

## ***Induction and Recursive Definition:***

1. Let  $P(n)$  be the statement that  $1^2 + 2^2 + \dots + n^2 = n(n+1)(2n+1)/6$  for the positive integer  $n$ .
  - a) What is the statement  $P(1)$ ?
  - b) Show that  $P(1)$  is true, completing the basis step of the proof.
  - c) What is the inductive hypothesis?
  - d) What do you need to prove in the inductive step?
  - e) Complete the inductive step, identifying where you use the inductive hypothesis.
  - f) Explain why these steps show that this formula is true
  - g) whenever  $n$  is a positive integer.
2. Prove that  $1^2 + 3^2 + 5^2 + \dots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$  whenever  $n$  is a nonnegative integer.
3. Give a recursive definition of the sequence  $\{a_n\}$ ,  $n = 1, 2, 3, \dots$  if
  - a)  $a_1 = 6$ .
  - b)  $a_n = 2n + 1$ .

## ***Counting:***

1. There are 18 mathematics majors and 325 computer science majors at a college.
  - a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
  - b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?
2. How many strings are there of lowercase letters of length four or less, not counting the empty string.
3. Suppose that there are nine students in a discrete mathematics class at a small college.
  - a) Show that the class must have at least five male students or at least five female students.
  - b) Show that the class must have at least three male students or at least seven female students

## ***Advanced Counting Techniques:***

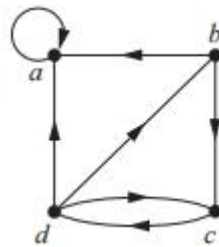
1. Issues:
  - a) Find a recurrence relation for the number of bit strings of length  $n$  that contain three consecutive 0s.
  - b) What are the initial conditions?
  - c) How many bit strings of length seven contain three consecutive 0s?

## ***Graphs (Part I):***

1. What kind of graph (from Table 1) can be used to model a highway system between major cities where.

- a) there is an edge between the vertices representing cities if there is an interstate highway between them?
- b) there is an edge between the vertices representing cities for each interstate highway between them?
- c) there is an edge between the vertices representing cities for each interstate highway between them, and there is a loop at the vertex representing a city if there is an interstate highway that circles this city?

2. Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.



3. Represent following graph with an adjacency matrix.

