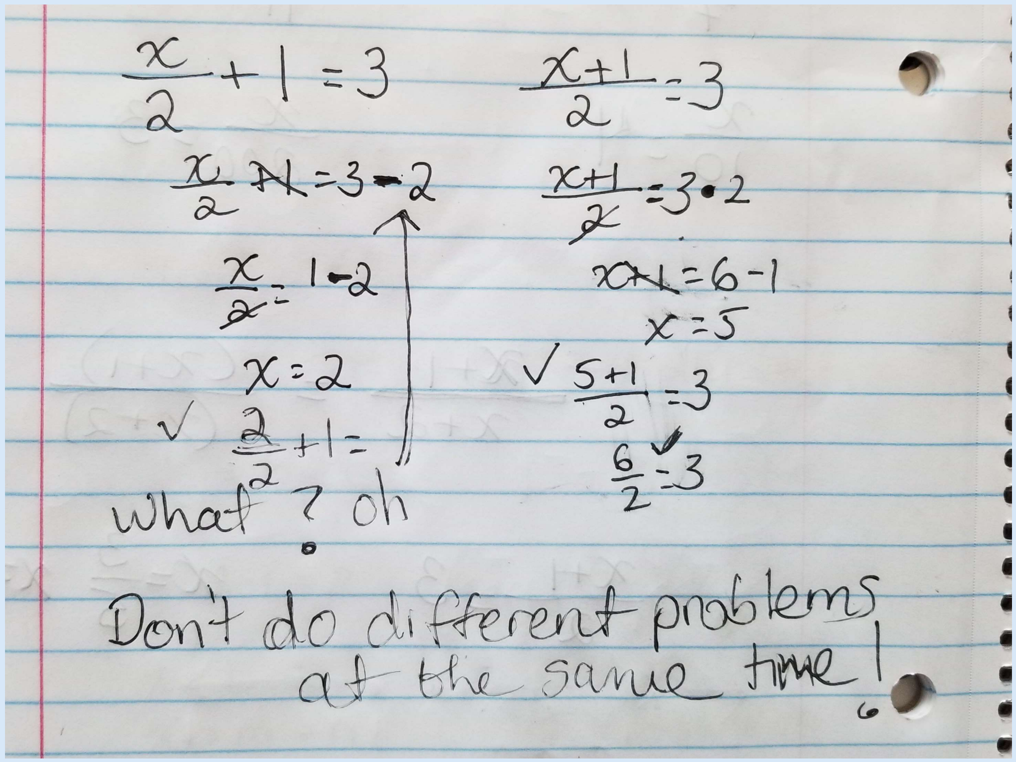
Different equations so solve these by undoing in a different order:

$$\frac{x}{2} + 1 = 3 \qquad \frac{x+1}{2} = 3$$

Aury was trying to help.



I was doing them together and got them confused. I blame Aury and Simba, haha.



Done correctly:

$$\frac{x}{2} \cancel{+1} = 3-1$$

$$\frac{x}{\cancel{2}} = 2\cdot 2$$

$$x=4$$

✓ $$\frac{4}{2}+1=3$$

$$2+1 \overset{\checkmark}{=} 3$$

$$\frac{x+1}{2}=3$$

$$\frac{x+1}{\cancel{2}} = 3\cdot 2$$

$$x\cancel{+1} = 6-1$$

$$x=5$$

✓ $$\frac{5+1}{2}=3$$

$$\frac{6}{2} \overset{\checkmark}{=} 3$$

Remember how division distributes and about equivalent expressions. Here the bottom three are all ways of writing the same thing. Notice that step 4 distributes the division across the terms.
a and b not equal to zero.

$$a(bx+c)=d \quad a,b \neq 0$$

$$bx+c=\frac{d}{a}$$

$$bx=\frac{d}{a}-c$$

$$x=\frac{d}{ab}-\frac{c}{b}$$

$$x=\frac{d}{ab}-\frac{ac}{ab}$$

$$x=\frac{d-ac}{ab}$$

a(bx+c)=d

abx+ac=d

abx=d-ac

x= $\frac{d-ac}{ab}$

x= $\frac{d}{ab}$ - $\frac{ac}{ab}$

x= $\frac{d}{ab}$ - $\frac{c}{b}$

Again, the bottom three are all equal. This way we got to the top one first and the other way we got to the bottom one first.

To check your work, you can plug the results back into the equation.

$$a(bx+c)=d \quad a,b \neq 0$$

$$bx+c=\frac{d}{a}$$

$$bx=\frac{d}{a}-c$$

$$x=\frac{d}{ab}-\frac{c}{b}$$

$$x=\frac{d}{ab}-\frac{ac}{ab}$$

$$x=\frac{d-ac}{ab}$$

$$a\left(\cancel{b}\left(\frac{d-ac}{a\cancel{b}}\right)+c\right)=d$$

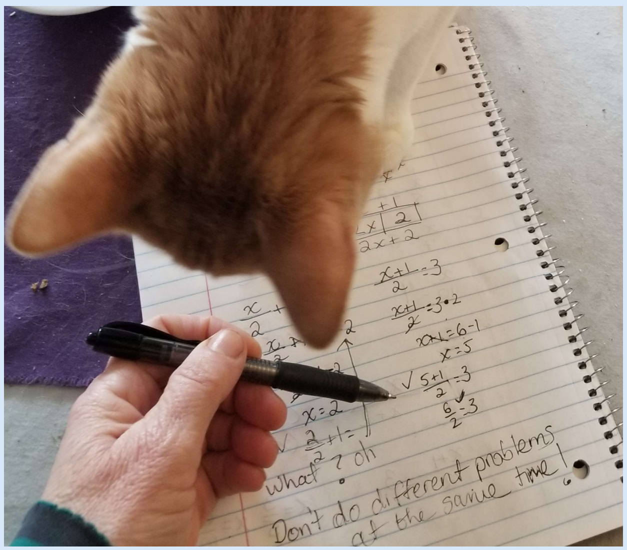
$$\cancel{a}\left(\frac{d-ac}{\cancel{a}}+c\right)=d$$

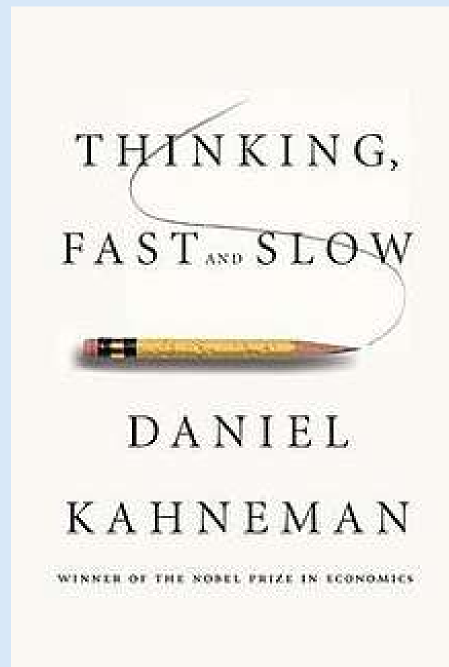
$$d-ac+ac=d$$

$$d=d$$

Write out enough that you can find errors.

Want to have ways of doing the steps systematically since they are slow processing. Write out enough that you can go back over it to find errors or have someone else check it over.





Cancel out identical terms that are on both sides.

$2x+x+3=2x+3+5$

$\cancel{2x}+x+\cancel{3}=\cancel{2x}+\cancel{3}+5$

Or cancel inverse terms on the same side

$2+x-2=1$

$\underline{2(5x)} = 3$

10

Division, fractions, and ratios can all be rates.

A rate is something per something else.

Dollars per hour is an hourly salary.

Dollars per year

Miles per hour

Meters per second

The bottom or denominator is the units of the input and is for one of those units. So, dollars per hour is how many dollars you make in **one** hour.

Factor-label method or unit conversion

In science, there are lots of formulas and you often need to convert between units of measurement. The units can cancel.

$$\frac{5\ \text{km}}{\text{hr}} \cdot \frac{1\ \text{hr}}{60\ \text{min}} \cdot \frac{1\ \text{min}}{60\ \text{sec}} \cdot \frac{1000\ \text{m}}{1\ \text{km}} = \frac{\text{m}}{\text{sec}}$$

amox $\frac{40\ mg}{Kg}$

$2.2\ Kg = 1\ pound$

$1000\ mg = 1\ kg$

$$\frac{x\ \cancel{lbs}}{1\ baby} \cdot \frac{2.2\ \cancel{kg}}{\cancel{lbs}} \cdot \frac{1000\ mg}{\cancel{Kg}} = \frac{mg}{1\ baby}$$

Fahrenheit to Celsius and vice versa

Formula to convert Fahrenheit to Celsius

$$C = (F - 32) \times \frac{5}{9}$$

Formula to convert Celsius to Fahrenheit

$$F = C \times \frac{9}{5} + 32$$

Some multistep problems can get very complicated. If you follow the rules and do them a step at a time, it should work out. I find it much easier to do with literal equations and then have a computer do the numbers, but it can depend on the context. There are lots of sites with practice problems and it is good to get fluent with the basics.

If you don’t want to practice at this point, it is ok to move on.

Depending on your goals, you can do lots of practice and get good at each skill. While the theory is basic and very important and being able to read the math symbols and follow the rules is important, these kinds of algebraic manipulation problems only really occur on math tests.

Just right challenge

If you understand the concepts, you may never run into problems that need multistep. If you are doing physics, you can review the type of problems then.

It is fine to not practice at this point, as long at you are comfortable with the concepts and the symbols.

Math skills build on each other so if you get to a point where you feel lost, back up and go back over the basic underlying concepts.

I will keep reviewing key skills as we go along.

Algebraic Manipulation Practice

Algebraic manipulation is a general term for moving variables and constants around to rearrange the pieces of the puzzle or solve problems.

The more you practice, the better and quicker you will get at doing it.

You can do one practice a day for the next year or you can do several problems once or twice a week.

Make up your own problems or get them from a book.

Multi-step equations review (article) | Khan Academy

IXL | Solve multi-step equations | 8th grade math

IXL - Rearrange multi-variable equations (Algebra 1 practice)

Algebra for physics and applied topics

How to Teach Physics to your Dog (Book) | King County Library System | BiblioCommons this is quantum physics so not helpful for algebra

Physics I for Dummies (Book) | King County Library System | BiblioCommons

The library doesn't have Superstrings and Other Things by Carlos I. Calle (goodreads.com)

I like the previous version of this book for algebra review: Painless Algebra (Book) | King County Library System | BiblioCommons