In algebra, we use letters to represent numbers or datapoints that can change. We call these variables.

We will use x as our standard input variable and y as our output variable. We will use other variables as needed. (I never use *e* as a variable as it is the constant irrational number that is Euler's number and the base for exponential functions in my brain.)

x input

y output

n a number a to represent answer

a, b, c, d used as numbers for multiplying the input variable.

Expressions

An expression is a way of writing something algebraically: x+1

We can plug in or **substitute** numbers for the variable and **evaluate** the expression.   If x is 1 then x+1 is 2 and if x is 2, then x+1 is 3.

x	$x+1$
1	$1+1=2$
2	$1+2=3$
n	$n+1$

Equations

An **equation** has an equal sign with something on both sides that are equal to each other. We usually mean that both sides equal each other but there can be times when that isn't true.

$x+1=3$

One-step equations in one variable

Equations in one variable can be represented on a line and in one dimension.

$x+1=3$ is an example

When solving, the goal is to get the variable on one side and a number on the other side.

If I have some cats, and one bunny, and that makes three pets total, how many cats do I have?

$$c + b = p$$

cats bunnies pets

$$c + 1 = 3$$

Did you look at this and figure out how many cats I have in this example?

If so, how did you think about it or figure it out?

It can be helpful to stop and think about how **you** think.

Figuring out how to solve very simple problems can help think about complex or hard problems.

As we do algebra, we will try and find patterns to help us solve any problem that fits the pattern.

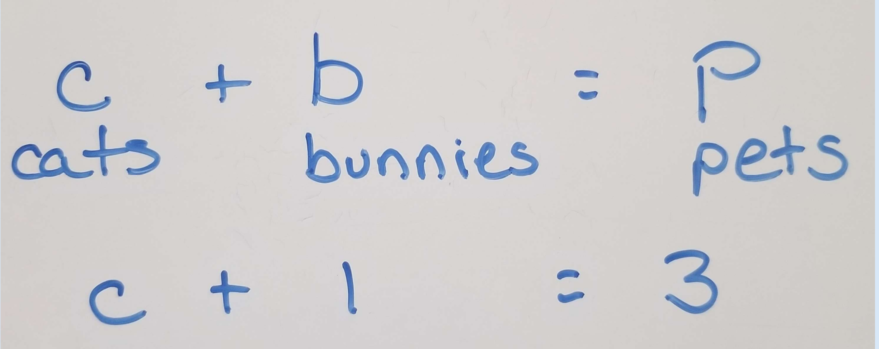
$$\underset{\text{cats}}{c} + \underset{\text{bunnies}}{b} = \underset{\text{pets}}{p}$$

$$c + 1 = 3$$

$$x + n = a$$

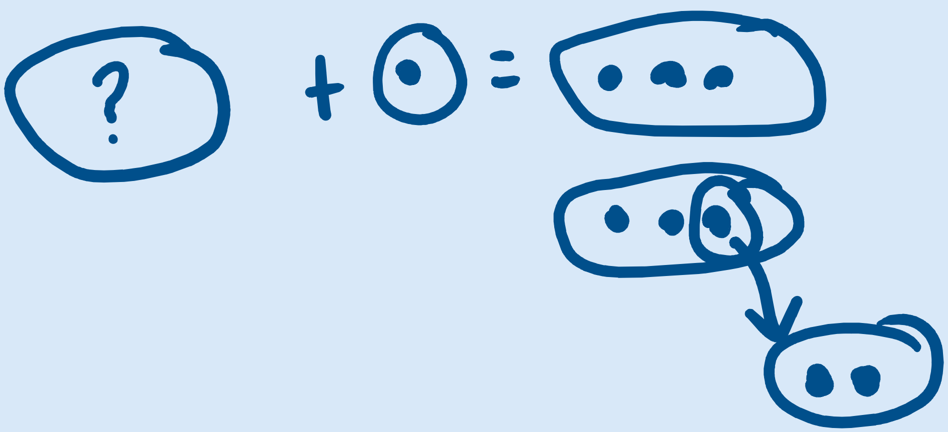
When we find patterns that help us solve any problem that fits the pattern, we can program a computer to do the calculations for us.

We will talk about different ways of thinking about how to solve this problem and then we will learn to program a computer to do the calculations for us.



With groups, if there is an unknown amount and you add one to it and get a total, then you would subtract one to find out how much you started with.

$$x+1=3$$

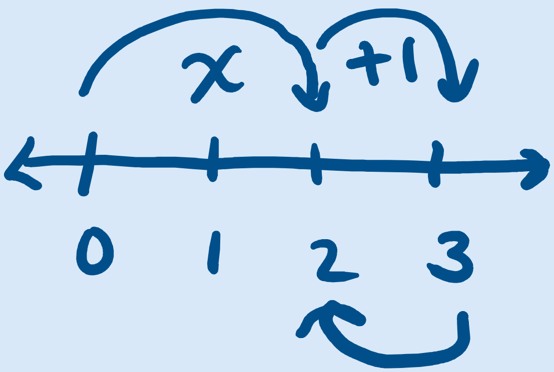


$$3-1=2$$

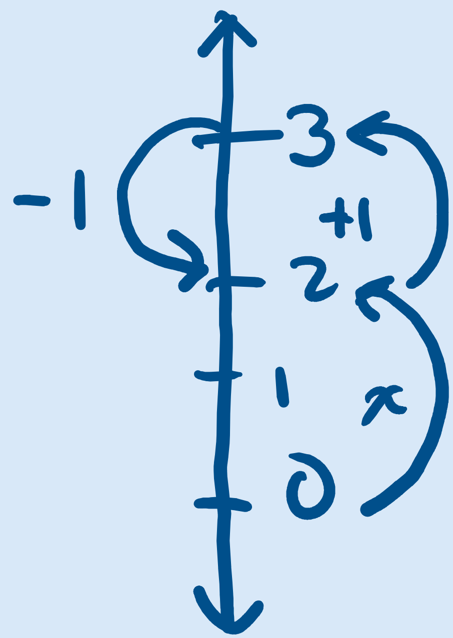
$$x=3-1$$

With a number line, if you start and then add one and end up at three, then you would subtract or go back one to get back to where you started.

$x+1=3$



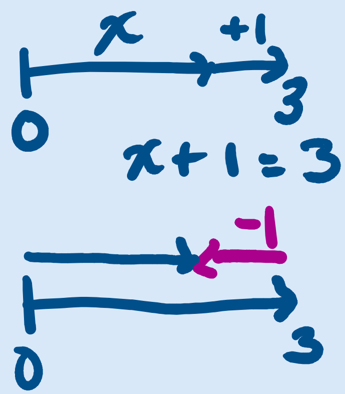
$3-1=x$



$x+1=3$

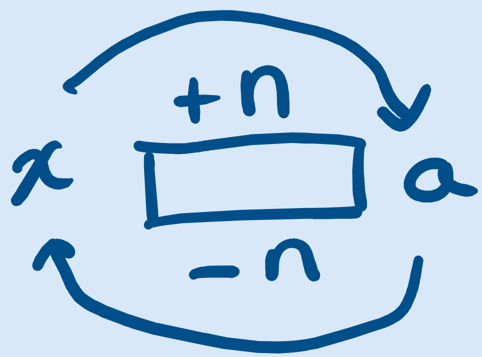
$x=3-1$

With vectors, if you add one unit to get a total length of 3, then you would subtract or go in the opposite direction to get back to the original length.



$$3-1=x$$

As a function, or process, if you add a number to an input, you would subtract that number from the output to get back to the input.



$$x + n = a$$
$$x = a - n$$

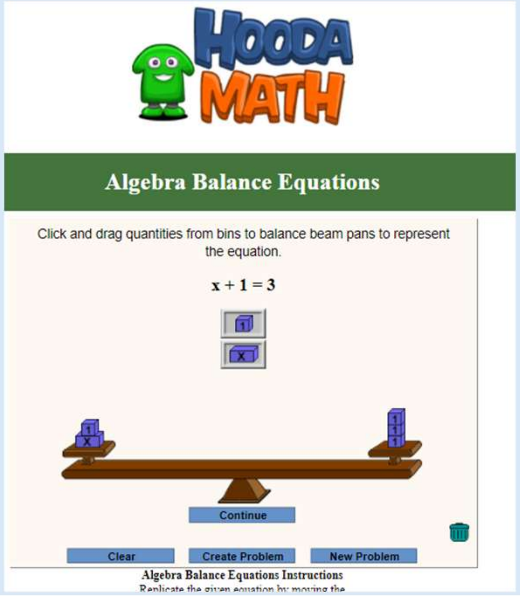
Balance scale

Some people visualize equations as a balance scale with both sides equal and you have to keep both sides equal. That means that whatever you do to one side, you have to do to the other.

With that model, if you have x+1=3 then you subtract 1 from each side to make the 1 go away and you are left with x=3-1 or x=2

[Algebra Balance Equations (hoodamath.com)](hoodamath.com)

$$\begin{array}{r} x+1=3 \\ \underline{\;\;-1=-1} \\ x+0=3-1 \\ x=2 \end{array}$$



Algebraically, we would write this with equations. The one on the right is a **literal equation** as it uses all letters to represent the general process or formula.

$$\begin{aligned} x + 1 &= 3 \\ -1 &= -1 \\ \hline x + 0 &= 3 - 1 \\ x &= 2 \end{aligned}$$

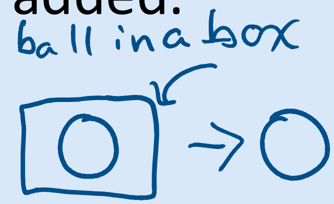
$$\begin{aligned} x + n &= a \\ -n &= -n \\ \hline x + 0 &= a - n \\ x &= a - n \end{aligned}$$

Unwrapping

You can also think of it as unwrapping a present by undoing what was done to the variable. To undo, you do the inverse operation on the other side. If someone took the x and added 1 to it, and got 3, to unwrap the x, you would take away the one that got added.

x+1=3 then x=3-1

$$x+n=a$$
$$x=a-n$$



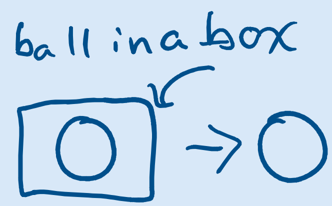
If I had on a t-shirt and then put on a jacket, to show what is on the t-shirt underneath, I would take off the jacket. That is a more generic way of thinking about undoing what was done to the input.

t + jacket = dressed
t = dressed - jacket

Unwrapping

If I had on a t-shirt and then put on a jacket, to show what is on the t-shirt underneath, I would take off the jacket. That is a more generic way of thinking about undoing what was done to the input.

$$t + \text{jacket} = \text{dressed}$$
$$t = \text{dressed} - \text{jacket}$$



$$x + n = a$$
$$x = a - n$$

Moving pieces

From that model of unwrapping a present, I think of it as moving the pieces to the side that I want them on to untangle the puzzle. When they move, I cross them out on the original side and write them as the inverse on the other side.

$x + \cancel{1} = 3 - 1$

$x + \cancel{n} = a - n$

$x = a - n$

$x+1=3$

$x\cancel{+1}=3-1$

By hand, I do a diagonal slash so that I can see it and don't forget that it moved already. This is useful when you are doing multiple step problems by hand.

How I visualize and write the step for solving the equation. This is useful when it is multistep.

$$x + \cancel{n} = a - n$$

$$x = a - n$$

Here x is our variable, in this case it is an unknown quantity. We can come up with a formula to tell the solution for any problem like this.

The unknown number plus a number equals an answer.

$$x + n = a$$

$$x = a - n \quad \text{or} \quad x = -n + a$$

Practice: You can use a calculator or just write it out without doing the calculation.

$x+1=5$ $\quad$ $x+2=6$ $\quad$ $x+32=53$ $\quad$ $x+28=345$

To see if you did it correctly, just plug in the result for the x and make sure it works.

$$x+n=a$$
$$x=a-n$$

$$4+1\overset{\checkmark}{=}5$$
$$x+1=5$$
$$x=5-1$$
$$x=4$$

$$x+2=6$$

$$x+32=53$$

$$x+28=345$$

```
Print("for x+n=a, please enter n")
Input(n)
Print("for x+n=a, please enter a")
Input(a)
Print("x+"n"="a", x=a-n)
```

Literal equations are very useful for programming a computer to do the calculations for us! Once we do the algebra and see the pattern, then we can enter the formula into the computer.

What about if we have our unknown number and we **subtract** a number to get an answer?

$$x - 1 = 2$$

Possible ways of thinking about or visualizing the problem.

Groups

Number line

Vectors

Balance scales

Function or process

Other?

If you have a stain on your shirt, you would wash it off to remove it.

Thinking about going up and down steps. If you are at step x and you go up n steps to step a. From step a you would go down n steps to get back to step x.

With subtraction, you do addition to undo the subtraction.

$x-1=2$

You would add 1 to the 2 to solve for x.

For $x - n = a$

$x=a+n$

Do you see why that happens?

Which of the models or ways of thinking about it did you use?

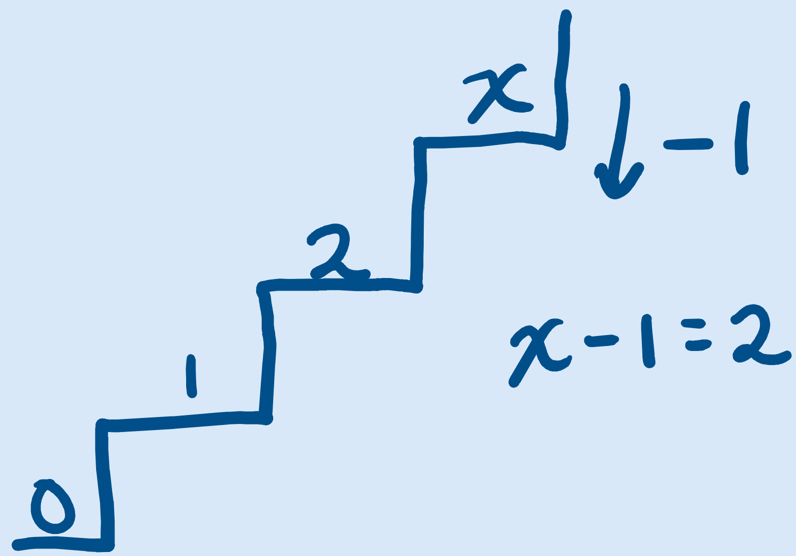
x-1=2 Can think of starting at step x and going down one step and getting to the second step. To find step x, we go back up one step from the second step.

step x down 1=step 2

x - 1 = 2

to get back to the original step from 2 we would go back up a step

x = 2 +1



With subtraction, you do addition to undo the subtraction.

For $x - n = a$

$x=a+n$

Do you see why that happens?

Which of the models or ways of thinking about it did you use?

Reach out if you need more help with this concept as we will move on.

Literal equations

Literal equations use letters or all variables to generalize what is happening. Most formulas are literal equations. Being able to solve literal equations means that you really understand the process. Then you can generalize any problem and program a computer to do the work for you.

One step linear literal equations

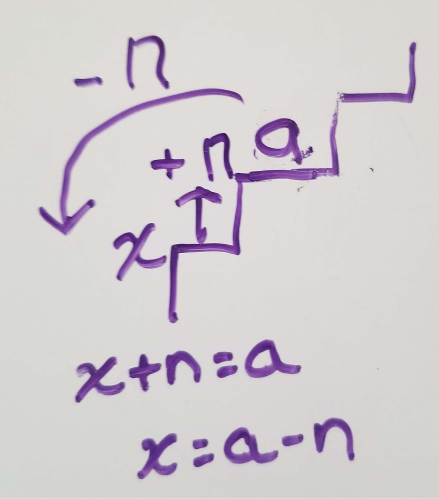
x + some number = the answer

x+n=a

x=a-n would be the formula for the solution

input + n = output

output – n = input



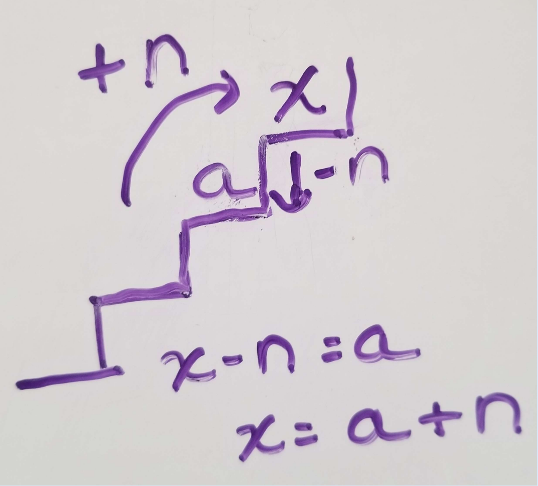
x – some number = the answer

x-n=a

x=a+n would be the formula for the solution

input – n = output

output + n = input



Literal equations

$x + n = a$

$x = a - n$

$x - n = a$

$x = a + n$

Checking the solution

If you have an equation in one variable, you can substitute the solution back into the original equation to make sure that it checks or works.

$x+2=3$

$x=1$

$1+2=3$ so that is correct

If I had gotten 5 by mistake because I added the 2 to the 3 instead of subtracting it, then:

$5+2$ does not equal 3 so I messed up.

To check and make sure you did it correctly, you can substitute the answer for x and make sure it works. x-1=2

$x = 2 + 1$

$x = 3$

$3 - 1 = 2$ ✓

If you have an equation in one variable, you can substitute the solution back into the original equation to make sure that it checks or works.

$x+n=a$

$x=a-n$

$a-n+n = a$ so that is correct

If I had gotten a+n by mistake because I added the n to the a instead of subtracting it, then:

$x+n=a$

$a+n+n = a + 2n$ does not equal a so I messed up.

Plugging in values to check literal equations.

$x+n=a$

$x=a-n$

let n be 2 and a be 3

$x=3-2=1$

$1+2=3$ ✓

going back to the original problem

$1+2=3$ and it checks

Solution sets

We say that the solutions to an equation are the solution set.

For a linear equation in one variable, you can have:

One unique solution

$x+2=3$, $x=1$

No solution or the null set $\emptyset$

$x+1=x+2$ or $1=2$

Infinitely many solutions or all Real numbers $\mathbb{R}$

$x=x$ or $0=0$ or $2=2$

Do you see a pattern for when you can get no solution or infinitely many solutions?

Checking literal equations by doing proofs

Doing out the steps to prove that what you are doing works is called a **mathematical proof**.

The different operations and steps that we follow have different definitions or properties. We can do out the steps and label the what we did to prove a literal equation.

x+n=a given

x+n-n=a-n subtraction property of equality (I can add the same thing to both sides and it is still equal)

x+0=a-n additive identity property of zero

x=a-n

In geometry or doing math theory, we write out the proofs. In algebra, we usually just write out steps.

Browser windows now will give you the solution. Try it. Type x+1=3 in your search engine and see what you get!

Here is one that lets you solve for a variable

[Solve For a Variable Calculator - Symbolab]