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Desarrollo matematico Parcial 1 SyS

① $x(t) = 20 \sin(7t - \pi/2) - 3\cos(5t) + 2\cos(10t)$

5 bits $[-3.3 \text{ a } 5]\,v \rightarrow$ Entrada ADC

$-20\cos(7t) - 3\cos(5t) + 2\cos(10t)$

$\omega_1 = 7$ $T_1 = (2/7)\pi$ $F_1 = (7/2\pi) = 1{,}114$ Hz
$\omega_2 = 5$ $T_2 = (2/5)\pi$ $F_2 = (5/2\pi) = 0.796$ Hz
$\omega_3 = 10$ $T_3 = (2/10)\pi$ $F_3 = (10/2\pi) = 1{,}592$ Hz $\rightarrow f_{max}$

$\text{Amplitud}_{max} = 20 + 3 + 2 = 25$ $F_s \geq 2 f_{max}$

$\text{Amplitud}_{min} = -20 - 3 - 2 = -25$ $F_s \geq 2(1{,}592)$

$F_s \geq 3{,}184$ Hz

Cero y pendiente:

$m = \dfrac{5 - (-3.3)}{25 - (-25)} = \dfrac{8.3}{50} \approx 0.166$

$-3.3 = \dfrac{8.3}{50}(-25) + b \quad \therefore \quad b = \dfrac{17}{20}$

$y = \dfrac{8.3}{50} x(t) + \dfrac{17}{20}$

Niveles de cuantizacion:

$N_c = 2^5 = 32$

El voltaje por nivel:

$V/_{niv} = \dfrac{8.3}{32} = 0.2594$

② $x(t) = 3\cos(1000\pi t) + 5\sin(2000\pi t) + 10\cos(11000\pi t)$ $f_s = 5$ KHz

$\omega_1 = 1000\pi$ $T_1 = \dfrac{2}{1000} = \dfrac{1}{500}$ [s] $F_1 = 500$ Hz

$\omega_2 = 2000\pi$ $T_2 = \dfrac{2}{2000} = \dfrac{1}{1000}$ [s] $F_2 = 1000$ Hz

$\omega_3 = 11000\pi$ $T_3 = \dfrac{2}{11000} = \dfrac{1}{5500}$ [s] $F_3 = 5500$ Hz $\rightarrow f_{max}$

$F_3 > F_s$ dado que la frecuencia f_3 es mayor que la frecuencia de muestreo f_s dada por el ejercicio no se cumple el teorema de Nyquist, por ende se necesita replantear el problema tal que:

$f_s \geq 2 f_{max}$ $f_s \geq 2(5500)$ $f_s \geq 11000$ Hz

4)

$$C_n = \frac{1}{T}\int_{t_1}^{tf} x(t)\, e^{-Jnw_0t}\, dt \longrightarrow x(t) = \sum_n C_n e^{Jnw_0t}$$

$$x'(t) = \frac{d}{dt}x(t) = \frac{d}{dt}\left\{\sum_n C_n e^{Jnw_0t}\right\} = \sum_n C_n e^{Jnw_0t}\, Jnw_0$$

$$x''(t) = \frac{\partial}{\partial t}\left\{\sum_n C_n e^{Jnw_0t}(Jnw_0)\right\} = \sum_n C_n e^{Jnw_0t}(Jnw_0t)^2$$

$$\tilde{C}_n = \frac{(x''(t) \cdot e^{Jnw_0t})}{\| e^{Jnw_0t}\|^2} = \int_{t_1}^{tf} \frac{x''(t)\, e^{-Jnw_0t}\, dt}{T} \; ; \; T = tf - t_1$$

$$\tilde{C}_n = C_n(Jnw_0)^2 = \int_{t_1}^{tf} \frac{x''(t)\, e^{-Jnw_0t}}{T}\, dt$$

$$C_n \frac{1}{(tf - t_1)(Jnw_0)^2}\int_{t_1}^{tf} x''(t)\, e^{-Jnw_0t}\, dt = \frac{1}{(t_1 - tf)\, n^2 w_0^2}\, x$$

$$\int_{t_1}^{tf} x''(t)\, e^{-Jnw_0t}\, dt$$

$$x(t) = a_0 + \sum_{n=1}^{N} a_n \cos(nw_0t) + b_n \operatorname{sen}(nw_0t)$$

$$x'(t) = \sum_{n=1}^{N} a_n(-nw_0)\operatorname{sen}(nw_0t) + b_n(nw_0)\cos(nw_0t)$$

$$x''(t) = \sum_{n=1}^{N} a_n(-nw_0)(nw_0)\cos(nw_0t) + b_n(nw_0)(-nw_0)\operatorname{sen}(nw_0t)$$

$$\tilde{a}_n = \frac{2}{T}\int_{t_1}^{tf} x''(t)\cos(nw_0t)\, dt \; ; \; \tilde{b}_n = \frac{2}{T}\int_{t_1}^{tf} x''(t)\operatorname{sen}(nw_0t)\, \partial t$$

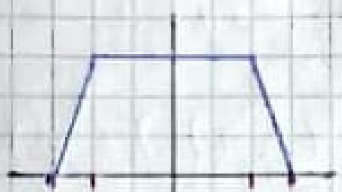
$$a_n(-n^2 w_0^2) = \frac{2}{T}\int_{t_1}^{tf} x''(t)\cos(nw_0t)\, \partial t$$

$$a_n = \frac{2}{-Tn^2 w_0^2}\int_{t_1}^{tf} x''(t)\cos(nw_0t)\, dt$$

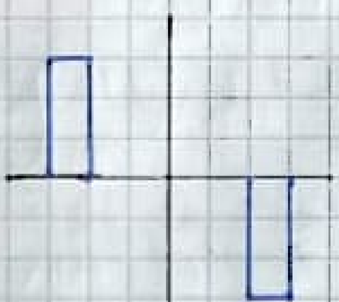
$$b_n(-n^2 w_0^2) = \frac{2}{T}\int_{t_i}^{t_f} x''(t)\,\text{sen}(n w_0 t)\,dt$$

$$b_n = \frac{2}{-T n^2 w_0^2}\int_{t_i}^{t_f} x''(t)\,\text{sen}(n w_0 t)\,dt$$

➤ Espectro de Fourier, su magnitud y error relativo para $n \in \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$ a partir de $x''(t)$ para la señal $x(t)$ en la figura 1. Compruebe el espectro obtenido con la estimación a partir de $x(t)$

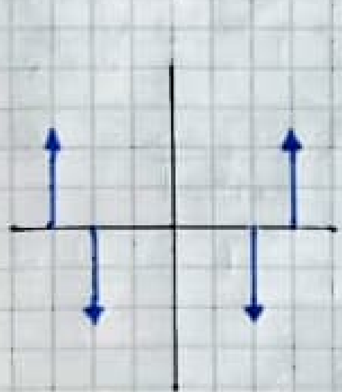


$$x''(t) = A\delta(t+d_2) - A\delta(t+d_1) - A\delta(t-d_1) + A\delta(t-d_2)$$



$$C_n = \frac{1}{-T n^2 w_0^2}\int_{-T/2}^{T/2} x''(t)\,e^{-jnw_0 t}\,dt$$

$$C_n = -\frac{1}{T n^2 w_0^2}\int_{-T/2}^{T/2} A[\delta(t+d_2) - \delta(t+d_1) - \delta(t-d_1) + \delta(t-d_1)]\,e^{-jnw_0 t}\,dt$$



$$C_n = \frac{A}{T n^2 w_0^2}\left(e^{-jnw_0(-d_2)} - e^{-jnw_0(-d_1)} - e^{-jnw_0 d_1} + e^{-jnw_0 d_2}\right)$$

$$C_n = \frac{-A}{T n^2 w_0^2}\left(e^{jnw_0 d_2} + e^{-jnw_0 d_2} - \left(e^{-jnw_0 d_1} + e^{-jnw_0 d_2}\right)\right)$$

$$C_n = \frac{-A}{T n^2 w_0^2}\left(2\cos(w_0 d_2) - 2\cos(n w_0 d_1)\right) = \frac{-2A}{T n^2 \frac{4\pi^2}{T}}\ x$$

$$\left(\cos\left(n\frac{2\pi}{T}d_2\right)-\cos\left(n\frac{2\pi}{T}d_1\right)\right)$$

$$C_n = -\frac{A}{T}\left(2\cos\left(\frac{n2}{T}\pi d_2\right)-\cos\left(n\frac{2\pi}{T}d_1\right)\right)$$

$$C_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt = \frac{1}{T}\int_{-d_2}^{-d_1}\frac{A}{d_2-d_1}(t+d_2)\,dt$$

$$+\frac{1}{T}\int_{-d_1}^{d_1}A\,dt+\frac{1}{T}\int_{d_1}^{d_2}-\frac{A}{d_2-d_1}(t-d_2)\,dt$$

$$= \frac{1}{T}\left[\frac{A}{d_2-d_1}\left(\frac{t^2}{2}+d_2t\right)\Big|_{-d_2}^{-d_1}+At\Big|_{-d_1}^{d_1}-\right.$$

$$\left.\frac{A}{d_2-d_1}\left(\frac{t^2}{2}-d_2t\right)\Big|_{d_1}^{d_2}\right]$$

$$= \frac{1}{T}\left[\frac{A}{d_2-d_1}\left(\frac{d_1^2}{2}-d_1d_2-\frac{d_2^2}{2}+d_2d_2\right)\right.$$

$$\left.+A(d_1+d_1)-\frac{A}{d_2-d_1}\left(\frac{d_2^2}{2}-d_2d_2-\frac{d_1^2}{2}+d_1d_2\right)\right]$$

$$= \frac{1}{T}\left[\frac{2A}{d_2-d_1}\left(\frac{d_1^2}{2}-d_1d_2-\frac{d_2^2}{2}+d_2^2\right)+2Ad_1\right]$$

Si $A=1$ $d_1=1$ $d_2=2$ y $T=2d_2=4$

$$C_n = -\frac{1}{\frac{2\pi^2}{2}n^2}\left(\cos\left(n\frac{2\pi}{4}2\right)-\cos\left(n\frac{2\pi}{4}\cdot 1\right)\right)$$

$= \frac{-1}{\frac{\pi 2}{2} n^2} \left(\cos(n\pi) - \cos\left(n \frac{\pi}{2}\right)\right)$

$C_0 = \frac{1}{4} \left[\frac{2 \cdot 1}{1} \left(\frac{1}{2} - 2 - 2 + 4 \right) + 2 \cdot 1 \cdot 1 \right]$

$= \frac{1}{4} \left[2 \left(\frac{1}{2} \right) + 2 \right] = \frac{3}{4}$

$P_x = \frac{1}{T} \int_{-T/2}^{T/2} (x(t))^2 dt = \frac{2}{T} \int_{-T/2}^{0} (x(t))^2 dt$

$= \frac{2}{T} \int_{-d_2}^{-d_1} \left(\frac{A}{d_2 - d_1} \right)^2 (t + d_2)^2 dt +$

$\frac{2}{T} \int_{-d_1}^{0} A^2 dt \rightarrow P_x = \frac{2}{T} \left(\frac{A}{d_2 d_1} \right)^2$

$\left(t^2 + 2 t d_2 + d_1^2 \right) \Big|_{-d_2}^{-d_1} + \frac{2}{T} A^2 t \Big|_{-d_1}^{0}$

$P_x = \frac{2}{T} \left(\frac{A}{d_2 - d_1} \right)^2 \left(d_1^2 - 2 d_2 d_1 + d_2^2 - d_2^2 \right.$

$\left. + 2 d_2 - d_2^2 \right) + \frac{2}{T} A^2 (0 + d_1)$

$P_x = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$

Demostracion Punto 3

Los intervalos de $x_2(t)$, se tiene que dividir la integral en 3 partes

$$\bar{P}_{p2-p2} = \lim_{T \to \infty} \frac{1}{T}\left[\int_0^{T/4} |A\cos(w_0 t) - 1|^2\, dt + \int_{T/4}^{3T/4} |A\cos(w_0 t) + 1|^2\, dt \int_{3T/4}^{T} |A\cos(w_0 t) - 1|^2\, dt\right]$$

Utilizamos factorizacion y los terminos de cada una

$$\bar{P}_{x1-x1} = \lim_{T \to \infty} \frac{1}{T}\left[\int_0^{T/4} (A^2\cos^2(w_0 t) - 2A\cos(w_0 t) + 1)\, dt + \int_{T/4}^{3T/A} (A^2\cos^2(w_0 t) + 2A\cos(w_0 t) + 1)\, dt\right.$$

$$\left. + \int_{3T/4}^{T} A^2\cos^2(w_0 t) - 2A\cos(w_0 t)\, dt\right]$$

Resolvemos primero para $0 \leq t < \frac{T}{4}$

$$\int_0^{T/4} (A^2\cos^2(w_0 t) - 2A\cos(w_0 t) + 1)\, dt \to \cos^2(w_0 t) = \frac{1 + \cos(2w_0 t)}{2}$$

$$A^2\cos^2(w_0 t) = \frac{A^2}{2}(1 + \cos(2wt))$$

$$\frac{A^2}{2}\int_0^{T/4} 1\, dt + \frac{A^2}{2}\int_0^{T/A} \cos(2w_0 t)\, dt - 2A\int_0^{T/A} \cos(w_0 t)\, dt + \int_0^{T/4} 1\, dt$$

Separamos la integral

$$\int_0^{T/4} 1\, dt = \boxed{\frac{T}{4}\ ;} \quad \bullet \int_0^{T/4} \cos(2w_0 t)\, dt = \left[\operatorname{sen}\frac{(2w_0 t)}{2w_0}\right]_0^{T/4} = \frac{1}{2w_0}$$

$\left(\operatorname{sen}\left(\frac{T}{2} w_0\right) - \operatorname{sen}(0)\right)$

$w_0 \to \frac{2\pi}{T}$

$$\frac{1}{2w_0}\left(\left(\operatorname{sen}\left(\frac{T}{2}\frac{2\pi}{T}\right) - \operatorname{sen}(0)\right)\right) \boxed{= 0}$$

$$\int_0^{T/4} \cos(w_0 t)\, dt = \left[\frac{\operatorname{sen}(w_0 t)}{w_0}\right]_0^{T/4} = \frac{1}{w_0}\left(\operatorname{sen}\left(w_0 \frac{T}{4}\right) - \sin(0)\right) = \frac{1}{w_0}$$

$$\left(\operatorname{sen}\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right)\right)$$

$$= \frac{1}{w_0}\left(\operatorname{sen}\left(\frac{\pi}{2}\right)\right) = \frac{1}{w_0} \to \boxed{\frac{T}{2\pi}}$$

$$\int_0^{T/4} 1\, dt = \boxed{\frac{T}{4}}$$

Sustituimos todos los terminos

$$\frac{A^2}{2} \cdot \frac{T}{4} + \frac{A^2}{2} \cdot 0 - 2A \cdot \frac{T}{2\pi} + \frac{T}{4} = \boxed{\frac{A^2}{8}T - \frac{AT}{\pi} + \frac{T}{4}}$$

Luego para $T/4 \leq t \leq 3T/4$

$$\int_{T/4}^{3T/4} \left(A^2 \cos^2(w_0 t) + 2A \cos(w_0 t) + T\right) dt \to A^2 \cos^2(w_0 t) \text{ la } \frac{A^2}{2}\left(1 + \cos(2w_0 t)\right)$$

Separamos la integral

$$\frac{A^2}{2}\int_{T/4}^{3T/4} 1\, dt + \frac{A^2}{2}\int_{T/4}^{3T/4} \cos(2w_0 t)\, dt + 2A\int_{T/4}^{3T/4} \cos(w_0 t)\, dt + \int_{T/4}^{3T/4} 1\, dt$$

Separamos la integral

$$\frac{A^2}{2}\int_{T/4}^{3T/4} 1\,dt + \frac{A^2}{2}\int_{T/4}^{3T/4} \cos(2w_0t)\,dt + 2A\int_{T/4}^{3T/4} \cos(w_0t)\,dt + \int_{T/4}^{3T/4} 1\,dt$$

Evaluamos cada integral

$$\int_{T/4}^{3T/4} 1\,dt = \frac{3T}{4} - \frac{T}{4} = \boxed{\frac{T}{2}} \; ; \quad \int_{T/4}^{3T/4} \cos(w_0t)\,dt = \left[\frac{\sin(2w_0t)}{2w_0}\right]_{T/4}^{3T/4}$$

$$w_0t = \frac{2\pi}{T}, \quad \frac{1}{2w_0}\left[\sin\left(\frac{4\pi}{\lambda}\cdot\frac{3T}{\lambda}\right) - \operatorname{sen}\left(\frac{4\lambda}{\lambda}\;\frac{\lambda}{\lambda}\right)\right] = \boxed{0}$$

$$\int_{T/4}^{3T/4} \cos(w_0t)\,dt = \left[\frac{\sin w_0t}{w_0}\right]_{T/4}^{3T/4} = \frac{1}{\frac{2\pi}{T}}\left[\operatorname{sen}\left(\frac{2\pi}{T}\cdot\frac{3T}{4}\right) - \operatorname{sen}\left(\frac{2\pi}{T}\cdot\frac{T}{4}\right)\right]$$

$$= \frac{1}{\frac{2\pi}{T}}\left[-1-1\right] = -\frac{2T}{2\pi} = \boxed{-\frac{T}{\pi}}$$

Ahora para $3T/4 \leq t \leq T$

$$\int_{3T/4}^{T} \left(A^2\cos^2(w_0t) - 2A\cos(w_0t + 7)\right)dt \rightarrow A^2\cos^2(w_0t) =$$

$$\frac{A^2}{2}\left(1 + \cos(2w_0t)\right)1$$

Separamos la integral

$$\frac{A^2}{2}\int_{3T/4}^{1} 1\,dt + \frac{A^2}{2}\int_{3T/4}^{T} \cos(2w_0t - 2A\int_{3T/4}^{T} \cos(w_0t) + \int_{3T/4}^{T} 1\,dt$$

Evaluamos cada integral

$$\int_{3T/4}^{T} 1\, dt = T - \frac{3T}{4} \boxed{= \frac{T}{4}} \quad ; \quad \cdot \int_{3T/4}^{T} \cos(2w_0 t)\, dt = \left[\operatorname{sen} \frac{(2}{2w_0} w_0 t \right]_{3T/4}^{T}$$