

# Machine Learning for Turbulence Modeling

Julia Ling

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# Sandia National Labs



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- Sites in Livermore, CA and Albuquerque, NM
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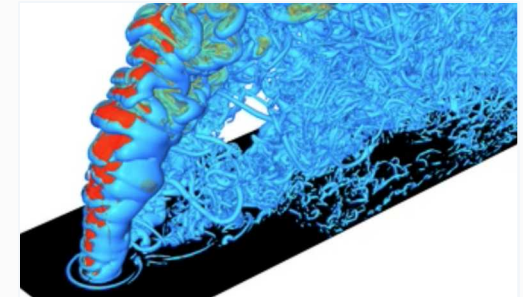
# Machine Learning on Engineering Systems



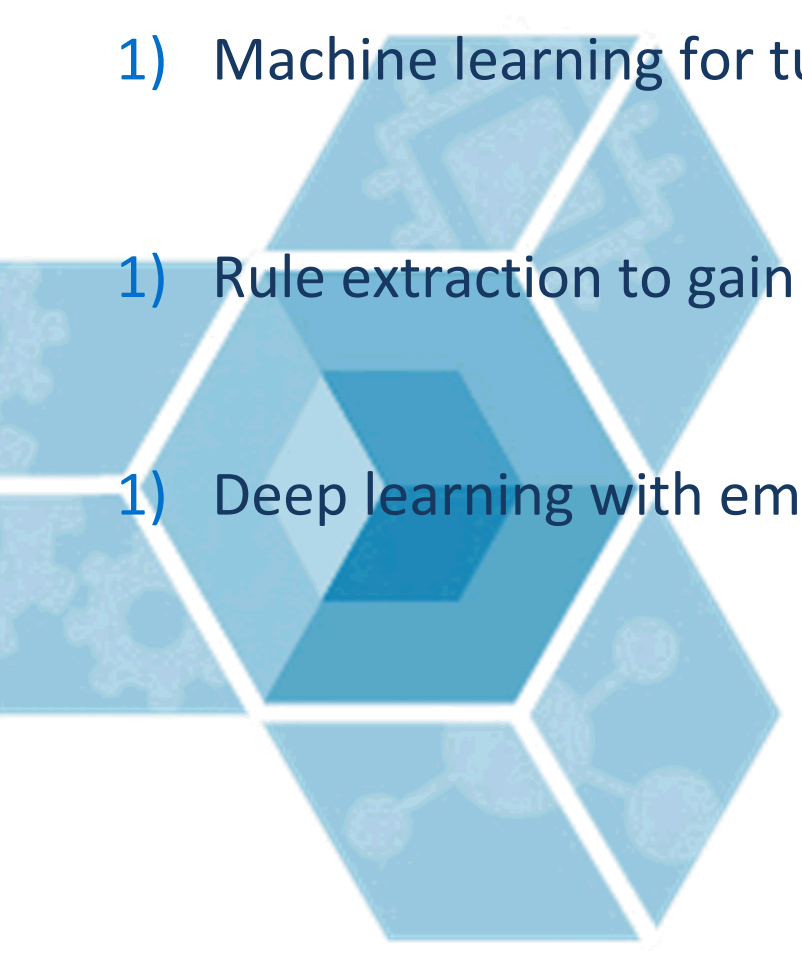
- How should scientific domain knowledge interact with data-driven models?

Physics  
Knowledge

Machine  
Learning  
Models





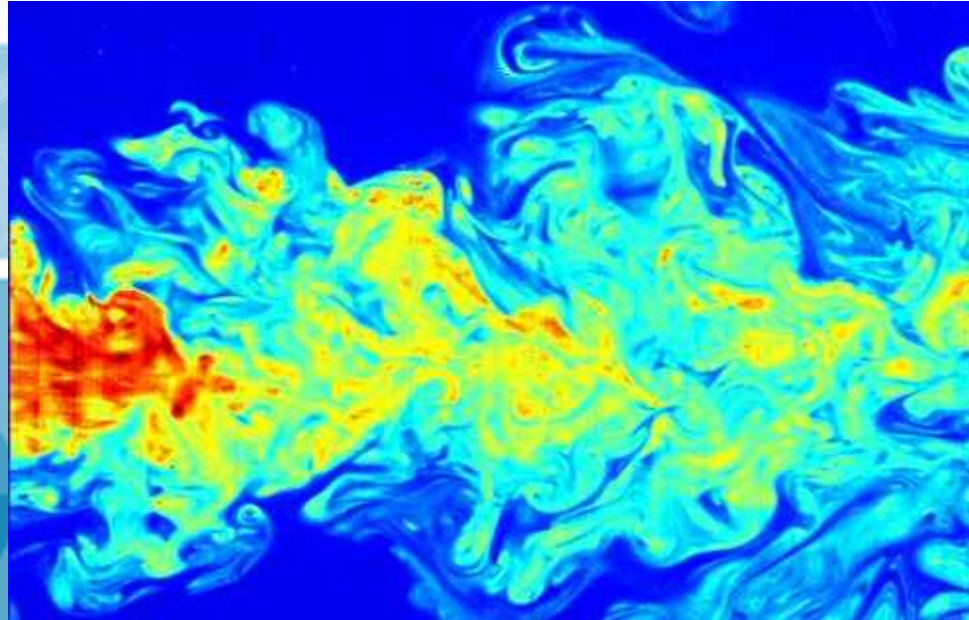
- 
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  - 1) Rule extraction to gain physical intuition
  - 1) Deep learning with embedded invariances



# Turbulence



Chaotic 3-D fluid motion at a continuum of scales



Fukushima et al.



Hokusai (c 1830)



<http://www.windturbinesyndrome.com/2011/wind-turbine-turbulence-what-are-the-micro-climate-effects/>

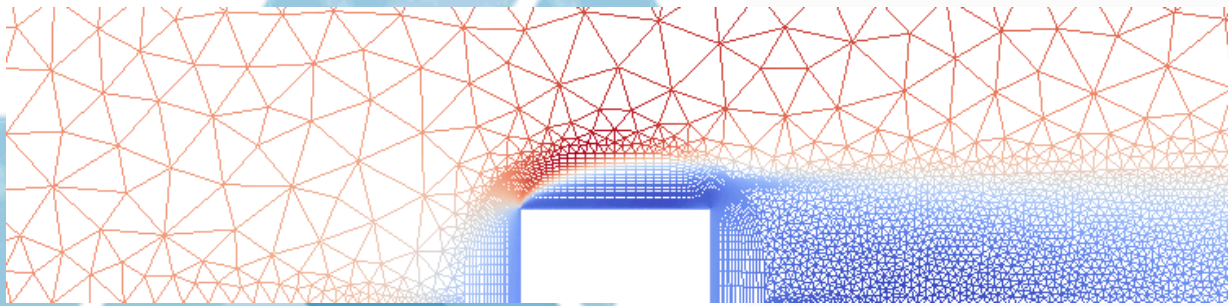


<https://brilliant.org/wiki/rocket-physics/>



# Turbulence Simulations

*“When I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic.” –Horace Lamb*



## Direct Numerical Simulation (DNS)

- Hundreds of millions of core hours for even simple flows
- Exact

## Reynolds Averaged Navier Stokes (RANS)

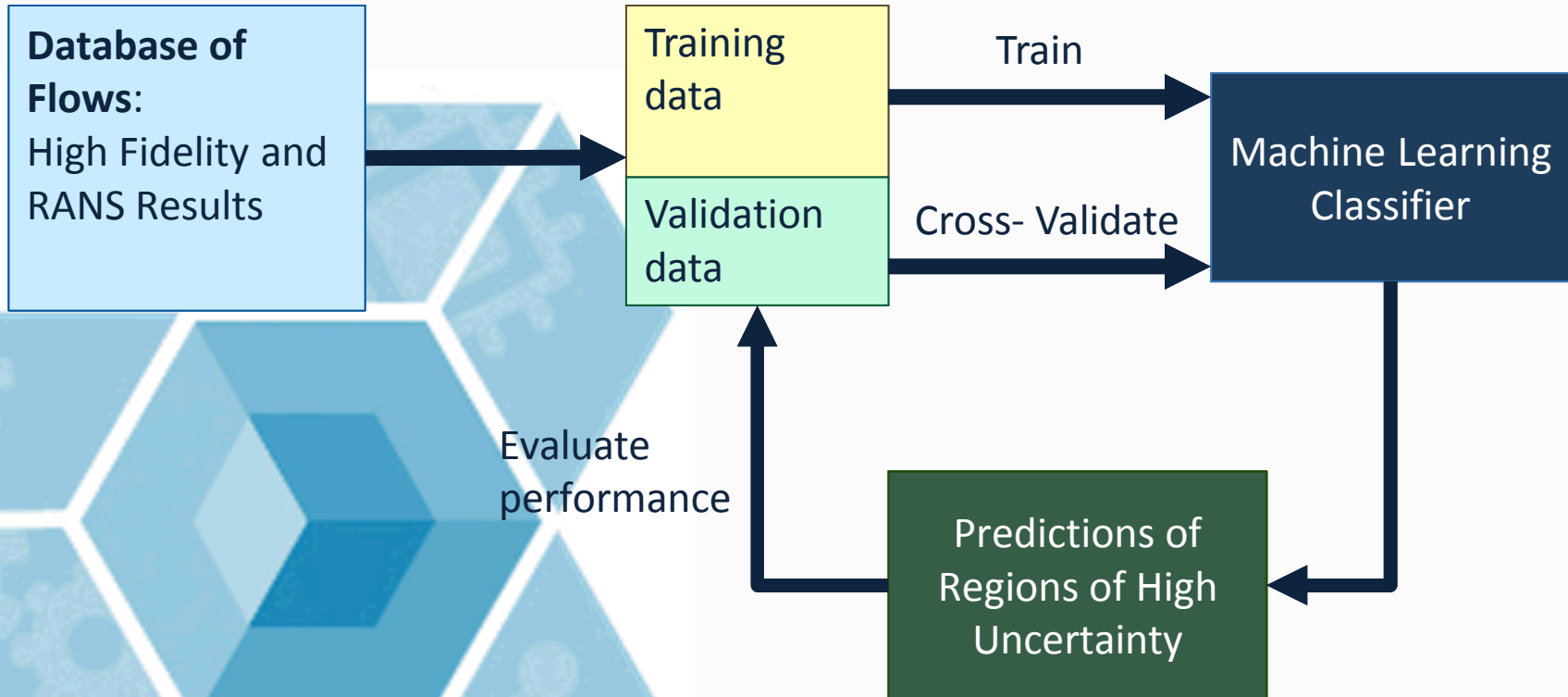
- Orders of magnitude less computationally expensive
- Approximate



# Machine Learning Problem Set-Up

- Assemble a database of flows for which both DNS and RANS results are available
- Use the DNS as “truth” data for supervised machine learning
- Build a binary classifier to predict when RANS has high uncertainty based on when specific model assumptions are violated
- Inputs are local flow variables from RANS (velocity, pressure, density, ...)
  - Each point in the mesh represents a separate data point

# Classifier Development





# Classifier Development



## Database of Flows:

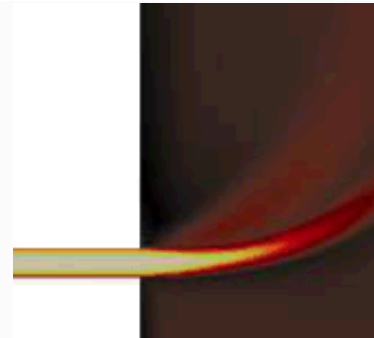
High Fidelity and  
RANS Results

### Contours of velocity magnitude

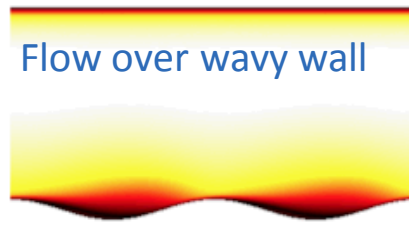
Angled jet in crossflow



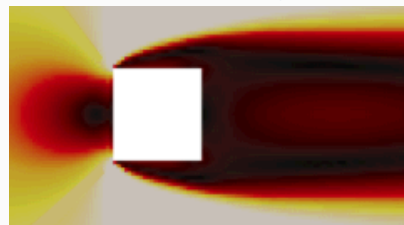
Jet in crossflow



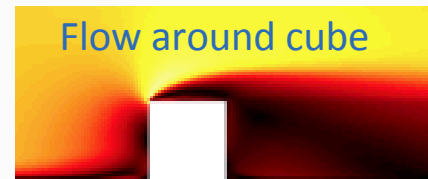
Flow over wavy wall



Flow around square

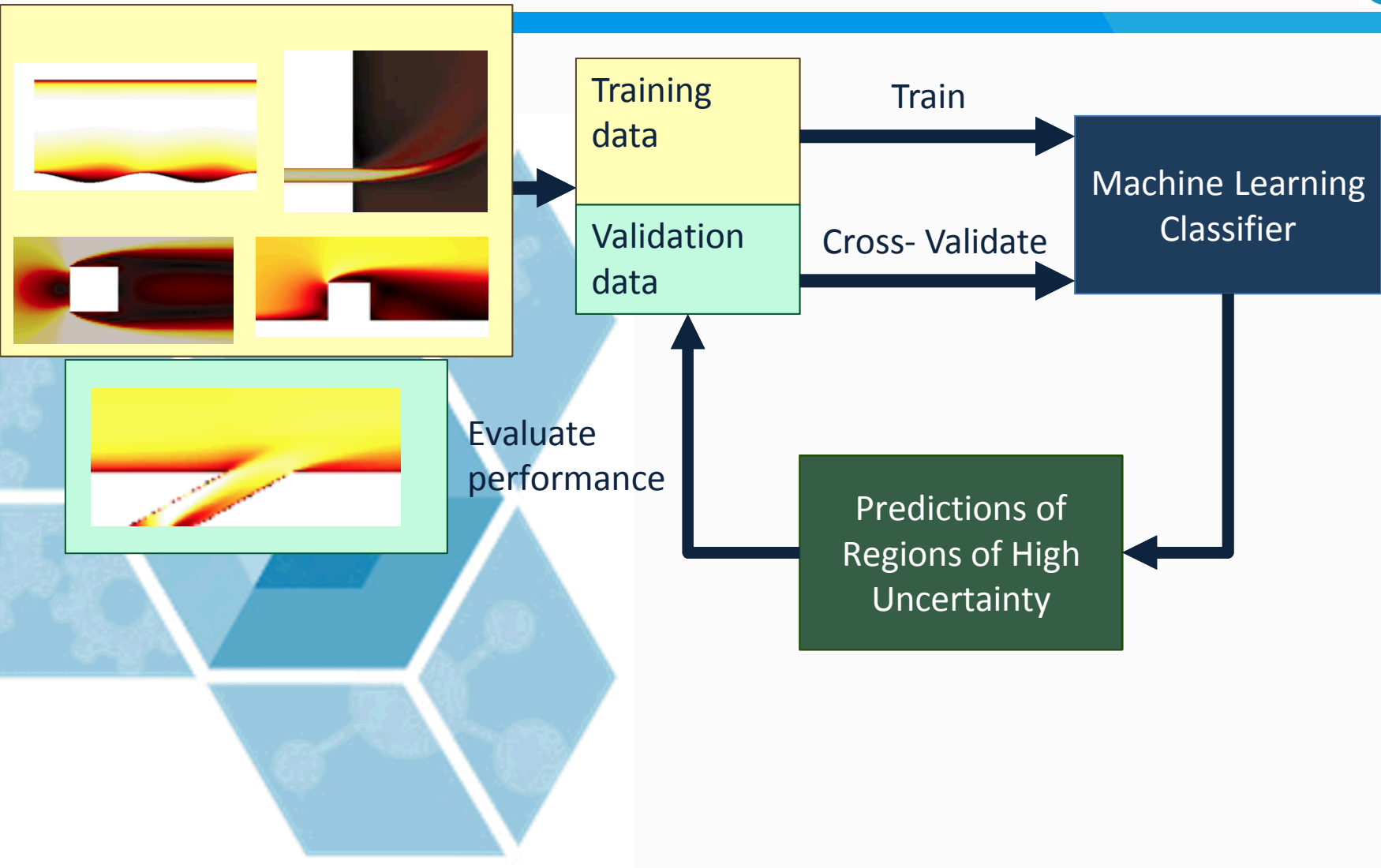


Flow around cube

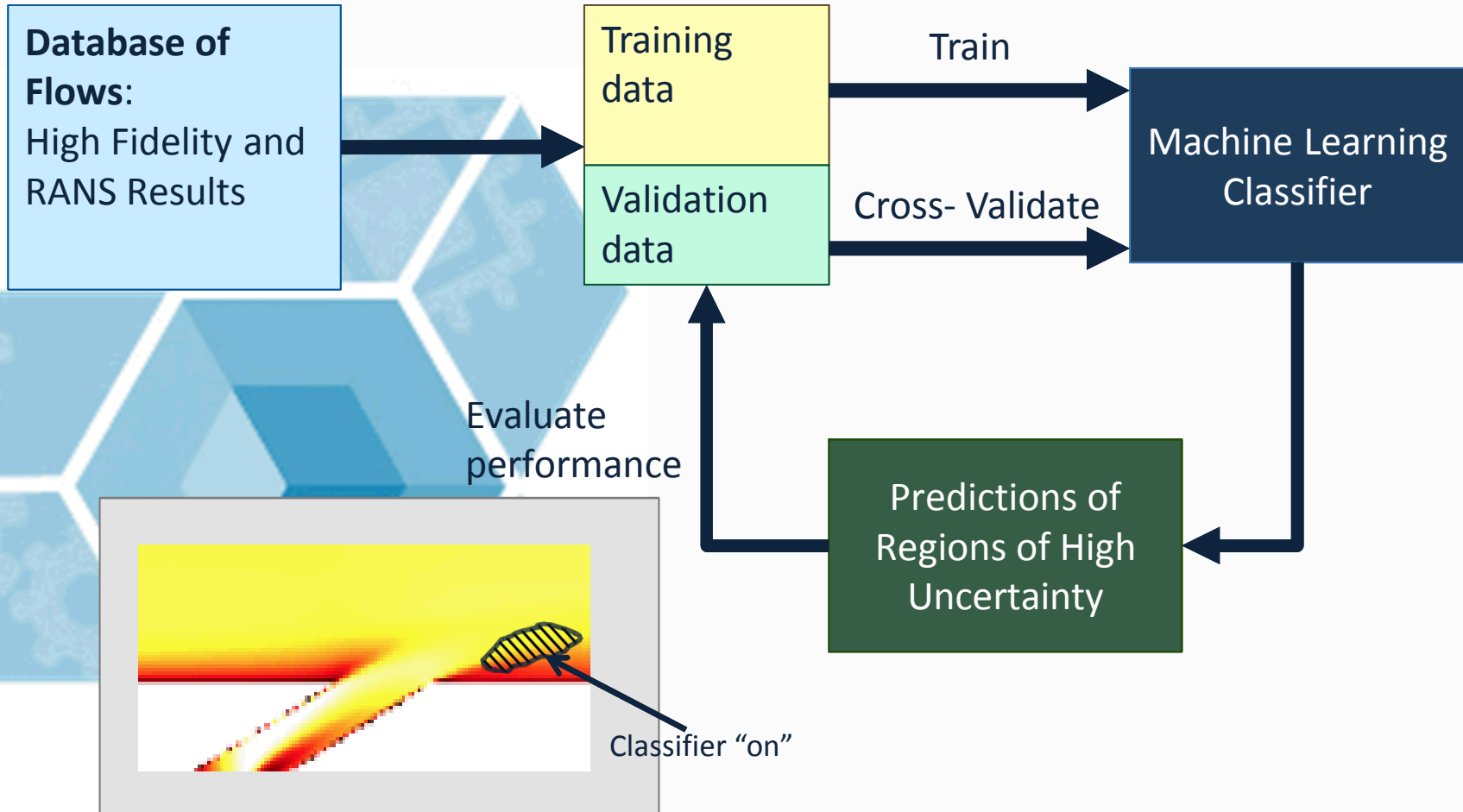


Machine Learning  
Classifier

# Classifier Development

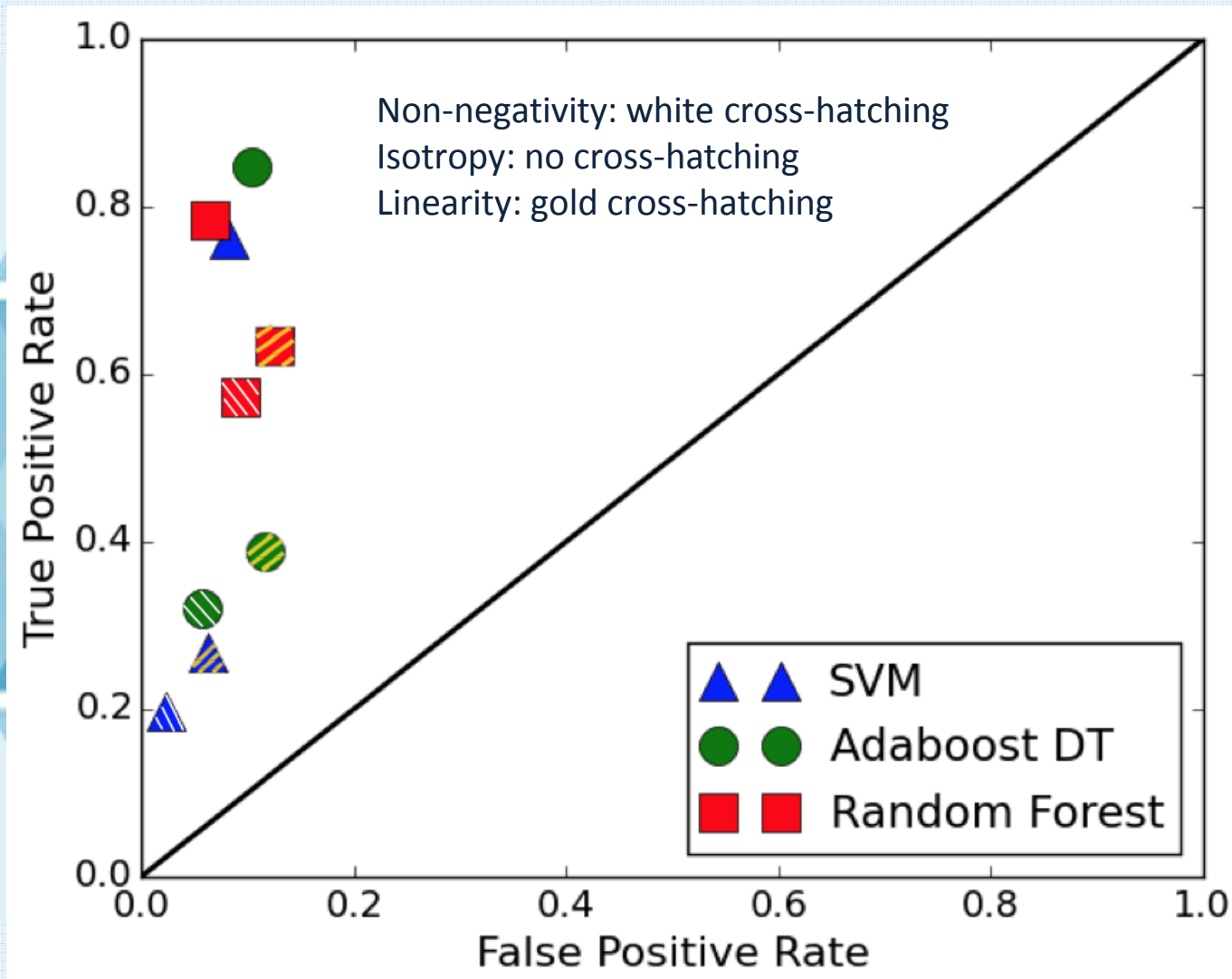


# Classifier Development



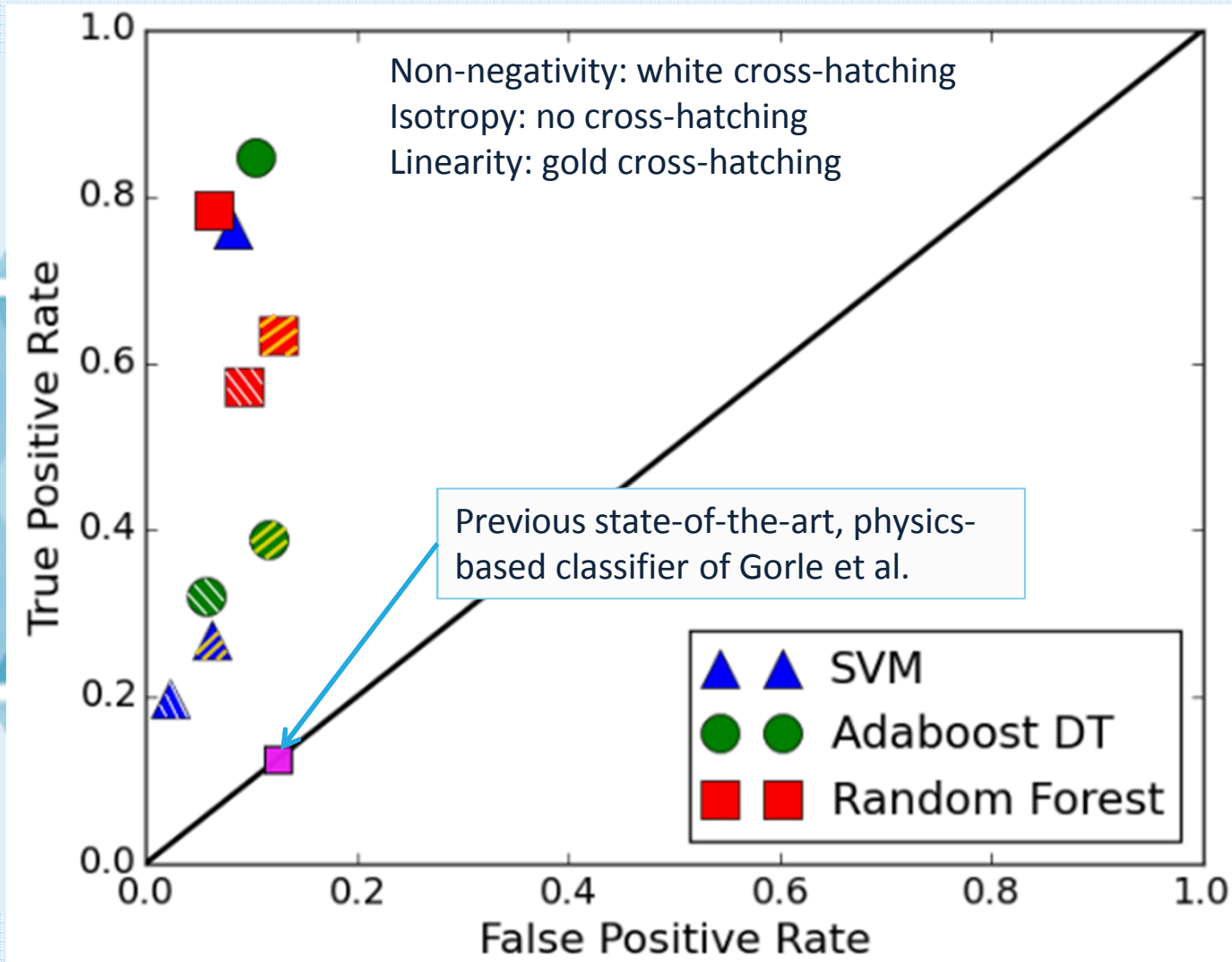
- Use classifier to make predictions on validation set
- Evaluate classifier by comparing to high fidelity results
- Leave-one-out cross-validation

# Classifier Performance

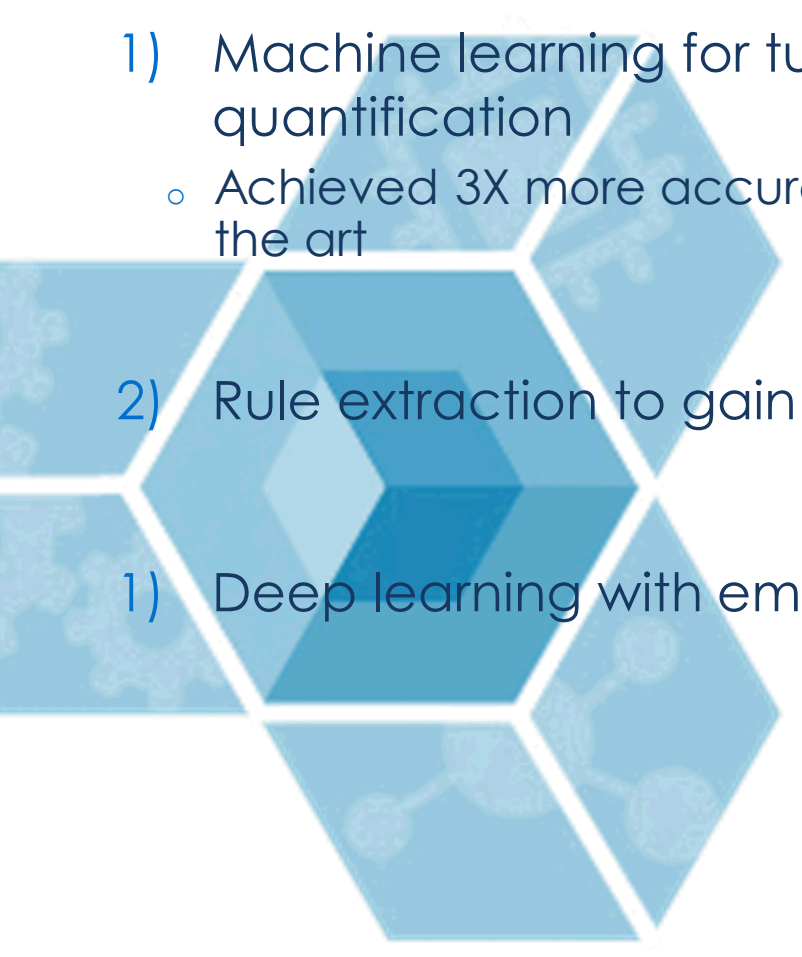




# Classifier Performance

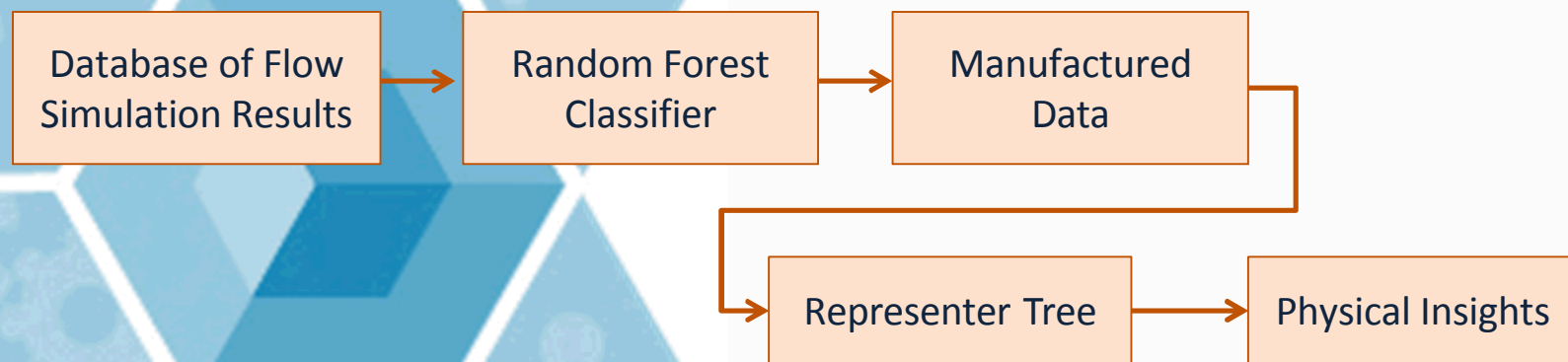




- 
- 1) Machine learning for turbulence model form uncertainty quantification
    - Achieved 3X more accurate error detection than previous state of the art
  - 2) Rule extraction to gain physical intuition
  - 1) Deep learning with embedded invariances



- Random Forests are much more robust and high-performance than single decision trees, but what have we lost?
  - Clarity—how can we understand these machine learned models?
- Representer Trees

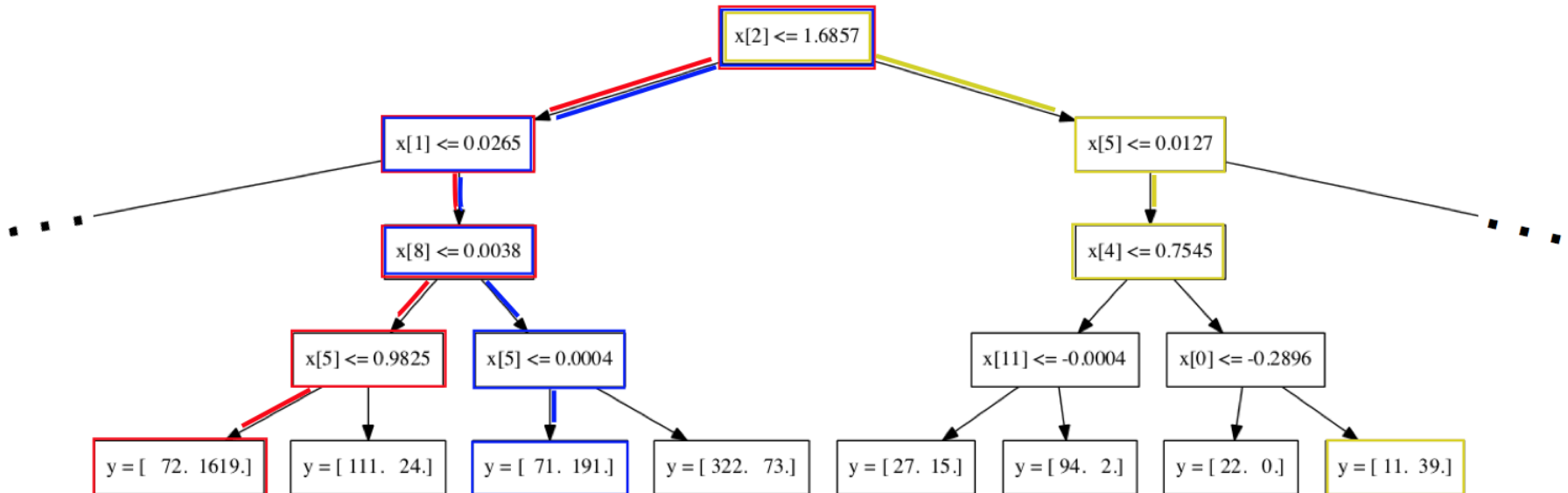


- Trained a representer decision tree based on Random Forest that predicted when the RANS isotropy assumption was invalid

# Analyzing the Representer Tree



- Look for consistent branches



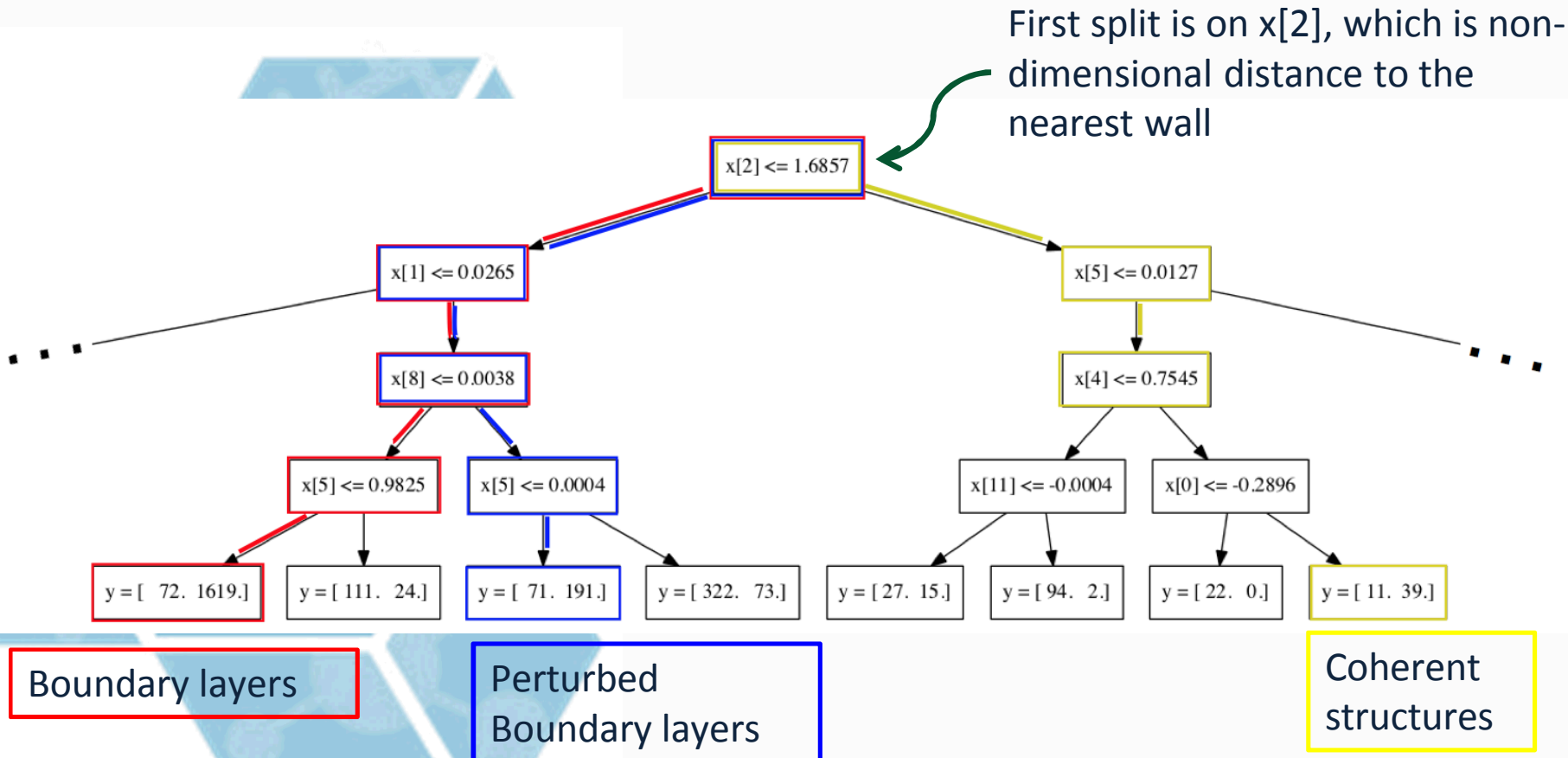
Boundary layers

Perturbed  
Boundary layers

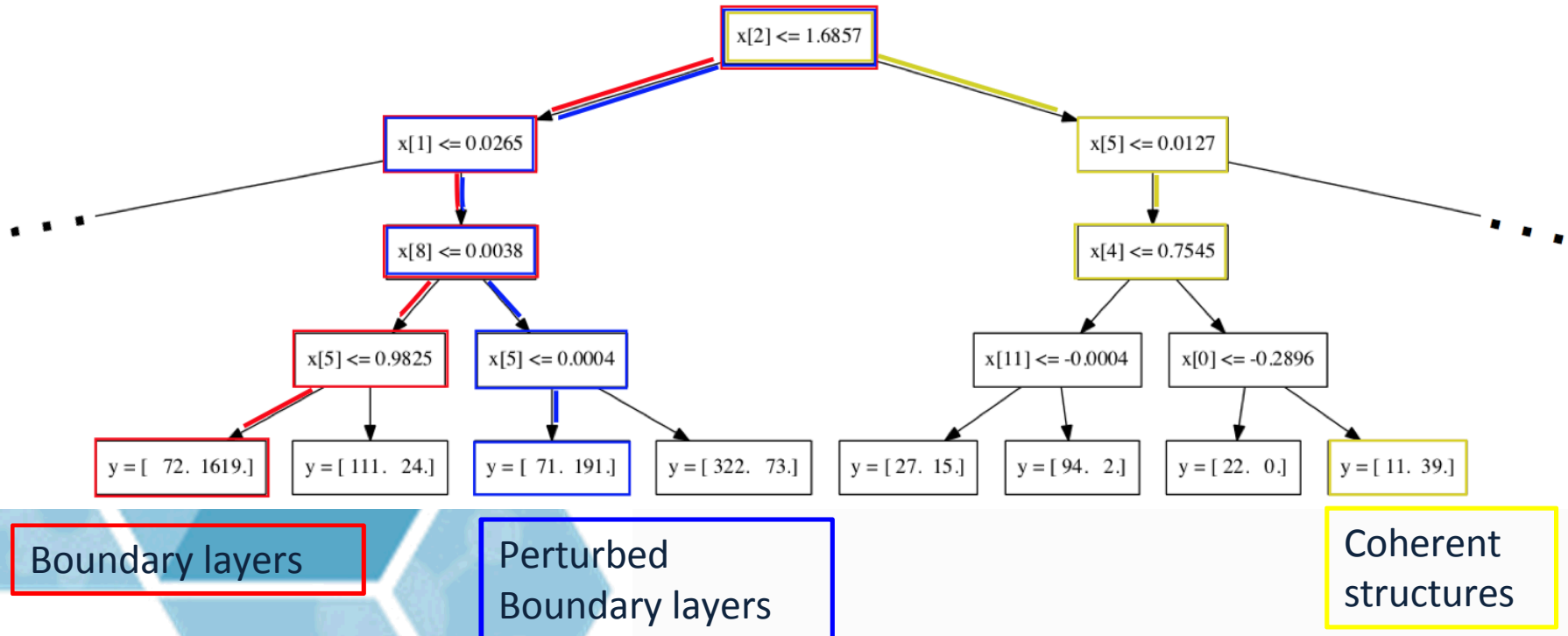
Coherent  
structures



# Analyzing the Representer Tree



# Analyzing the Representer Tree



- Can determine physical regimes where assumptions are violated
- Can see that different mechanisms cause assumption to break down in near wall region than in free stream



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  - Used rule extraction to understand how the complex machine learning algorithm was making its predictions
  - Increases confidence in ML predictions
  - Provides feedback to turbulence modeler
- 3) Deep learning with embedded invariances

# Deep Learning for Turbulence Modeling



- RANS simulations rely on empirical models to model the turbulence transport
- These models were proposed based on theory + sparse experimental data
- We want to use a deep neural network to model the turbulence transport

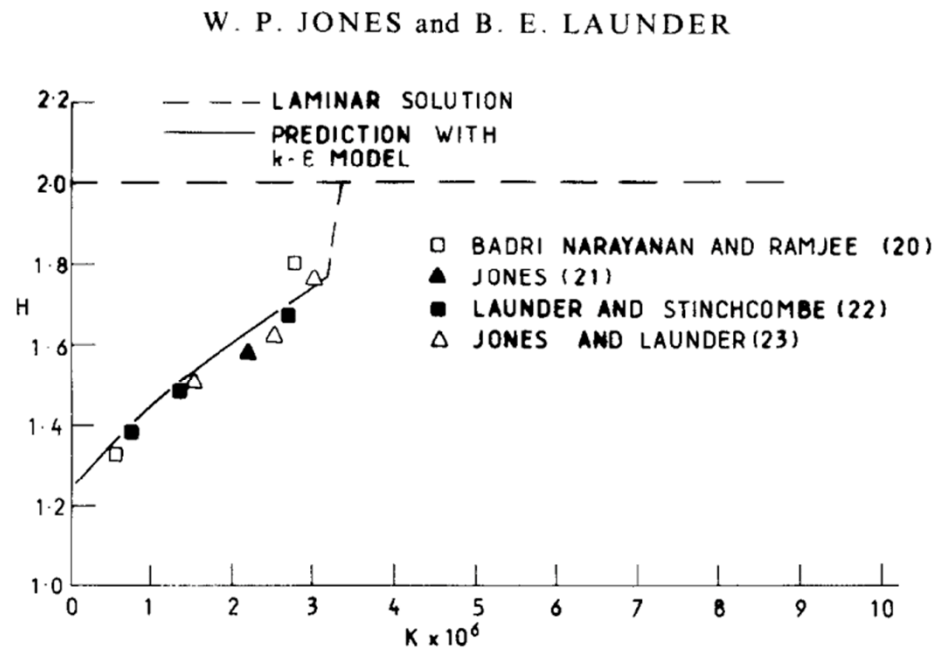


FIG. 2. Sink flow turbulent boundary layers.



# Deep Learning for Turbulence Modeling



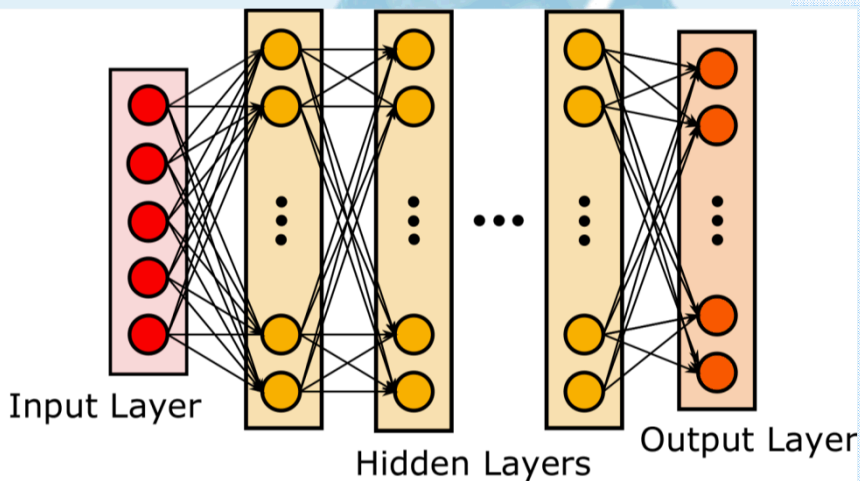
- Inputs: Tensors  $\mathbf{x}$ ,  $\mathbf{y}$
- Output: Tensor  $\mathbf{b}$
- Would like to enforce Galilean invariance
  - Invariance to inertial coordinate frame transformations
  - Borrow some ideas from group theory, representation theory
  - All Galilean invariant tensors that are a function of  $\mathbf{x}$  and  $\mathbf{y}$  lie on a tensor basis: the *integrity basis* of  $\mathbf{x}$  and  $\mathbf{y}$  for the orthogonal group

$$\mathbf{b} = \sum_{n=1}^{10} g^{(n)}(\lambda_1, \dots, \lambda_5) \mathbf{T}^{(n)}$$

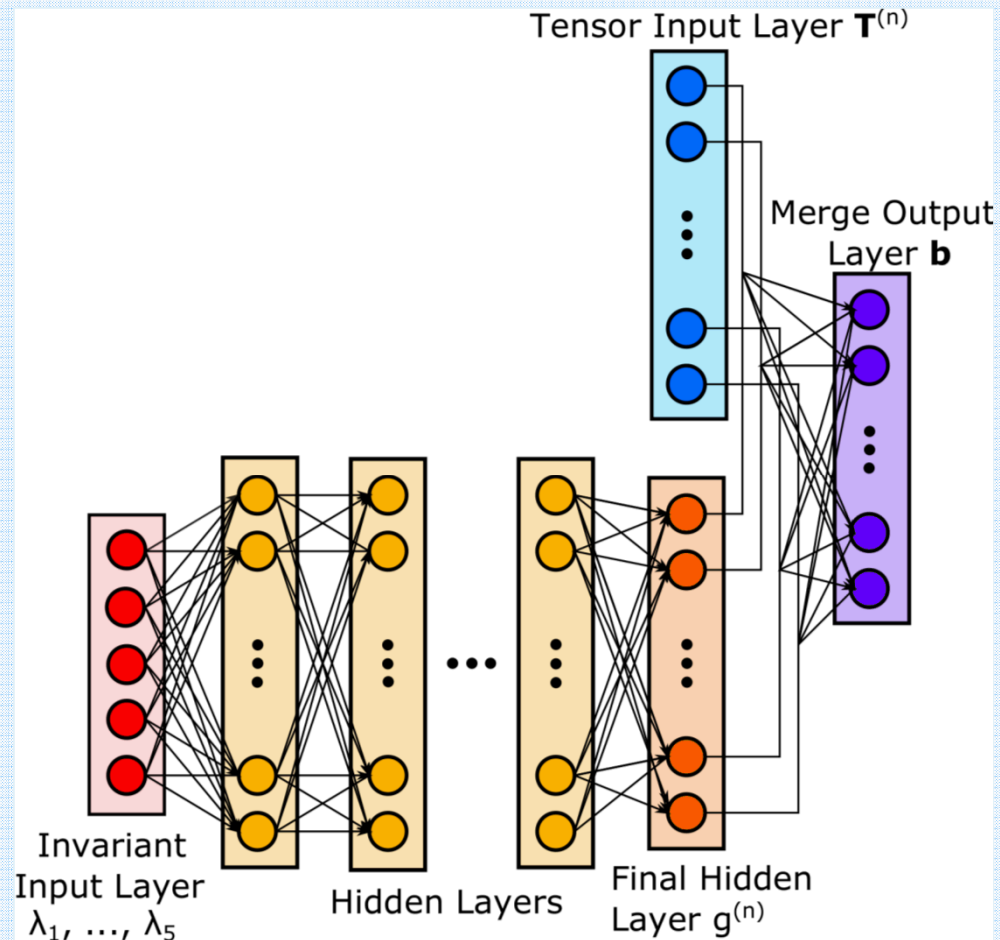
# Embedding Galilean Invariance



## Generic MLP



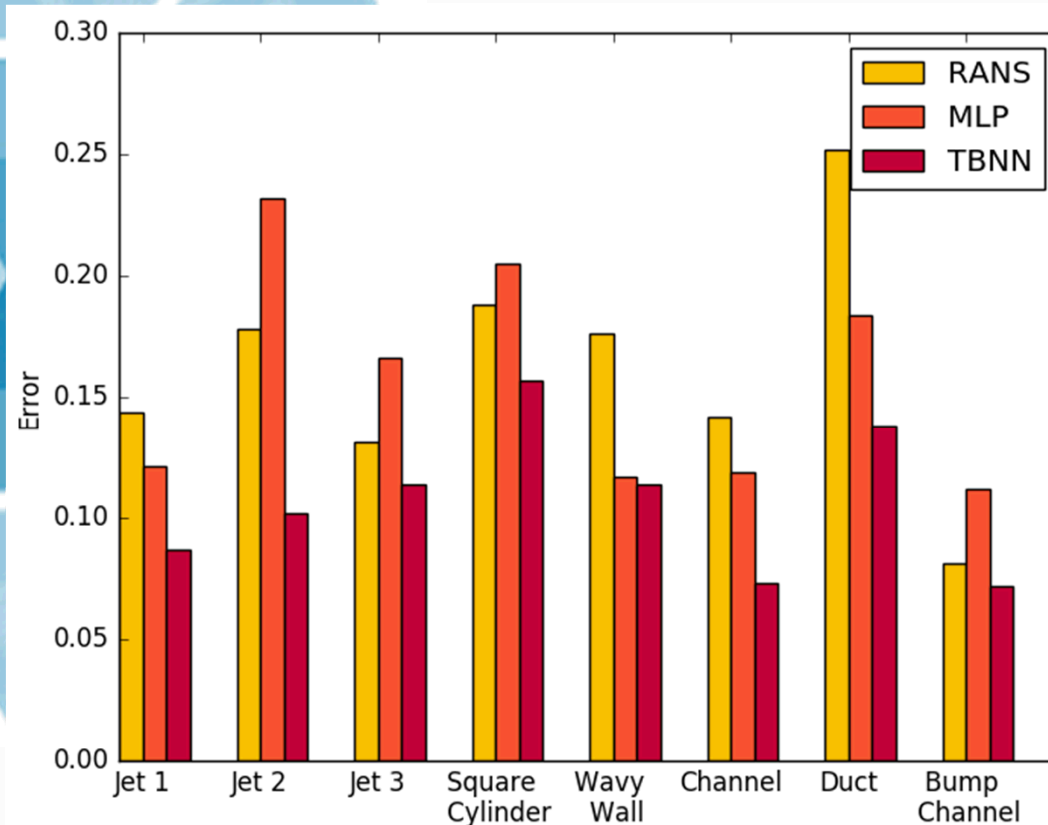
## Tensor Basis Neural Network



# Deep Learning for Turbulence Modeling



- Deep neural networks implemented using the Lasagne library: built on Theano
- Bayesian optimization used to tune the network hyper-parameters
- Trained an ensemble of networks to provide regularization





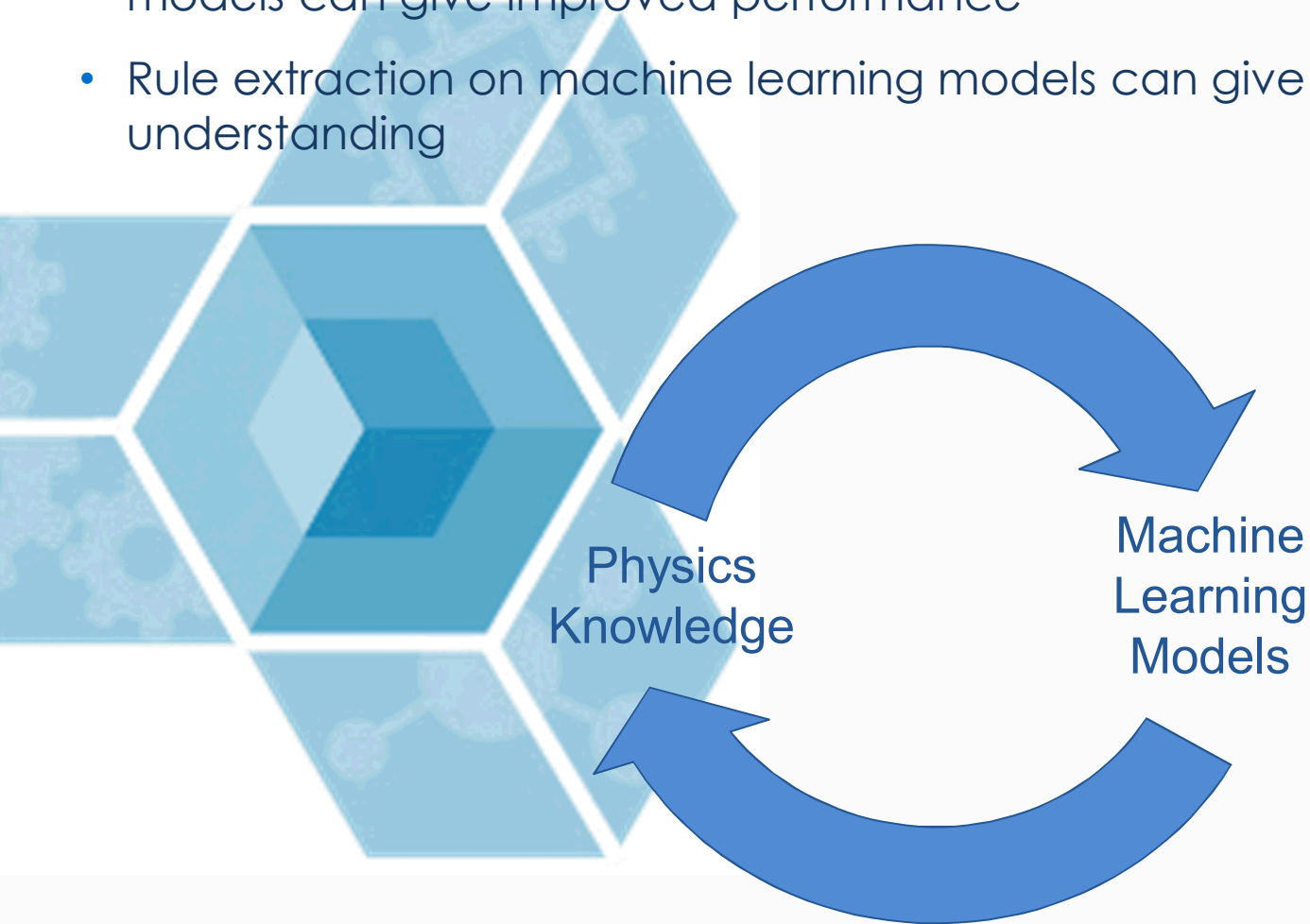
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- 3) Deep learning with embedded invariances
  - Embedded known invariance property into network architecture
  - Achieved significantly improved Reynolds stress anisotropy predictions
  - First application of deep learning to turbulence modeling



# Big Picture



- Directly embedding scientific domain knowledge into machine learning models can give improved performance
- Rule extraction on machine learning models can give improved physical understanding





- **J. Ling**, A. Kurzawski, and J. Templeton, “Reynolds Averaged Turbulence Modeling using Deep Neural Networks with Embedded Invariance,” *Journal of Fluid Mechanics*, (2016).
- **J. Ling** and J. Templeton, “Evaluation of machine learning algorithms for prediction of regions of high Reynolds averaged Navier Stokes uncertainty,” *Physics of Fluids*, (2015).
- **J. Ling**, “Using Machine Learning to Understand and Mitigate Model Form Uncertainty in Turbulence Models,” *IEEE ICMLA*, (2015).
- **J. Ling**, A. Ruiz, G. Lacaze, and J. Oefelein, “Uncertainty analysis and data-driven model advances for a Jet-in-Crossflow,” *Journal of Turbomachinery*, (2016).
- **J. Ling**, R. Jones, and J. Templeton, “Machine Learning Strategies for Systems with Invariance Properties,” *J. Comp. Phys.*, (2016).
- **J. Ling**, F. Coletti, S. Yapa, J. Eaton, “Experimentally informed optimization of turbulent diffusivity for a discrete hole film cooling geometry,” *International Journal of Heat and Fluid Flow*, (2013).

# Questions

