Number systems

Different civilizations and cultures had and have different number systems.

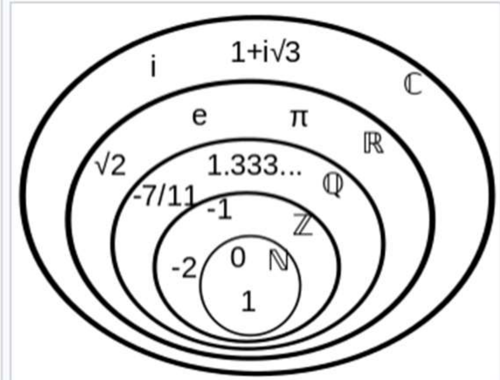
Our number system is base ten or mod ten. This means that we go from zero to nine and then we put a one in the tens place and start over again up to 2 tens then up to one hundred etc.

We think we started with counting numbers historically, whole or natural numbers, then we needed the negative counting numbers with zero in the middle.

We talked about this with mathcounting and place value in UAM 1.4 slides 4 to 8.

A is a subset of B

There are sets of such mathematical importance, to which mathematicians refer so frequently, that they have acquired special names and notational conventions to identify them.



The natural numbers $\mathbb{N}$ are contained in the integers $\mathbb{Z}$, which are contained in the rational numbers $\mathbb{Q}$, which are contained in the real numbers $\mathbb{R}$, which are contained in the complex numbers $\mathbb{C}$

Many of these important sets are represented in mathematical texts using bold (e.g. $\mathbf{Z}$) or blackboard bold (e.g. $\mathbb{Z}$) typeface.[38] These include:[14]

- $\mathbf{N}$ or $\mathbb{N}$, denoting the set of all natural numbers: $\mathbf{N} = \{0, 1, 2, 3, \ldots\}$ (some authors exclude 0)[38]
- $\mathbf{Z}$ or $\mathbb{Z}$, denoting the set of all integers (whether positive, negative or zero): $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$[38]
- $\mathbf{Q}$ or $\mathbb{Q}$, denoting the set of all rational numbers (that is, the set of all proper and improper fractions): $\mathbf{Q} = \{a/b \mid a, b \in \mathbf{Z}, b \neq 0\}$, for example, $1/4 \in \mathbf{Q}$ and $11/6 \in \mathbf{Q}$, and since every integer n can be expressed as the fraction $n/1$, all integers are members of this set ($\mathbf{Z} \subsetneq \mathbf{Q}$)[38]
- $\mathbf{R}$ or $\mathbb{R}$, denoting the set of all real numbers, including all rational numbers, together with all irrational numbers (that is, algebraic numbers that cannot be rewritten as fractions such as $\sqrt{2}$, as well as transcendental numbers such as π, e)[38]
- $\mathbf{C}$ or $\mathbb{C}$, denoting the set of all complex numbers: $\mathbf{C} = \{a + bi \mid a, b \in \mathbf{R}\}$, for example, $1 + 2i \in \mathbf{C}$[38]

Each of the above sets of numbers has an infinite number of elements, and each can be considered to be a proper subset of the sets listed below it.

Sets of positive or negative numbers are sometimes denoted by superscript plus and minus signs, respectively. For example, $\mathbb{Q}^+$ represents the set of positive rational numbers.

Set (mathematics) - Wikipedia

Historically, with our base ten system, new types of number systems get added.

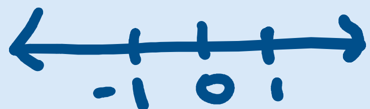
Counting numbers: 1,2,3…

Natural numbers: counting numbers often with zero

Integers: positive and negative counting numbers $\ldots -2, -1, 0, 1, 2 \ldots$

Rational numbers: can be written as a fraction of two integers $\frac{1}{2}$ or $\frac{2}{1}$

Irrational numbers: decimals that go on forever $\pi \approx 3.14\ldots$

Real numbers 

$\mathbb{R}$

Complex numbers

Surreal numbers

With algebra, we usually use the set of Real numbers.

Counting numbers: 1,2,3…

Natural numbers: counting numbers often with zero

Integers: positive and negative counting numbers $\ldots -2, -1, 0, 1, 2 \ldots$

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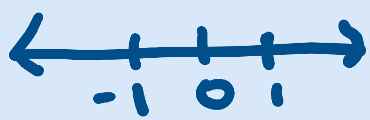
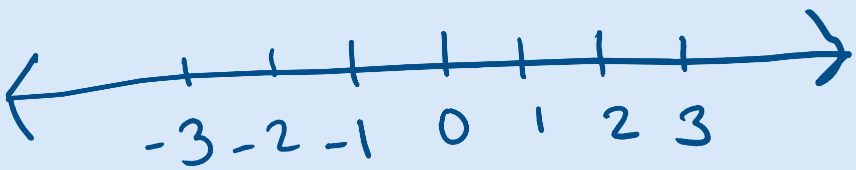
Real numbers 

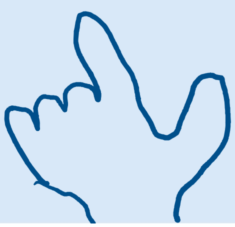
$\mathbb{R}$

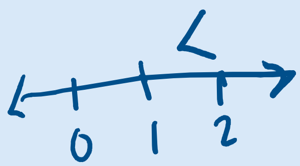
When we do algebra, we usually use the set of Real numbers. We can represent the Real numbers on a number line.

A horizontal number line has zero in the middle and then the positive numbers get bigger to the right and the negative numbers get bigger to the left.

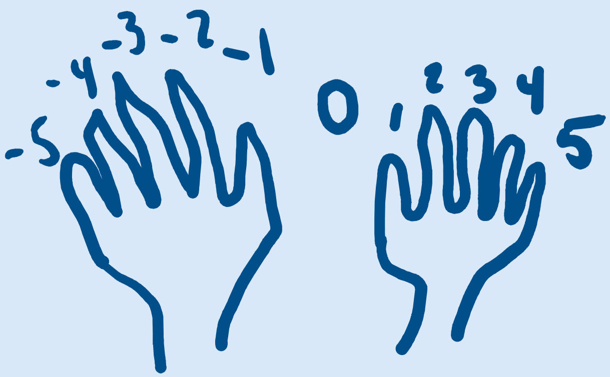


Warning! The base 10 number system writes the larger numbers to the left, but the number line has the larger numbers to the right. With my brain wiring I get my right and left confused. If this is an issue for you, there is a trick. Left, L less than.

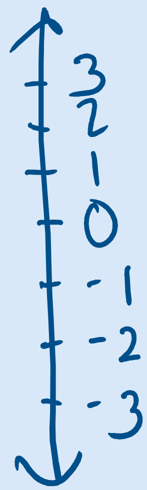




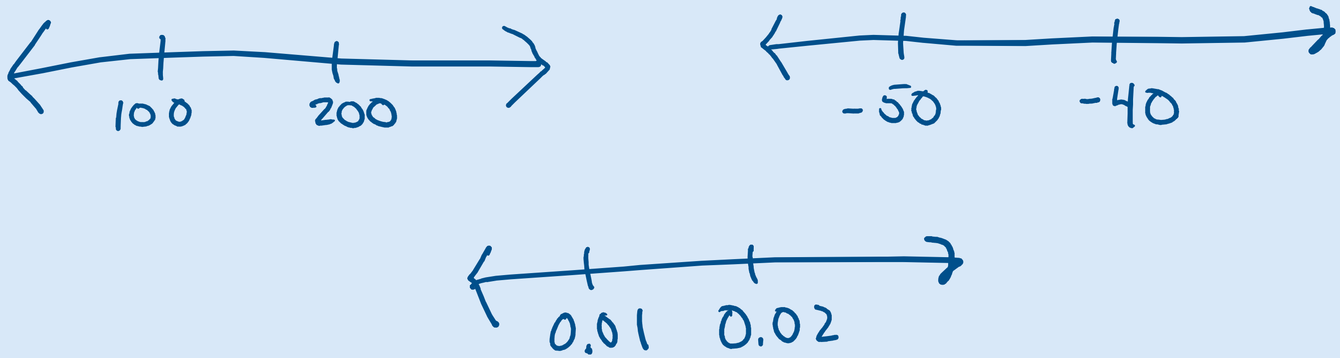
You can use your fingers to practice integers in the direction of the number line.



On a vertical number line, the positive numbers get bigger going up and the numbers get more negative going down. This is easier for people like me.

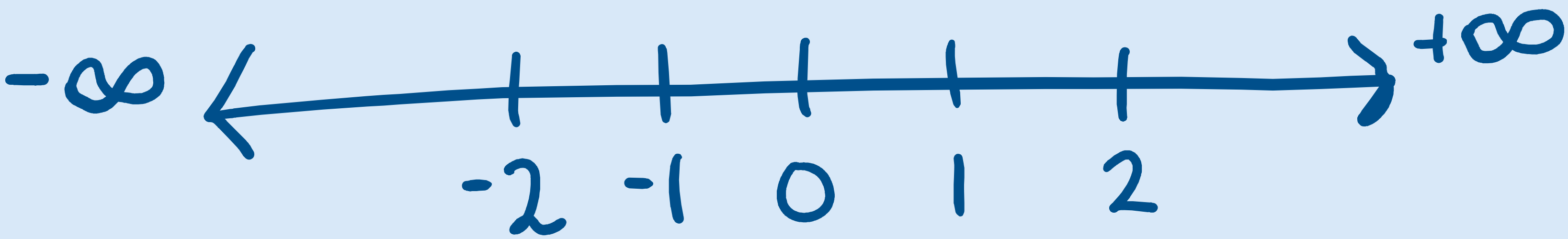


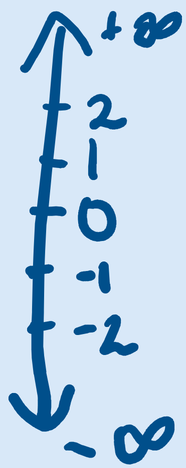
A number line can be zoomed in or zoomed out and you can show a piece of a number line. I will usually show the integers right around zero as that is much easier for the brain to process.



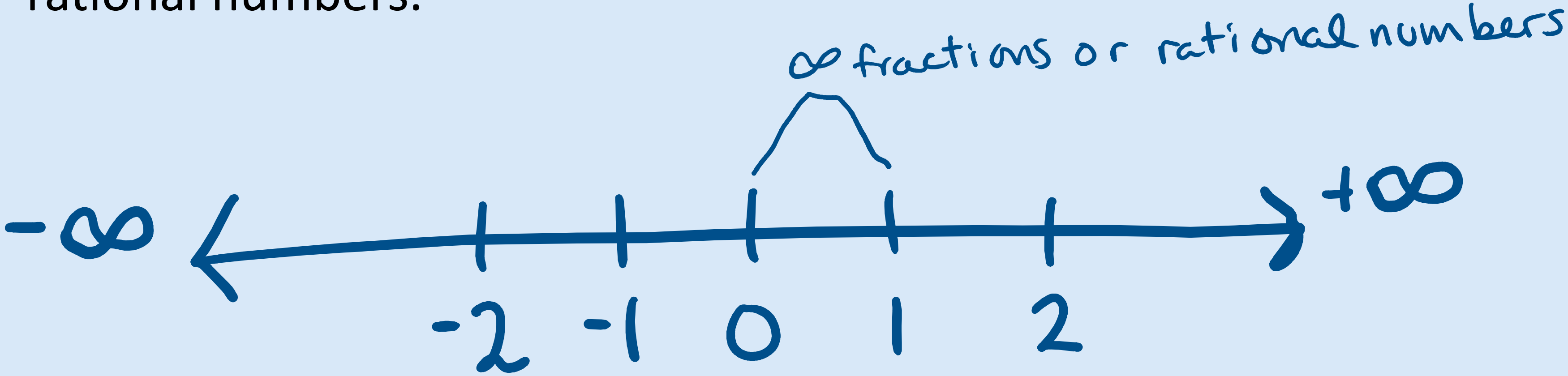
On a number line there is infinity in both directions, and we say positive infinity and negative infinity.

$+\infty$ $-\infty$

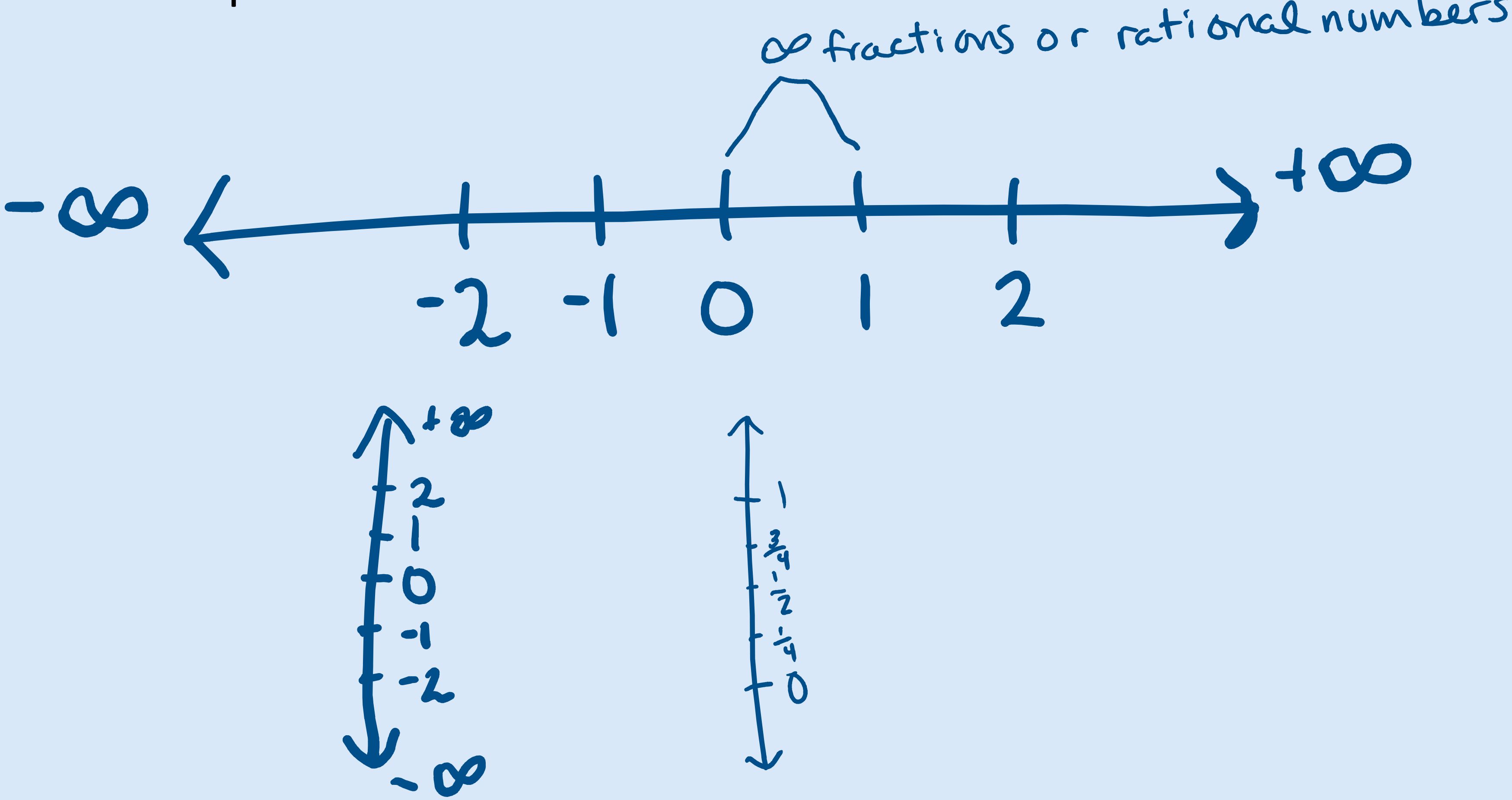




In between any two of the numbers shown, there are infinitely many rational numbers.



Rational numbers can be written as a ratio of two integers. We also call them simple fractions.



In between any rational numbers, there are infinitely many irrational numbers. These can’t be written as a fraction or a ratio of integers.

We will talk about these irrational numbers as they come and the ones that we care about are pi, the square roots, other roots, and e.

π $\sqrt{2}$ $\sqrt{5}$ $\sqrt[3]{10}$ $e \approx 2.7$

Because we can’t write them as a fraction and they keep going on infinitely as decimals, we have special names and symbols for them.

You may have seen pi and square roots.

e (mathematical constant) – Wikipedia is the base for the exponential function which is the fixed point for the derivative and integral in calculus. So we will learn about *e* then.