Proof:

1. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If Socrates is human, then Socrates is mortal.
Socrates is human.

∴ Socrates is mortal.

2. What rule of inference is used in each of these arguments?
   a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
   b) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

3. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.
   a) "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirlpool."
   b) "Every student has an Internet account." "Homer does not have an Internet account." "Maggie has an Internet account."

4. What is wrong with this argument? Let $H(x)$ be "x is happy." Given the premise $\exists x H(x)$, we conclude that $H($Lola$)$. Therefore, Lola is happy.

5. Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1. $\forall x(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal instantiation from (1)
3. $P(c)$	Simplification from (2)
4. $\forall x P(x)$	Universal generalization from (3)
5. $Q(c)$	Simplification from (2)
6. $\forall x Q(x)$	Universal generalization from (5)
7. $\forall x(P(x) \vee \forall x Q(x))$	Conjunction from (4) and (6)

Relations and Their Properties:

1. List the ordered pairs in the relation R from A={0, 1, 2, 3, 4} to B= 0, 1, 2, 3}, where (a, b) ∈ R if and only if (*lcd:* least common multiple)

a) $a = b$. b) $a + b = 4$. c) $a>b$. d) $a \mid b$. e) $gcd(a, b) = 1$. f) $lcm(a, b) = 2$.

2. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive. (Kennith 2011 Eng. Ver. p.581 No. 3)

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}

3. List the 16 different relations on the set {0, 1}.

4. Which of the 16 relations on {0, 1}, which you listed in Exercise 4, are

a) reflexive?
b) irreflexive?
c) symmetric?
d) antisymmetric?
e) asymmetric?
f) transitive?

5. Represent each of these relations on {1, 2, 3, 4} with a matrix (with the elements of this set listed in increasing order).

a) {(1, 2),(1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}
b) {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)}

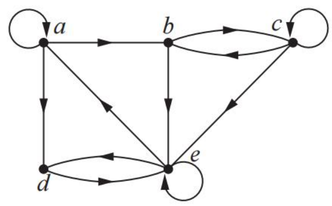
6. Draw the directed graphs representing each of the relations from Exercise 5.

7. Let R be the relation on the set {0, 1, 2, 3} containing the ordered pairs (0, 1), (1, 1), (1, 2), (2, 0), (2, 2), and (3, 0). Find the

a) reflexive closure of R. b) symmetric closure of R.

8. Determine whether these sequences of vertices are paths in this directed graph.

a) a, b, c, e
b) b, e, c, b, e
c) a, a, b, e, d, e
d) b, c, e, d, a, a, b
e) b, c, c, b, e, d, e, d
f) a, a, b, b, c, c, b, e, d



9. Which of these relations on {0, 1, 2, 3} are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) {(0, 0), (1, 1), (2, 2), (3, 3)}
b) {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}

10. Which of these relations on {0, 1, 2, 3} are partial orderings? Determine the properties of a partial ordering that the others lack.

a) {(0, 0), (1, 1), (2, 2), (3, 3)}

b) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}