pset 2*

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https://yale.instructure.com/courses/113840/assignments/558950

1 Problem 1

Show how to come to the conclusion v given the following premises. Show which rule of inference you used at each step and which premises and/or previous conclusions you applied them to.

$p \vee \neg q \vee r$
$p \to s$
$s \to v$
$\neg q \to \neg u$
u
$r \to v$

1.1 solution

we are given u=true. from this, using negation, we can derive that $\neg u$ is false. From this we can further derive that, because $\neg q \to \neg u$ is true and since $\neg u$ is false, we are forced to conclude that $\neg q$ is also false.

In other words, if we were to entertain that $\neg q$ is true while knowing $\neg u$ is false, that would result in us concluding that the given predicate $\neg q \to \neg u$ is not true, which is not the case.

*made in Overleaf

From here, we arrive at the rule that $p \vee \neg q \vee r$. However, since we know $\neg q$ to be false, we must assume that either r is true or p is true. In other words, we know that $r \vee p$ must be true.

From here, we may consider the case that p is true. Since we are given the rule that $p \rightarrow s$, to abide by this rule in this scenario s is true. From this, we are also given that $s \rightarrow v$, so to abide by *that* rule, v must also be true.

Now let us consider the other case. In the case that r is true, we are given the rule that $r \rightarrow v$. Abiding by this given predicate, we can arrive at the conclusion that v must also be true.

As demonstrated, regardless of whether either r or p is true (or both), it can be concluded that v must be true from the above conclusions. if one were to demonstrate this in a predicate-formative manner, we could say that the statement

$$(p \vee r) \rightarrow v$$

is true, thus making v true.

2 Problem 2

You meet four people on the Island of Knights and Knaves. Given the following statements they make, determine for each individual whether they are a knight or a knave. Show your reasoning. (Knights' statements are always true and knaves' statements are always false, but consider each utterance as a single (possibly compound) statement, so if a knave's statement is a conjunction, then at least one of the conjuncts must be false, but other conjuncts may be true).

- A: I am a knight and C is a knave.
- B: A is a knave.
- C: There is only one knight and it is not me.
- D: A and myself are the only knights among us.

2.1 solution

Let us assume that B is a knight. We can conclude from this that A is a knave. As A says that A is a knight and C is a knave, we can further derive that C is a knight. However, C states that there is only one knight and it is not C. We have arrived at a contradicting statement. C, despite being a knight in this scenario, is saying something that conflicts with our established conclusions.

However, let us entertain A's compound statement, that A being a knight is false, but C being a knave is true (we cannot assume the inverse in this scenario). We can conclude that there is either multiple knights or that C is not a knight, or both. Seeing as C must be a knave, the only possible true predicate in this compound statement is that there are multiple knights.

We thus arrive at the conclusion that D must be a knight. However, D says a compound statement that D *AND* A are knights. Since both of these things must be true, and that in this scenario, A must be a knave due to B's assumed knighthood and statements, we are forced to conclude that B cannot be a knight.

Now, let us assume that B is a knave. We can conclude that A is a knight. A declares themselves a knight (true) and C is a knave. C says there is only one knight, and it is not C. The latter half already having been proven true, we can turn to the former and say that D is the only remaining option as the other knight. D's statement agrees with this, saying A and D are the only knights; a conclusion that reasons well with our arrived conclusion.

Thus, A and D are knights

3 Problem 3

Let P be the set of players, T be the set of professional teams, and C be the set of college teams. Let $M(x, y)$ be the predicate "player x plays for pro team y" and $N(x, y)$ be the predicate "player x played for college team y"

Write each of the following in predicate logic. Be sure to indicate the domain of each quantified variable using P, T, or C. Assume that the specific nouns mentioned are members of the appropriate sets.

- No player played for Harvard. [note: Harvard is a college team]
- Every player who plays for Golden State played for Virginia. [note: Golden State is a pro team and Virginia is a college team]

- Some player didn't play for any college team.
- For every college team, there is a player who played for them and plays for a pro team.
- Every player played for a unique college team. [note: use logical connectives, the predicates defined above, and the two standard quantifiers $\forall$ and $\exists$; you may also use $=$ (which is really a two-place predicate written using infix notation rather than function notation, so $x = y$ can be thought of as the predicate $=(x, y)$ meaning "x and y are the same element")]

3.1 solution

statement	statement	domain
No player played for Harvard. [note: Harvard is a college team]	$\neg\forall x[N(x, harvard)]$	$x \in P$
Every player who plays for Golden State played for Virginia. [note: Golden State is a pro team and Virginia is a college team]	$\forall x[M(x, GoldenState) \to N(x, Virginia)]$	$x \in P$
Some player didn't play for any college team.	$\exists x\forall y[N(x, y)]$	$x \in P, y \in C$
For every college team, there is a player who played for them and plays for a pro team.	$\exists x\forall y\exists z[N(x, y) \wedge M(x, z)]$	$x \in P, y \in C, z \in T$
Every player played for a unique college team. [note: use logical connectives, the predicates defined above, and the two standard quantifiers $\forall$ and $\exists$; you may also use $=$ (which is really a two-place predicate written using infix notation rather than function notation, so $x = y$ can be thought of as the predicate $=(x, y)$ meaning "x and y are the same element")]	$\forall x\exists y[(N(x, y) \wedge \neg N(z, y)) \vee \neg(x = z)]$	$x \in P, z \in P, y \in C$

note that this table notation is for ease of reading