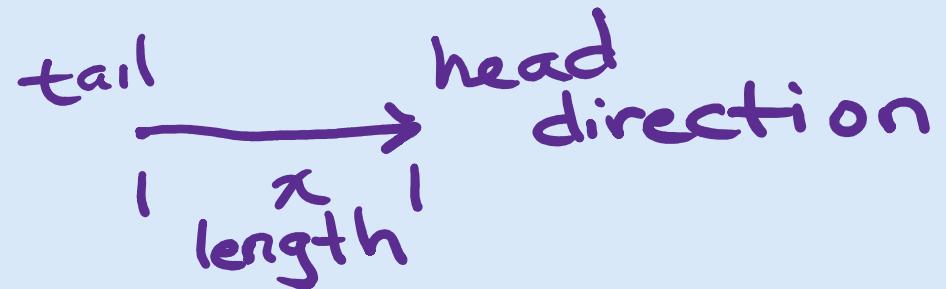


Vectors on a two-axis system

A Vector has **magnitude** or length
and **direction**

We can draw a vector as an arrow or ray with a starting point, an ending point, and use an arrowhead to show direction.

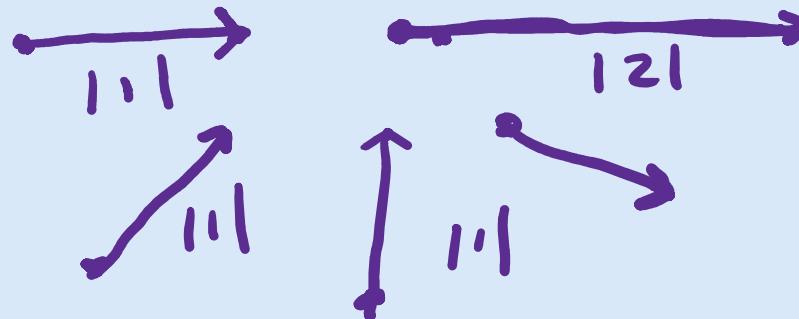


Starting with the basics: a Vector has **magnitude** or length and **direction**

We can draw a vector as an arrow or ray with a starting point, an ending point, and use an arrowhead to show direction.

A unit vector has a length of one unit $|1|$.

We use the $|x|$ symbol to represent the magnitude or length.



We can model all kinds of different concepts as vectors.

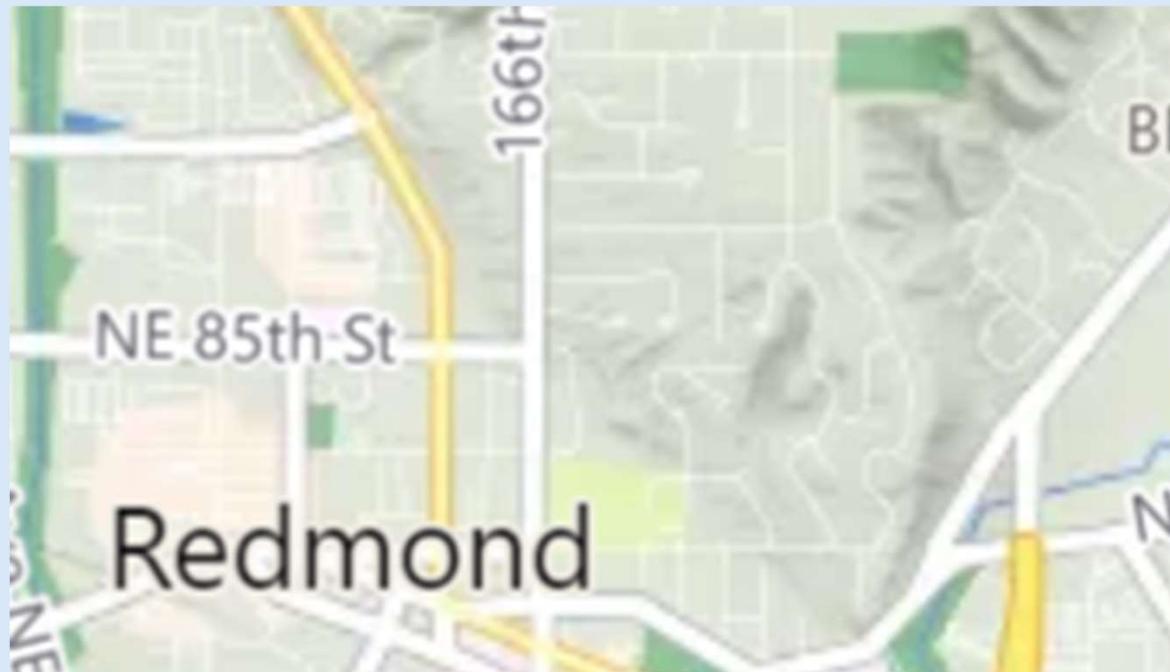
Let's start with looking at an object having a position or location on a two-dimensional plane (a flat surface). Then we can model the object traveling around on the plane.

Here is my object, Arnie, my frimily dog. My two-dimensional plane is the ground of our neighborhood. Our neighborhood isn't flat, but we are going to pretend it is for our modeling.



Our neighborhood isn't flat, but we are going to pretend it is for our modeling.

Dimension reduction is when we go from the three-dimensional space of our neighborhood and ignore the height dimension to reduce it to two-dimensions. [Dimensionality reduction - Wikipedia](#)



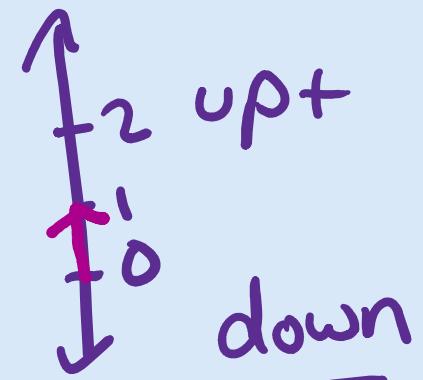
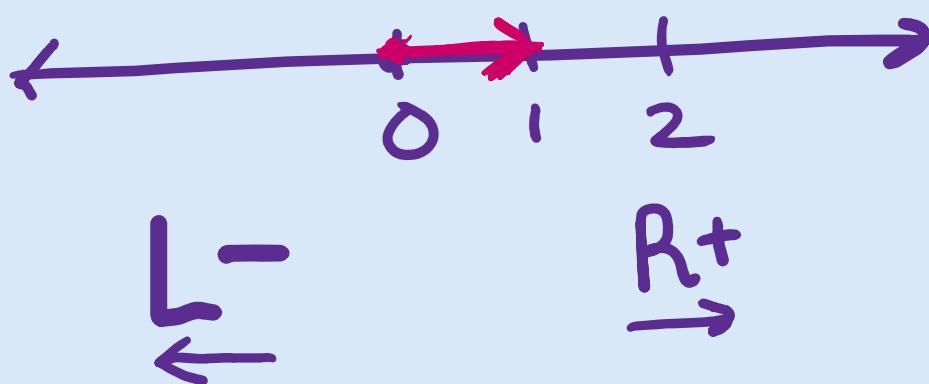
A Vector has **magnitude** or length and **direction**

A unit vector has a length of one unit $|1|$.

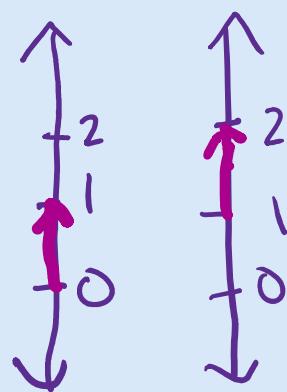
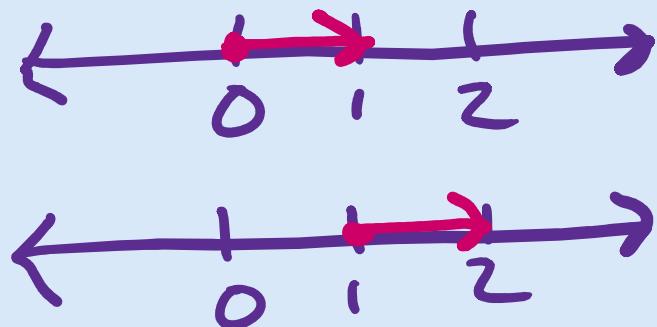
We can map or graph it in one dimension on a number line.

Horizontal number lines go positive to the right and negative to the left.

Vertical number lines go positive up and negative down.

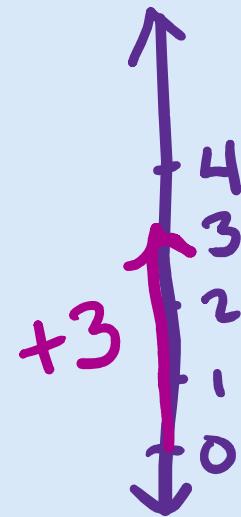
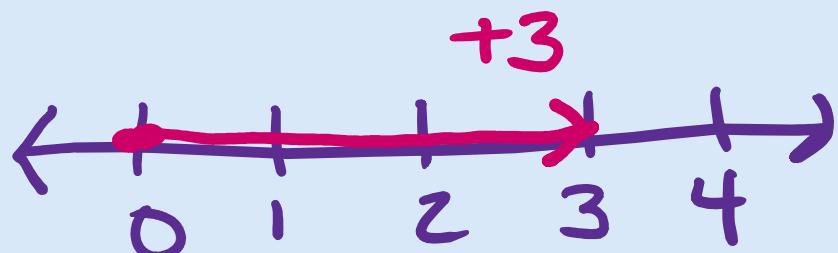


We can map or graph a vector in one dimension on a number line. We usually start at zero, but we could start at any location. For now, we will start from zero.



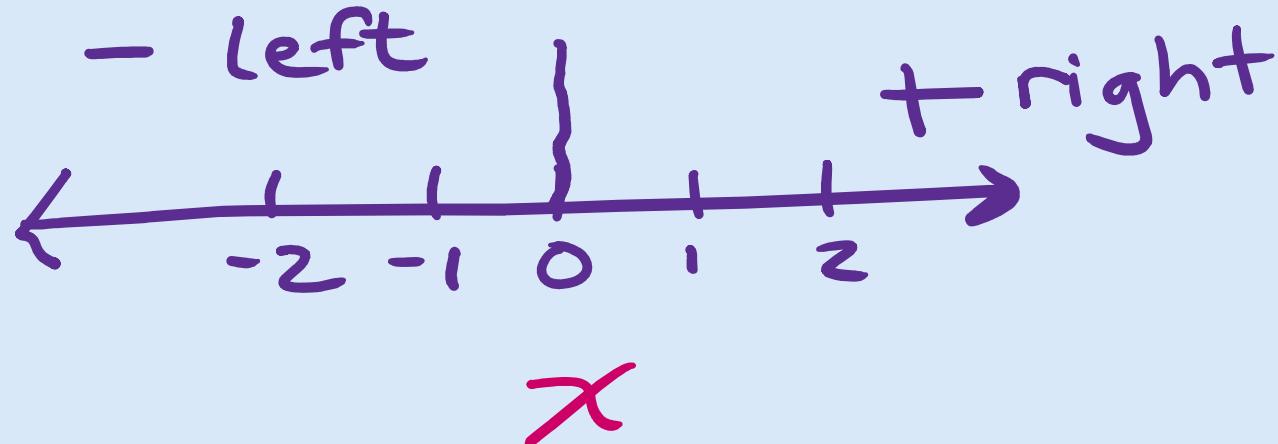
A Vector has **magnitude** or length and **direction**

We can map or graph it in one dimension on a number line. Starting at zero, we can map a length of x. This goes in the positive direction.

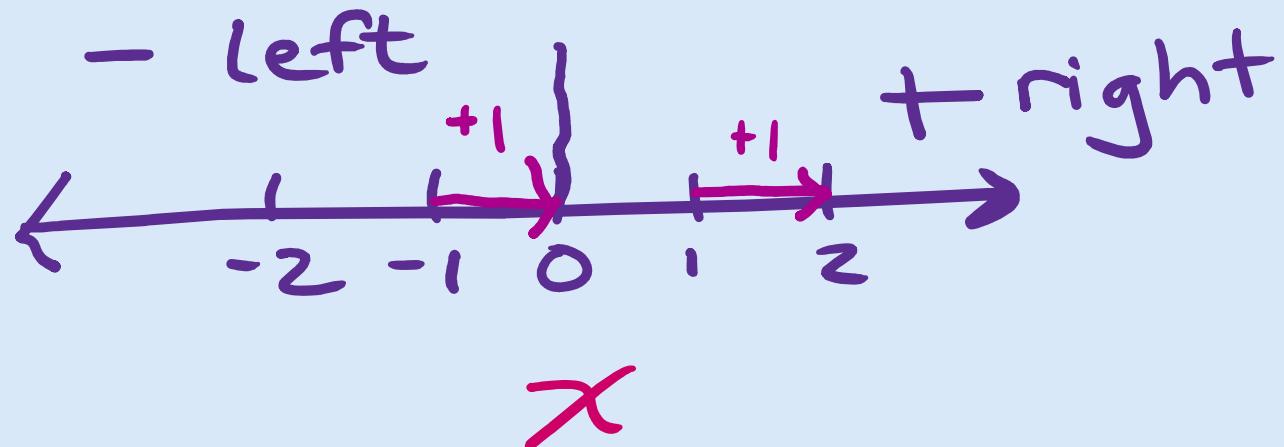


A Vector has **magnitude** or length and **direction**

We can map or graph the vector in one dimension on a horizontal number line. For direction on a horizontal number line, we use positive with the symbol + to mean right and negative with the symbol – to mean left. And we use x for the length.



These vectors are both +1 because they both go to the right and have a length of one. (Thanks Annabelle)



We can map or graph the vector in one dimension on a number line.

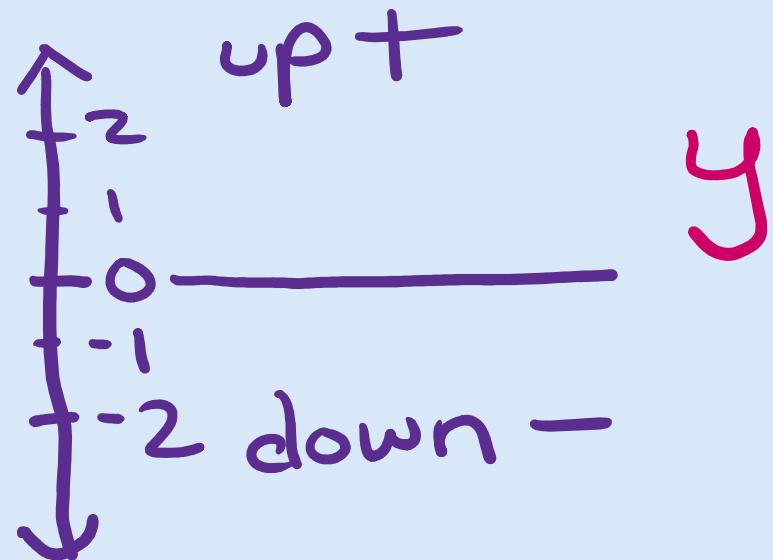
On a number line, we use positive with the symbol + to mean right and negative with the symbol – to mean left.

A regular x is used to represent the point or x-coordinate on the number line and a bold **x** to represent the vector.

A unit vector in the x direction is often represented as



We can map or graph the vector in one dimension on a vertical number line. For direction on a vertical number line, we use positive with the symbol + to mean up and negative with the symbol – to mean down. We use y for the length.



A regular x is used to represent the point or y -coordinate on the number line and a bold \mathbf{y} to represent the vector.

A unit vector in the y direction is often represented as



A unit vector is defined as being +1 so R for \hat{x} and up for \hat{y}

Let's start with looking at an object having a position or location on a two-dimensional plane (a flat surface). Then we can model the object moving around on the two-dimensional plane.

Here is my object, Arnie, my frimily dog. My two-dimensional plane is the ground of our neighborhood.



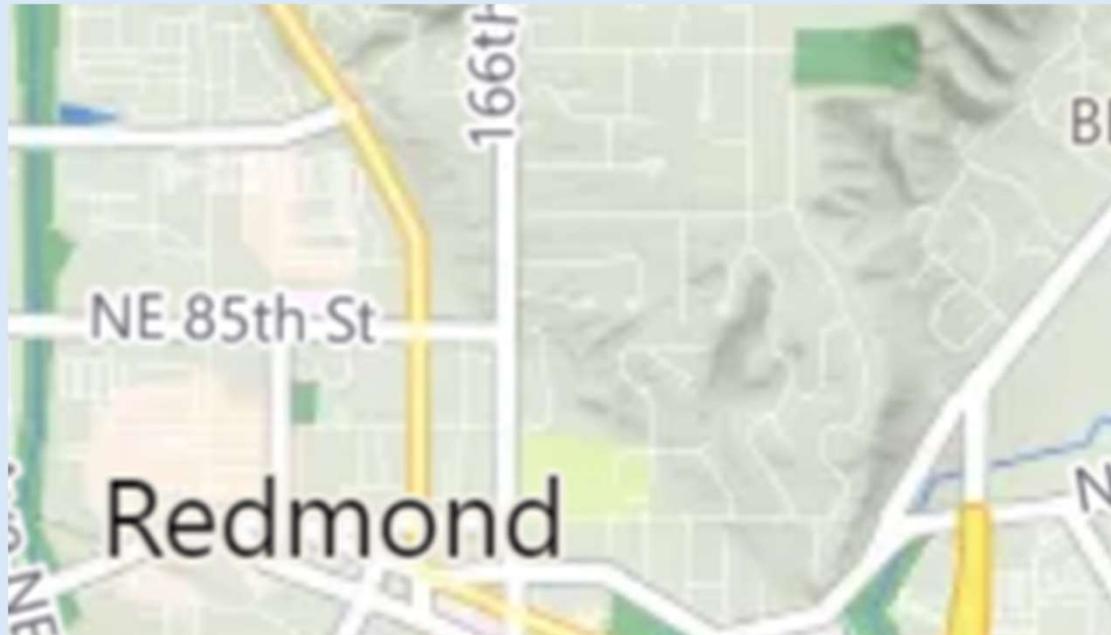
2024



(c) Crainix

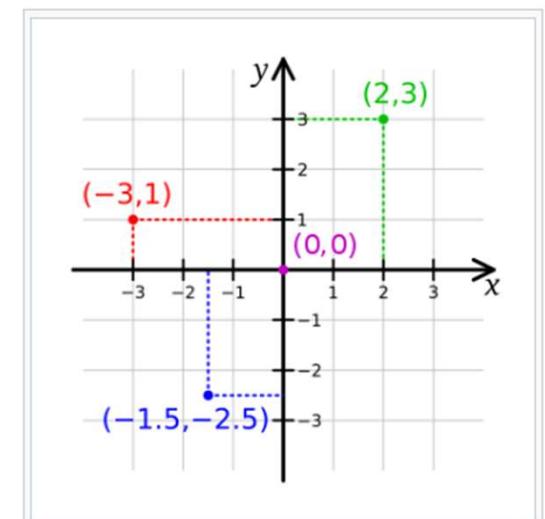
13

Here is a map of our neighborhood.



For this, we want to move from going back and forth or up and down on a line to a two-dimensional plane. We do this using a grid called a cartesian coordinate system or rectangular coordinate system.

In mathematics and geometry, the **Cartesian coordinate system** is a coordinate system used to give the location of points on a plane by using two numbers for each point. It is also called the *rectangular coordinate system*. The numbers are usually called the *x coordinate* and the *y coordinate*. To find the coordinates of a point, two perpendicular lines, called **axes** (singular: *axis*), are drawn. The point where the axes meet is the coordinate **origin**, written $(0,0)$. The *x coordinate* gives the **position** of a point measured along the *x axis*, and the *y coordinate* gives the position along the *y axis*. Cartesian coordinates can be used in three dimensions (3D), by adding a third number, the *z coordinate*. In four dimensions (4D), a fourth number, the *w coordinate*, is added. Each coordinate represents a dimension of space.



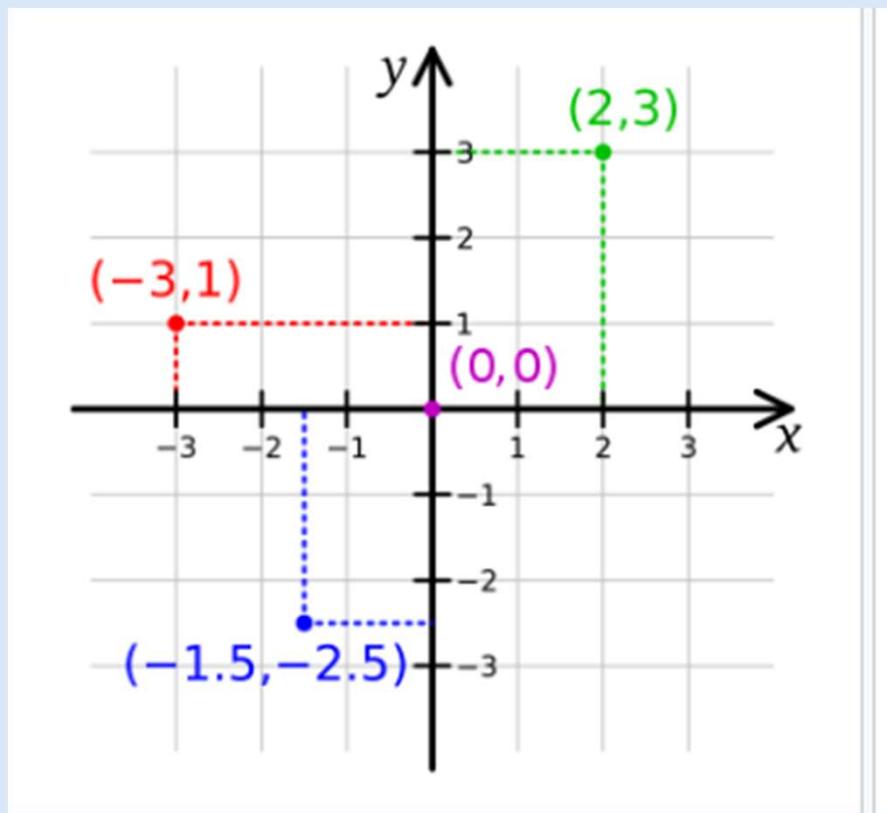
Two dimensional Cartesian coordinate system. Four points are marked: $(2,3)$ in green, $(-3,1)$ in red, $(-1.5,-2.5)$ in blue and $(0,0)$, the origin, in violet

We do this using a grid called a [cartesian coordinate system](#) which is named after a French mathematician [René Descartes - Wikipedia](#)



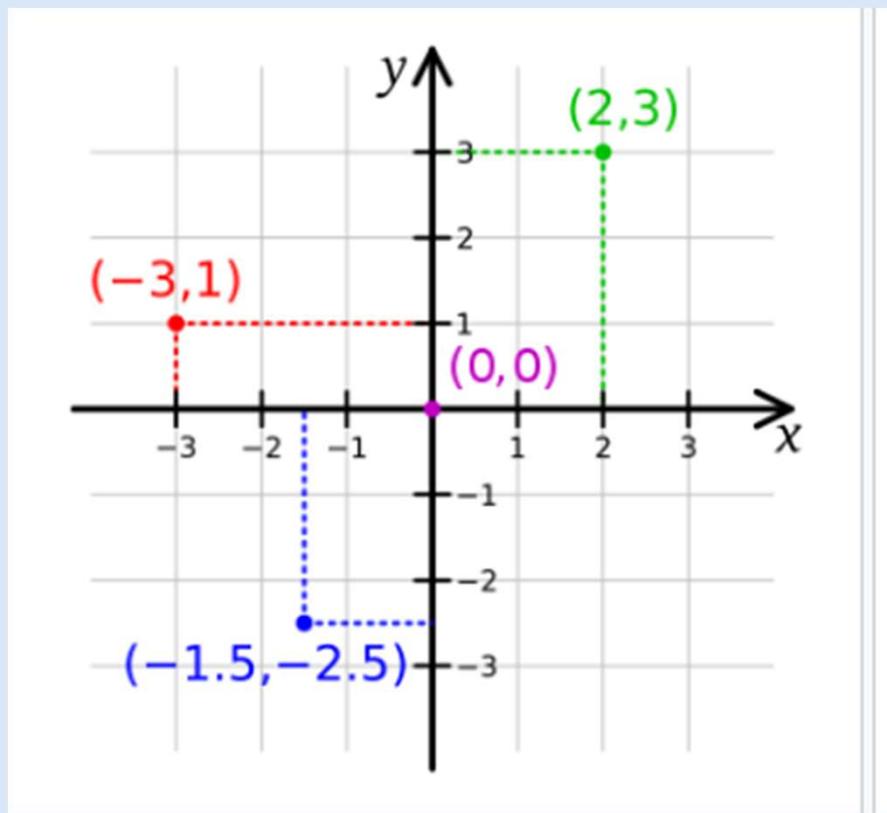
Rectangular coordinate system refers to the fact that the grids form rectangles. It is also called a two-dimensional axis system or graph.

Graphic from the Wikipedia page [cartesian coordinate system](#)



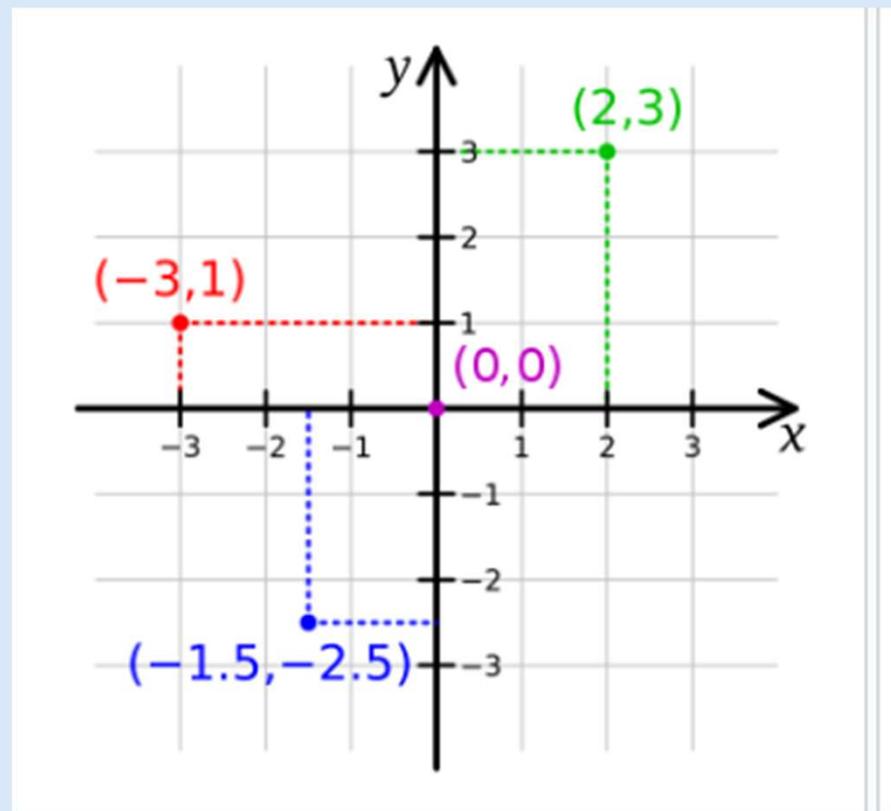
For now, we will be talking about length in the x and y directions with both axes having the same scale and units, like meters or feet.

Graphic from the Wikipedia page [cartesian coordinate system](#)



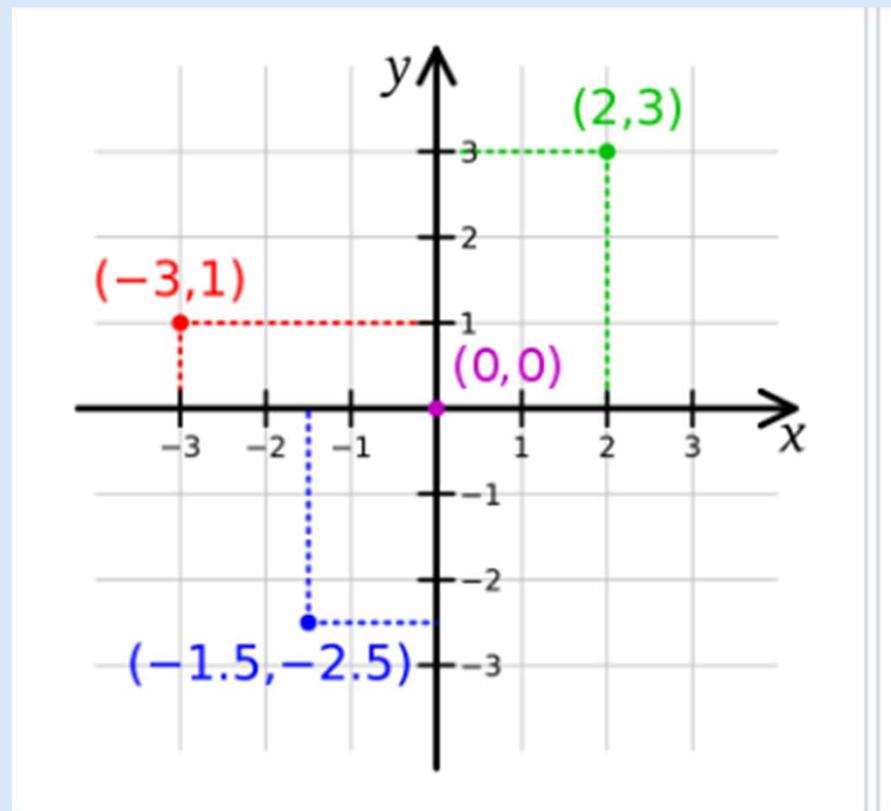
We label points on the system (plane, grid, axis system, graph) as an ordered pair with the location relative the x axis first, then a comma, then the location relative to the y axis second. Graphic from [cartesian coordinate system](#)

(x, y)
 $(\text{horz}, \text{vert})$



Unlike axis systems when we do functions, the units of the x and y for vectors are usually the same unit. For our examples in space, they would be lengths. Graphic from [cartesian coordinate system](#)

(x, y)
 $(\text{horz}, \text{vert})$

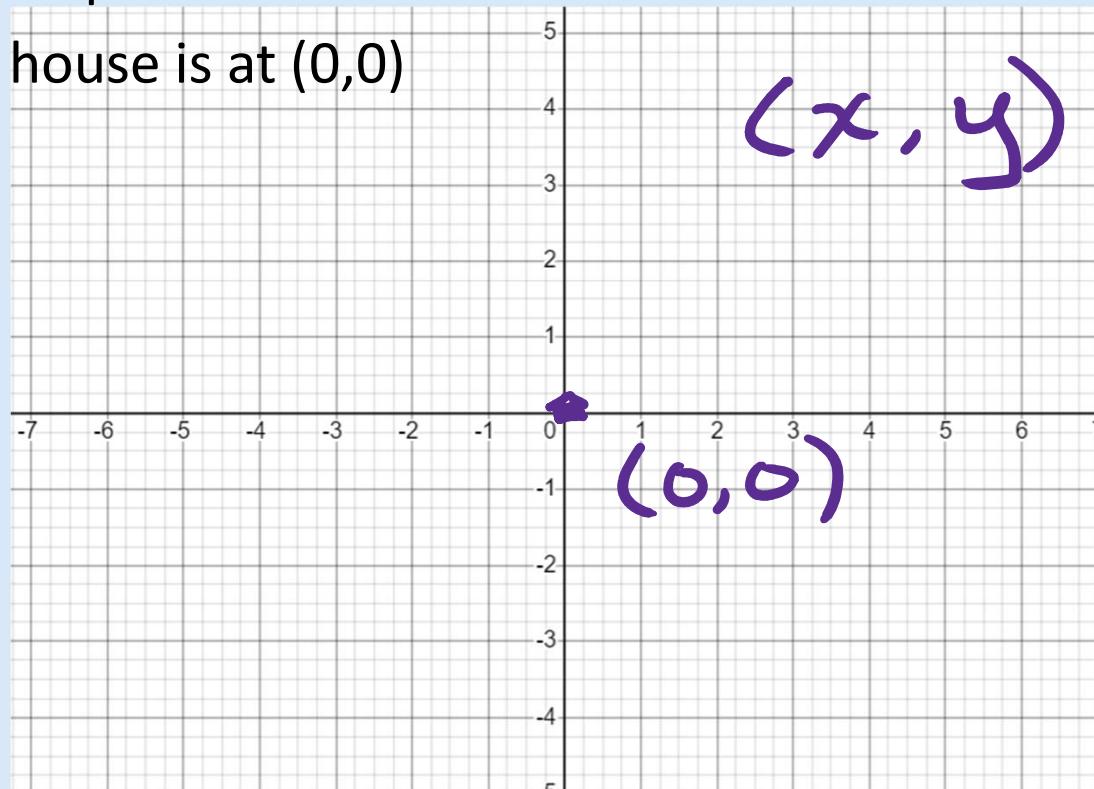


If Arnie is at his house at $(0,0)$ then he can go visit friends or the off-leash dog park. Do you prefer thinking in terms of a story?

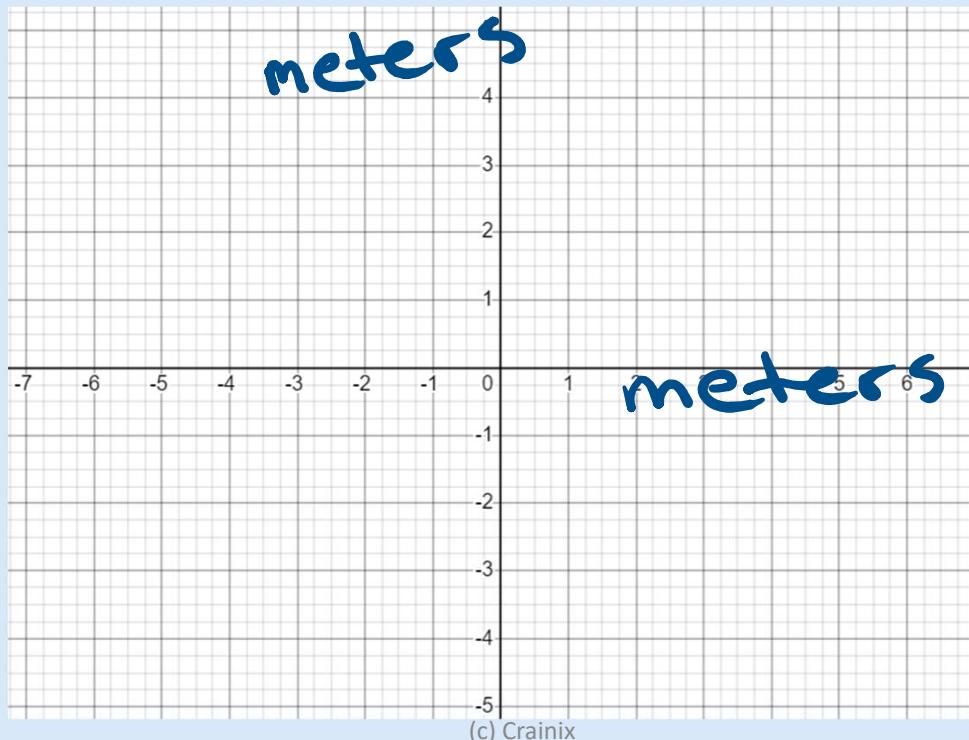


Points represent infinitely small locations. We can describe locations by giving the x and y coordinates of the points. We always have the x coordinate first, a comma, and then the y coordinate. Points or locations are in parentheses.

Here Arnie's house is at $(0,0)$

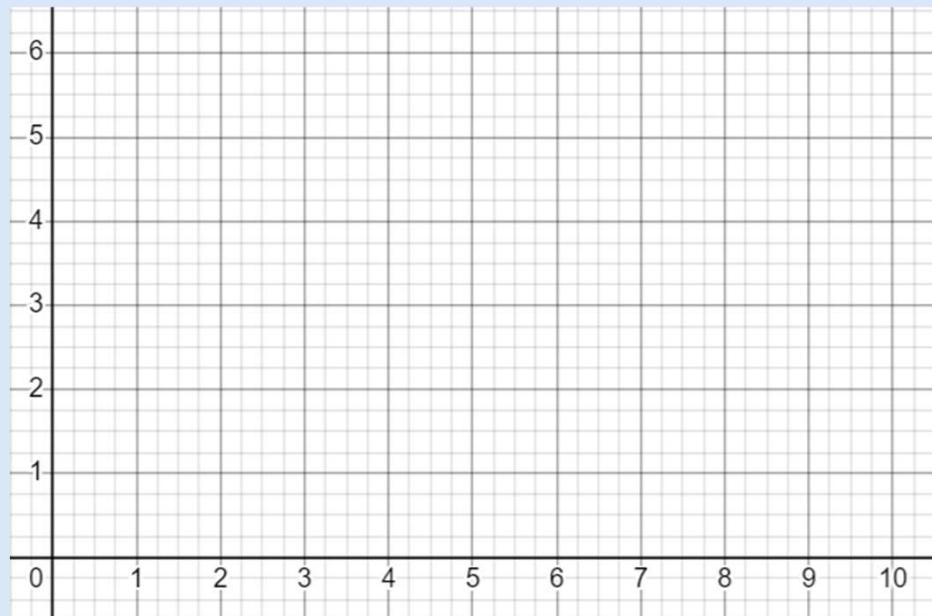


We can label locations as points on the axis system. For vectors, analytic geometry, and trigonometry, we use lengths or distances for the axes, and they are both in the same units. This makes the whole grid be in the same units. Miles, meters, or Arnie strides, the whole grid is in terms of the same units. The metric or measure is the term for the units.

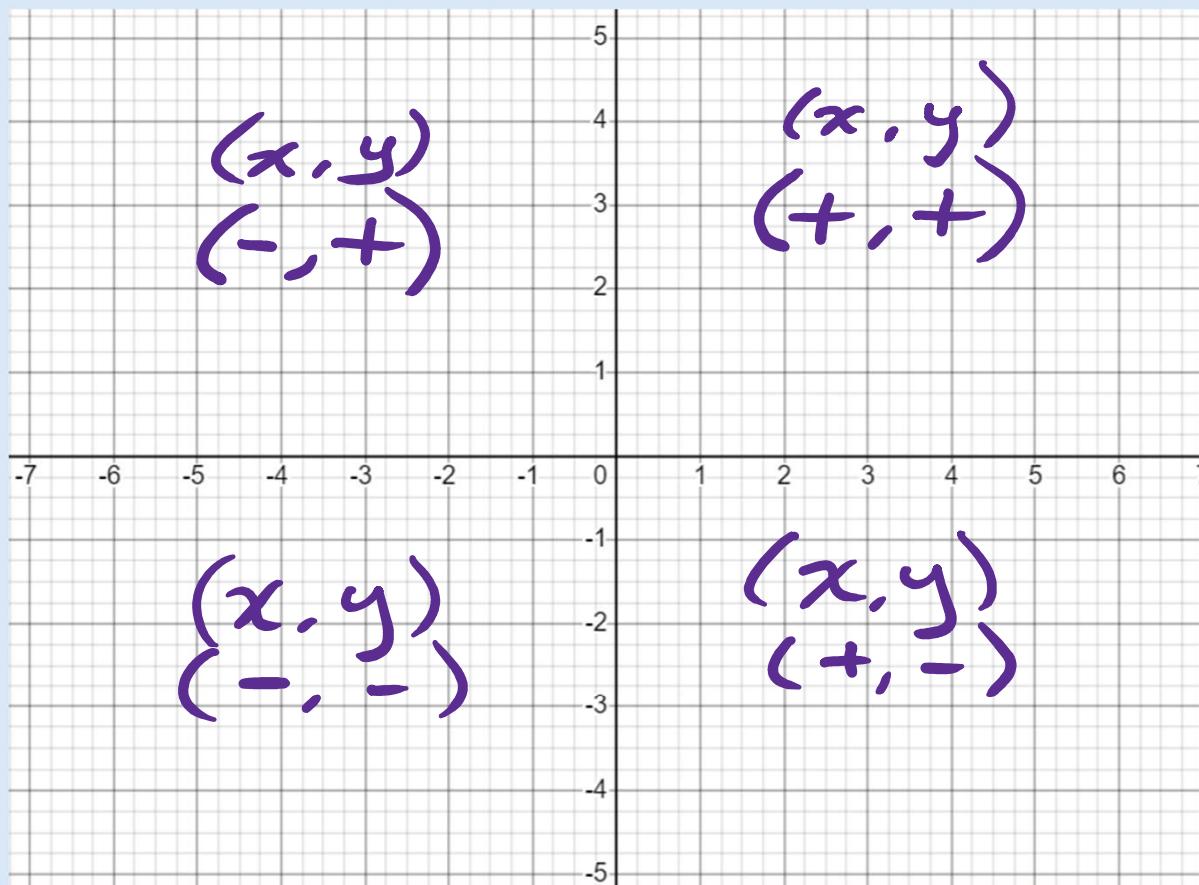


We can shift the axes to zoom in or out and to focus on a particular area.

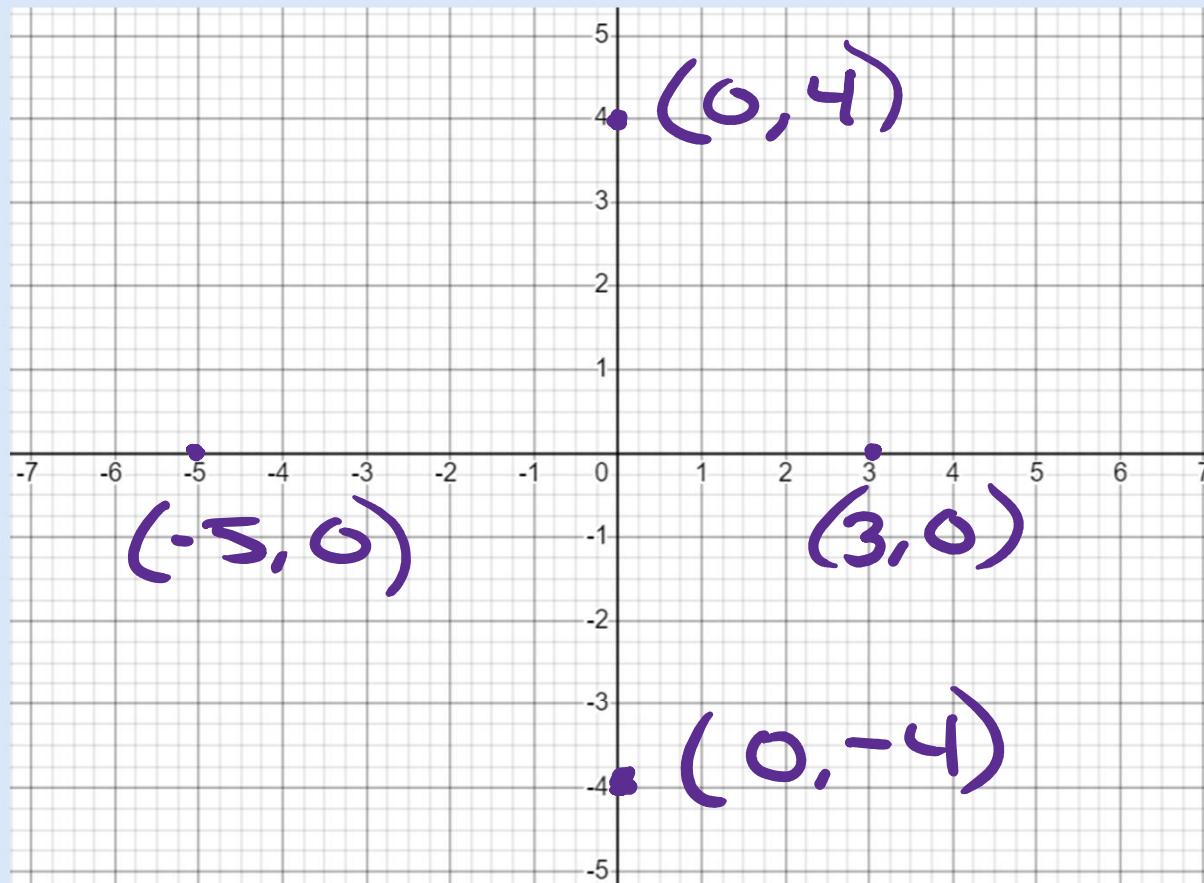
Here the graph just focuses on the quadrant where x and y are both positive, and it is zoomed to a scale with labels at the whole numbers.



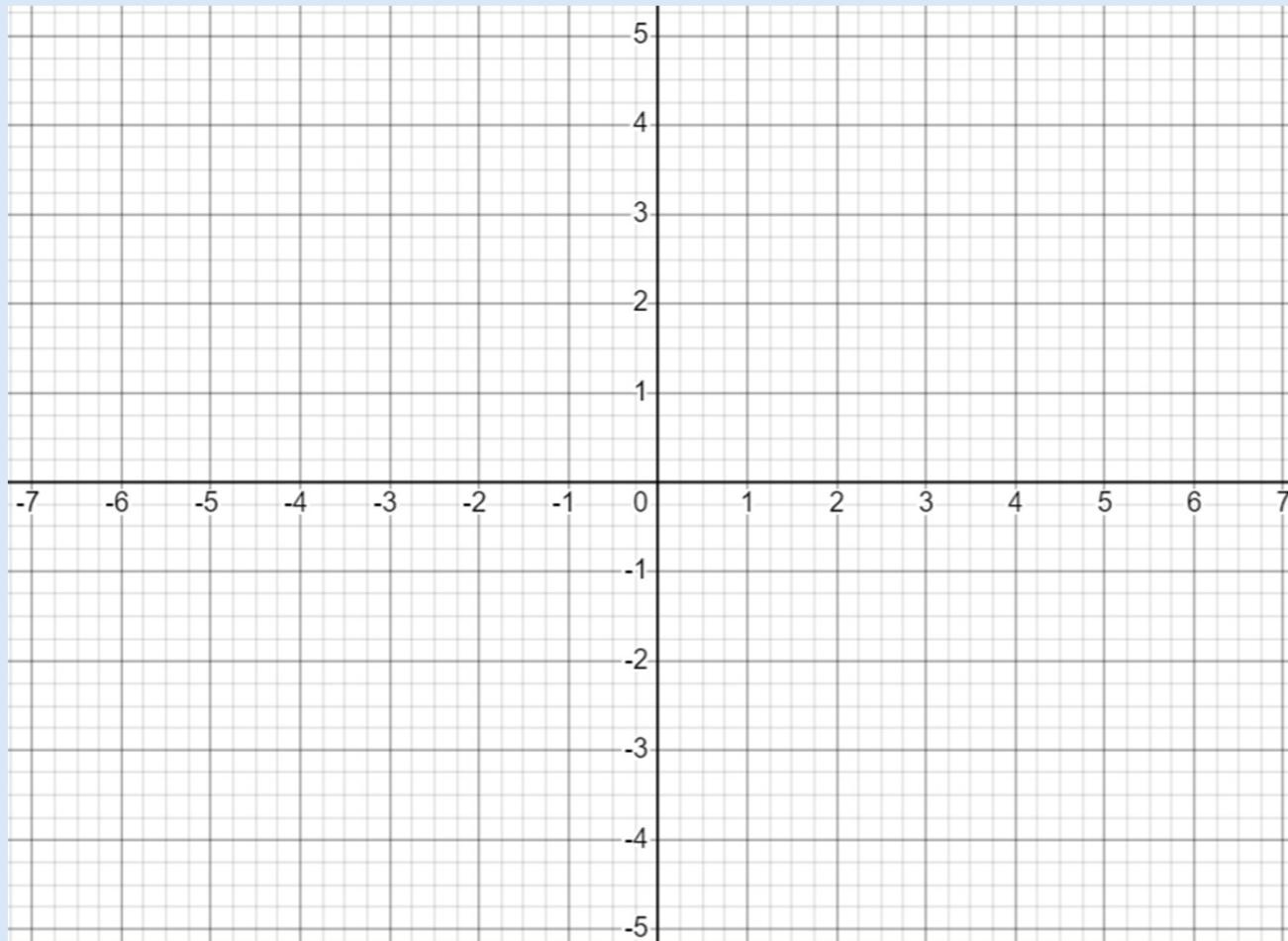
For now, we will use all four quadrants with this zoom. Notice the positive and negative signs or directions for each quadrant.



Notice that on the x axis, the y is zero and on the y axis, the x is zero.



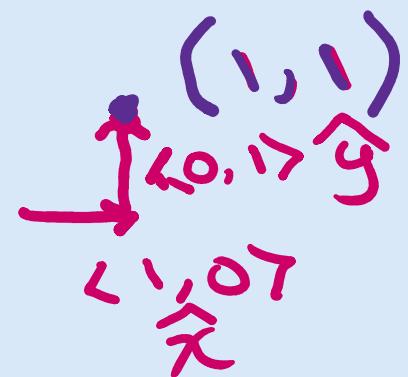
Here is an axis system if you want to practice plotting points.



You can draw in a scale and practice labeling the points that represent Arnie's friends and the dog park entrance. Or create your own game.

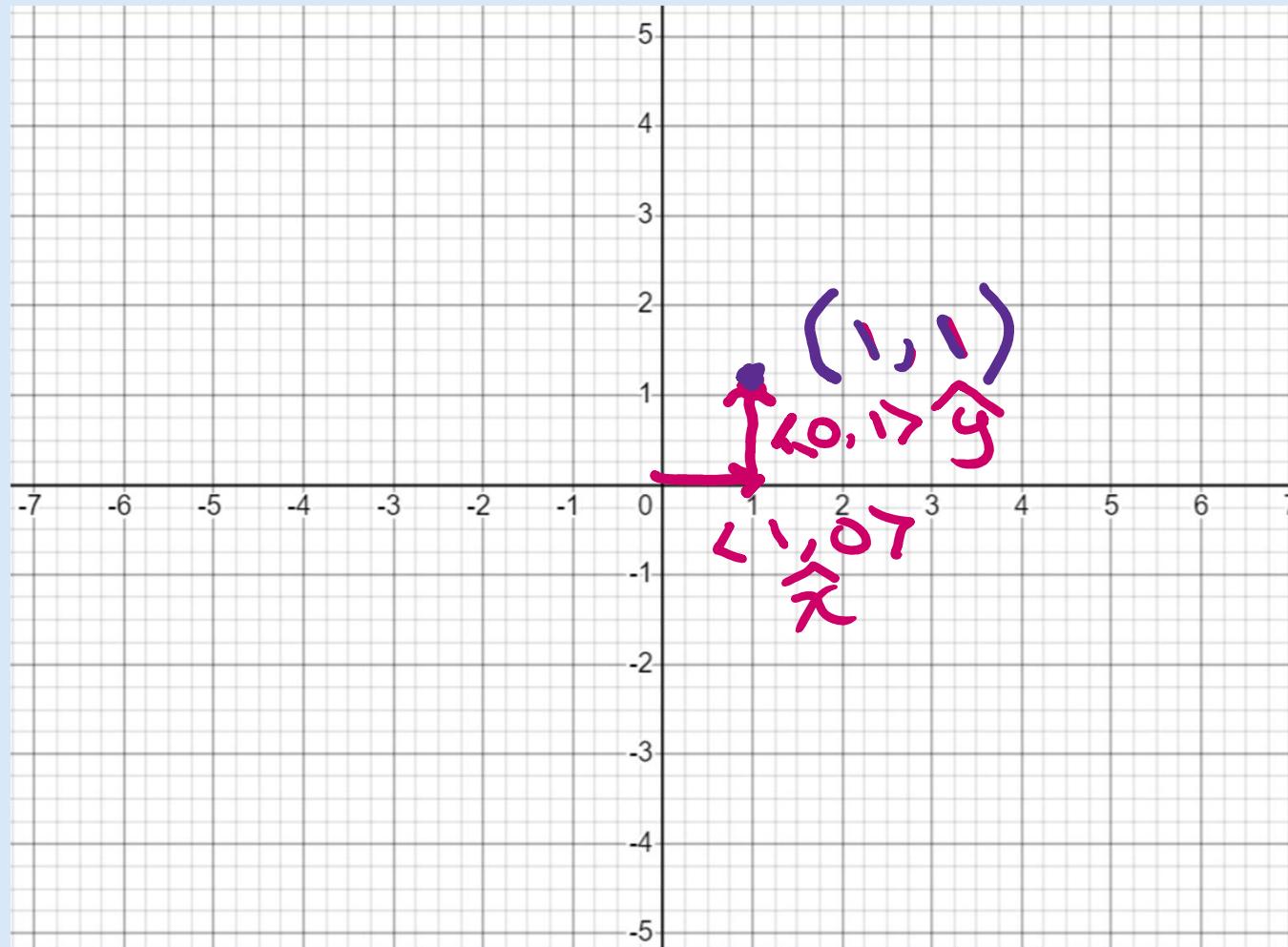


We use vector symbols for points that are the endpoints of vectors that start at the origin or $(0,0)$.



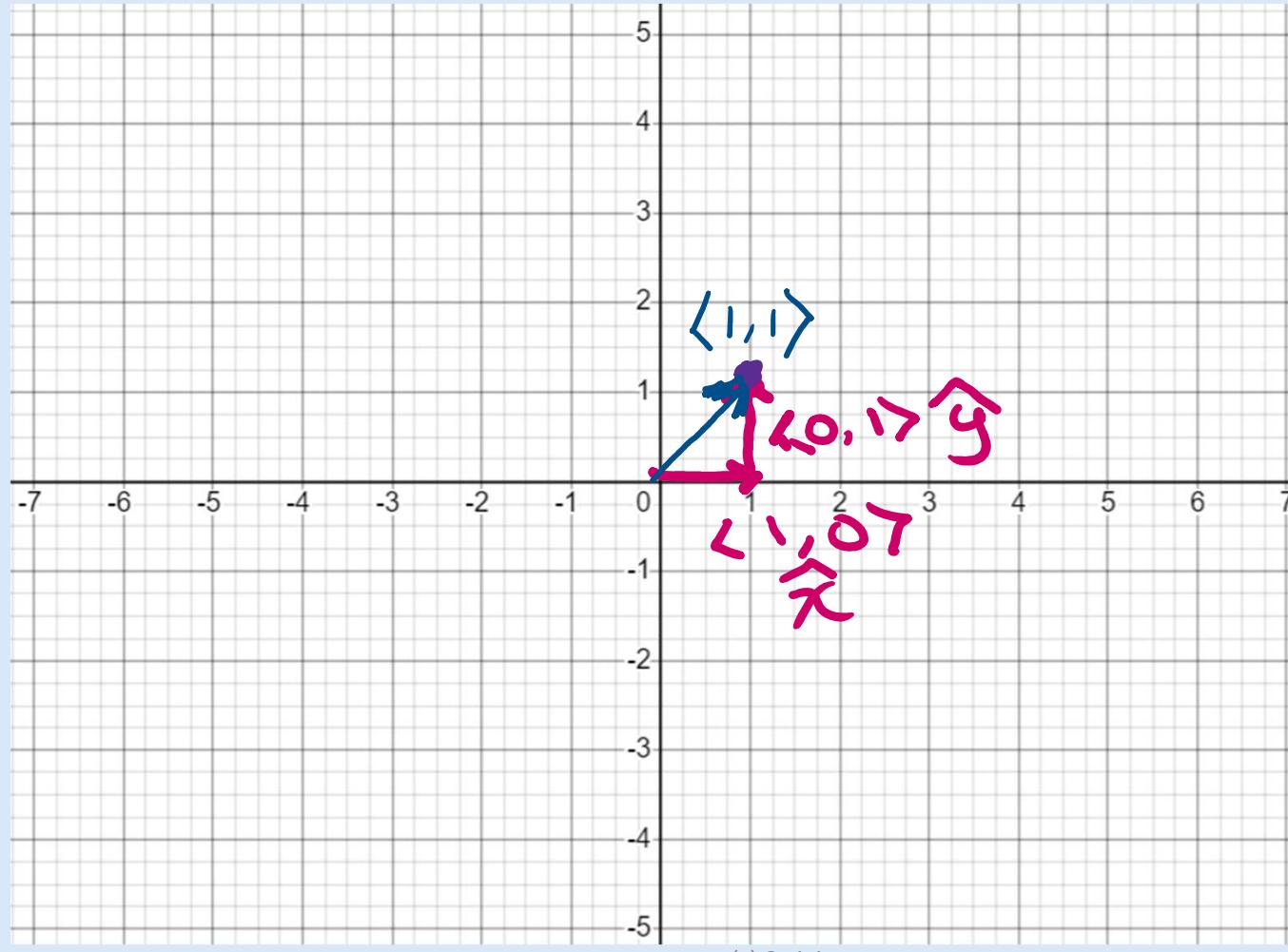
$$\begin{aligned}\hat{x} &= +1 \langle 1, 0 \rangle \\ \hat{y} &= +1 \langle 0, 1 \rangle\end{aligned}$$

If we go R one unit, \hat{x} and up one unit \hat{y} then we make the vector $\langle 1, 1 \rangle$

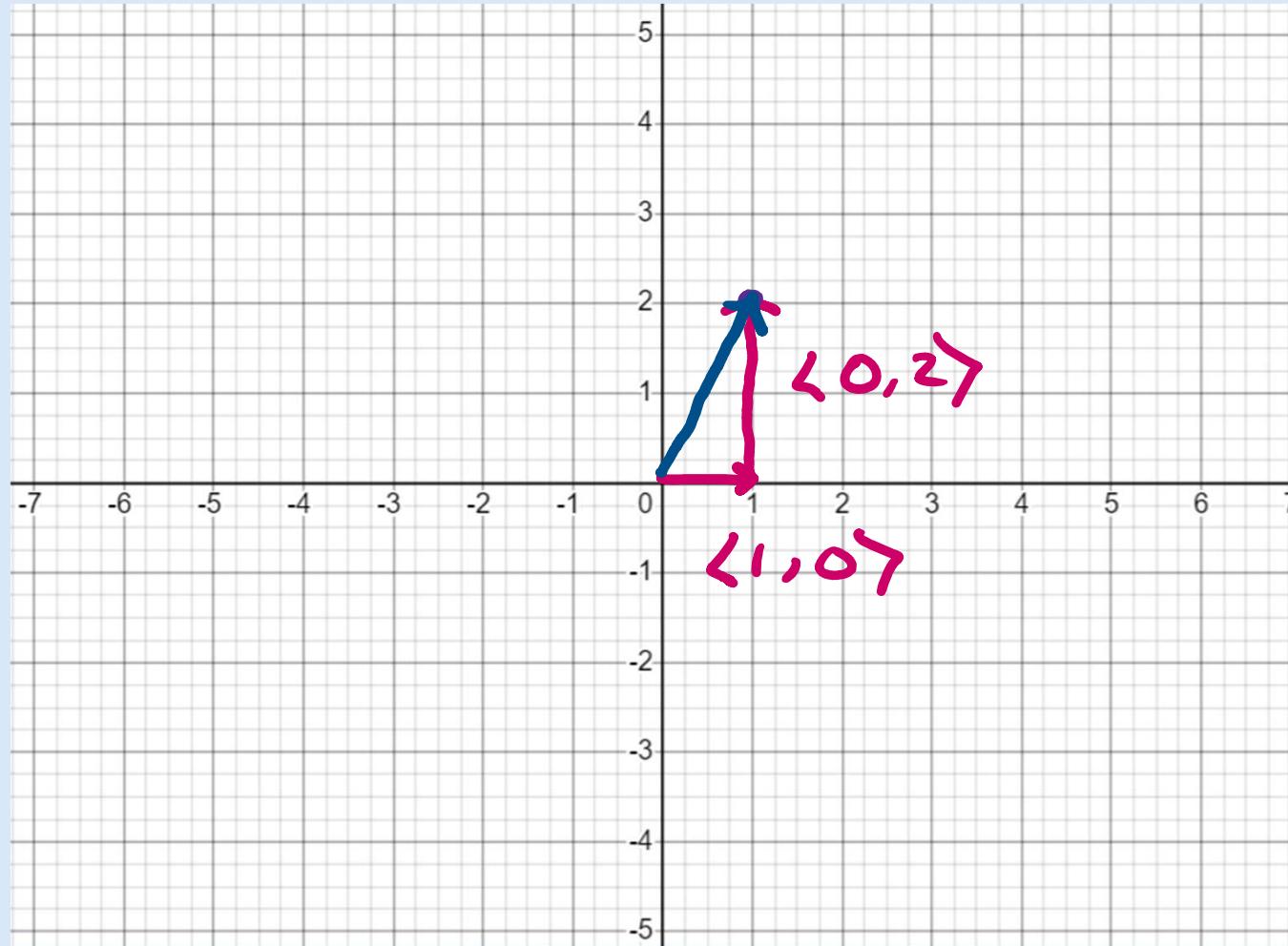


$$\begin{aligned}\hat{x} &= +1 \langle 1, 0 \rangle \\ \hat{y} &= +1 \langle 0, 1 \rangle\end{aligned}$$

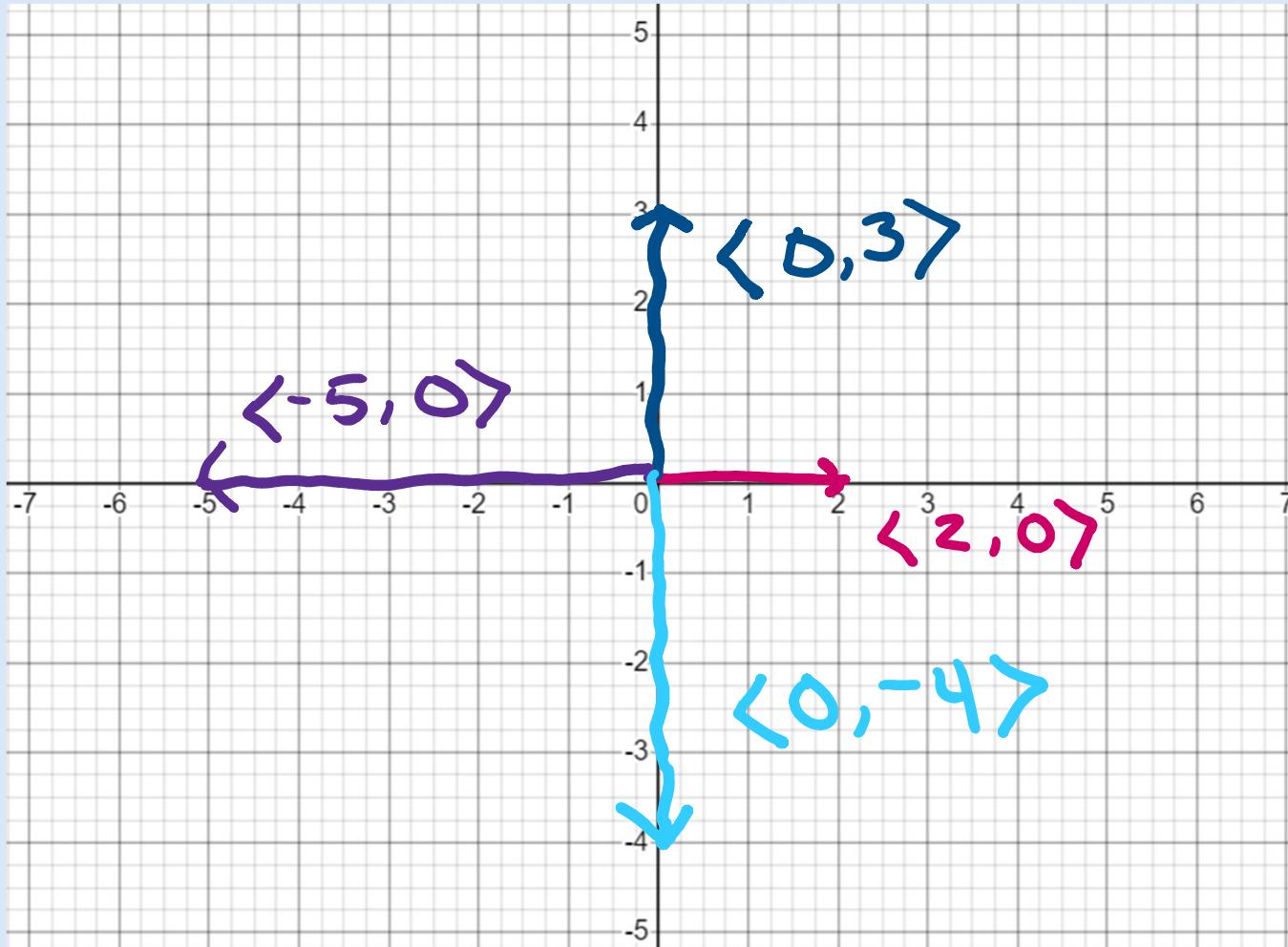
Here the vector $\langle 1, 1 \rangle$ is the vector from point $(0, 0)$ to the point $(1, 1)$.



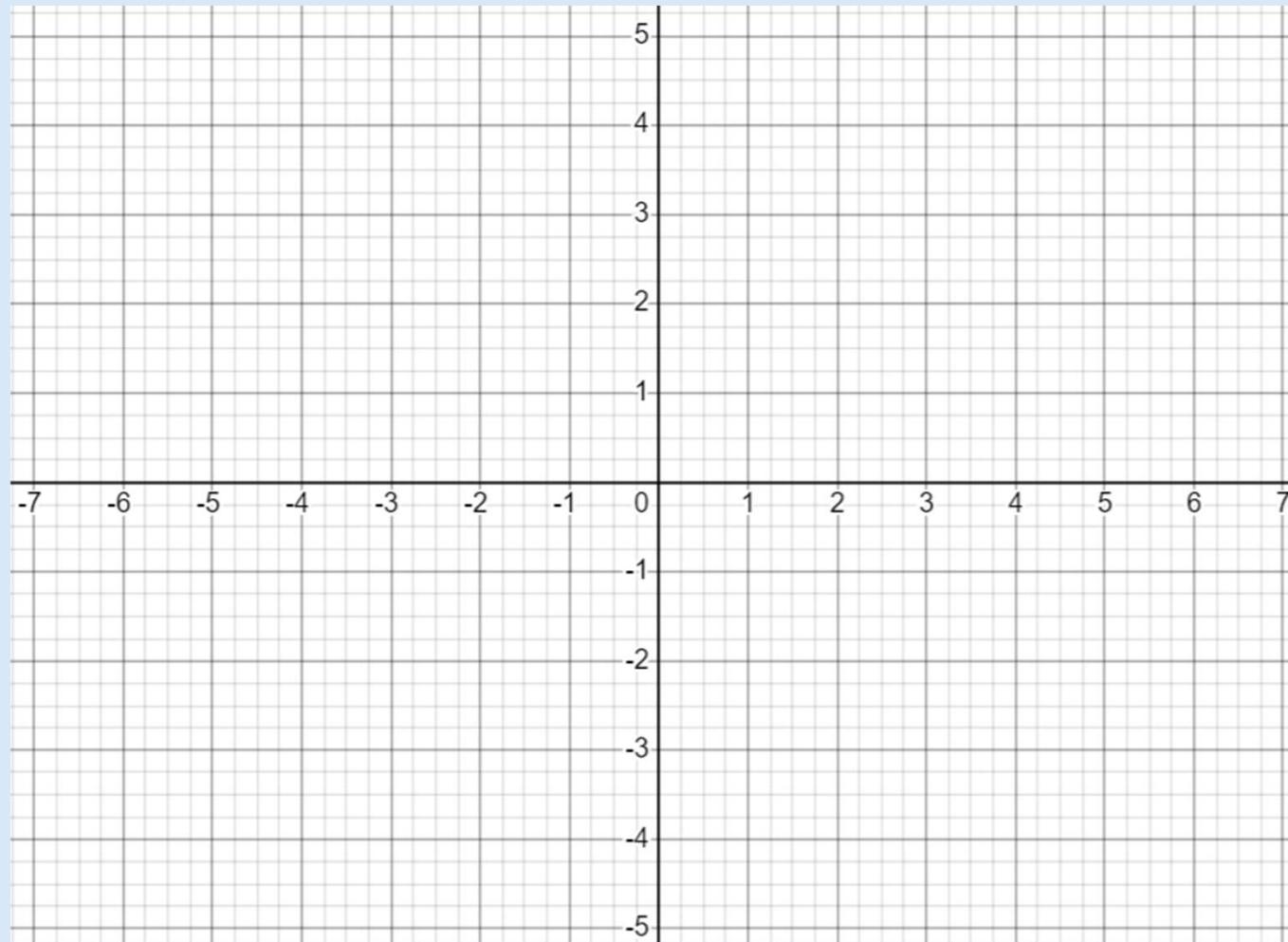
We can draw some vectors:



We can draw some vectors:



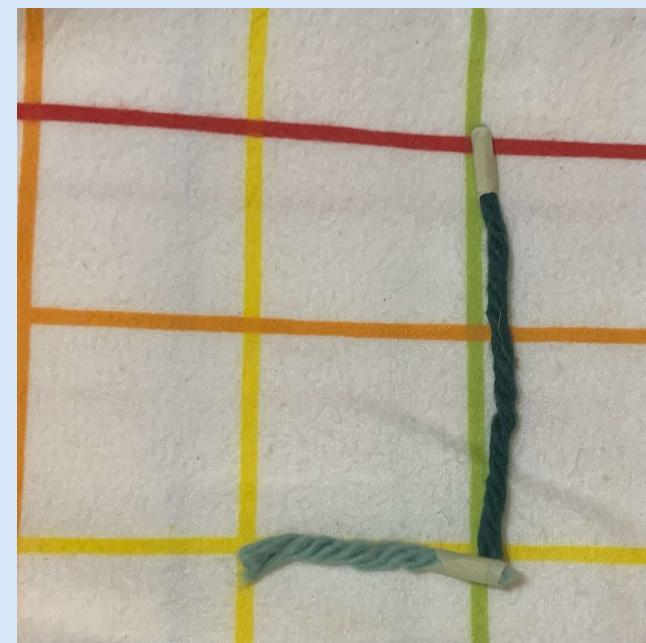
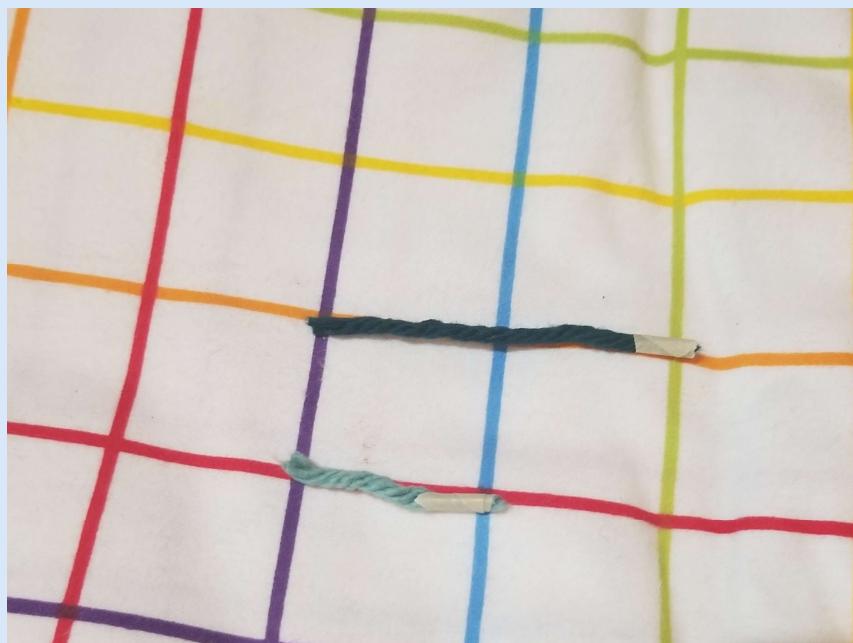
Here is a grid if you want to practice.



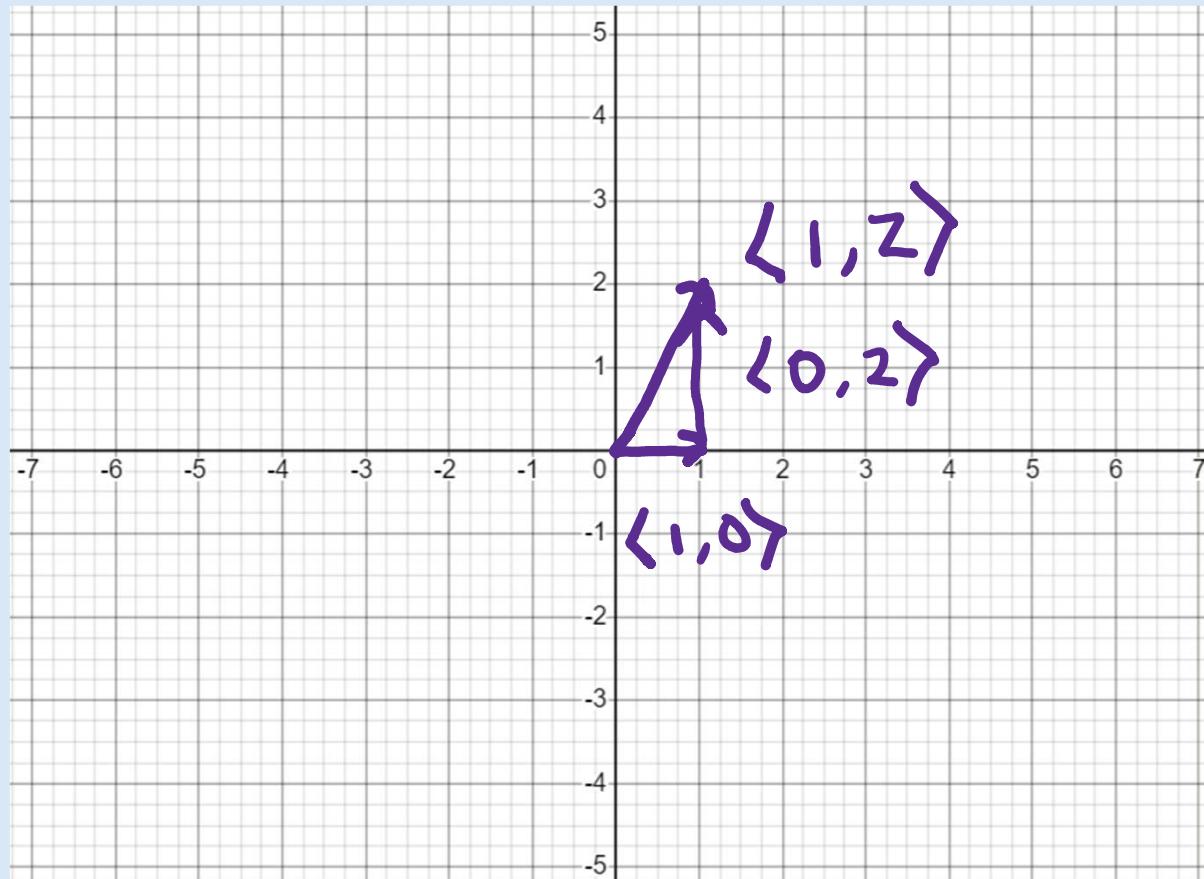
If Arnie is at his house at $(0,0)$ then he can go visit friends or the off-leash dog park. Here is the diagram if you want to practice.



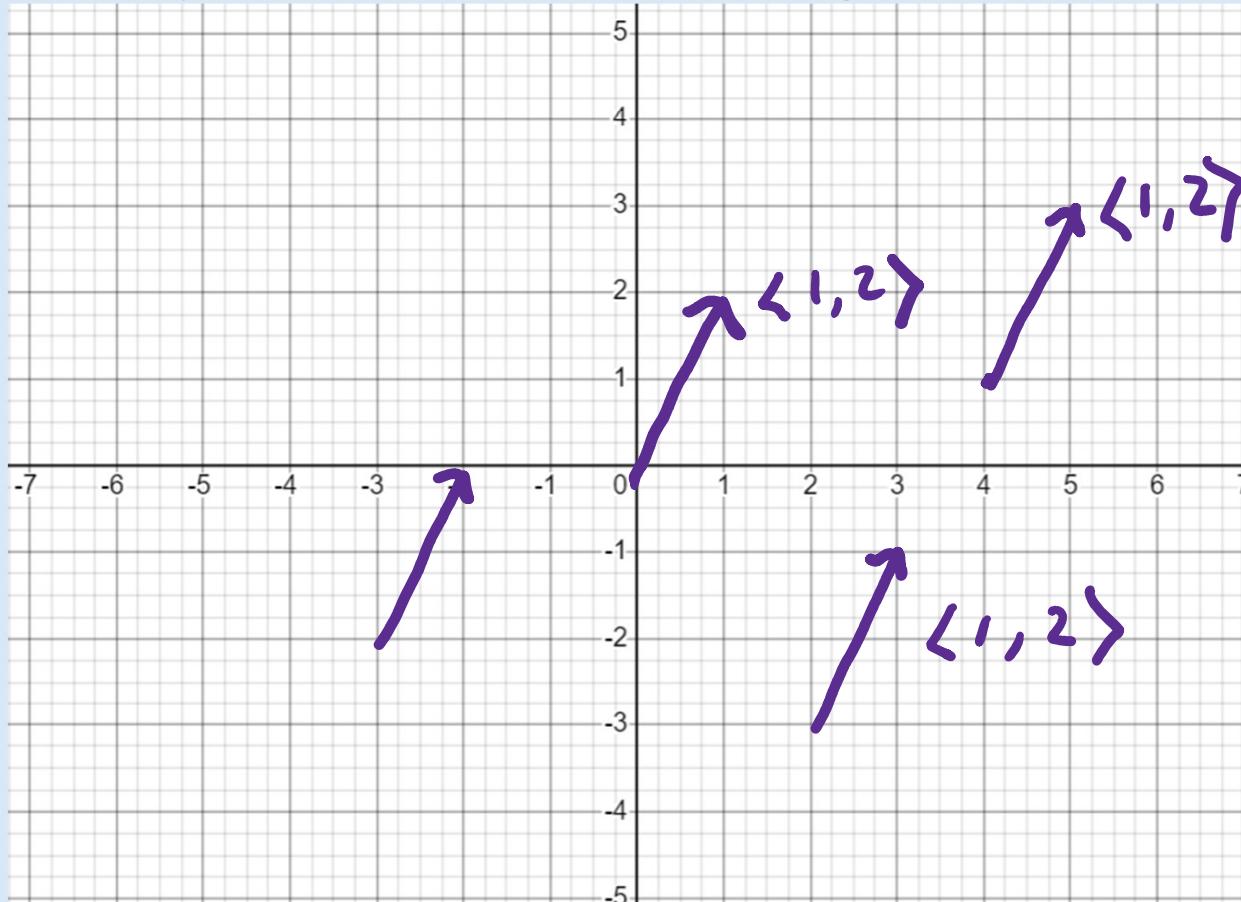
We can make vectors that we can use to play with or just draw them.
Here are string vectors, with tape to represent the heads, on grid fabric.
Do you want to make manipulatives or use sidewalk chalk to play with vectors?



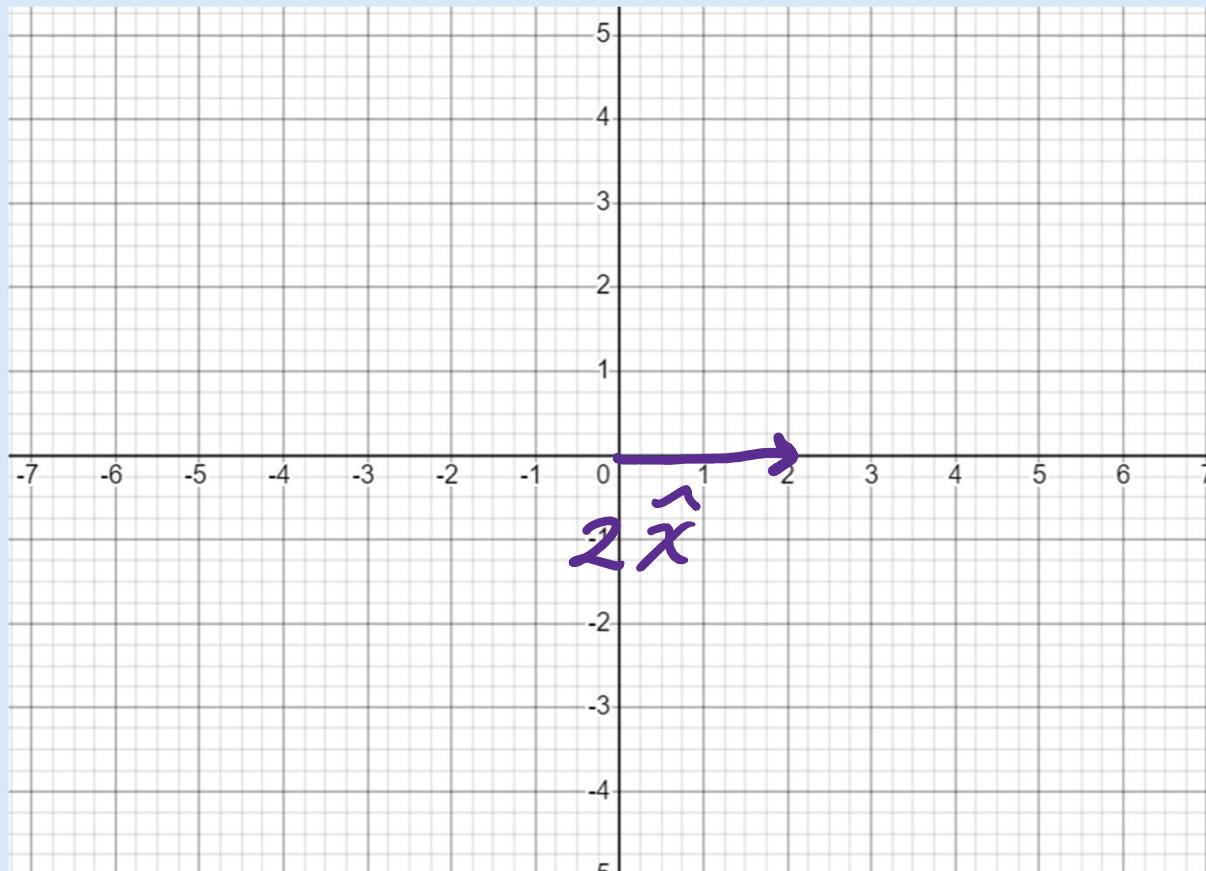
With rectangular coordinates, we view the vectors in terms of components, which are the x direction and y direction.



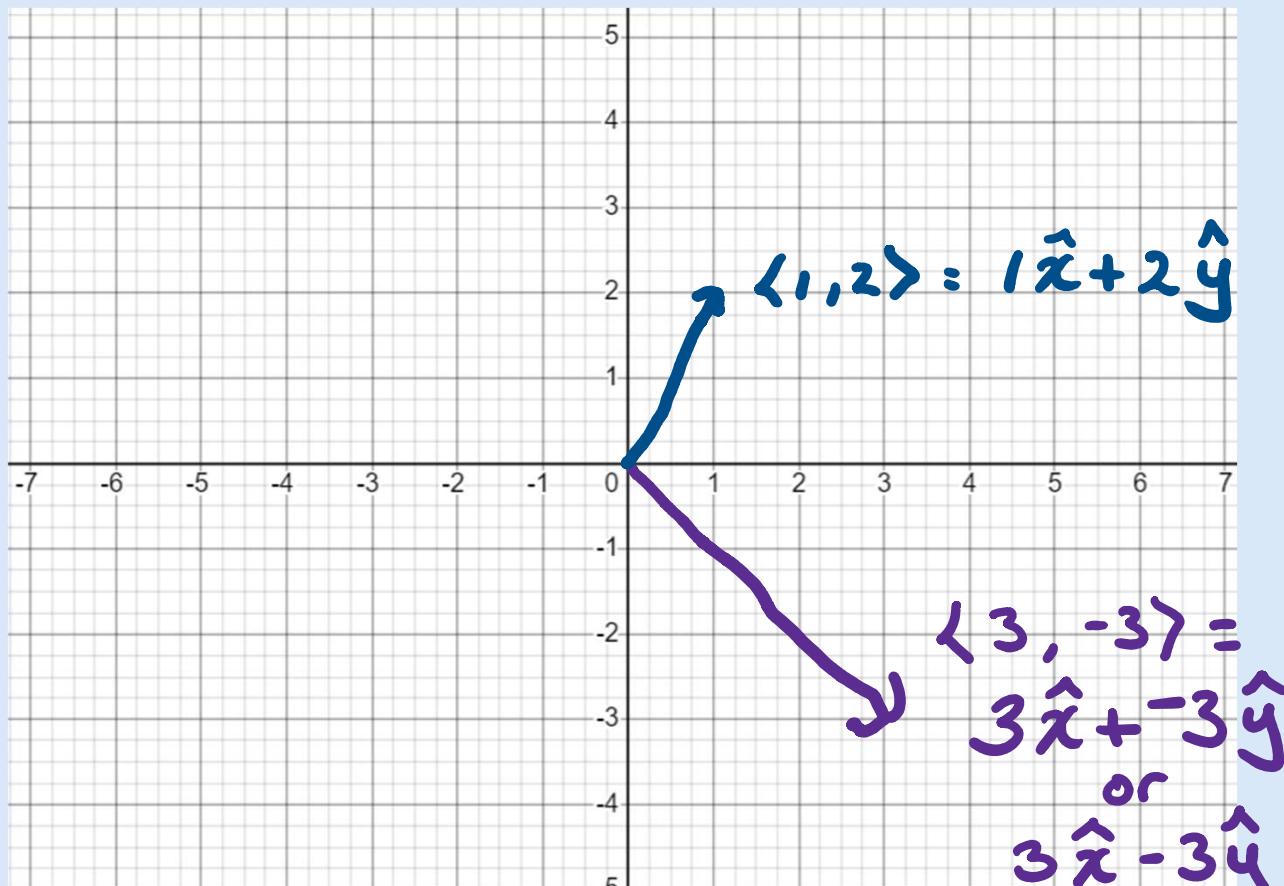
Notice that the vector that goes to the point $(1,2)$ is written with the vector symbol. It could be anywhere, but we are starting at $(0,0)$ to make it easier.



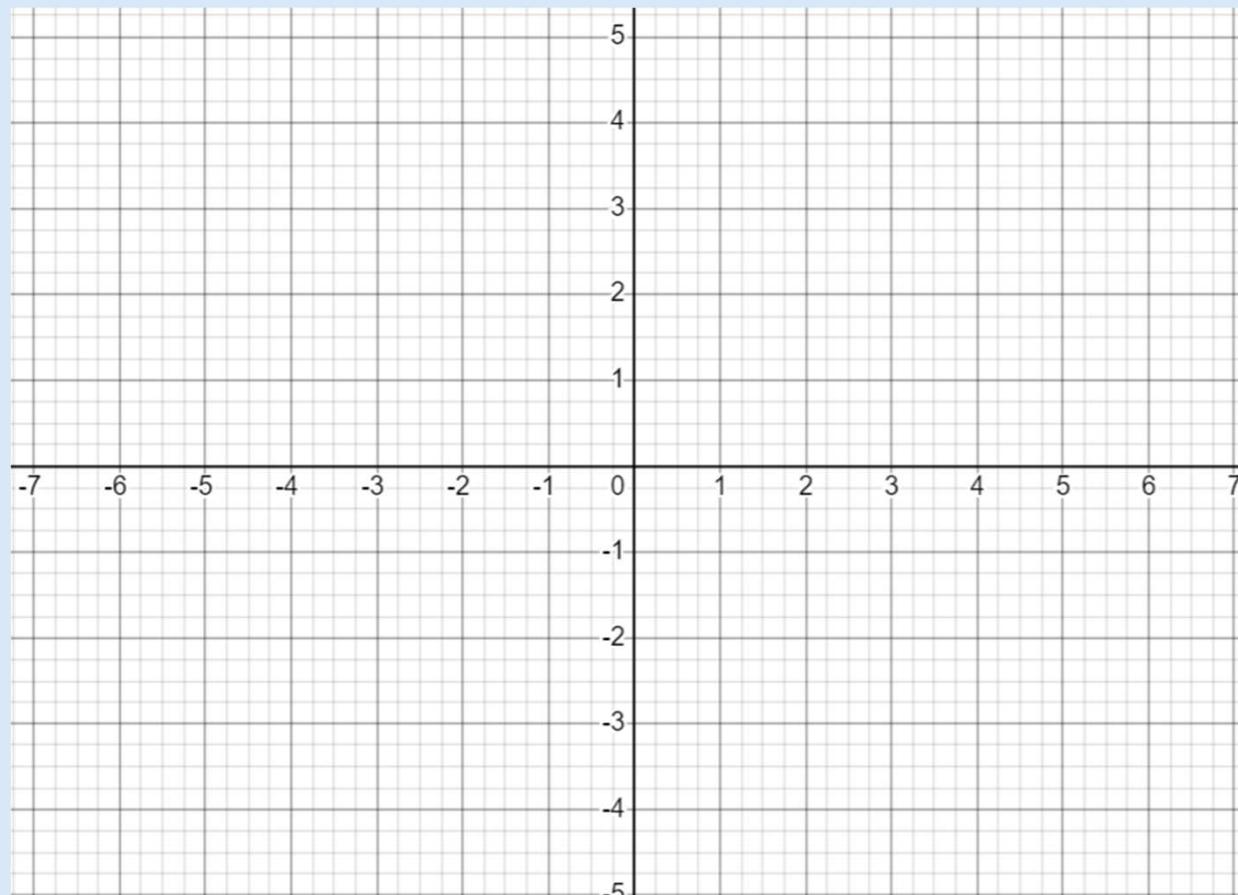
A unit vector can be multiplied by a nonvector number, called a scalar (because it scales the vector).



We can write and use vectors in terms of x and y unit vector components.



Here is a grid if you want to practice writing vectors in terms of their unit vector components.



Turtle Programming seems to use some concepts of vectors for programming a turtle to move around to create a game.

[Self Paced Introduction to Turtle Programming In App Lab - Code.org](#)

[Turtle Academy](#) Turtle lessons

For iPad [Move the Turtle - Programming for Kids on the iPhone and iPad](#)