pset 1

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link to problem: https://yale.instructure.com/courses/113840/assignments/558946

1 Problem 1

(16 points): Show that $(\neg p \vee q) \rightarrow (r \wedge \neg q)$ and $(\neg q \wedge (p \vee r))$ are logically equivalent

- using truth tables (show your intermediate work)
- using the list of logical equivalences including the conversion from $\rightarrow$ to $\neg$ and $\vee$ labelling each step with the rule(s) you are using.

1.1 solution

$$\begin{aligned}
(\neg p \vee q) \rightarrow (r \wedge \neg q) &\equiv (\neg p \vee q) \rightarrow (r \wedge \neg q) && \text{(given)} \\
&\equiv \neg(\neg p \vee q) \vee (r \wedge \neg q) && \text{(substitution**)} \\
&\equiv (p \wedge \neg q) \vee (r \wedge \neg q) && \text{(De Morgan)} \\
&\equiv \neg q \wedge (p \vee r) && \text{((reverse) distributive)}
\end{aligned}$$

** since we know that $p \rightarrow q \equiv \neg p \vee q$

2 problem 2

(16 points): Let a be the statement "Alex teaches CS", b be the statement "Buwan teaches CS", and c be the statement "Chichima teaches physics."

[*]made in Overleaf

- For each of the following, write the statement form that most closely follows the natural language, and a logically equivalent statement form that does not use $\wedge$
  - It is not the case that both Alex teaches CS and Chichima teaches physics.
  - Neither Alex nor Buwan teaches CS.
- For each of the following, write the statement form that most closely follows the natural language, and a logically equivalent statement form that uses only $\neg$ $\wedge$ (and parentheses to force order of operations when necessary).
  - It is not the case that at least one of Alex or Buwan teaches CS.
  - If Chichima teaches physics then Alex does not teach CS.

2.1 solution to part 1

where it is instructed to not use $\wedge$ (and), or $\rightarrow$ (if/then)

2.1.1 question 1

"It is not the case that both Alex teaches CS and Chichima teaches physics." translates literally to

$$\neg(a \wedge c)$$

however, since we cannot use $\wedge$ (and), we must translate this to an equivalent expression. using distributive law, we find that

$$\neg(a \wedge c) \equiv \neg a \vee \neg c$$

2.1.2 question 2

"Neither Alex nor Buwan teaches CS." translates directly to

$$\neg a \vee \neg b$$

However, since this does not use $\wedge$ (and), and since we are using inclusive or (not XOR) this problem is already complete.

2.2 solution to part 2

where it is instructed to not use $\vee$ (or), or $\rightarrow$ (if/then)

2.2.1 question 1

"It is not the case that at least one of Alex or Buwan teaches CS." translates literally to

$$\neg(a \wedge c)$$

note that this does not use $\vee$, and is thus complete.

2.2.2 question 2

"If Chichima teaches physics then Alex does not teach CS;" note the presence of the if/then statement. this then can be translated as

$$c \rightarrow \neg a$$

from here, we can replace the if/then ("$\rightarrow$")

$$\equiv \neg c \vee \neg a$$

we can see here the $\vee$ (or) has appeared. However, using the distributive law in reverse, we find this expression equivalent to

$$\equiv \neg(c \wedge a)$$

which thus completes the problem

3 Problem 3

(4 points): In addition to and, or, not, and if/then, we can define other logical operators. For example, the binary operator NAND ("not and"), with p NAND q (written p $\uparrow$ q) logically equivalent to $\neg(p \wedge q)$.

the truth table for NAND is then

Find a statement form logically equivalent to $\neg(p \wedge \neg p) \vee r$ that uses (possibly more than once) $\uparrow$ as its only logical operator. You will find it helpful to complete the Canvas quiz before attempting this question.

p	q	p $\uparrow$ q
t	t	f
t	f	t
f	t	t
f	f	t

3.1 solution

Given that

$$\neg(p \land \neg p) \lor r$$

and that given arbitrary a and b:

$$\neg(a \land b) \equiv a \uparrow b$$

we can use this definition to say this expression is equivalent to

$$(p \uparrow \neg q) \lor r$$

specifically, using p as a and $\neg q$ as b.

Furthermore we can simplify further, saying that this is equivalent by negation:

$$\neg(\neg r \land \neg(p \uparrow \neg q))$$

which, using the identity mentioned earlier, we can substitute in $\uparrow$s (nand):

$$\neg r \uparrow \neg(p \uparrow \neg q)$$

to complete this solution, we can make one final identity: that

$$\neg a \equiv a \uparrow a$$

(this truth table will be expounded later). For now, however, we can use this identity to complete the solution with only $\uparrow$ (nand):

$$(r \uparrow r) \uparrow ((p \uparrow (q \uparrow q)) \uparrow (p \uparrow (q \uparrow q)))$$

as you can see, a section is duplicated to account for the double input of $\uparrow$ (nand)

4 truth tables and figures

here is a proof of the substitution used earlier:

4.1 solution 3.1

$$\neg a \equiv a \uparrow a$$

a	¬ a	a ∧ a	a ↑ a
1	0	1	0
0	1	0	1