Functions

We have been looking at operations on a number where we have an input number, do the operation and get an output number.

input x $\xrightarrow{\text{operation}}$ output y

Now we are going to shift to look at the relationship between groups of inputs and groups of outputs.

x: 1 2 3

y: 2 4 6

A **relation** is when/how elements of one set (or group) relate to elements of another set (or group).

In algebra, we say 1+1=2 meaning the real number one, the operation of addition of real numbers, and that it equals 2. These have precise mathematical definitions.

In real life, we can say that one pile of leaves and one pile of leaves is one pile of leaves. (Example from Ellen Langer, Psychologist)

We want to be clear and precise in our definitions when we are doing algebra.

Functions [Function (mathematics) - Wikipedia]

A **function** is a relation where there is a **predictable** output for any given input.

For a function, every element of the first set relates to one specific element of the second set. This means that the output is predictable if you know the input and the function that you are doing.

$$x \xrightarrow{+2} y \qquad x \xrightarrow{*2} y$$

$$x \xrightarrow{-2} y \qquad x \xrightarrow{\cdot\frac{1}{2} \text{ or } /2} y$$

A **function** is a relation where there is a **predictable** output for any given input. If we know the input, we can figure out the output. This can be true for other types of data, including nominal data but we will be focusing on Real number inputs and Real number outputs for now.

$x \xrightarrow{+2} y$

$2 \xrightarrow{+2} 4$

$x \xrightarrow{*2} y$

$x \xrightarrow{-2} y$

$x \xrightarrow{\cdot\frac{1}{2} \text{ or } /2} y$

We can model and graph functions in various different ways.

We talked about modeling with a function machine or diagram where we have the input (usually x) and an output (usually y) and we do an operation or series of operations on the input to get a predictable output.

$$x \xrightarrow{+2} y \qquad\qquad x \to f(x) \to y$$

$$x \xrightarrow{-2} y \qquad\qquad x \; \overset{\longrightarrow}{\boxed{f(x)}} \; y$$

We call the function or formula being done to the input “f of x” and use a fancy f symbol to represent the function.

With typing, we just use the regular typed letter. F, G, and H are the most common letters used to name functions.

Warning! The parentheses here are not multiplication. They mean that you are doing the function or formula on the input variable that is x.

We can also write these as equations with the equal sign showing that both sides are equal to each other.

$x \to f(x) \to y$

$f(x) = y$

$x \; \boxed{f(x)} \; y$

Warning! The parentheses here are not multiplication. They mean that you are doing the function or formula on the input variable that is x (or t or a number). $f(x)$ function on input x

We will come back to function notation, but I use it so much that I wanted to show it first and then explain it more.

$x \to f(x) \to y$

$t \to f(t) \to y$

$1 \to f(1) \to y$

$f(x) = x + 2 = y$

$f(t) = t + 2 = y$

$f(1) = 1 + 2 = 3$

We can think about functions with different data types or with numbers. Here we are talking about an input set or group and an output set or group.

There are several ways of modeling functions:

Function diagram

Mapping diagram

Tables

Graph on a coordinate axis

Algebraic formula

Use function notation

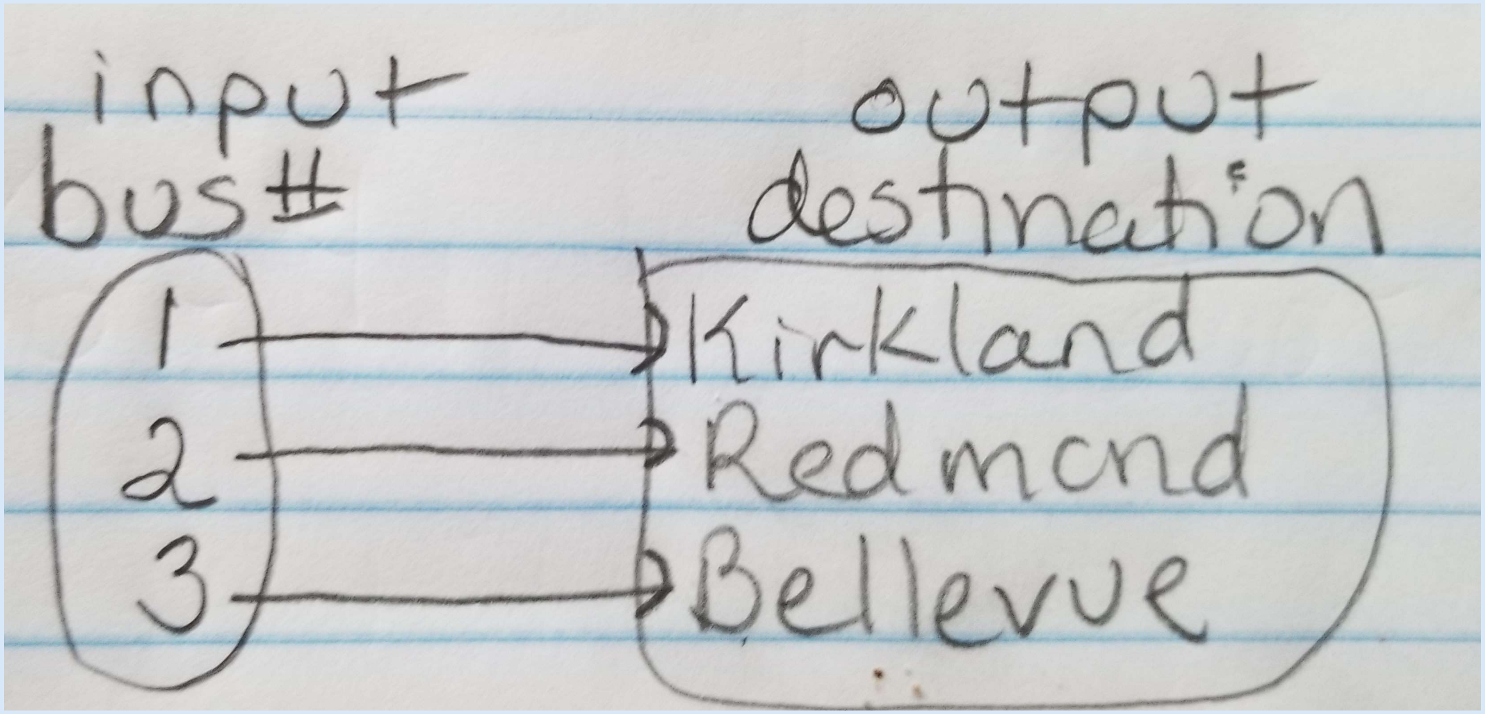
Function diagram

$$x \xrightarrow{f(x)} y$$

function

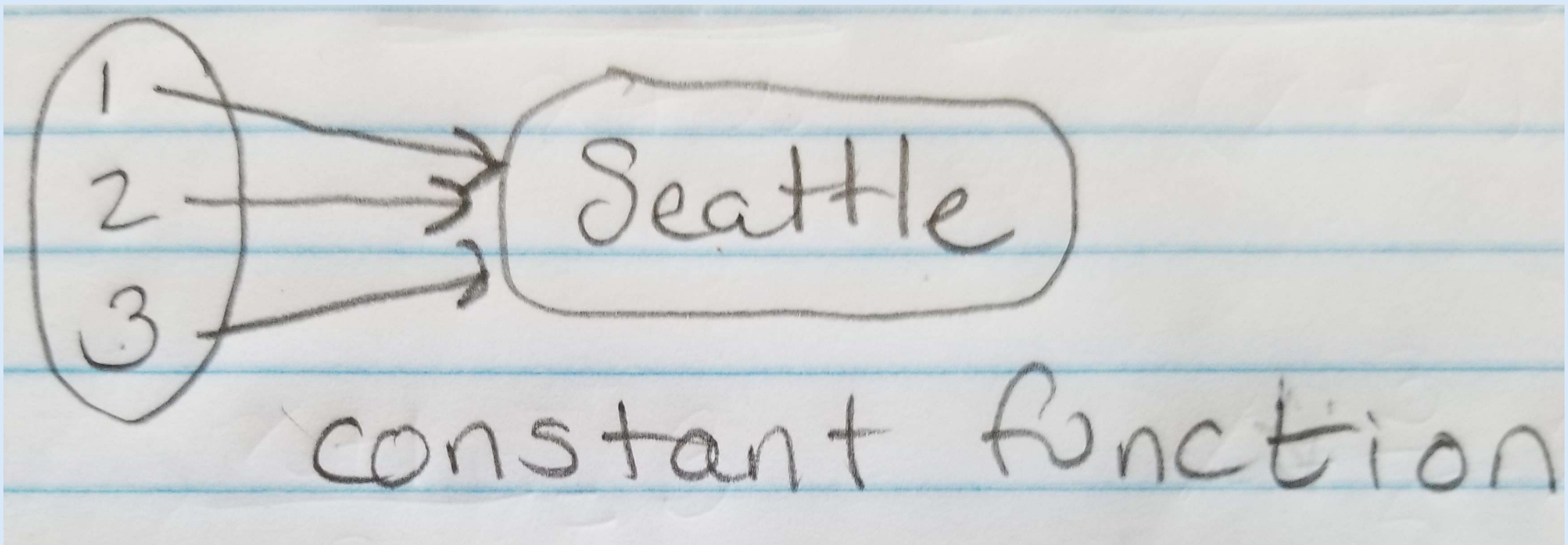
input $\longrightarrow$ $+2$ $\longrightarrow$ output

Mapping diagram

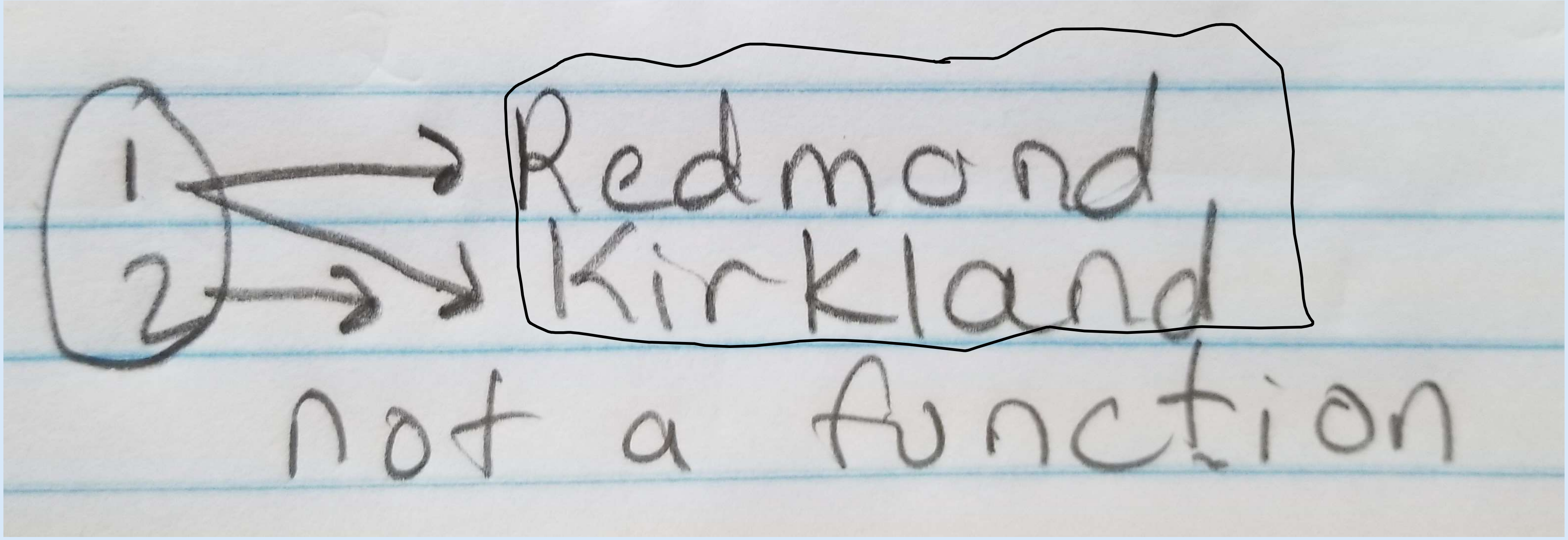


This is a one-to-one function.

Constant function: all inputs give the same output



Not a function! Do you see why not?



Mapping

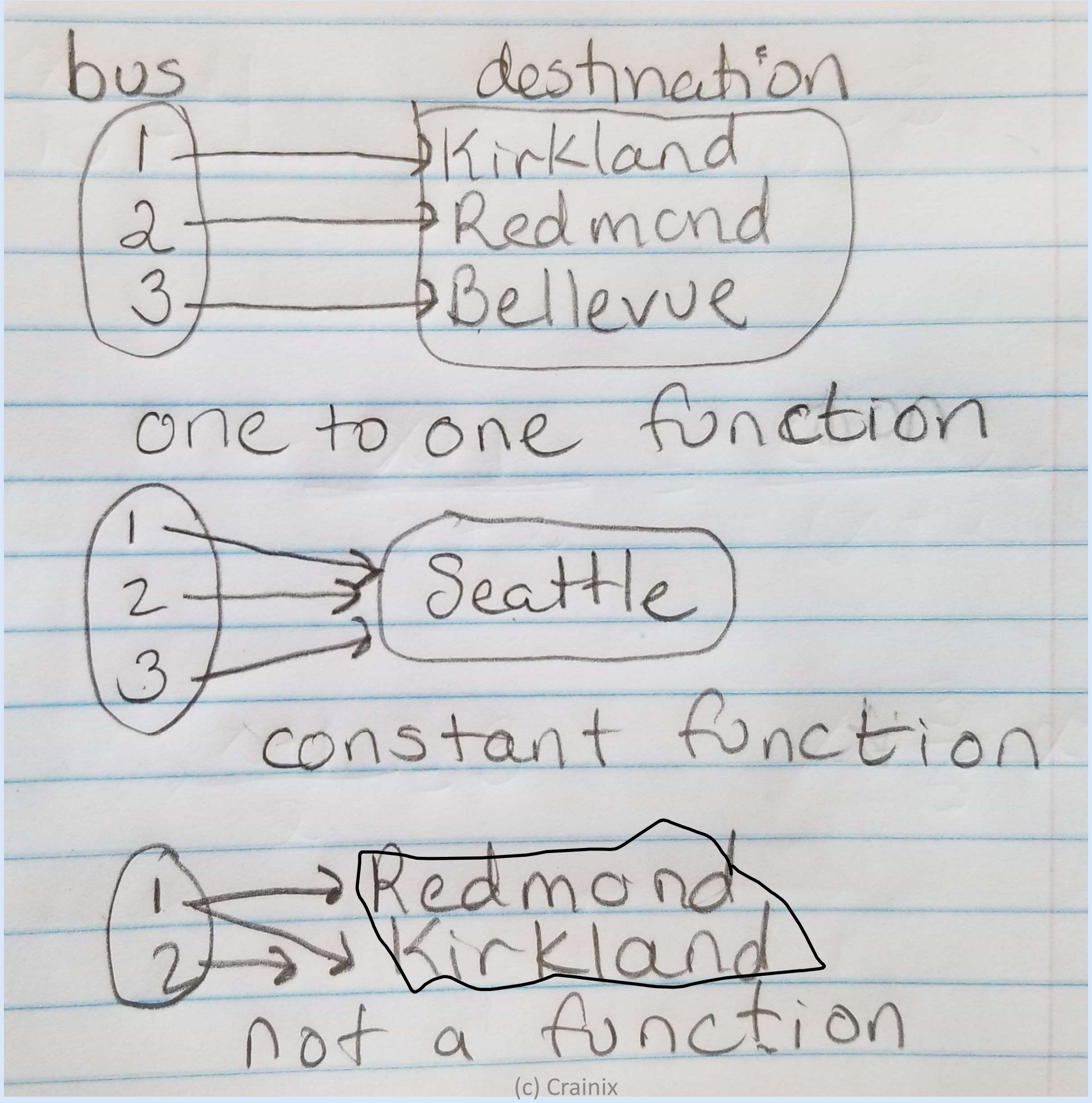

bus
destination
1
2
3
Kirkland
Redmond
Bellevue
one to one function
1
2
3
Seattle
constant function
1
2
Redmond
Kirkland
not a function

Table: We can use these with lots of data or can use them to track a few key points.

input x	output y
-1	-2
0	0
1	2
2	4

	A	B	C	D	E
1	x	x+2	3x	3x+2	
2	-1	1	-3	-1	
3	0	2	0	2	
4	1	3	3	5	
5	2	4	6	8	
6					

Done in Microsoft Excel

We can write the information from tables as ordered pairs with an input and an output. We use parentheses for an ordered pair and the order matters!
We always put the x or the input first and the y or the output second.

input x	output y
-1	-2
0	0
1	2
2	4

(x, y)

$(-1, -2)$

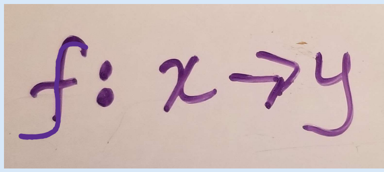
$(0, 0)$

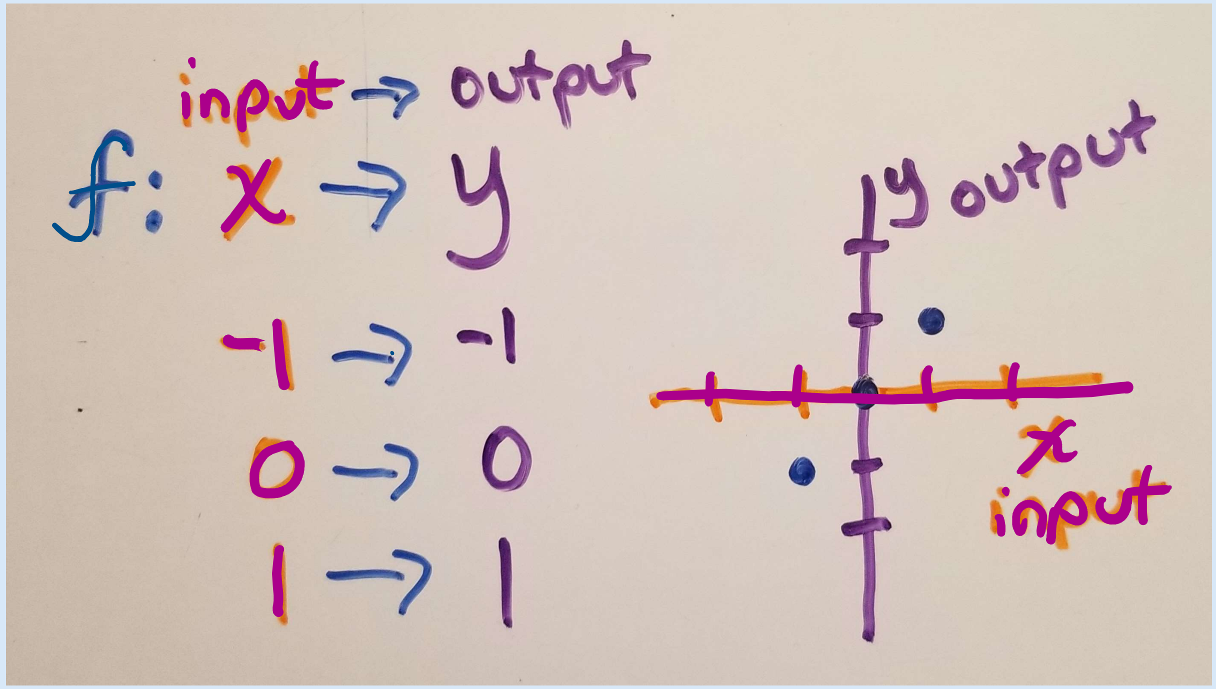
$(1, 2)$

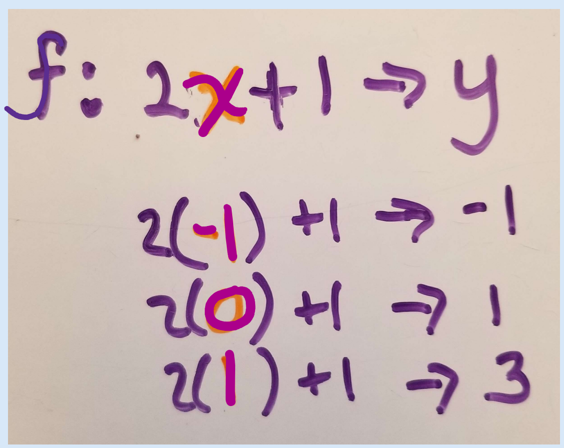
$(2, 4)$

(input, output)

Function mapping







Some examples with numbers.

$x \quad y$

$1 \rightarrow 4$

$2 \rightarrow 6$

$3 \rightarrow 8$

x	y
1	4
2	6
3	8

$(1,4)\ (2,6)\ (3,8)$

not a function

$1 \rightarrow 4$

$1 \rightarrow 6$

$1 \rightarrow 8$

$2 \rightarrow 6$

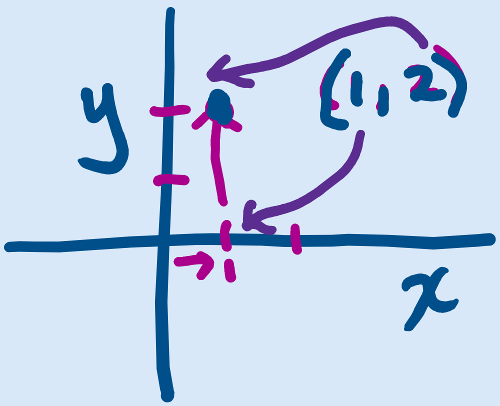
$3 \rightarrow 8$

x	y
1	4, 6, 8
2	6
3	8

$(1,4)\ (1,6)\ (1,8)$
$(2,6)\ (3,8)$

Ordered pairs represent points on a graph. (input, output) The input is always first and then the output is second. (x,y) are usually used with x as the input and y as the output.

(x, y)



$(x \quad , \quad y)$

(input , output)

$(\overset{+}{\text{across}}/\overset{-}{\text{back}}, \overset{+}{\text{up}}/\overset{-}{\text{down}})$

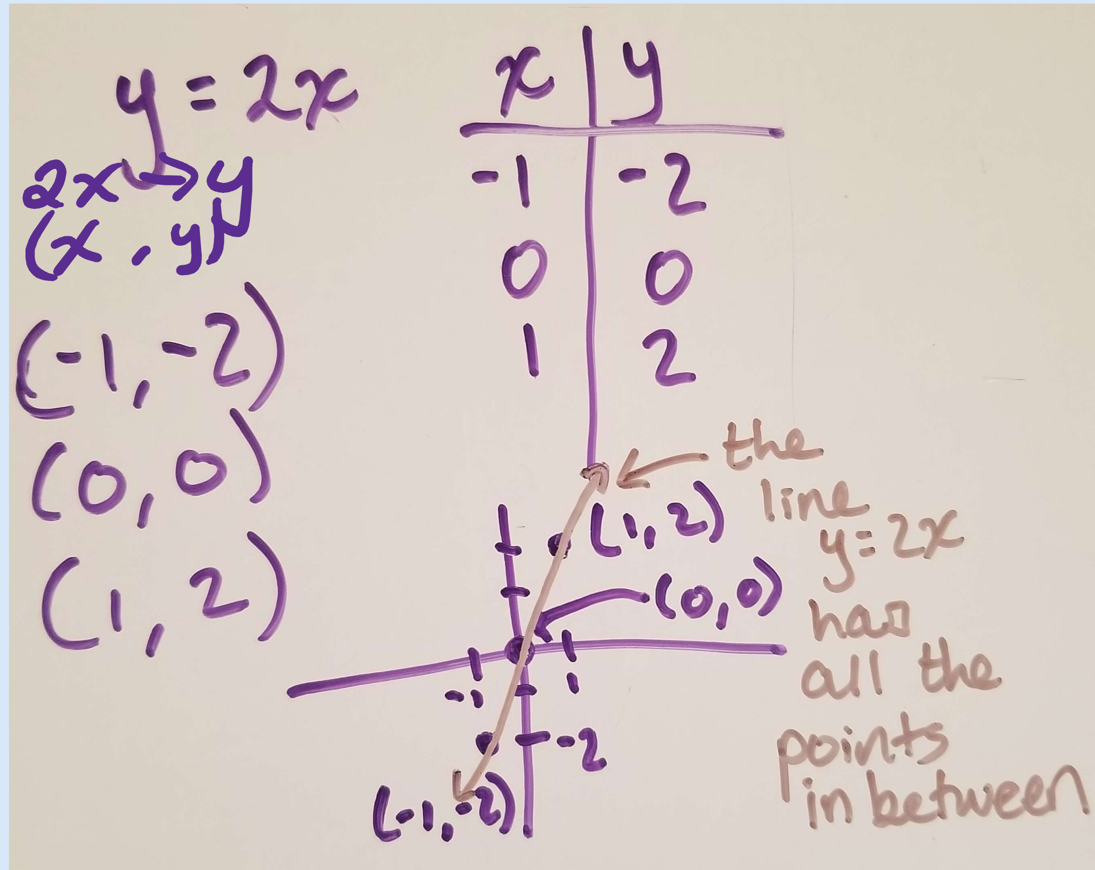
$({}^{+}R/L^{-}, +\text{up}/-\text{down})$

$(+1 \quad , +2)$

(R1 , up 2)

If we have the formula f(x)=2x then the line contains all the points that satisfy the formula.

$\{(x, y): y = 2x, x \in \mathbb{R}\}$



If we have the formula $f(x)=2x+1$ then the line contains all the points that satisfy the formula. Want to try with this or your own formula for a function?

Is an ordered pair, which is a point on a graph, the simplest function?

I asked Alan, the expert, and he pointed out (pun intended) that the empty set is really (another pun?) the simplest function.

$x \in$ domain $\{x_1, x_2, x_3 \ldots\}$ $y \in$ range $\{y_1, y_2, y_3 \ldots\}$

The domain is the fancy term for the set of all possible input values x and the range is the set of all possible output values. So, the empty set has no domain and no range. $\underset{\text{domain}}{x \in \emptyset}$ { } $\underset{\text{range}}{y \in \emptyset}$ { }

One point on a graph, which is one ordered pair, has one possible domain element and therefore one possible range element.

{(x,y) : x=1, y=2} has a domain of 1 and a range of 2.

Notice that we use set notation for the members or elements of the input and output sets.

$x \in$ domain $\{x_1, x_2, x_3 \ldots\}$ $y \in$ range $\{y_1, y_2, y_3 \ldots\}$

We use set notation for the domain and range.

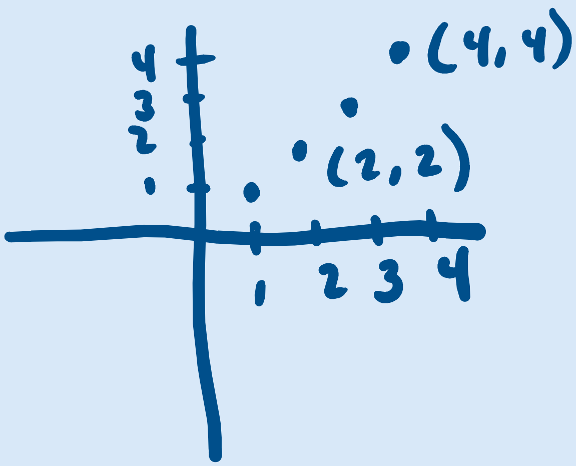
$\underbrace{x \in \emptyset \ \{\}}_{\text{domain}}$ $\underbrace{y \in \emptyset \ \{\}}_{\text{range}}$

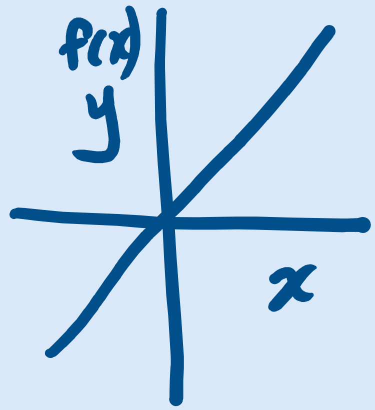
One point on a graph, which is one ordered pair, has one possible domain element and therefore one possible range element.

When the domain is a discontinuous set, like the set of integers, then we get a discontinuous function on the set of Real numbers.

Discontinuous means we pick up our writing utensil to draw the graph.

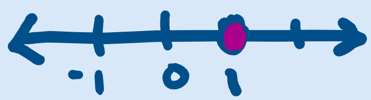
$$f(x) = x = y$$





One variable on a number line vs two variables on a coordinate axis.

With only one variable, there is just an input and no output so no relation and no function.



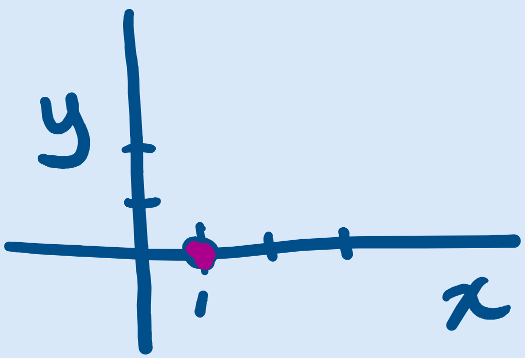
$x = 1$

x

1

no $f(x)$

no y



$(1, 0)$

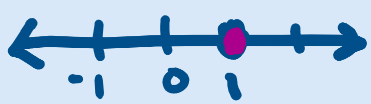
(x, y)

$x \quad y$

$1 \to 0$

x	y
1	0

With only one variable, there is just an input and no output so no relation and no function.

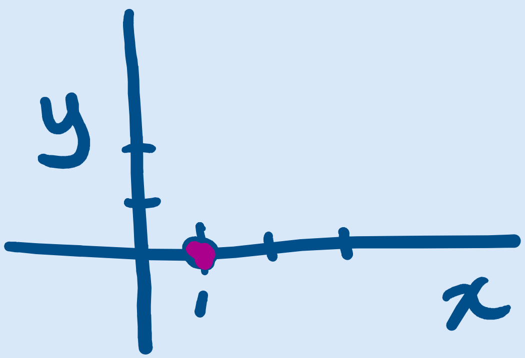


$x = 1$

x

1

no mapping
no table
no output
not a function



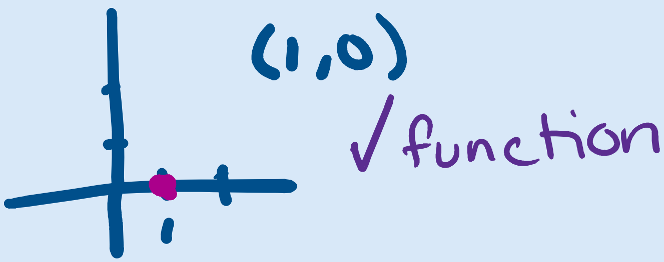
$(1, 0)$

$x \quad y$

$1 \to 0$

x	y
1	0

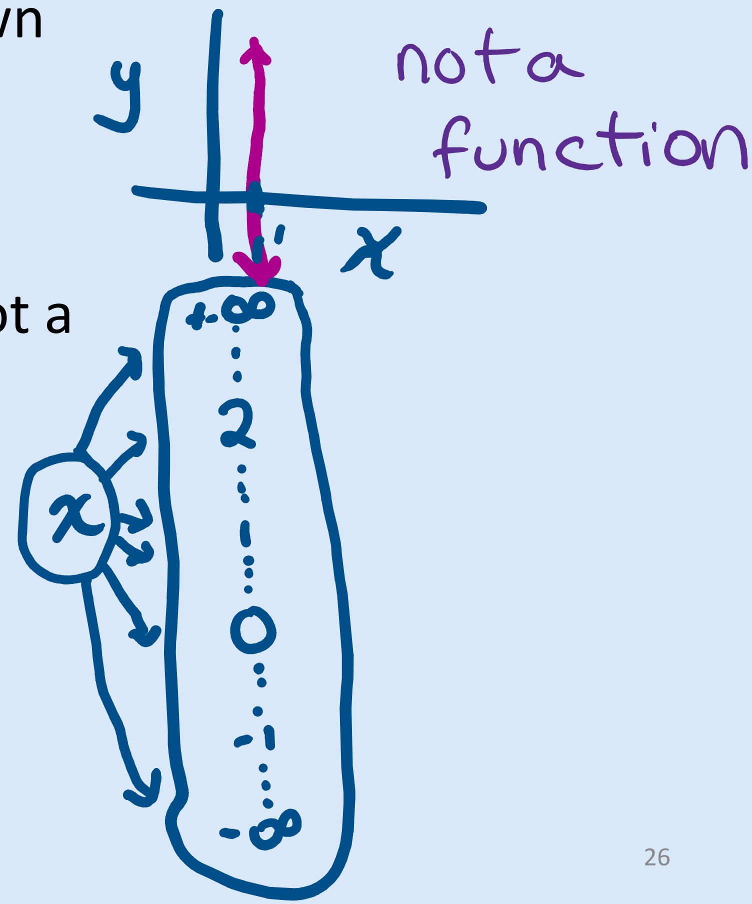
x=constant is not a function



If you go right one and don't go up or down, you know where you end up. This is a function.

x=1

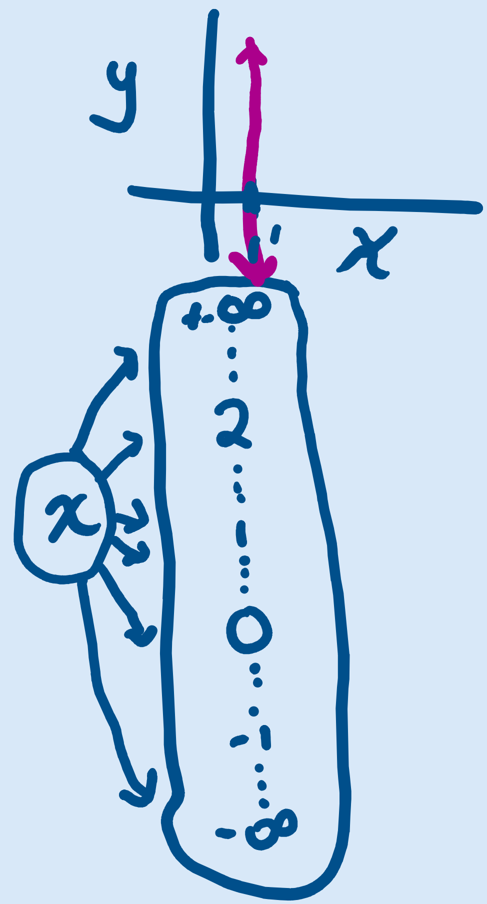
If you go right one and go up or down as much as you want, you don't know where you end up. This is not a function.



x=constant on a coordinate graph is a vertical line and not a function

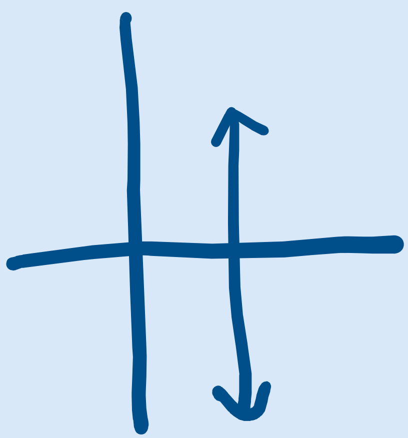
x=1

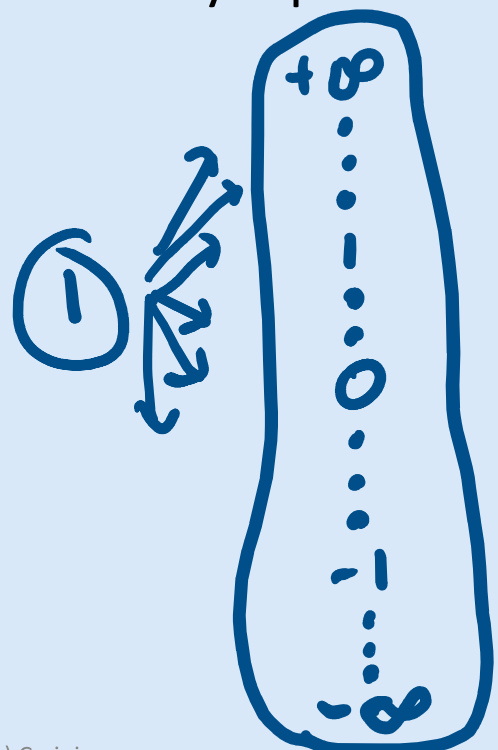
If you go right one and go up or down as much as you want, you don't know where you end up. This is not a function.



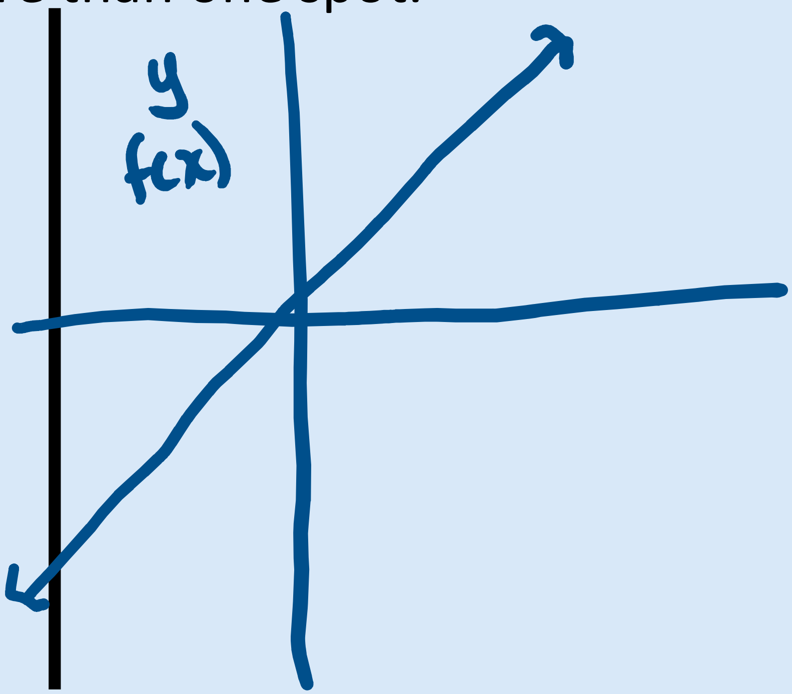
A vertical line is not a function

Because there are infinite outputs for every input.



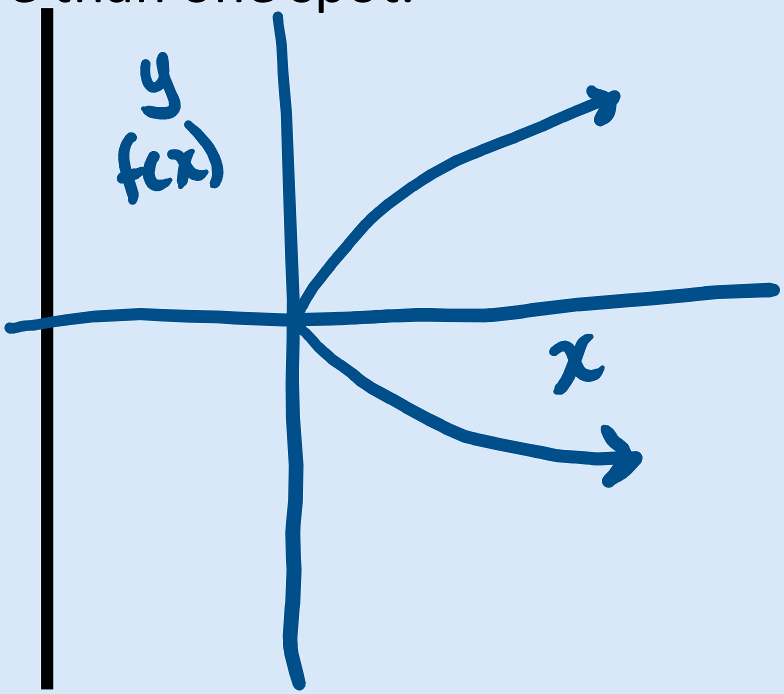


The vertical line test is a way of checking a graph to see if a vertical line crosses at more than one spot.



Here we are fine and don't double cross.

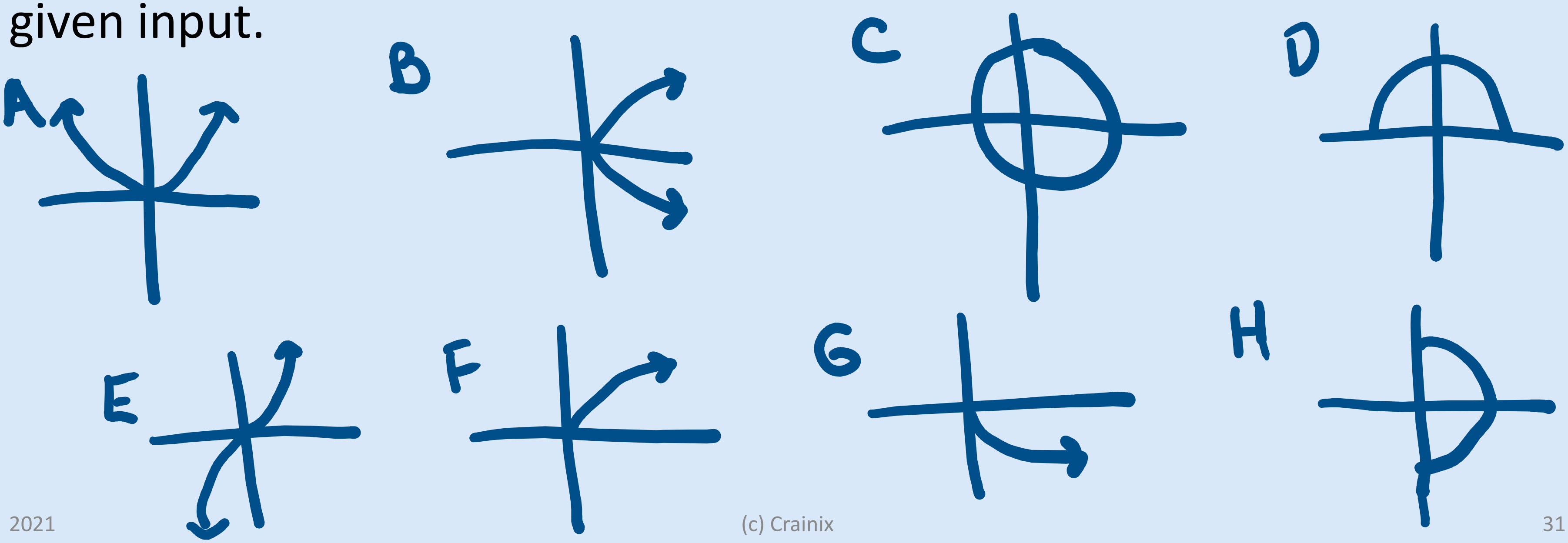
The vertical line test is a way of checking a graph to see if a vertical line crosses at more than one spot.



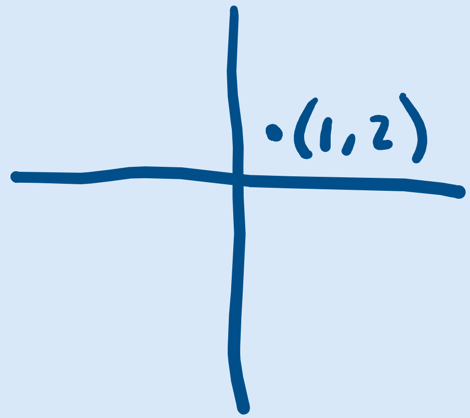
Here we don't cross, then we cross once at one point, then we double cross for the rest of the function. So this is not a function.

The vertical line test checks to see if a graph is a function

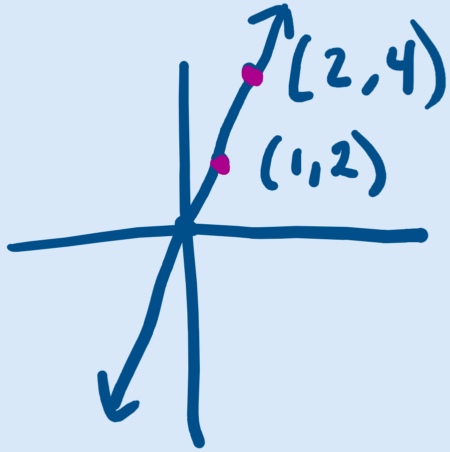
Run an imaginary vertical line, or use a pencil, and see if a curve or graph crosses the vertical line at more than one point. If it does, then it is not a function because there are more than one outputs for any given input.



If the function is just a point, then one ordered pair tells you what the function is.



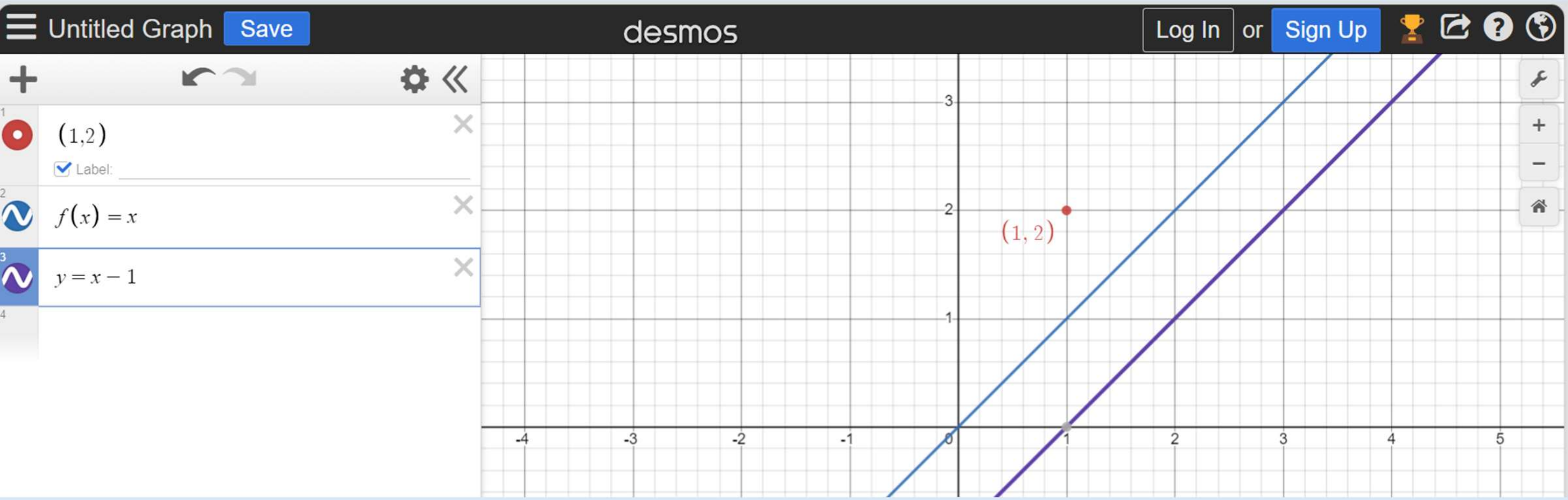
If the function is a line, then two points tell you what the function is.



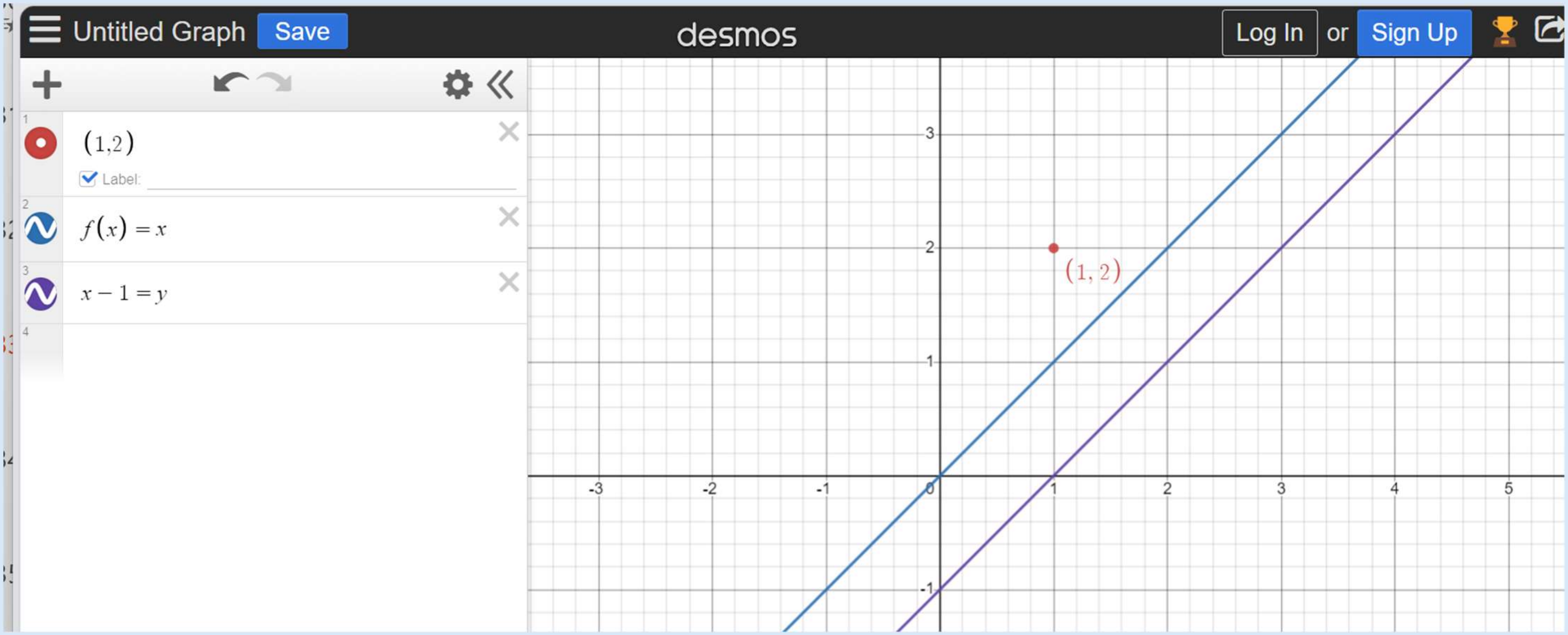
From geometry, we define a point as having location but no size. A line is a set of points that all fall along a straight path. Line (geometry) – Wikipedia

Any two points can define a line. All the points that fall along that line are called colinear.

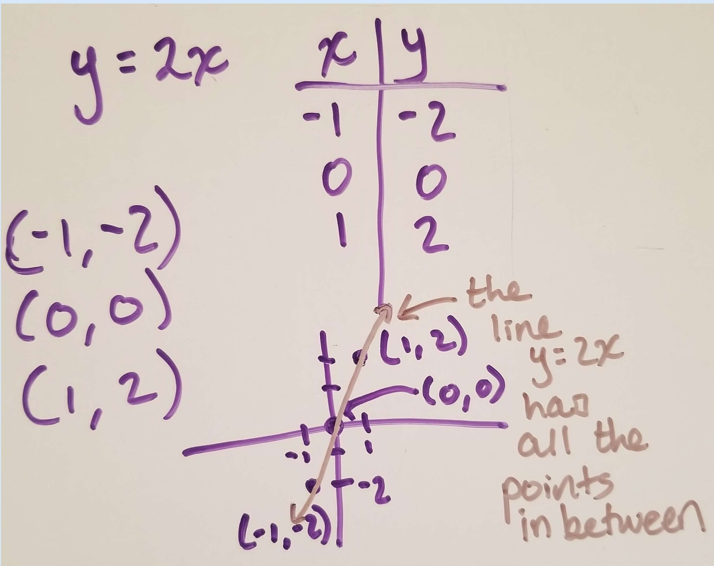
Desmos is great for graphing and all graphs that look like they are from Desmos are from Desmos | Graphing Calculator. Notice that you can use f(x) or y.

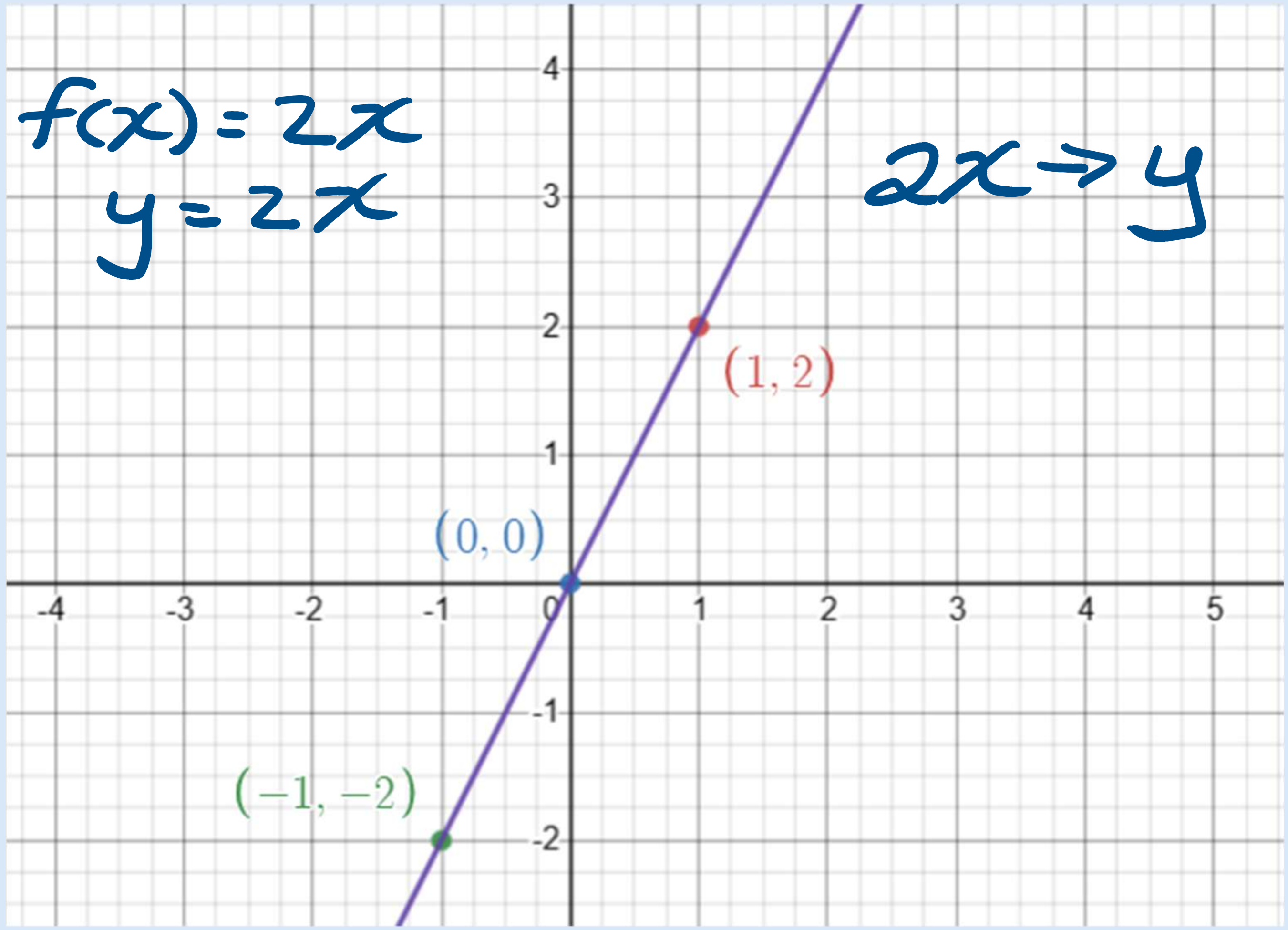


Because the x comes before the y in the ordered pair, it is easier for some of use who mix up right and left to think of f(x)=x=y. Desmos is ok with y=x-1 or x-1=y either order.



Colinear is when points are all on the same line.

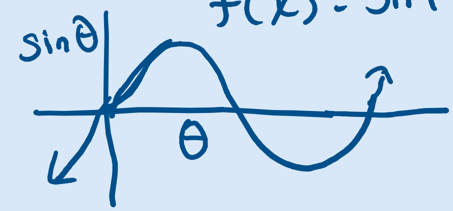


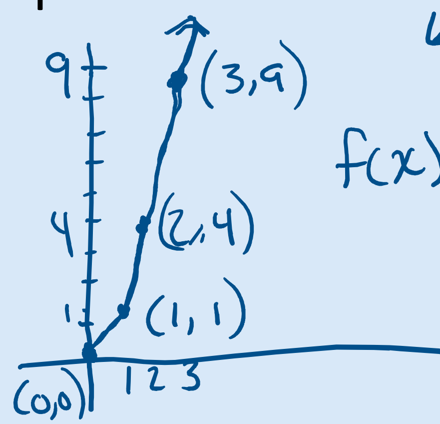


Beyond a line, if you have a curve, you need more than just a couple of points to figure out what the function is.

If you have a **formula**, then you can go back and put an input into the formula, to get the output for that particular input. Those pairs of inputs and outputs can be written as ordered pairs and are points on the graph of the function.

$f(x) = \sin\theta$

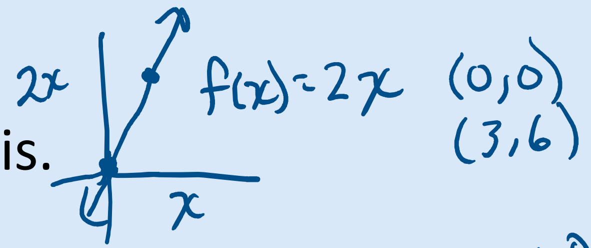




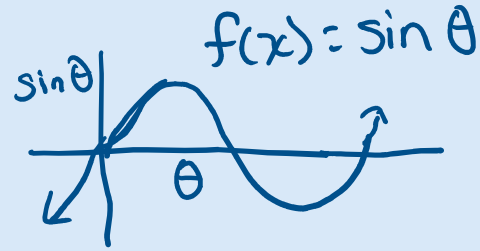
$f(x) = x^2$

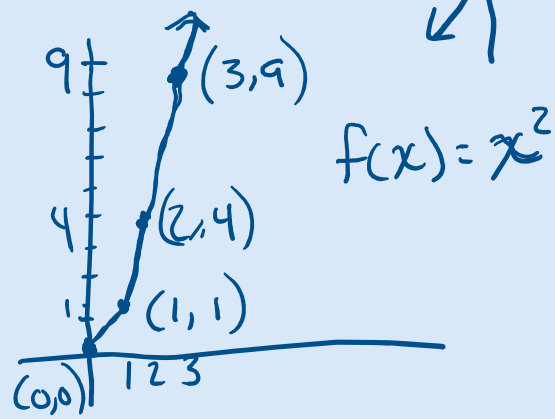
If the function is just a **point**, then one ordered pair tells you what the function is.

If the function is a **line**, then two points tell you what the function is.



Beyond a line, if you have a **curve**, you need more than just a couple of points to figure out what the function is.





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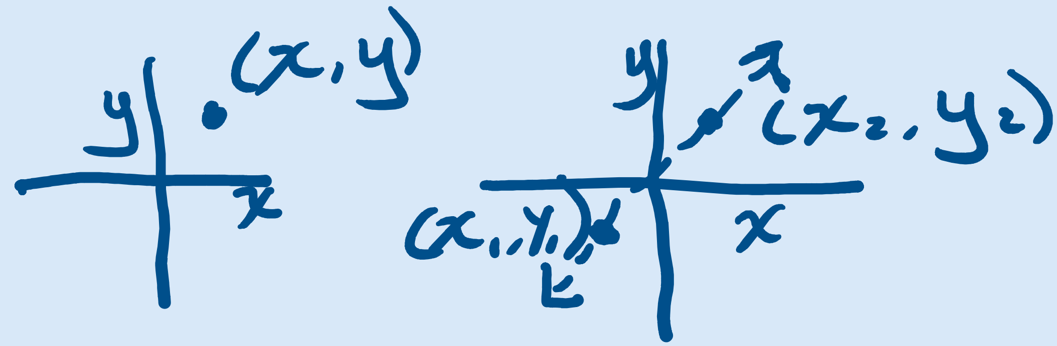
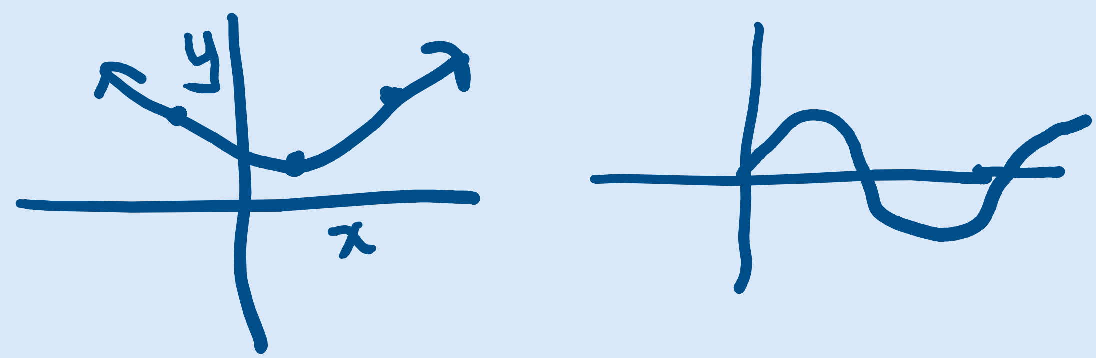




Function notation $f(x)$

We use function notation to indicate that it is a function:

$f(\text{input})=\text{output}$ we usually use x for the input and y for the output.

$f(x)=x+1$

$f(x)$

$g(x)=2x$

We use the function notation to indicate that it is a function.

When we are programming computers, manipulating functions, or doing calculus, we use the function notation.

Set builder notation

When we are solving equations, it is easier to use x and y.

Really, the full set notation is:

$\{(x,y): y=x\}$ or $\{(x,y): y=2x+3\}$

Read as: the set of all ordered pairs x and y such that y equals x or y equals 2 times x plus 3.

Also called curly bracket notation. Usually, it is implied that y=x means the full explanation. $\{(x,y): y=x\}$

$\{(x,f(x)), x \in R: f(x)=x\}$

Functions as equations in two variables

y=x+1 or f(x)=x+1

y=2x or f(x)=2x

Are just alternative ways of writing the same thing but f(x) means that it is a function.

What is the difference between a function and an equation in two variables? y^2= x (4, 2) and (4,-2)

An equation doesn't have to be a function. It is often easier to write functions in the two variable equation form when solving them.

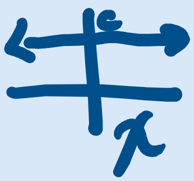
We study the behavior of functions.

Functions have patterns and so it is easier to study the behavior of functions by learning the patterns of the parent function and then seeing how they change when we do stuff to them. The parent function is the most basic function for that type of function.

Parent functions

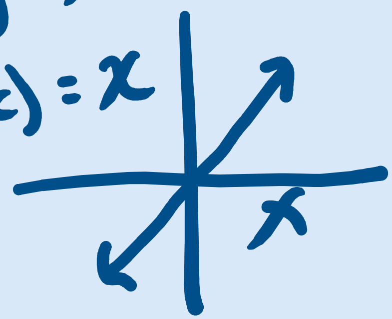
$y = c$

$f(x) = c$



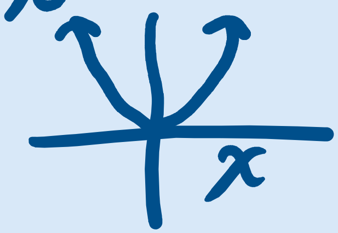
$y = x$

$f(x) = x$



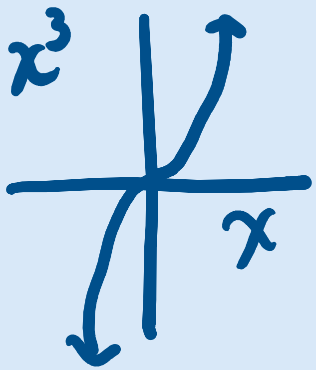
$y = x^2$

$f(x) = x^2$

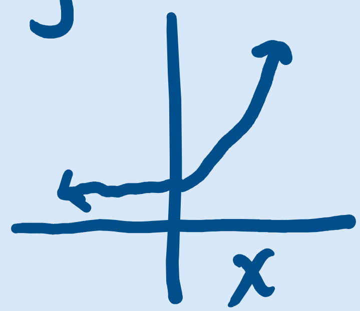


$y = x^3$

$f(x) = x^3$

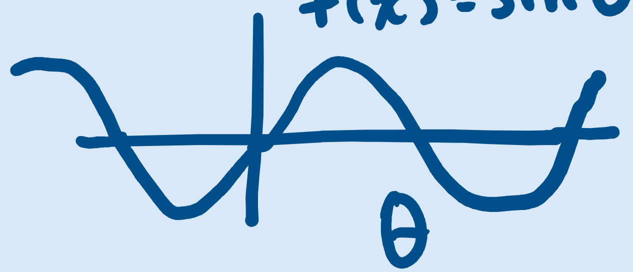


$y = e^x$

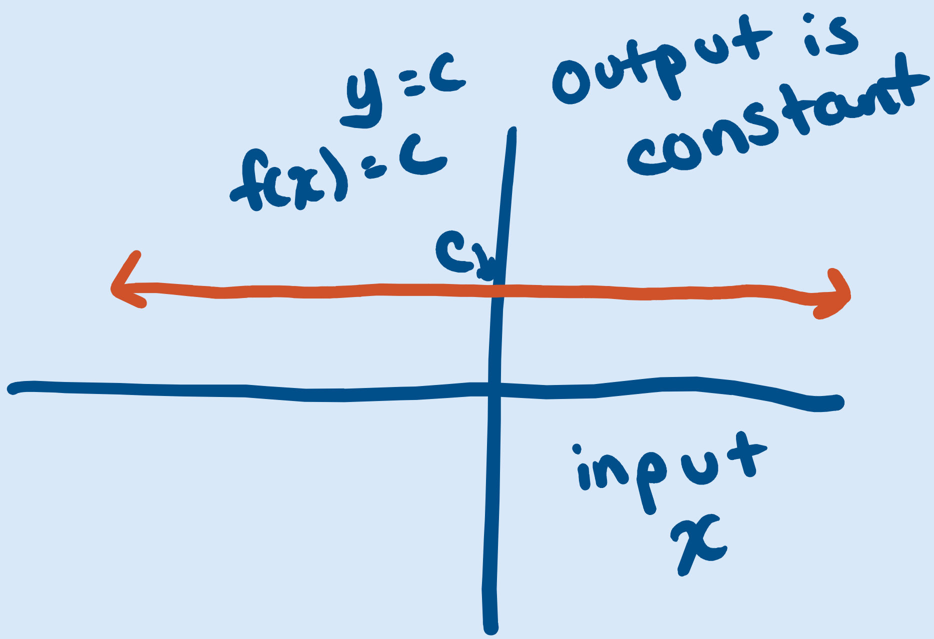


$y = \sin\theta$

$f(x) = \sin\theta$

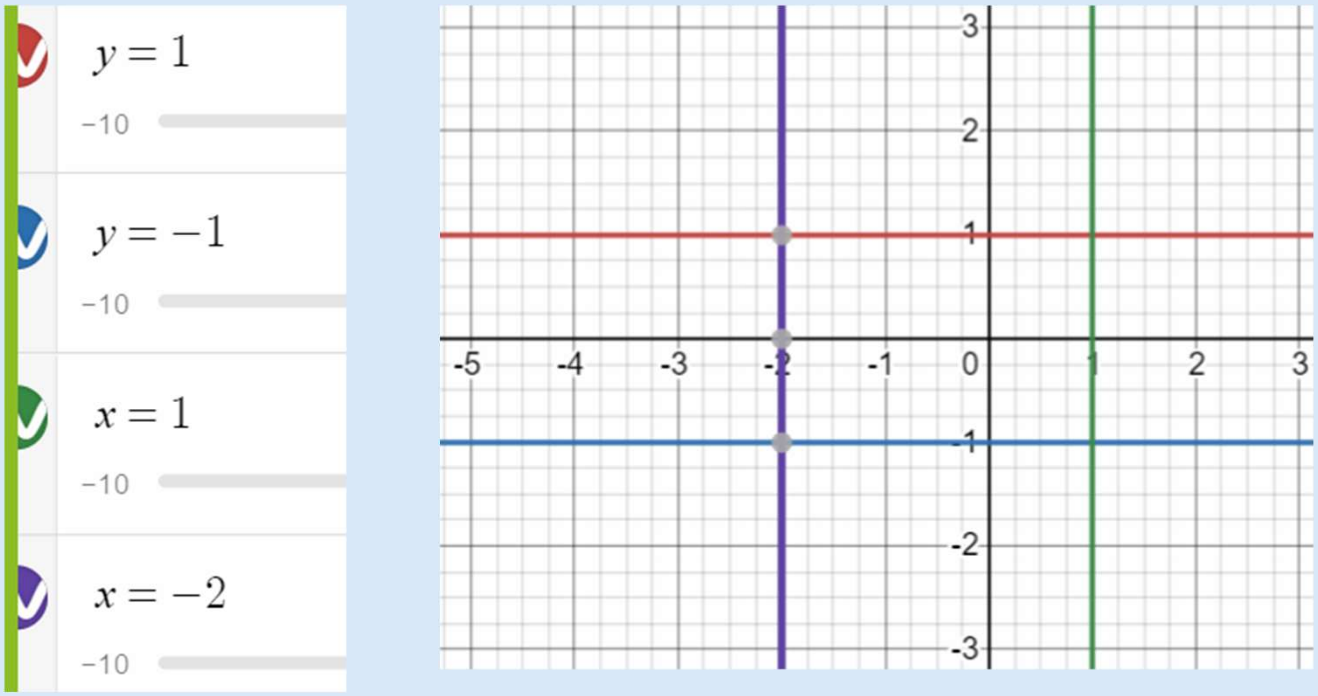


Graph of a constant function: no matter what x is, y or the output is always the same number which we are calling c.



Vertical lines are not functions and horizontal lines are constant functions.

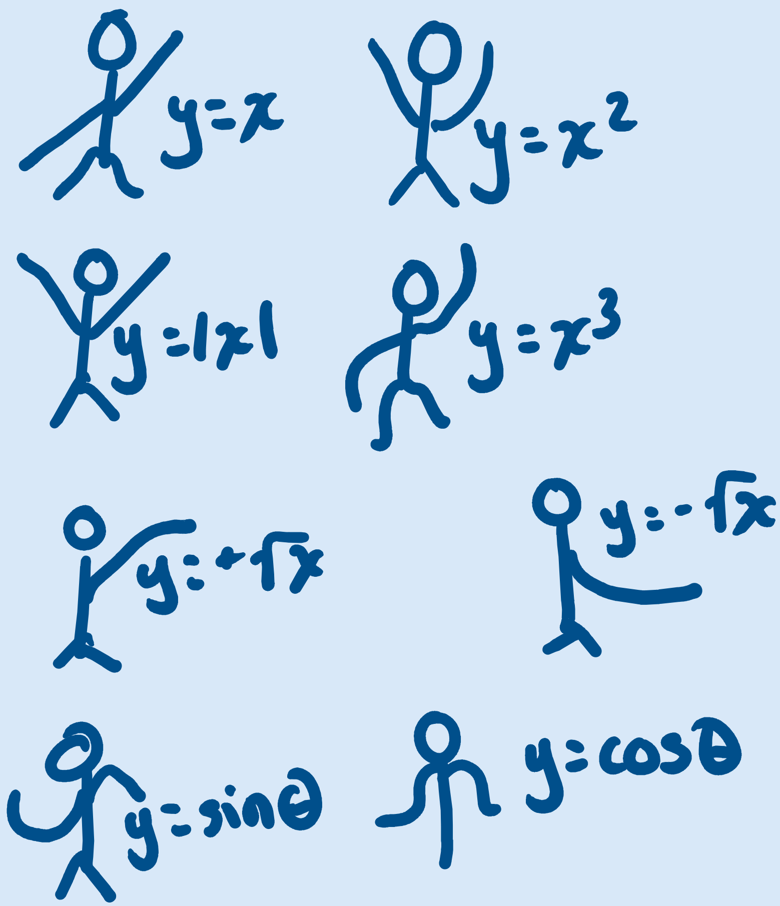
$x=1$

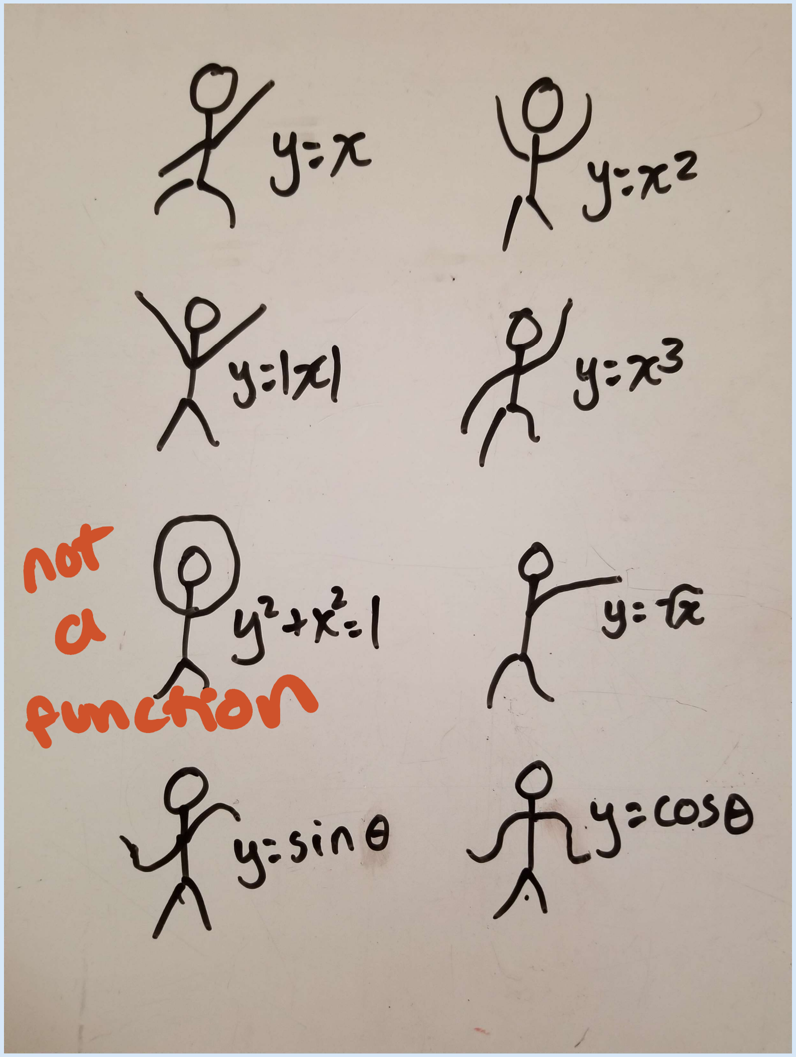


$y=1 \quad \{(x,y): y=1\}$

$y=0x+1$

Here we have the function movement dance.




y=x
y=x2
y=|x|
y=x3
not a function
y2+x2=1
y=√x
y=sinθ
y=cosθ