How can you represent a line algebraically?

The line that contains two named points $(x1, y1)$ and $(x2,y2)$

Graph on a coordinate axis system

Function notation $f(x)=x$ or $g(x)=2x+3$ $f(x)=mx+b$, $f(x)=ax+b$

Linear equation in two variables $y=mx+b$ (slope-intercept form)

$y=ax+b$ (polynomial form)

$Ax+By=C$ (Standard Form)

$(y-y1)=m(x-x1)$ (point slope form)

Linear equations with only one variable are not functions.

$x=2$ or some constant is a vertical line and not a function

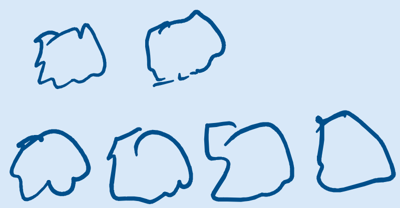
If we have a relationship where there are two variables and one depends on the other with a constant rate of growth, then we get a linear function.

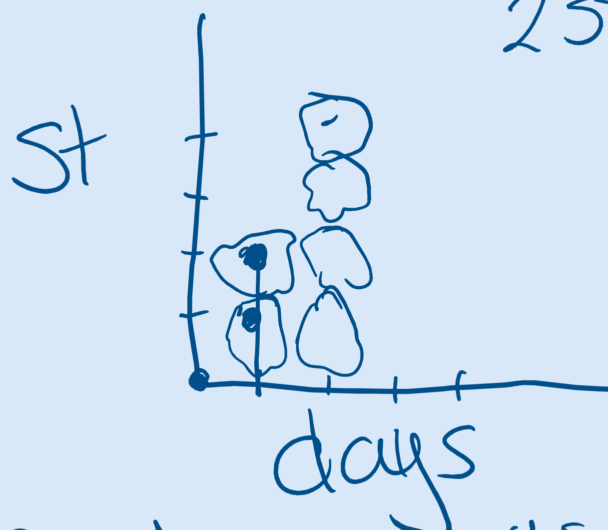
days stones

0

1

2





2 stones per day

$$\#\text{ stones} = \frac{2\text{ stones}}{\text{day}} \cdot \text{days} + \#\text{ stones starting}$$

$$s = r \cdot d + s_0$$

Slope-intercept form of a linear equation

y=mx+b is called the slope-intercept form of a line because the m is the slope, and the b is the y-intercept.

I am not sure why they use m for slope and b for y-intercept, but the French word for slope is “monter”…

Any y value on the line = the slope times the x value plus the y-intercept

y=slope*x+y-intercept

f(x)=mx+b is the simplified form of a linear function.

In math we use the terms coefficient, slope, or rate for a in $y=ax+b$

Since math is abstract, we use the term slope to mean how much y changes per how much x changes.

Rate implies different units for x and y.

Slope is used generically for the same units or different units on each axis.

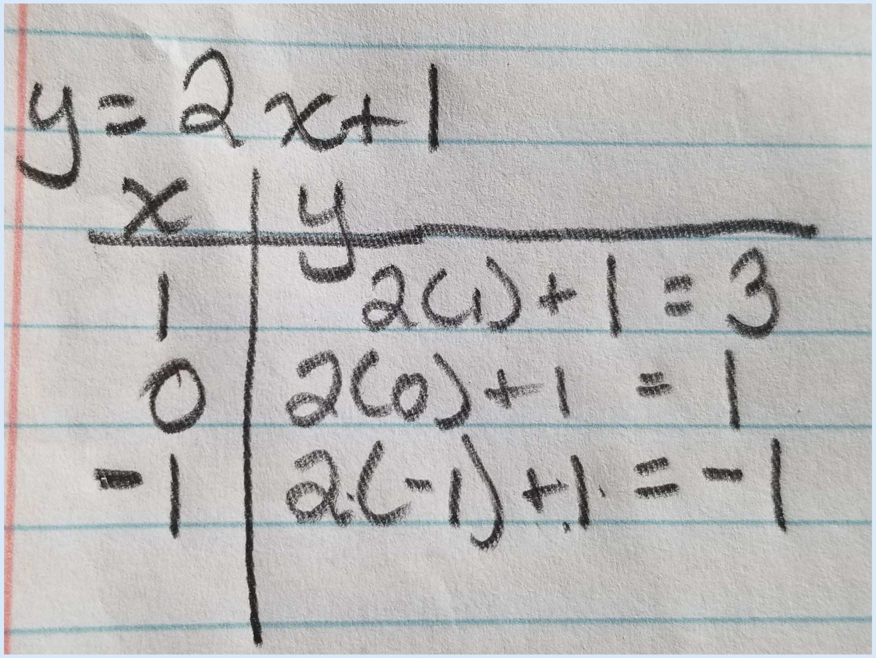
When we talk about polynomials, we use the term coefficient as the multiplier of x.

For any equation of a line, you can **substitute** in the x value to get the y value.

For y=2x+1 you can see that when x is zero, y=1. This is why (0,1) is the y-intercept of this line.

(1,3), (0,1), (-1,-1)

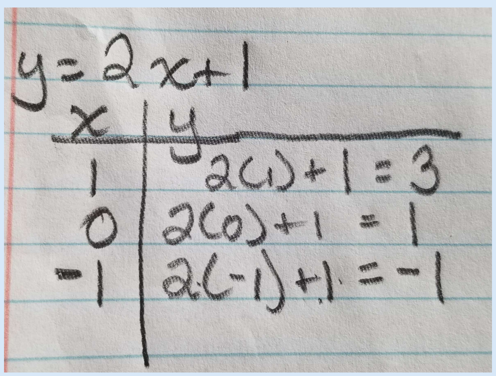
are all points on the line

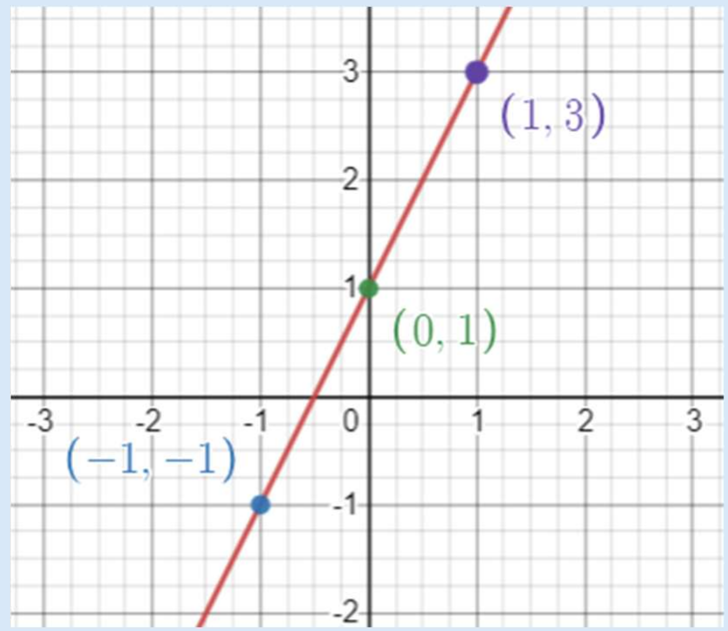


You can then plot the points on a graph to see the graph of the function.

In geometry, two points define a line so you only need two points to graph a linear function.

(1,3), (0,1), (-1,-1) are all points on the line





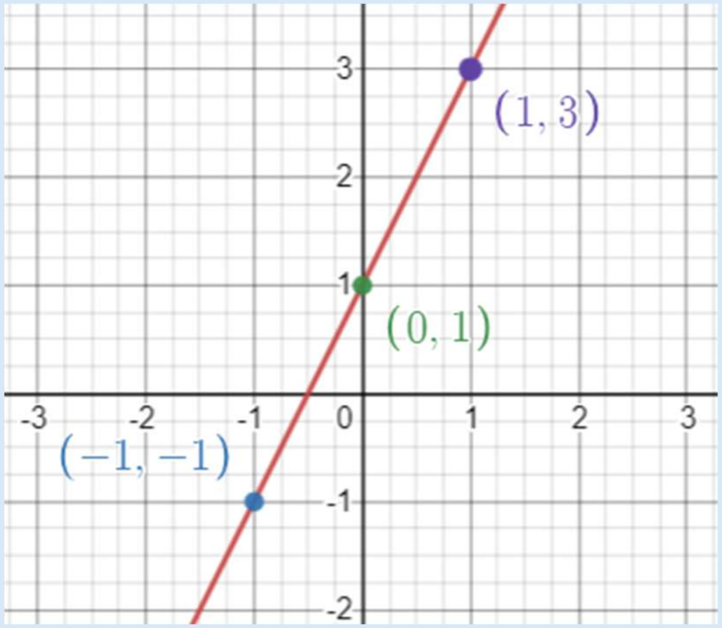
You can recognize a linear function or equation because there is y to the first power and x to the first power and you can put them in a form where the output is based on the input to the first power.

$$f(x) = ax + b$$

$$f(x) = 2x + 1$$

$$y = ax + b$$

$$y = 2x + 1$$



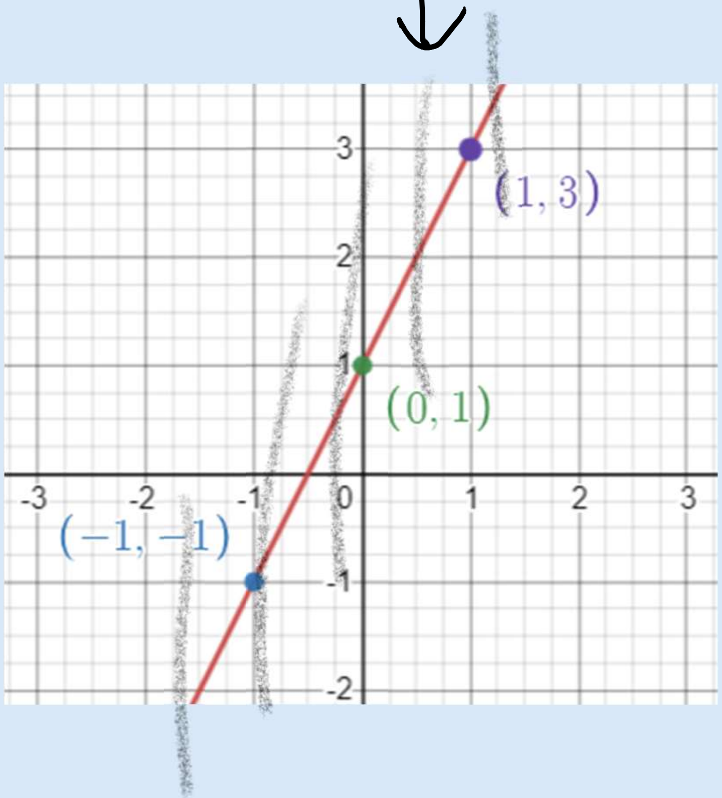
It doesn't matter if it is in equation form or function notation form. Nonvertical lines are all functions because there is a predictable output for any given input. On a graph, that means it passes the vertical line test.

$f(x) = ax+b$

$f(x) = 2x+1$

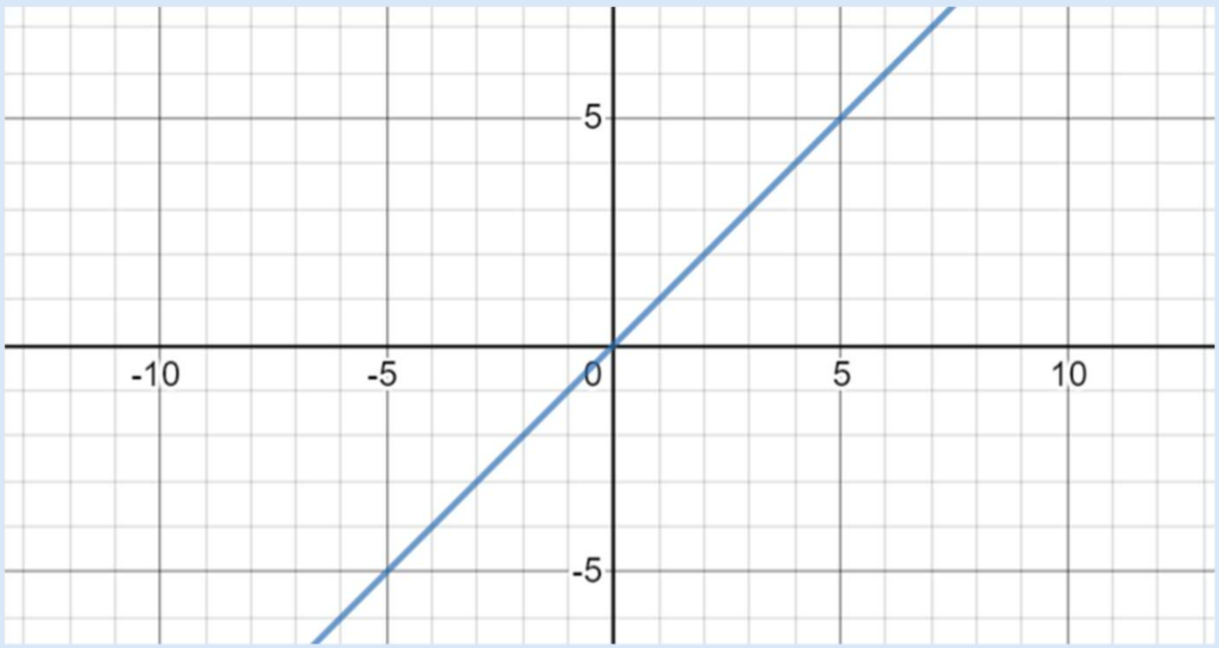
$y = ax+b$

$y = 2x+1$

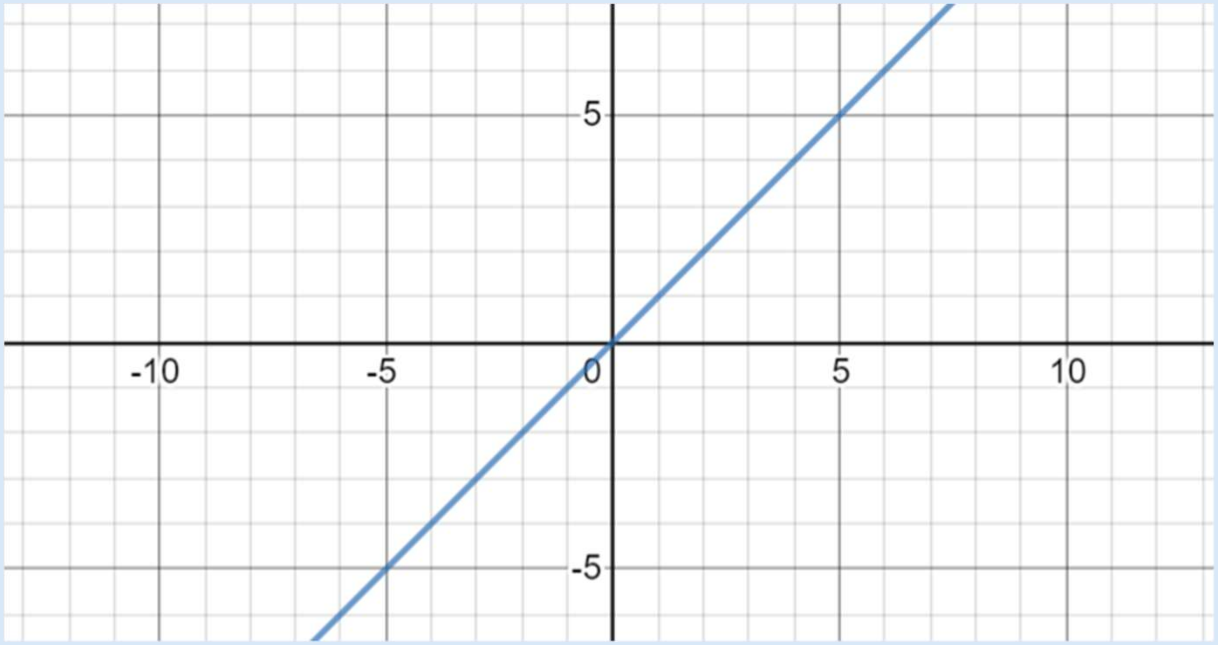


I like to think of functions in terms of their "parent" functions. These are the most basic of a catagory of functions.

y=x or f(x)=x is the parent linear function.

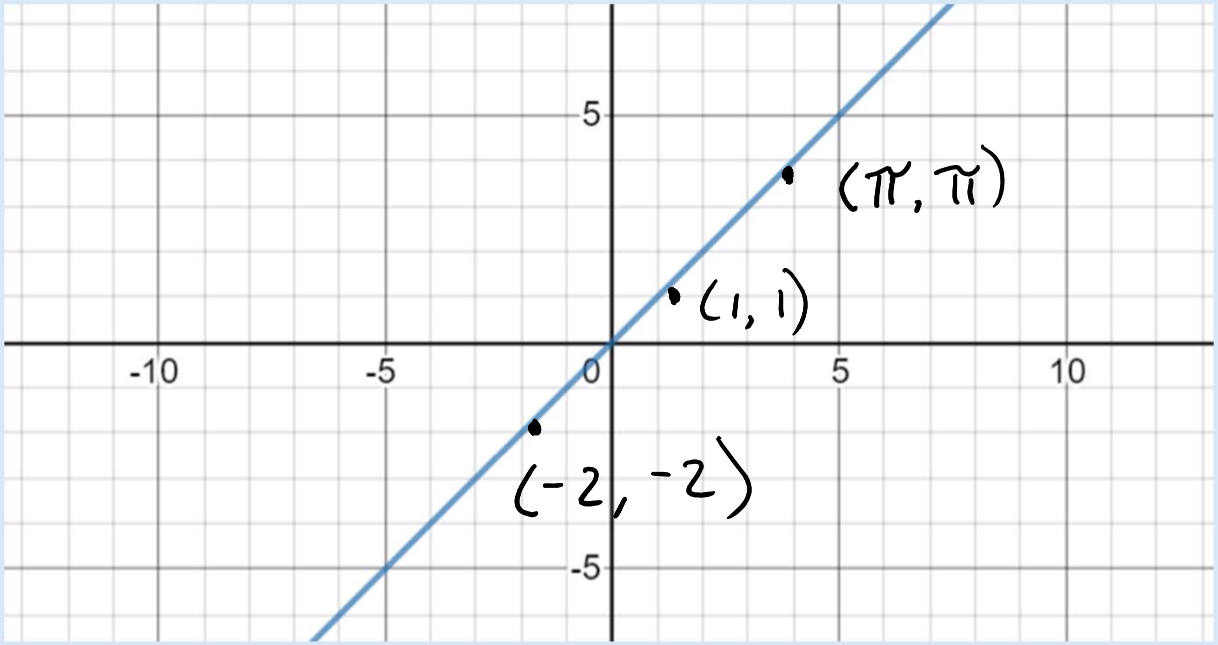


y=x or f(x)=x is the parent linear function.
It is the same as y=1x+0.
It has a slope of 1 and a y-intercept of zero.

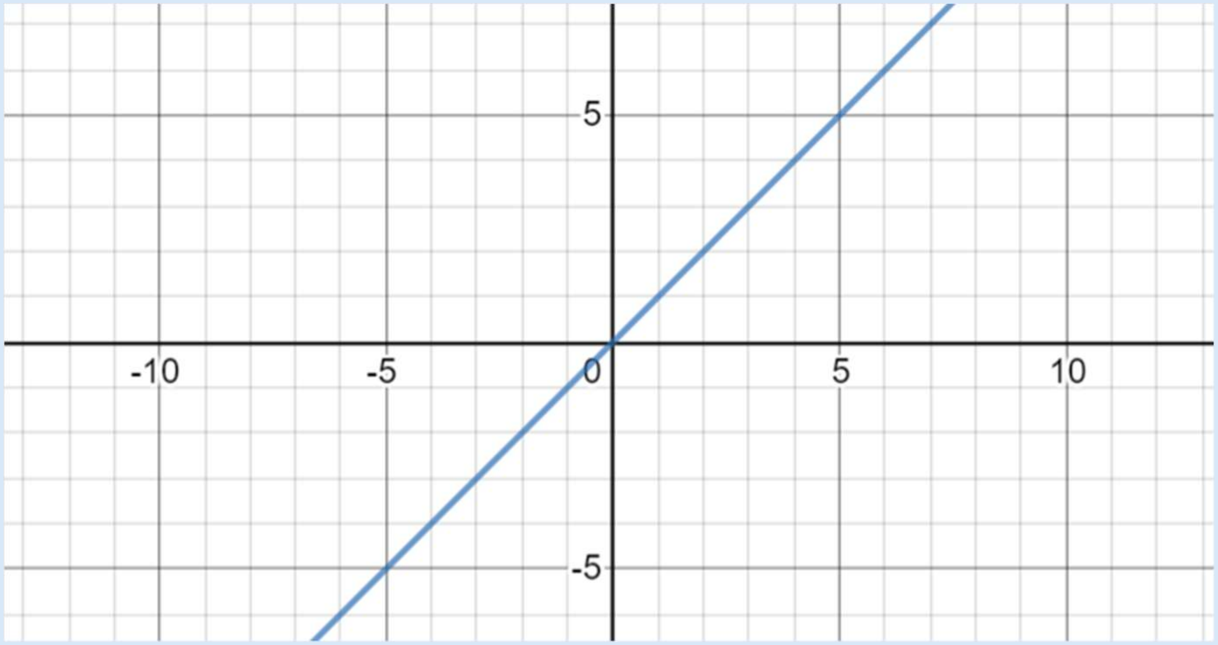


You can see that all points where the input and the output are equal are on the line. (-2,-2), (1,1), (π , π)

Because they are real number lines, all the irrational numbers are included in the function.



We can tilt and shift our parent function to transform it to a new function.

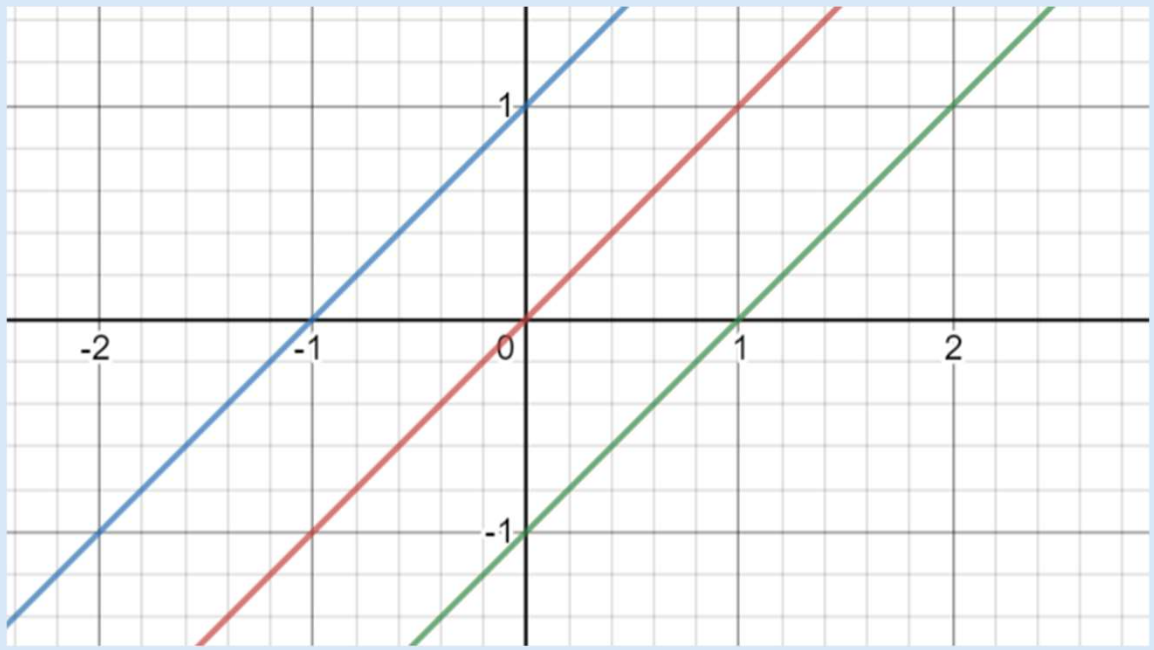


Notice what happens to the intercepts when we add a constant to our parent function of y=x

$y = x$

$y = x + 1$

$y = x - 1$

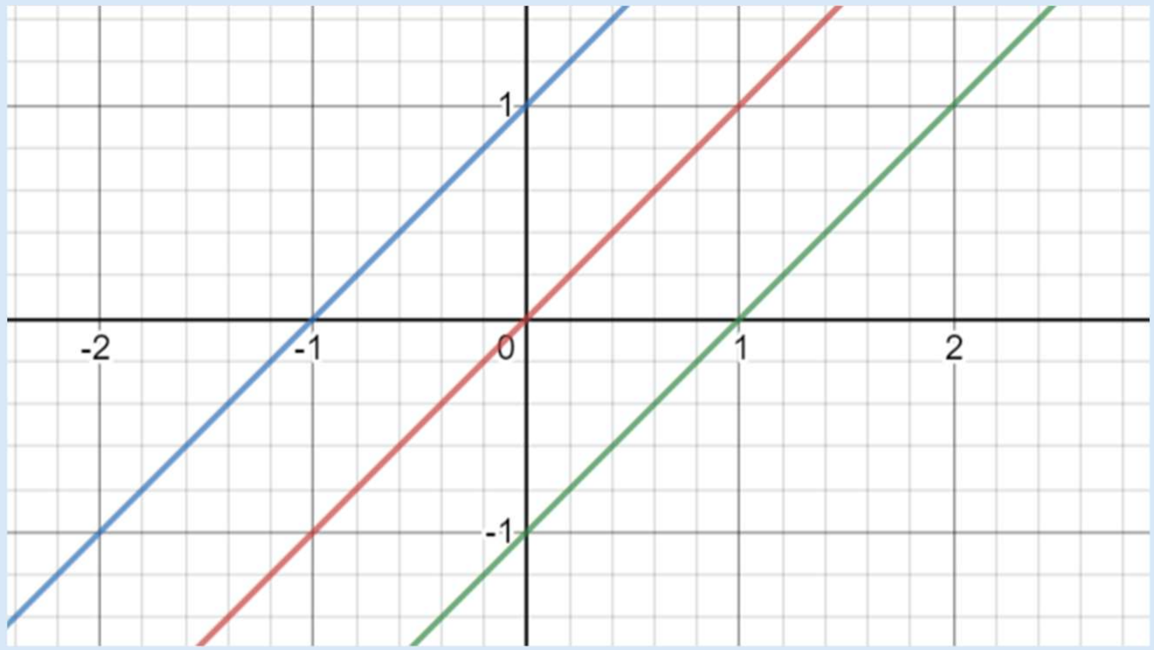


Adding a positive constant to the parent function shifts it up by that amount. Adding a negative constant shifts it down by that amount.

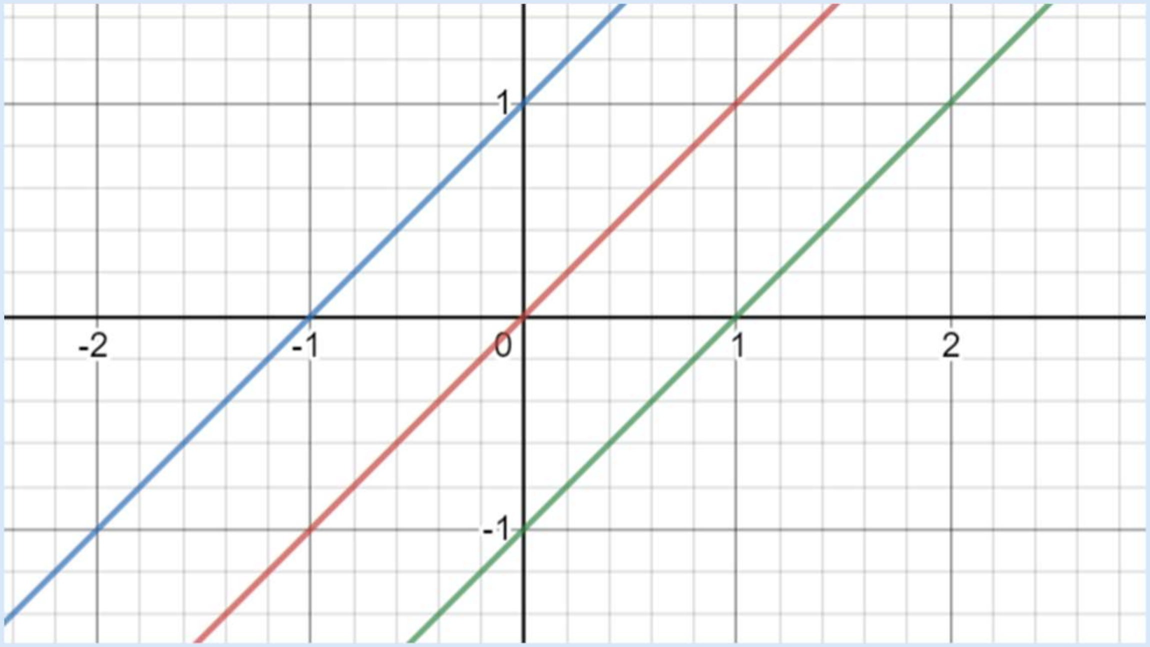
$y = x$

$y = x + 1$

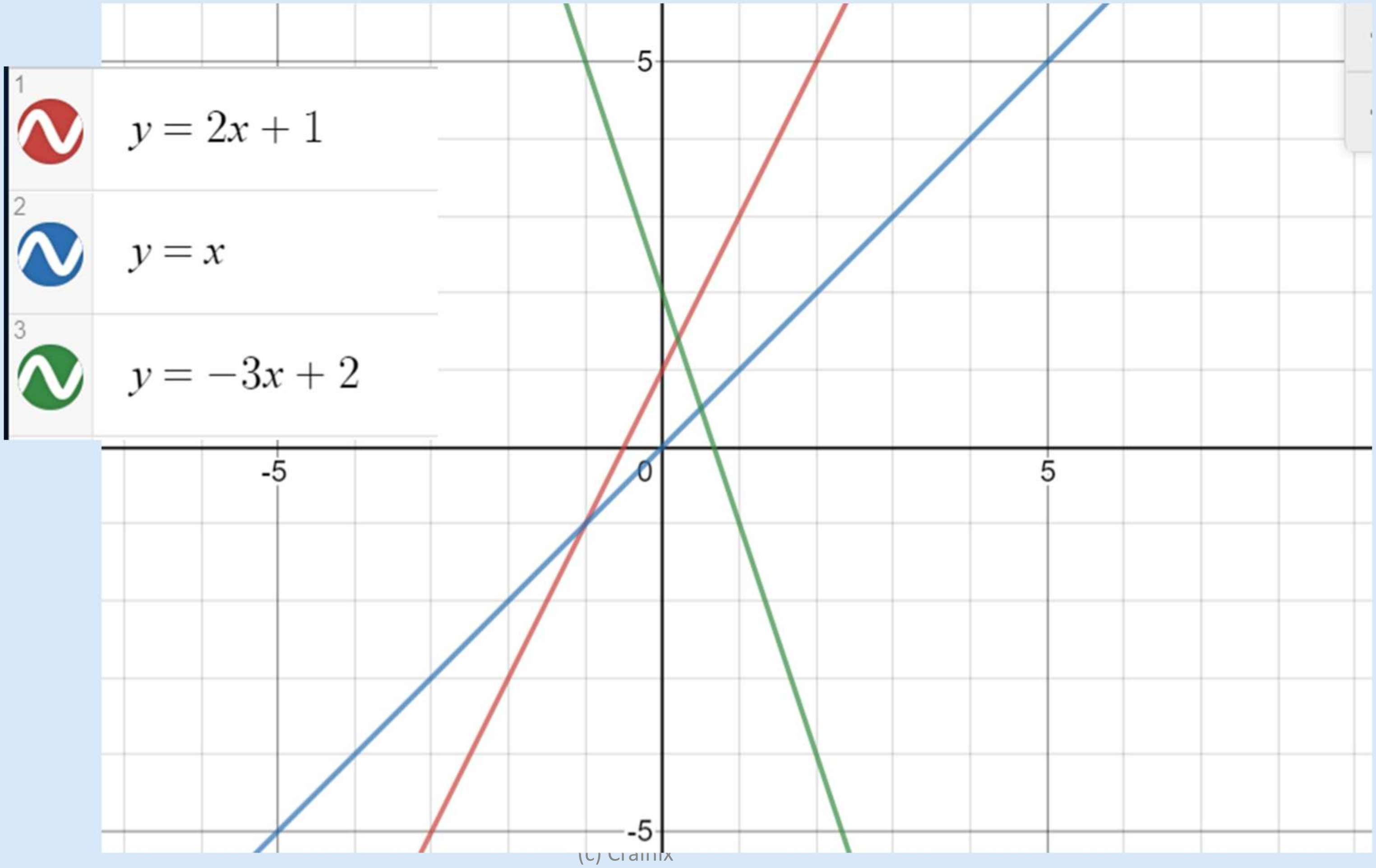
$y = x - 1$



We saw how adding a constant shifted up and subtracting a constant shifted down our parent function and changed the y-intercept. For y=ax+b,
b is the y value when x is zero, which is the y-intercept.
(0,b) is the point of the y-intercept.



The slope doesn't matter as b is the y value when x is zero.



A linear function that goes through the origin, (0,0) is called a **direct variation** as the y-intercept is zero. The output changes as a direct multiple of the input.

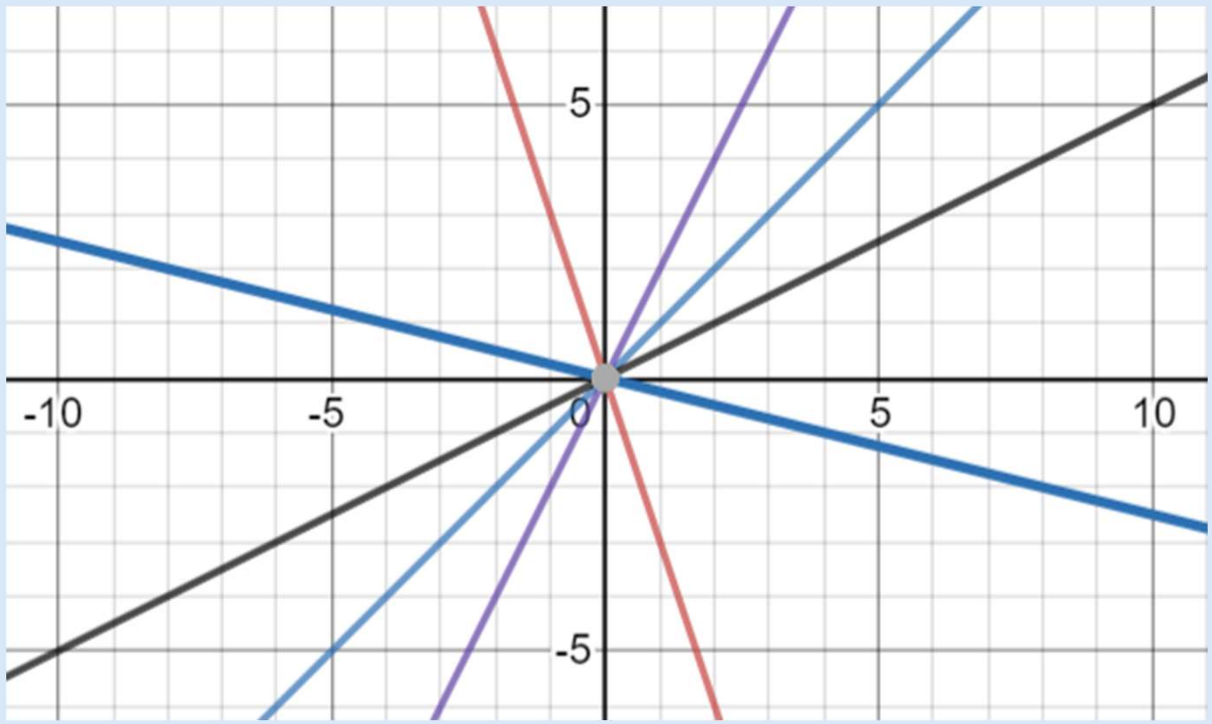
1 $y = x$

2 $y = 2x$

3 $y = \frac{1}{2}x$

4 $y = -3x$

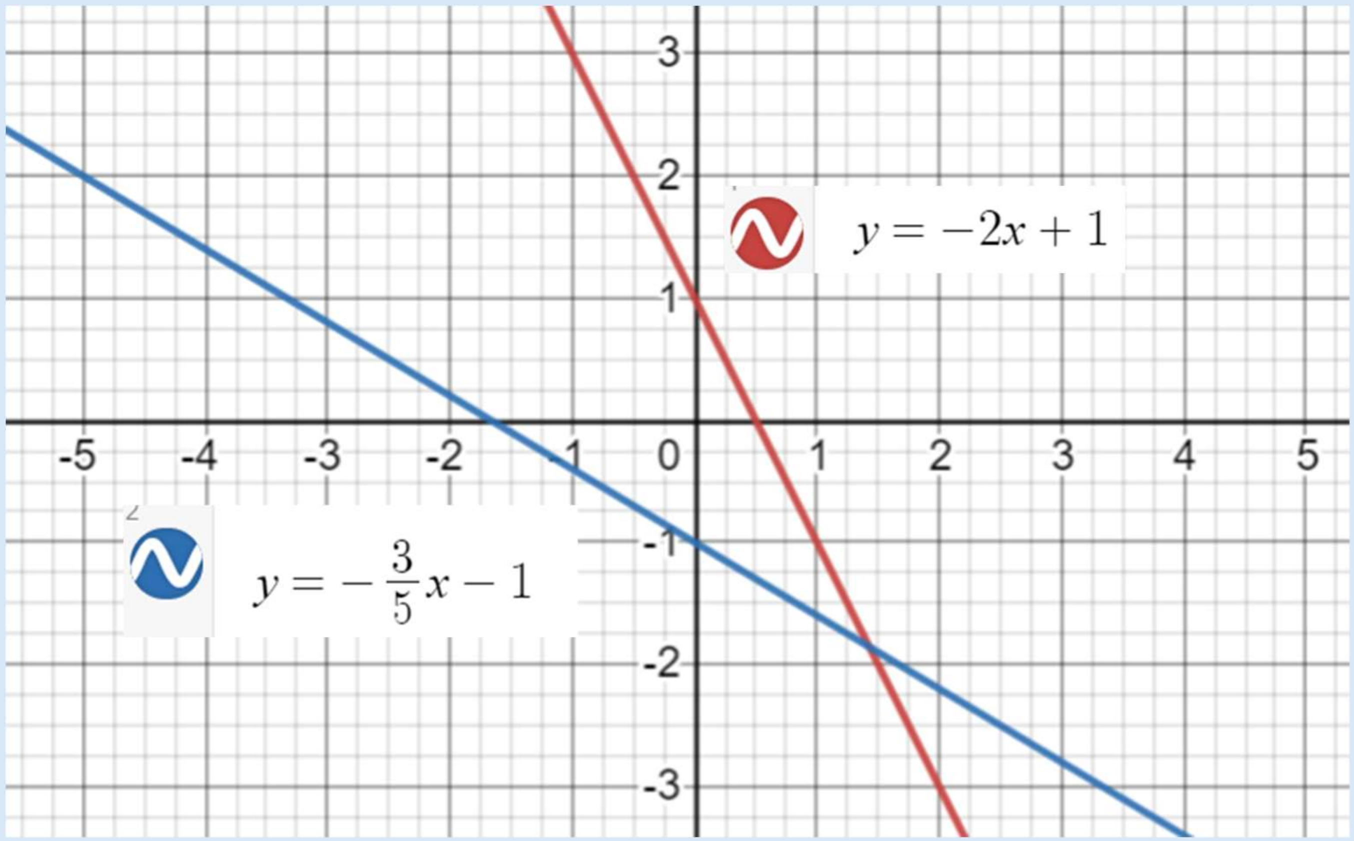
5 $y = -\frac{1}{4}x$



Negative slope

$y = -2x + 1$

$y = -\frac{3}{5}x - 1$



It could be anything that grows at a constant rate.

Dollars per hour

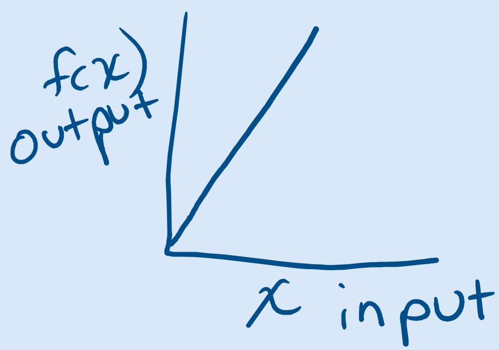
Inches per week

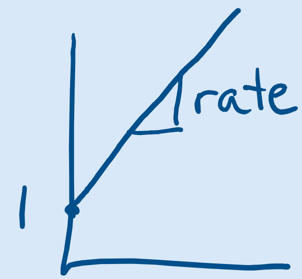
Meters per year

Distance up per steps hiked

Total amount = rate * time + starting amount

f(x)= rate * x + f(0) rate will have units of $\frac{\text{output}}{\text{input}}$





$\frac{\Delta f(x)}{\Delta x}$

Δ is change

The output equals the rate times the input plus the starting output.

output
total
distance

input

starting
output
point
location

$$f(x) = r\,x + f(0)$$
$$y = m\,x + b$$
$$y = a\,x + b$$

slope
rate
multiplier
coefficient

a) $y = 3x + 5$

b) $y = -2x + 3$

c) $y = 10x - 6$

d) $y = \frac{3}{4}x + 1$

e) $y = x + 2$

f) $y = 37.5x$

g) $y = x$

Practice seeing the slope and y-intercept.

The rate, slope, multiplier, or coefficient is equal to the change in output over the change in input. It is a ratio with units of the output per the input.

$$y = rx + b$$

$$y - b = rx$$

$$rx = y - b$$

$$r = \frac{y-b}{x}$$

$$\text{rate} = \frac{y_x - y_0}{x}$$

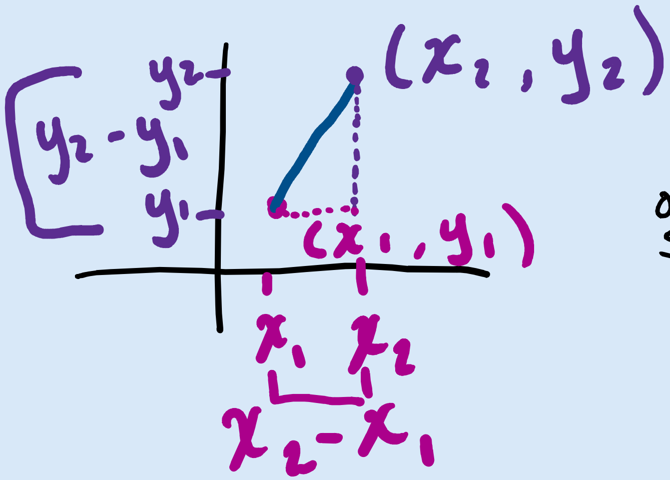
$$f(x) = ax + f(0)$$

$$ax = f(x) - f(0)$$

$$a = \frac{f(x) - f(0)}{x}$$

What if we want to know the rate or the slope between two point?

(x_1, y_1) (x_2, y_2)

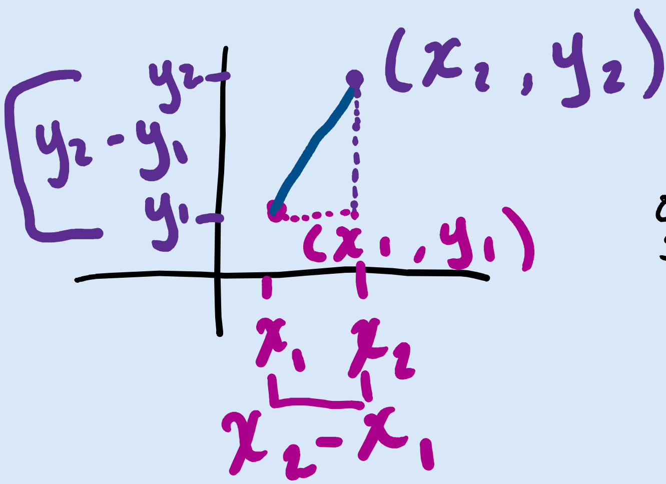


$$\text{rate or slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

We say that the slope or rate is the change of y over the change of x.
We use this symbol Delta from the Greek alphabet for change of: Δ
You will hear me say delta y over delta x as a shorthand.

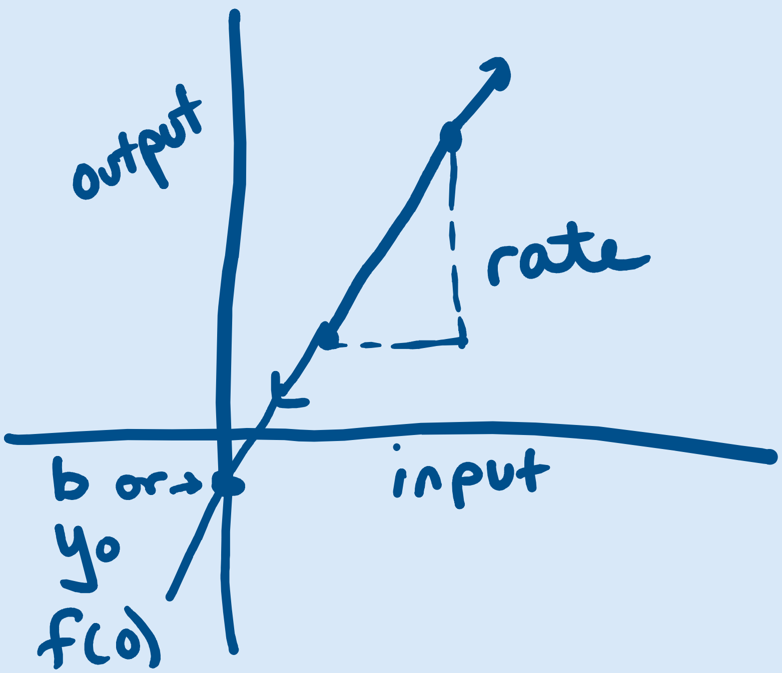
$$a = \frac{\Delta y}{\Delta x} \qquad r = \frac{\Delta y}{\Delta x} \qquad m = \frac{\Delta y}{\Delta x}$$

$$(x_1, y_1) \quad (x_2, y_2)$$



$$\text{rate or slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

The slope intercept form gives us information about the slope and the y-intercept or starting point. It doesn't tell us where the line crosses the x axis. For linear functions, we are usually starting with the input and don't care where it crosses the x axis.



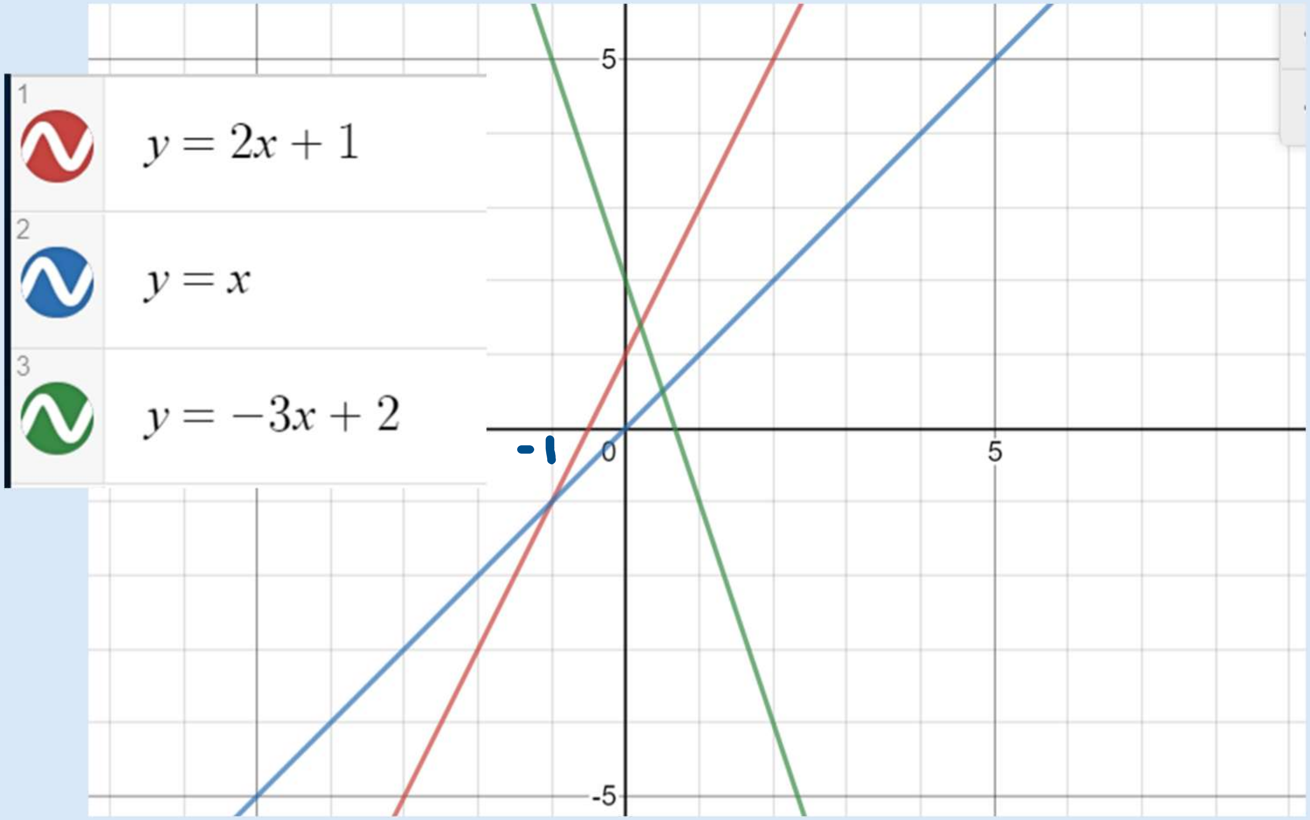
Linear functions are lines that have a slope and a y-intercept.

y=-3x+2

Y int is 2

(0,2)

Slope is -3

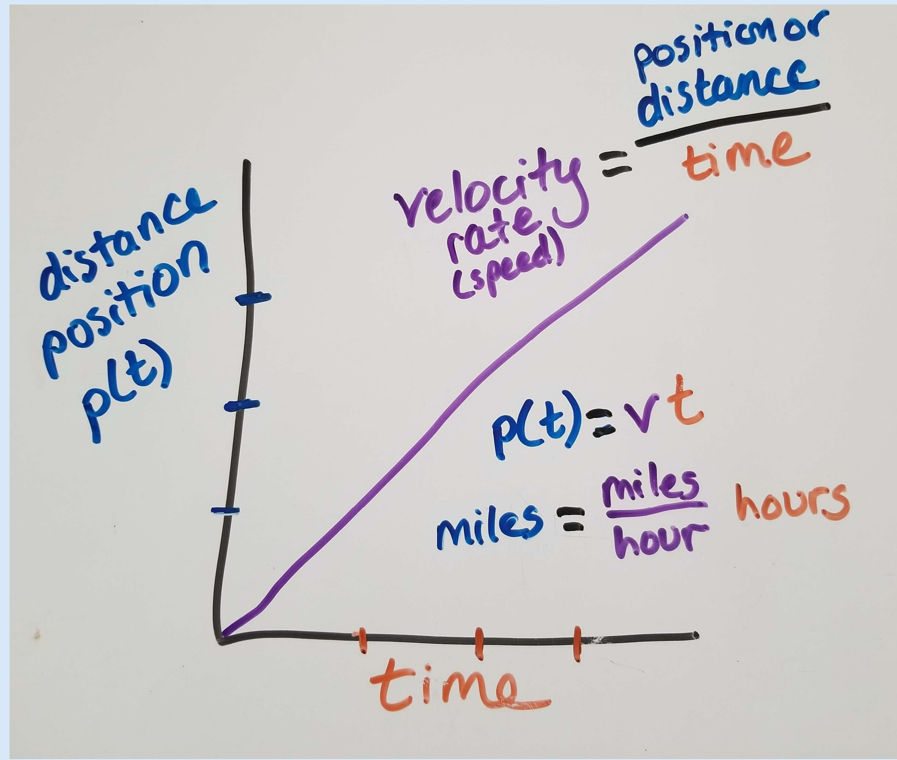


With labels

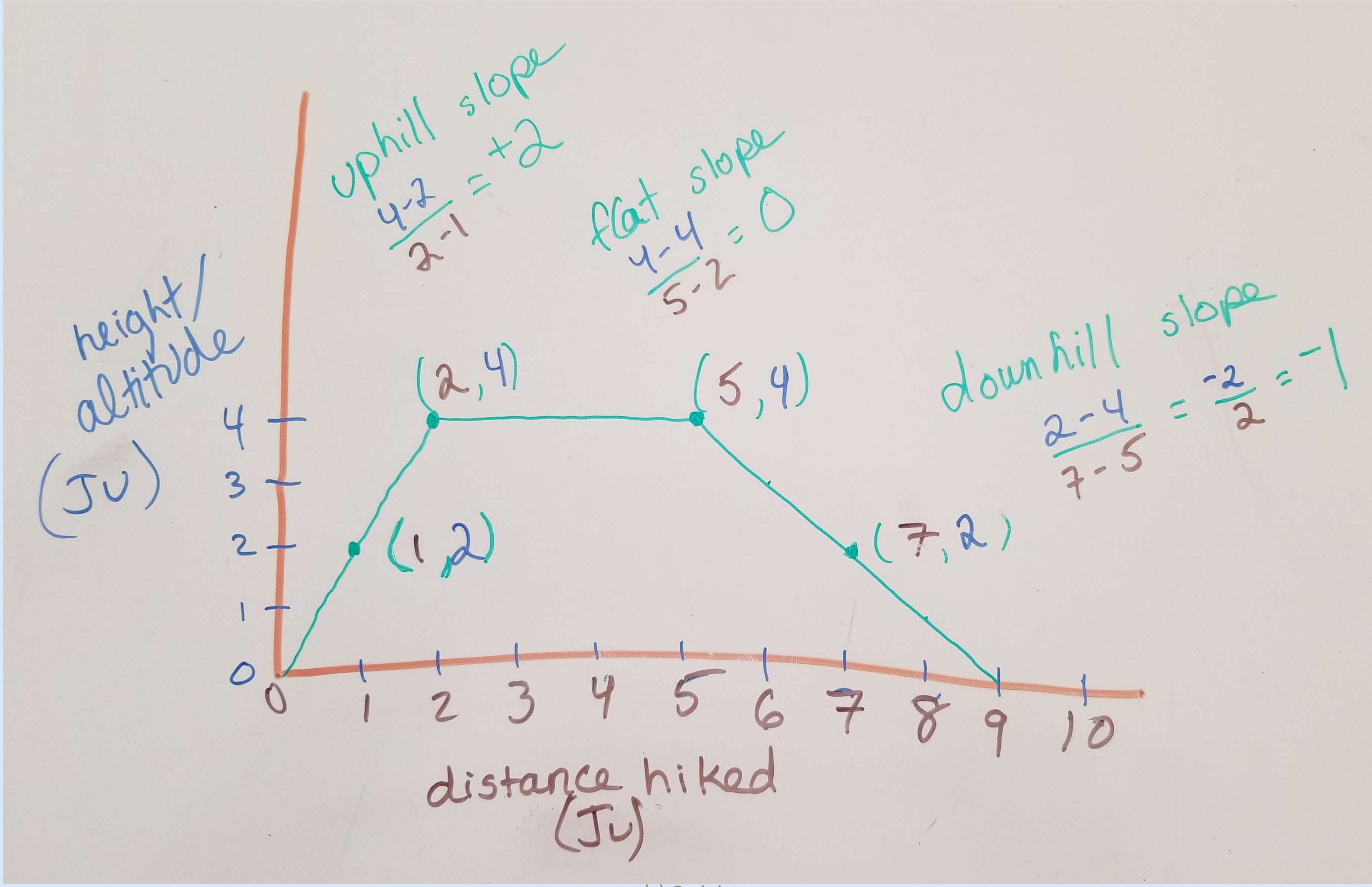
p = rate t + starting position

position = rate · time + starting position
velocity

$p = vt + p_0$



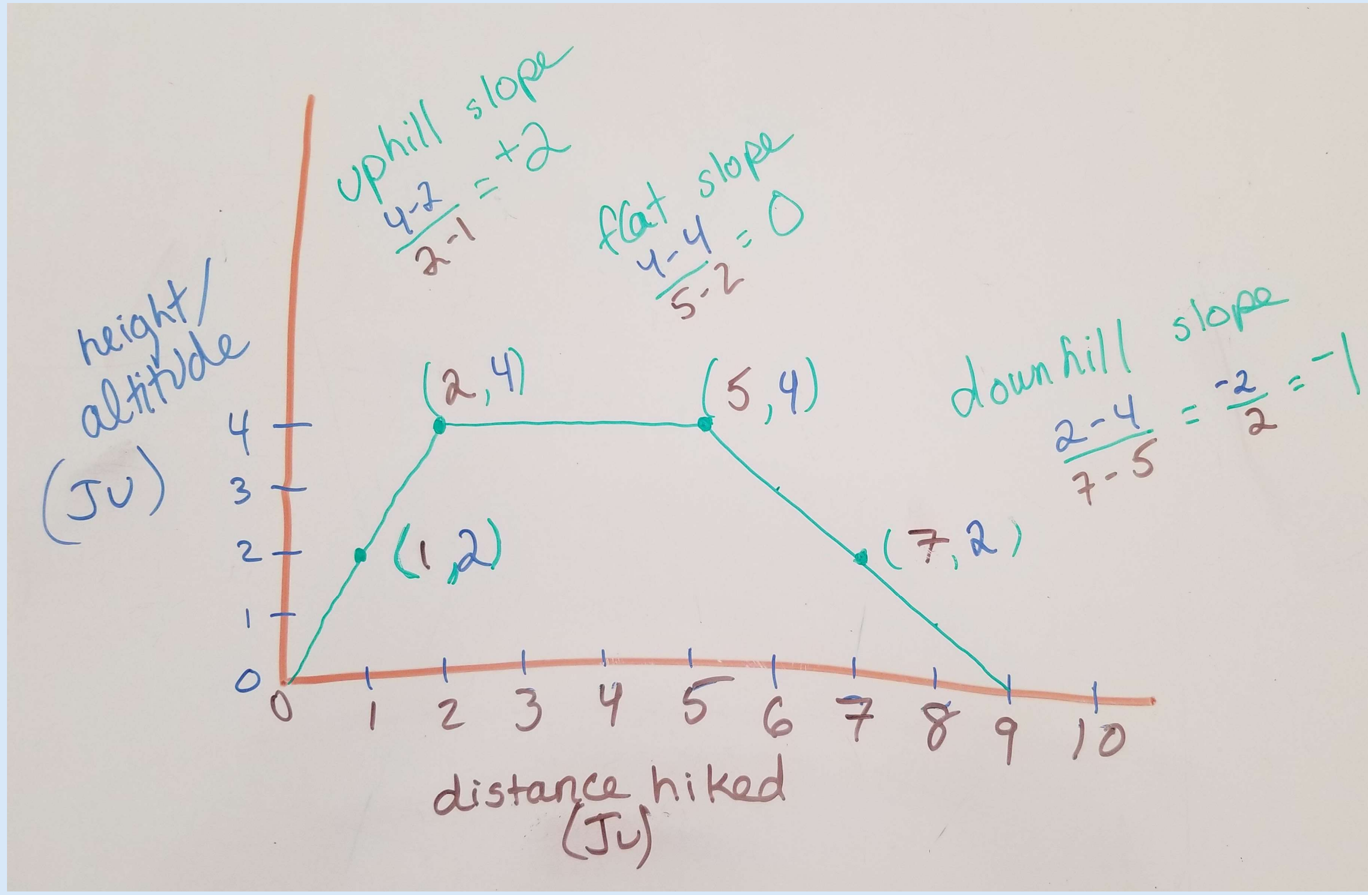
Hiking with height or altitude vs distance hiked in Jae units



Start with a slope of positive 2 so going up twice as much as hiking.

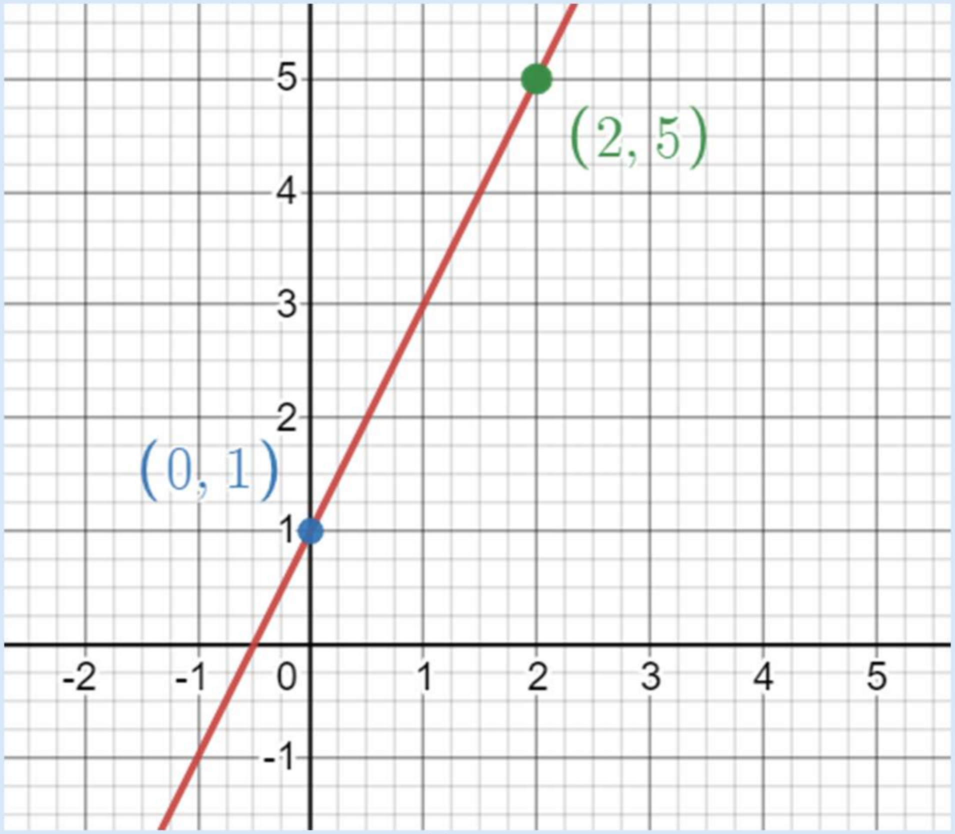
Then going with a slope of 0 along a flat ridge so still hiking but not going up or down.

Then coming down a shallower grade with a slope of -1



Example f(x)=2x+1=y (2,5) and (0,1) are points on the line. The rate or slope is 2

$y = 2x + 1$



Knowing the slope and the y-intercept, you can write the equation of any line.

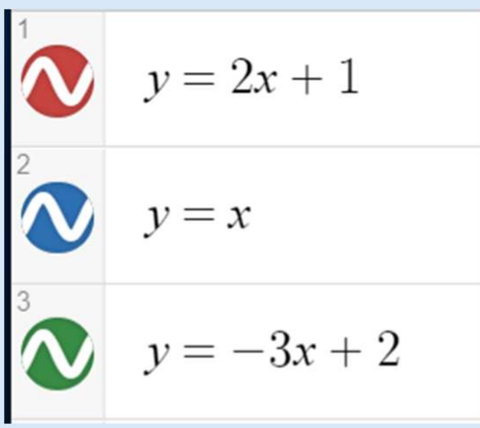
Any y coordinate on the line will equal the slope times the corresponding x coordinate plus the y-intercept.

$y=ax+b$ a is rate or slope or multiplier or coefficient

b is the y-intercept, where the line crosses the y axis, the constant term or starting output

You can also read the slope and the y-intercept from this form.

$y=1x+0$



Write the equation

Slope	y-intercept
-3	7
a	5
4	b
-2.1	45.3
2	-1

Slope intercept form

$y=ax+b$

Point slope form

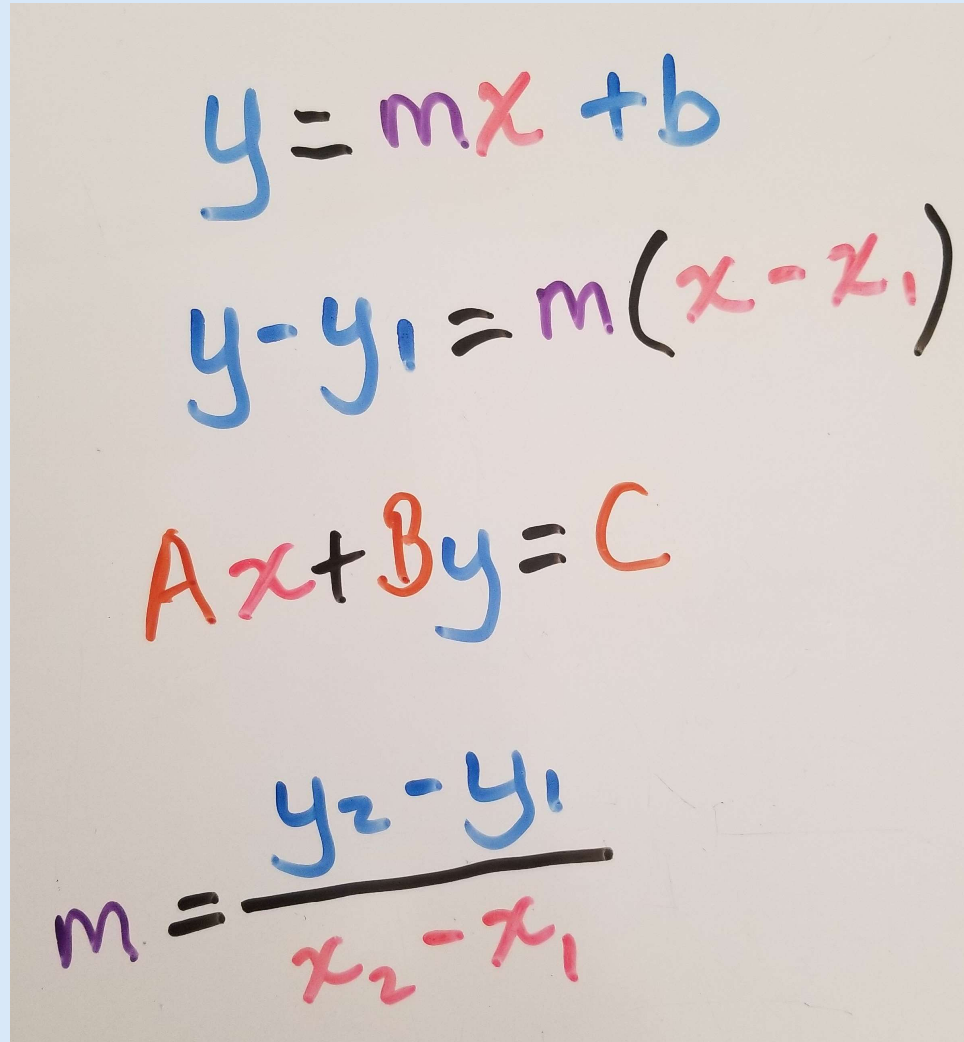
$y-y1=a(x-x1)$

Standard form

$Ax+By=C$

Slope formula

$a= (y-y1)/(x-x1)$



Ax+By=C is called the standard form of a linear equation. Notice it is not in function format. Standardized tests love making students convert and it is easy to write a program to do the conversion.

This is an equation in two variables and is not in function form, even though it is a function. All linear equations are functions because there is a predictable output for every input.

This AxBy=C form is useful for quickly figuring out the x and y intercepts. It comes up a lot in finance and other fields so is useful to be able to convert to function or slope intercept form.

Ax+By=C

This AxBy=C form is useful for quickly figuring out the x and y intercepts.

$x=0$

$$Ax+By=C$$
$$By=C$$
$$y=\frac{C}{B}$$

$y=0$

$$Ax=C$$
$$x=\frac{C}{A}$$

Ax+By=C

This AxBy=C form is useful for quickly figuring out the x and y intercepts.

$$2x+3y=6$$

$$(0,\ \ )$$

$$3y=6$$

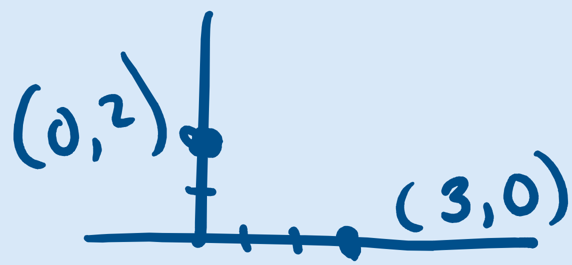
$$y=\frac{6}{3}=2$$

$$2x+3y=6$$

$$(\ \ ,0)$$

$$2x=6$$

$$x=\frac{6}{2}=3$$

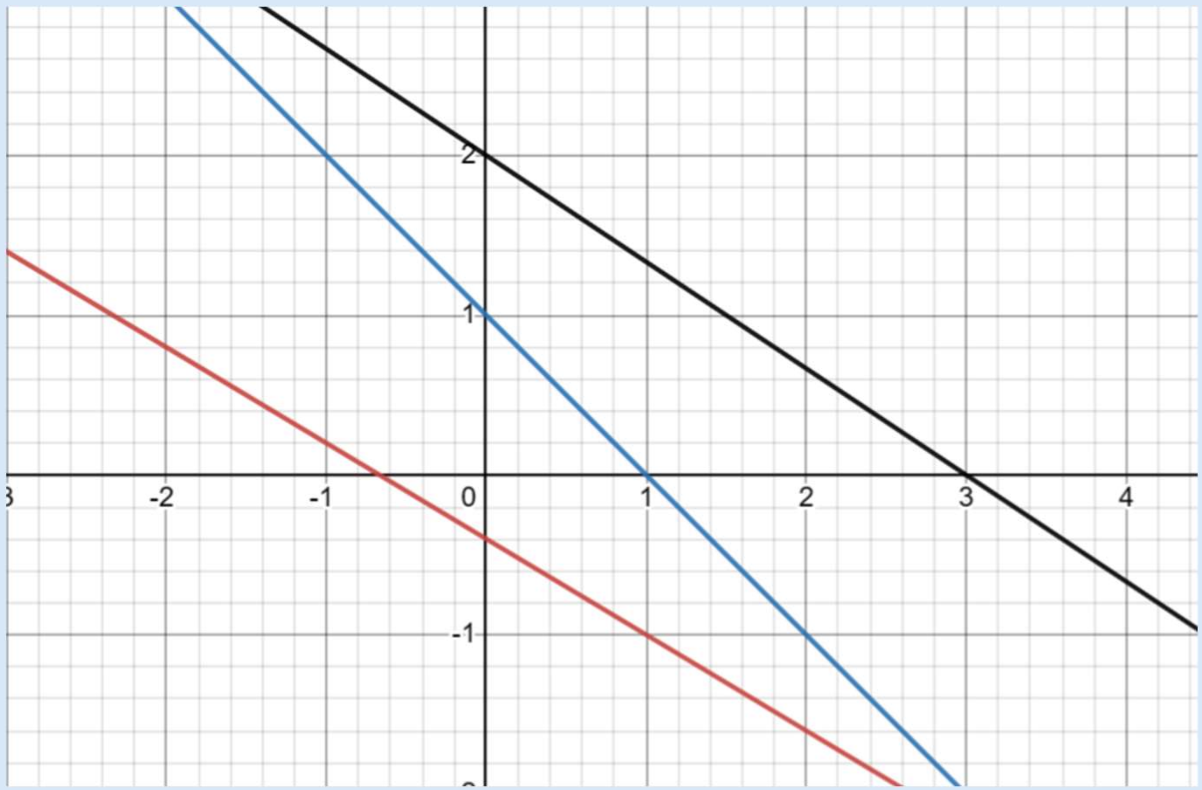


Ax+By=C

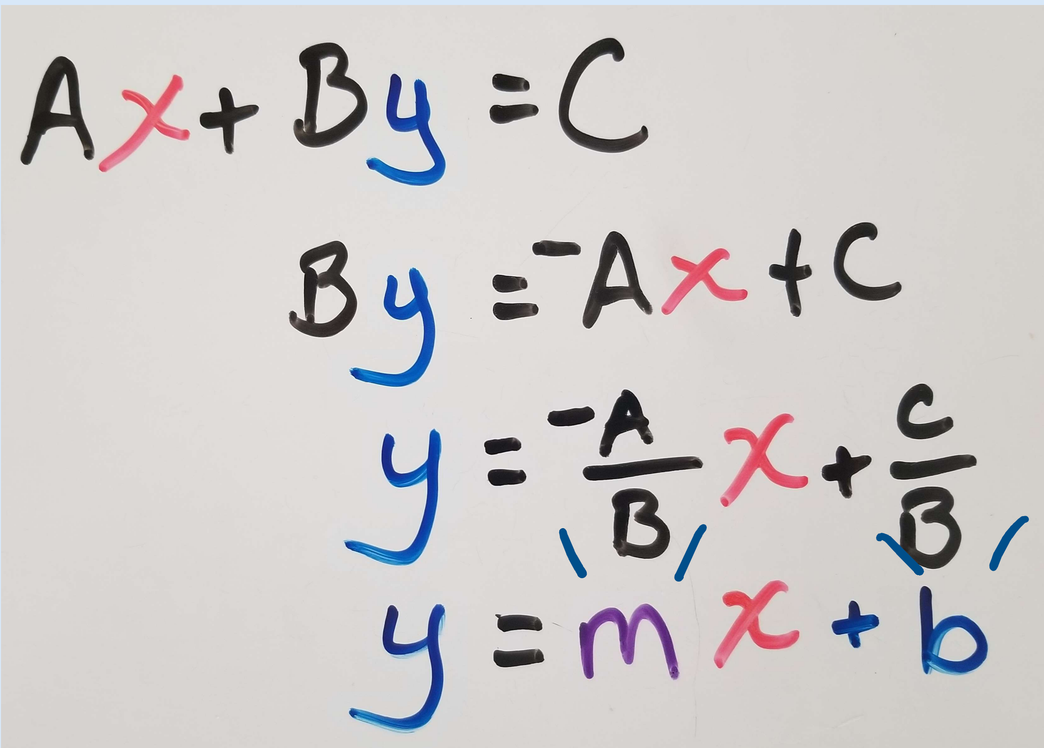
Notice that when A and B are the same sign, the slope is negative.

Can you match the equations to the lines?

1. $-3x - 5y = 2$
2. $x + y = 1$
3. $2x + 3y = 6$



Slope intercept form from Standard form of a linear equation.



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Want **y=mx+b**

Ax+By=C

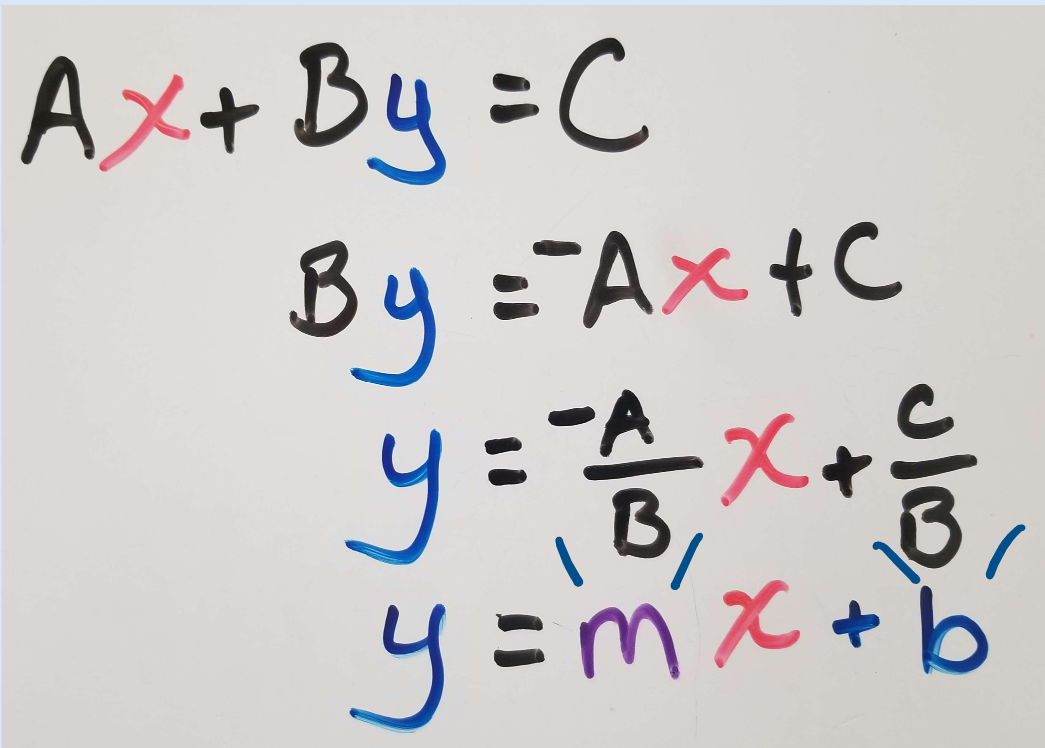
By=C-Ax

By= -Ax+C

$$y=\frac{-Ax+C}{B} \qquad y=\frac{-A}{B}x+\frac{C}{B}$$

The slope m is $\frac{-A}{B}$ and b is $\frac{C}{B}$

Slope intercept form from Standard form of a linear equation.



$$Ax + By = C$$

$$m = \frac{-A}{B}$$

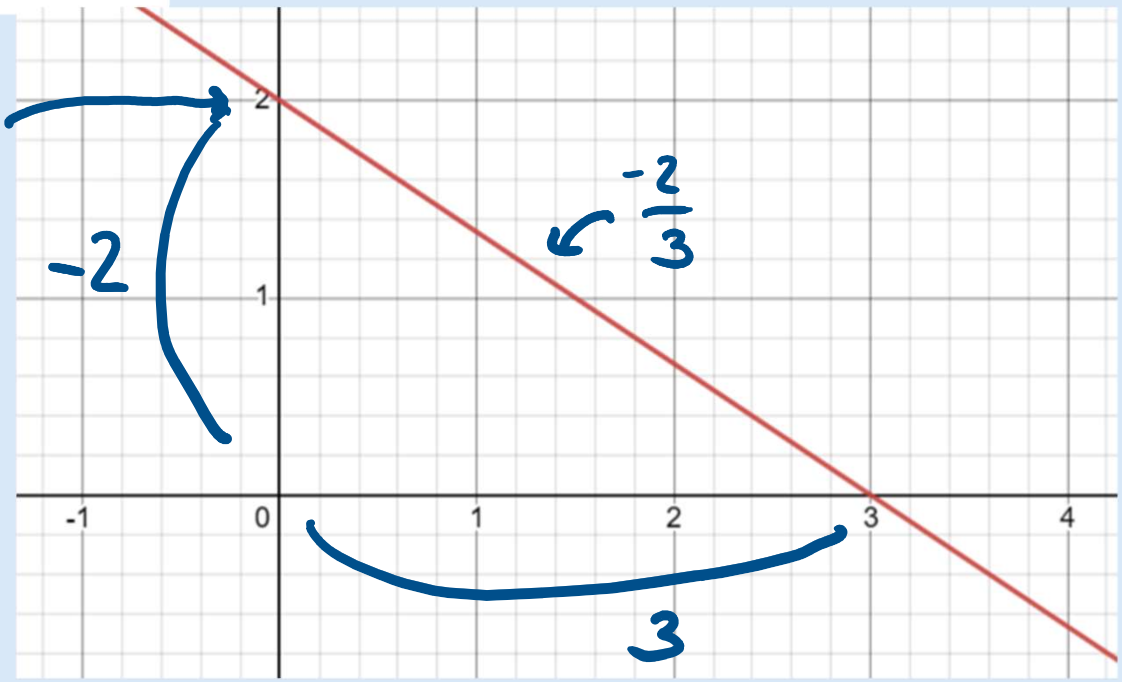
$$b = \frac{C}{B}$$

$$y = \frac{-A}{B} x + \frac{C}{B}$$

Slope intercept form from Standard form of a linear equation. You can program a computer to do the conversion for you. Check out how Desmos does it for you.

$2x + 3y = 6$

$y = \frac{-2}{3}x + 2$



Slope intercept form

$$y = mx + b$$

Point slope form

$$y - y_1 = m(x - x_1)$$

Standard form

$$Ax + By = C$$

Slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$