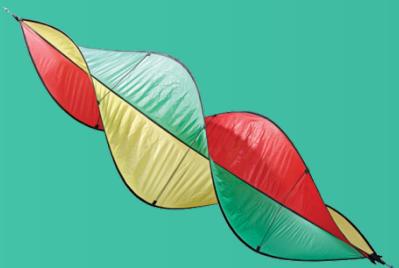


Heuristics: recent trends and real world applications



Dr. Tony Wauters
CODES research group

Overview

- Recent trends
 - Hyperheuristics
 - Hybrid heuristics
 - Matheuristics
 - Example: Personnel Task scheduling
- Real world applications
 - Production scheduling in the food industry
 - Prisoner transportation
 - Lock scheduling
 - Personnel scheduling
 - Patient admission scheduling

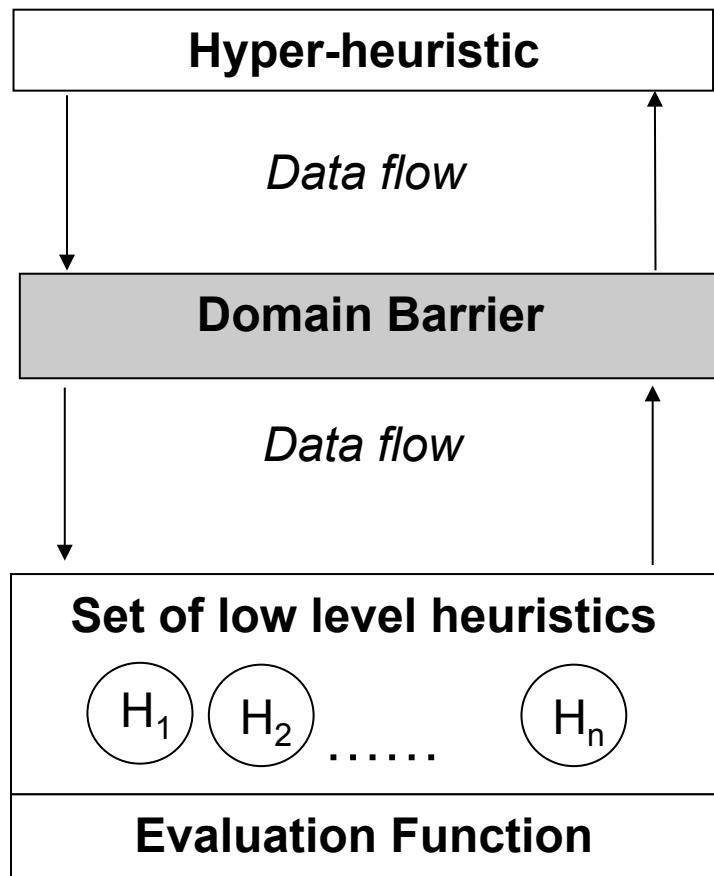
Recent trends

Hyperheuristics

- Simple Idea:
 - Heuristics to choose heuristics
 - Or heuristics to generate heuristics
- Operates on a search space of heuristics rather than directly on a search space of solutions

Hyperheuristics

- Metaheuristics tackle specific problems
- Are often tailored
- Can we increase the level of generality?
- Good enough, cheap enough, fast enough?

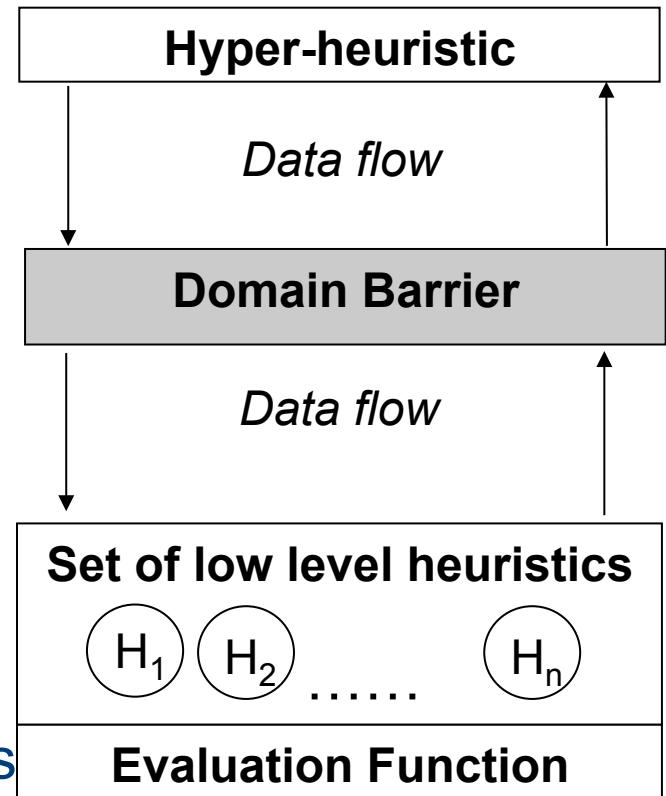


Hyperheuristics

- No ‘Superalgorithms’ (No Free Lunch)
- Intelligent schemes that function well on a number of problems
- Give the developer extra handles to bring in his domain expertise without requiring him to study the details of an advanced optimisation strategy

Hyperheuristic

- Above the domain barrier
 - Genetic algorithm
 - Genetic programming
 - Tabusearch
 - Simple random
- Room for machine learning
 - Learn the weights of the low level heuristics
 - Relate them to solution space features (> expert?!)
 - Reduce the number of parameters
- Hyper heuristic competition



Cross-domain Heuristic Search Challenge

<http://www.asap.cs.nott.ac.uk/external/chesc2011/>

- 6 problem domains
 - Boolean satisfiability (MAX-SAT)
 - One dimensional bin packing
 - Permutation flow shop
 - Personnel scheduling
 - Traveling salesman problem (TSP)
 - Vehicle routing (VRP)
- Given for each problem domain:
 - low-level heuristics or operators (swaps, crossover, hillclimber,...)
 - Objective function to evaluate obtained solutions.
- Winner:
 - Best performing over all problem domains
 - Adaptive/learning hyperheuristic by Mustafa Misir(CODES)



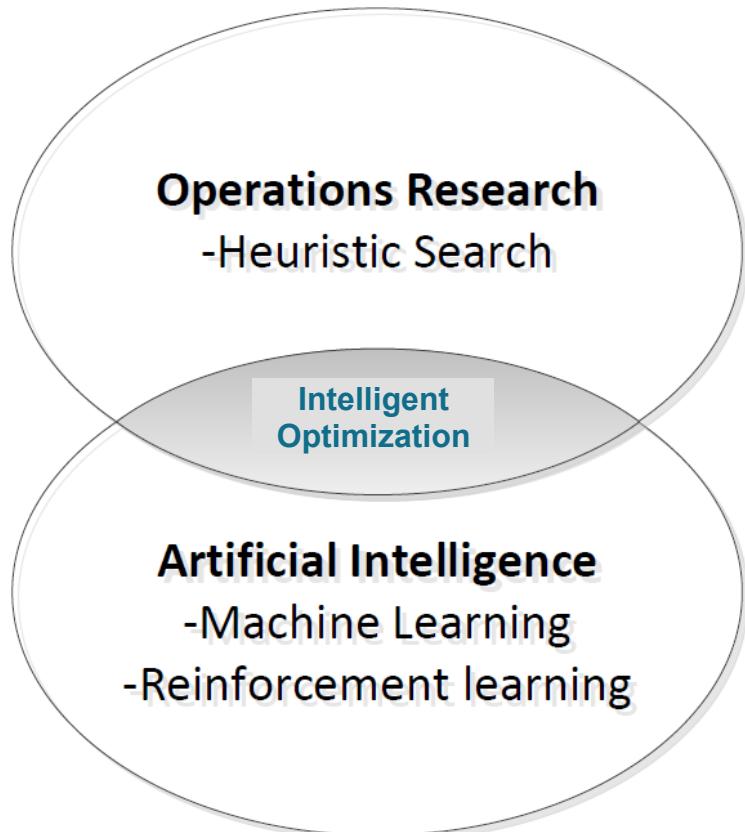
Hybrid heuristics

- Hybridizations of:
 - (meta)heuristics with (meta)heuristics
 - (meta)heuristics with exact methods (mathematical programming (MIP)) → **Matheuristics**
 - (meta)heuristics with constraint programming
 - (meta)heuristics with machine learning and data mining
 - Intelligent Optimization

Matheuristics

- Hybridization of **mathematical methods** and **heuristics**
 - Use MIP as metaheuristic neighbourhood
 - Use MIP to initialize metaheuristic
 - Use metaheuristic to initialize MIP
 - Exchange solution between MIP and metaheuristic
 - ...

Reactive search – Intelligent optimization



BOOK:

ROBERTO BATTITI, MAURO BRUNATO AND FRANCO MASCIA, **Reactive Search and Intelligent Optimization**, *Operations Research/Computer Science Interfaces Series*, Vol. **45**, Springer, November 2008.

Matheuristic example:

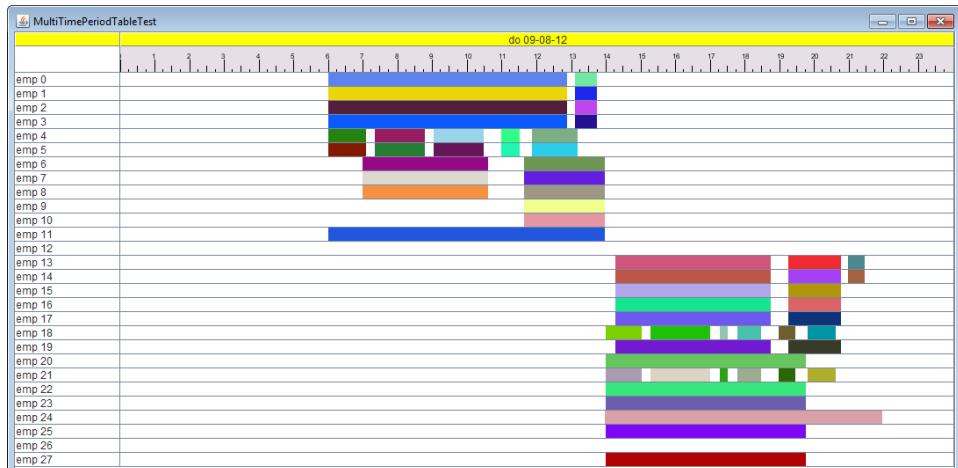
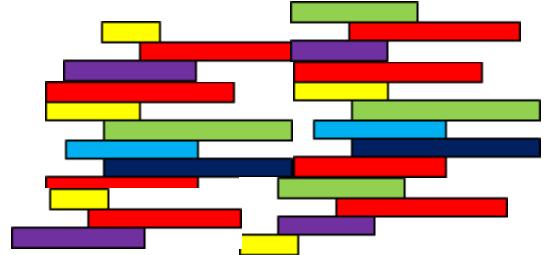
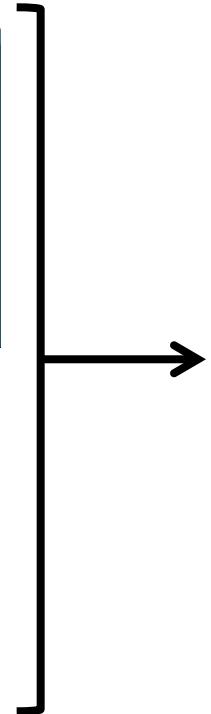
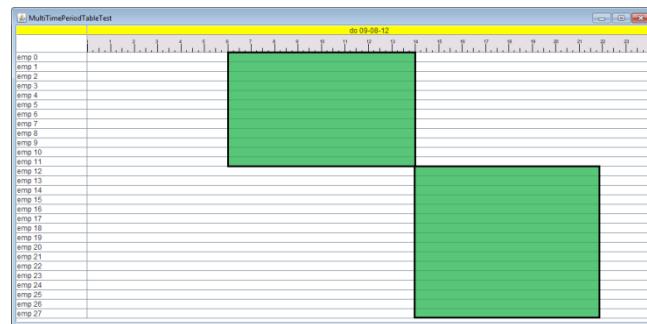
Hardness analysis and a new approach for the shift minimisation personnel task scheduling problem

Pieter Smet
Tony Wauters
Greet Vanden Berghe

CODES, KAHO Sint-Lieven

Introduction

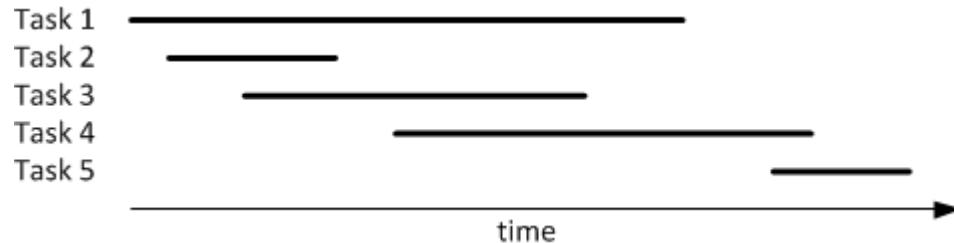
- Assigning tasks to employees
 - Fixed working times of employees
 - Fixed start and end times of tasks



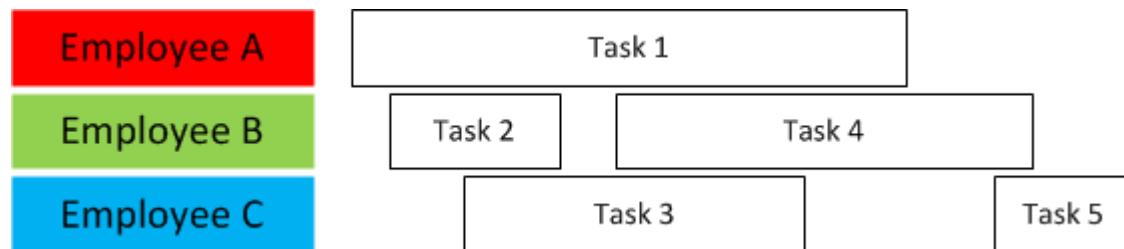
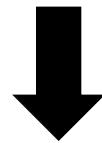
Problem description

- Shift minimisation personnel task scheduling problem (SMPTSP)
(Krishnamoorthy et al, 2011)
- Objective: minimise number of employees used
- Constraints
 - Each task is performed by one employee
 - No preemption
 - No overlapping tasks assigned to one employee
 - Respect qualification requirements
 - Assign only tasks to working employees

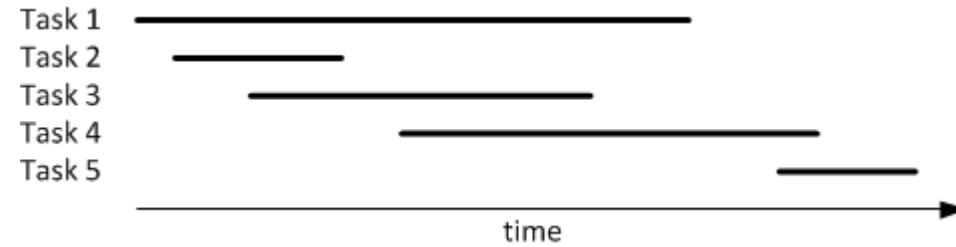
Example



	Task 1	Task 2	Task 3	Task 4	Task 5
Employee A	X	X	X		
Employee B		X	X	X	
Employee C			X	X	X

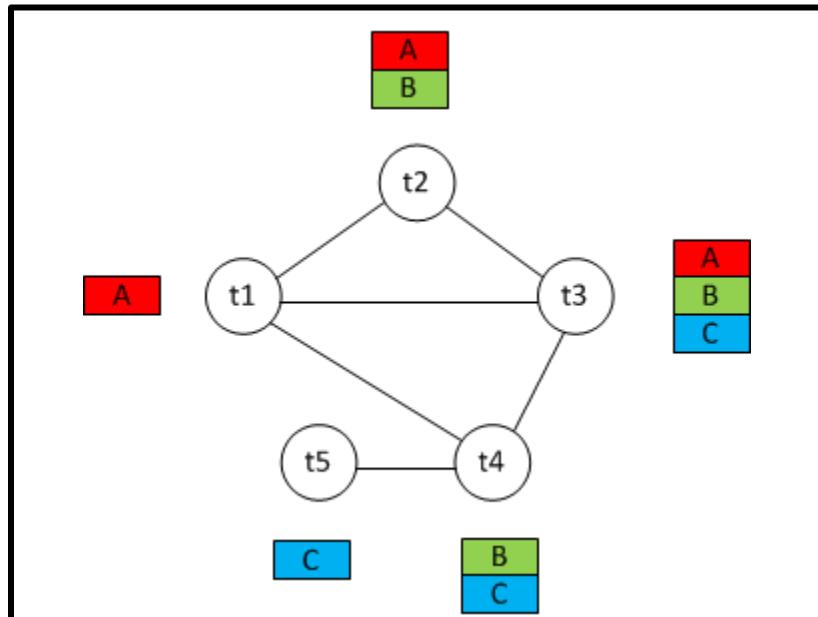


SMPTSP

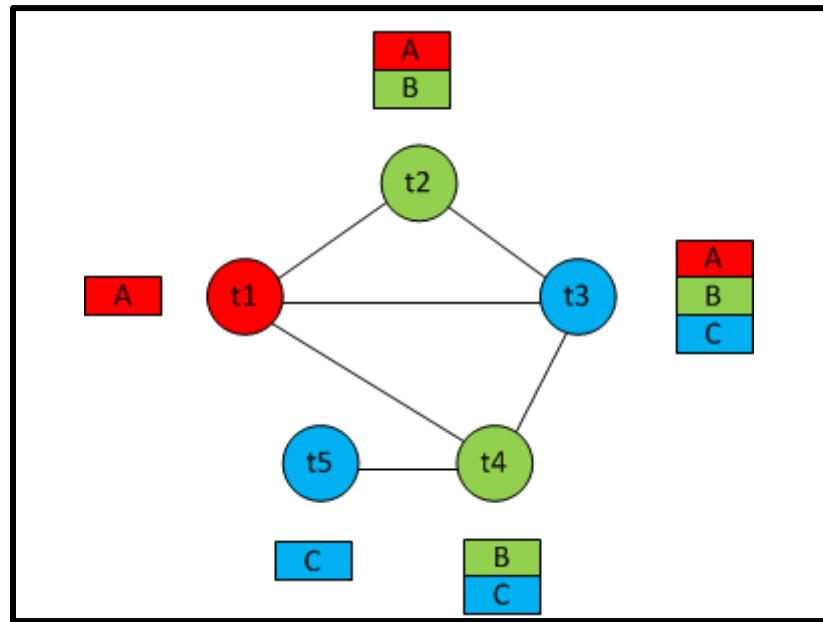


	Task 1	Task 2	Task 3	Task 4	Task 5
Employee A	X	X	X		
Employee B			X	X	X
Employee C				X	X

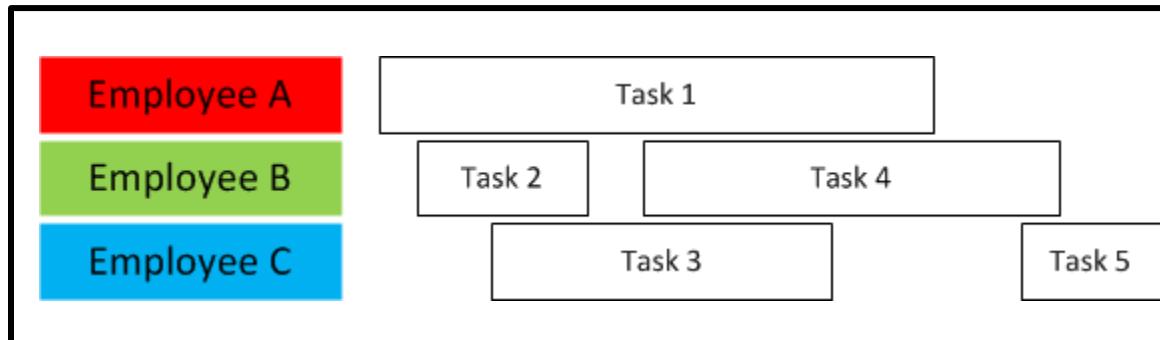
As a list colouring problem



List colouring solution



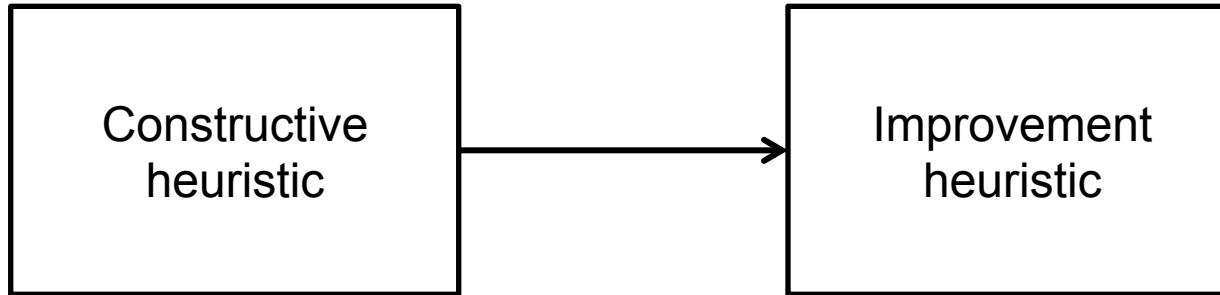
As a SMPTSP
solution



State of the art

- Krishnamoorthy et al (2011)
 - Lagrangean relaxation based heuristic
- Smet and Vanden Berghe (2012)
 - Fix-and-optimize heuristic

Solution approach: overview



- Both phases use a general purpose MIP solver

MIP model

- **Decision variables**

$$x_{jw} = \begin{cases} 1 & \text{if task } j \in J \text{ is assigned to employee } w \in W \\ 0 & \text{otherwise} \end{cases}$$

$$y_w = \begin{cases} 1 & \text{if employee } w \in W \text{ is active, meaning that } w \text{ has at least one task} \\ 0 & \text{otherwise} \end{cases}$$

- **MIP model**

$$\min \sum_{w \in W} y_w \tag{1}$$

$$s.t. \sum_{w \in P_j} x_{jw} = 1 \quad \forall j \in J \tag{2}$$

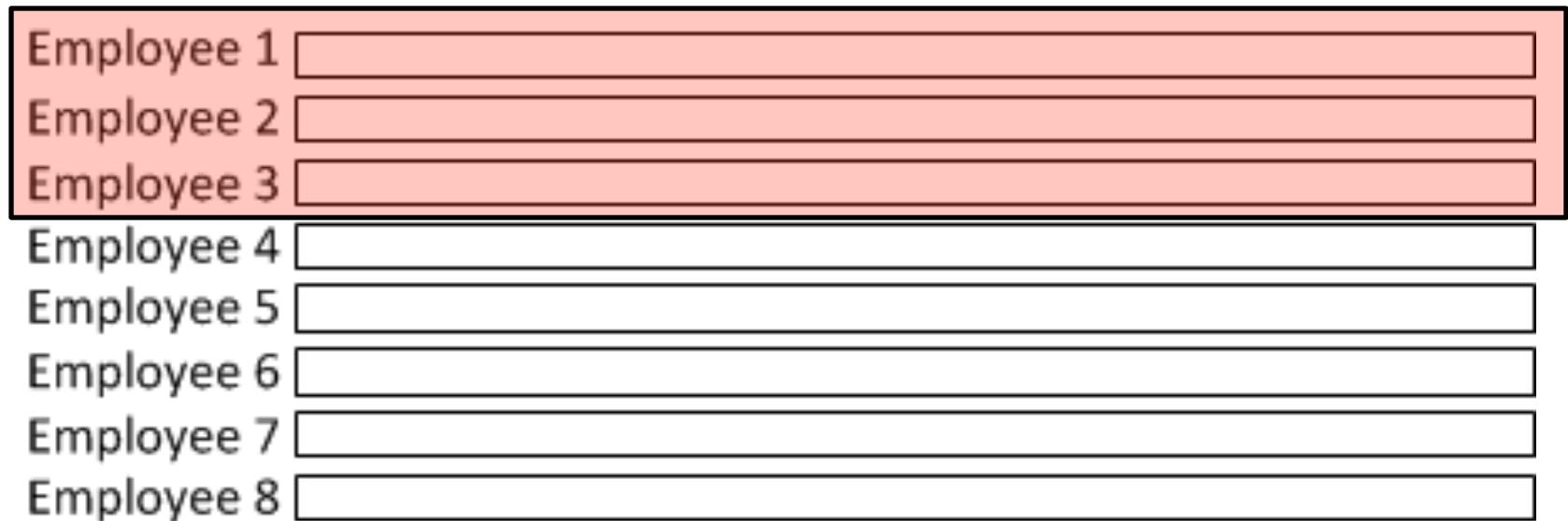
$$\sum_{j \in K_t^w} x_{jw} \leq y_w \quad \forall w \in W, K_t^w \in C^w \tag{3}$$

$$0 \leq y_w \leq 1 \quad \forall w \in W \tag{4}$$

$$x_{jw} \in \{0, 1\} \quad \forall j \in J, w \in W \tag{5}$$

Solution approach: constructive heuristic

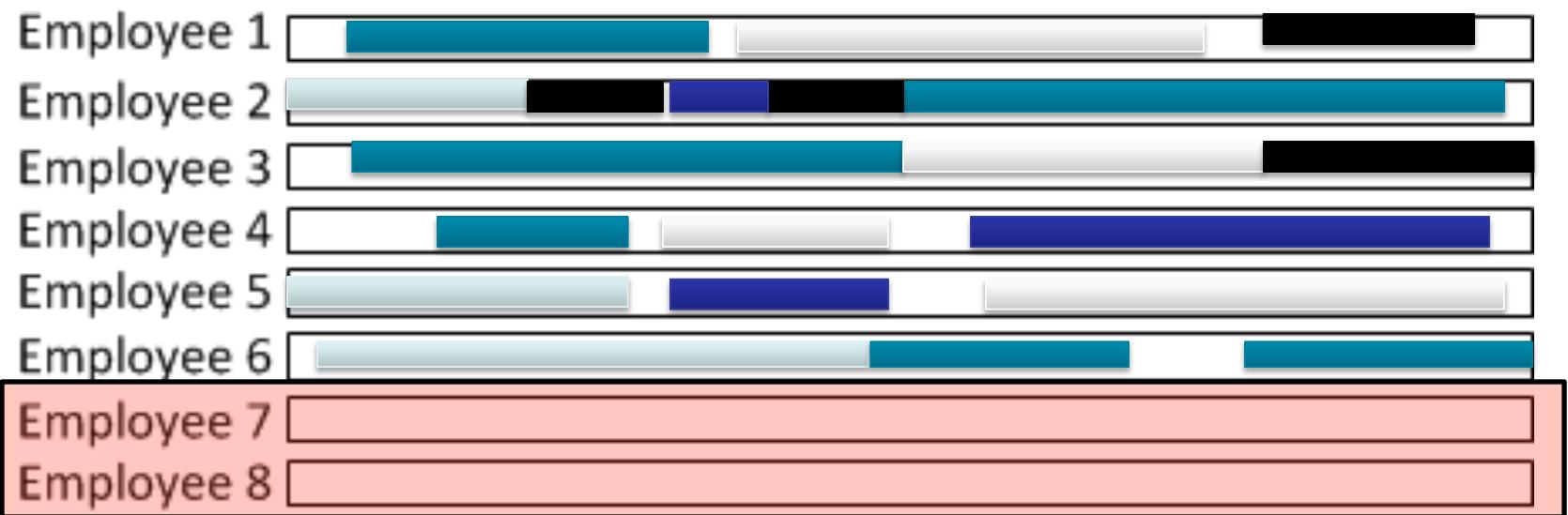
- Optimally assign tasks to a subset of b employees, maximizing the sum of assigned task durations



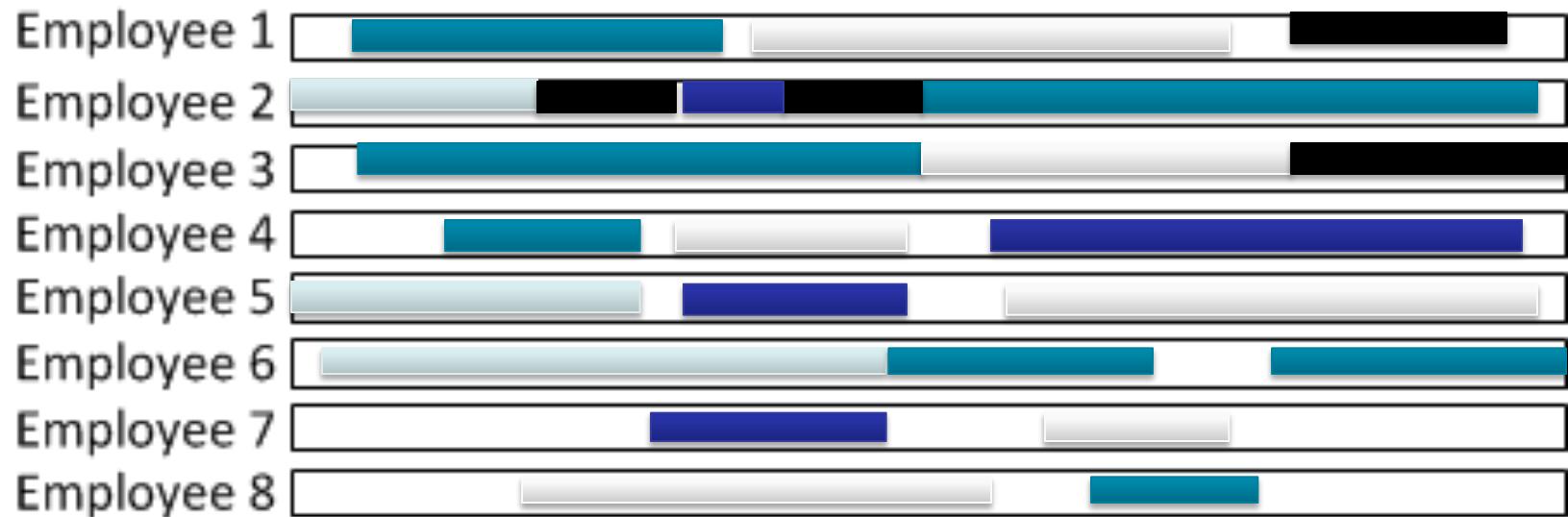
Solution approach: constructive heuristic



Solution approach: constructive heuristic



Solution approach: constructive heuristic



- It is possible that tasks remain unassigned

Solution approach: constructive heuristic

Algorithm 1 Constructive matheuristic (CMH)

Input: J := set of tasks
 W := set of employees
 b := number of employees in one subproblem

Output: a (partial) solution for the SMPTSP

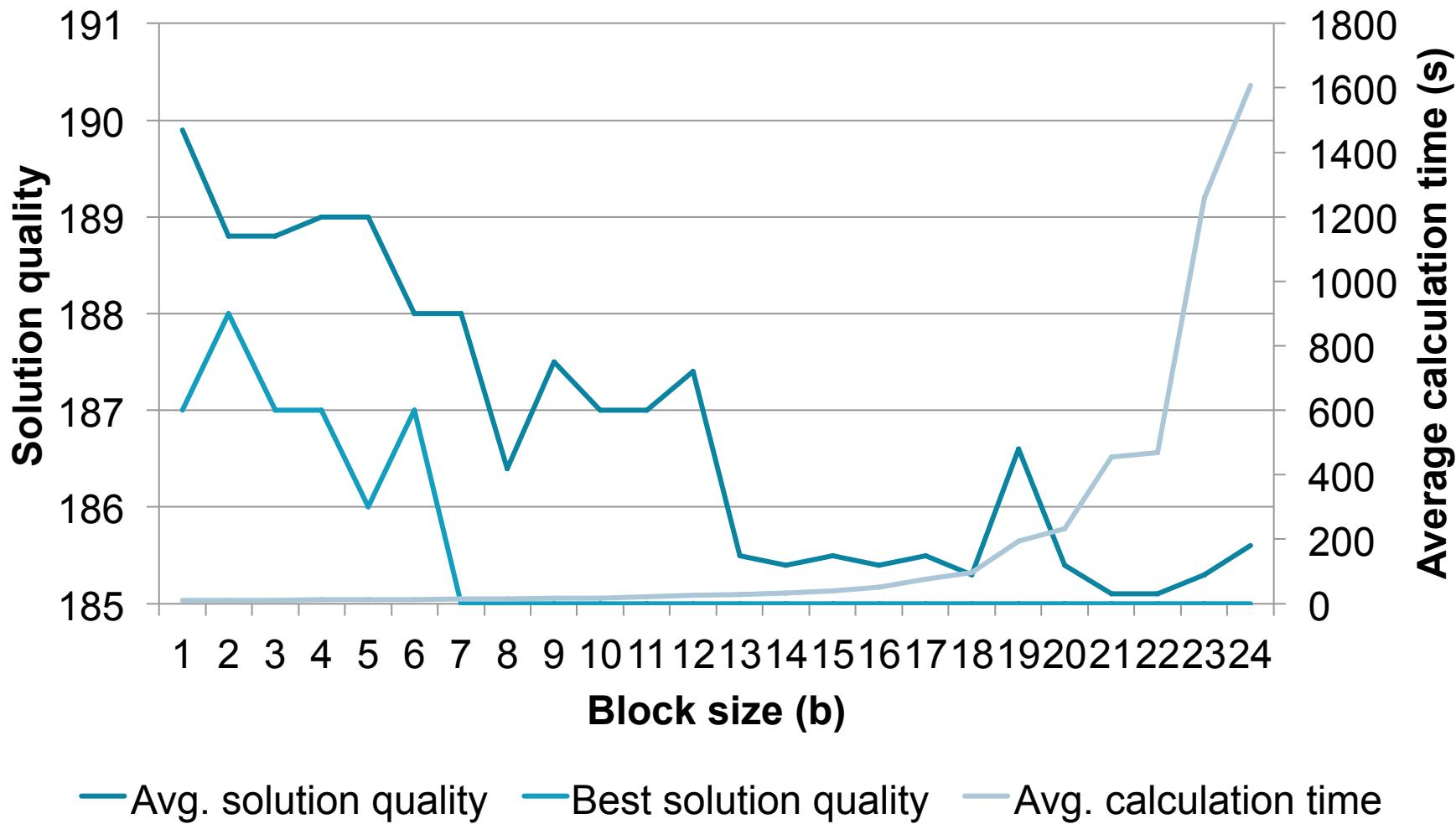
```
while  $W$  is not empty do
     $W'$  := sample and remove  $b$  employees from  $W$ 
    optimally assign tasks  $J' \subseteq J$  to  $W'$ 
    remove assigned tasks  $J'$  from  $J$ 
    if all tasks assigned then
        reoptimise  $W'$  using the original objective
    end if
end while
return employees with their assigned tasks
```

Fill employees' schedules by solving a subproblem iteratively until all employees have tasks assigned.

Solve the subproblem by maximizing sum of assigned task durations.

Reoptimize last block for maximum efficiency.

Solution approach: constructive heuristic



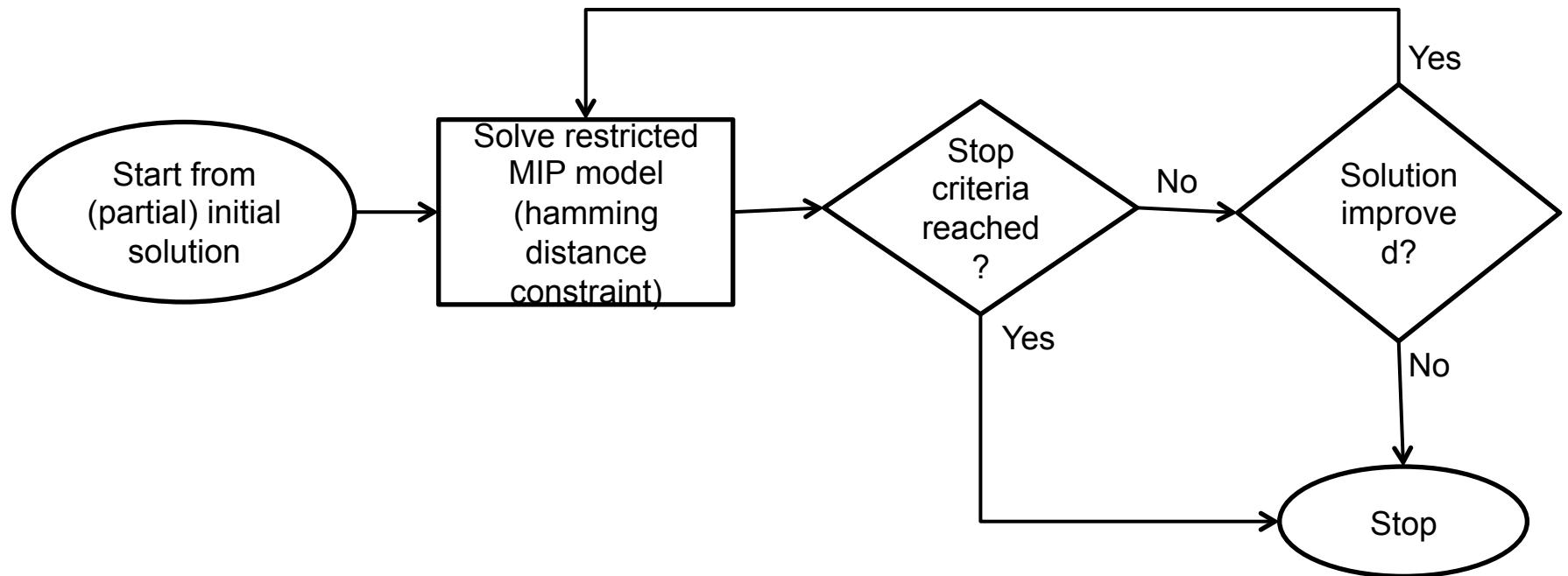
Solution approach: constructive heuristic

- Tested on 137 instances from literature
- # employees ranges from 23 to 245
- # tasks ranges from 40 to 2105

	CMH $b=10$	CMH $b=15$	First fit	Best fit
Number of optimal solutions	118	131	13	22
Average solution quality	123,00	122,6	125,40	125,80
Average calculation time (s)	12,26	44,93	0,02	0,11
Maximum calculation time (s)	169,30	889,80	0,21	1,10

Solution approach: improvement heuristic

- Iterative matheuristic procedure



Solution approach: improvement heuristic

Algorithm 2 Hybrid improvement heuristic

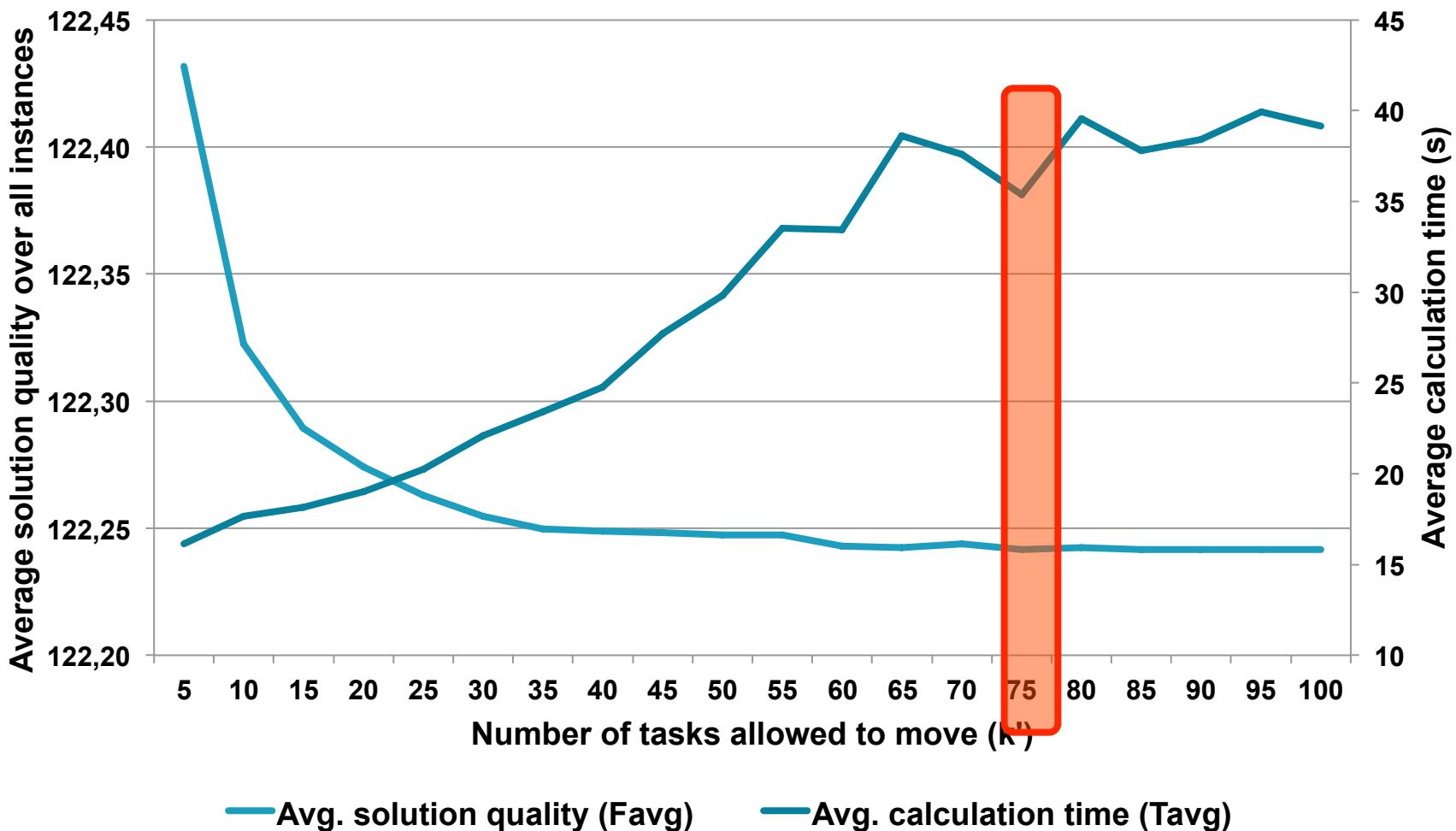
Input: $F(x)$ evaluation function value of x
 $LB :=$ lower bound on the evaluation function value
 $x' :=$ given reference solution
 $k' :=$ maximum number of tasks to reassign

Output: a solution for the SMPTSP

```
improved := true
while improved and  $F(x') \neq LB$  and stop criterion not met do
     $x := solverestricted(x', k')$ 
    if  $F(x) < F(x')$  then
        improved := true
    else
        improved := true
    end if
end while
return  $x'$ 
```

Starting from an initial solution, allow the MIP solver to only change a limited number of variables.

Solution approach: improvement heuristic



Solution approach: improvement heuristic



Solution approach: improvement heuristic

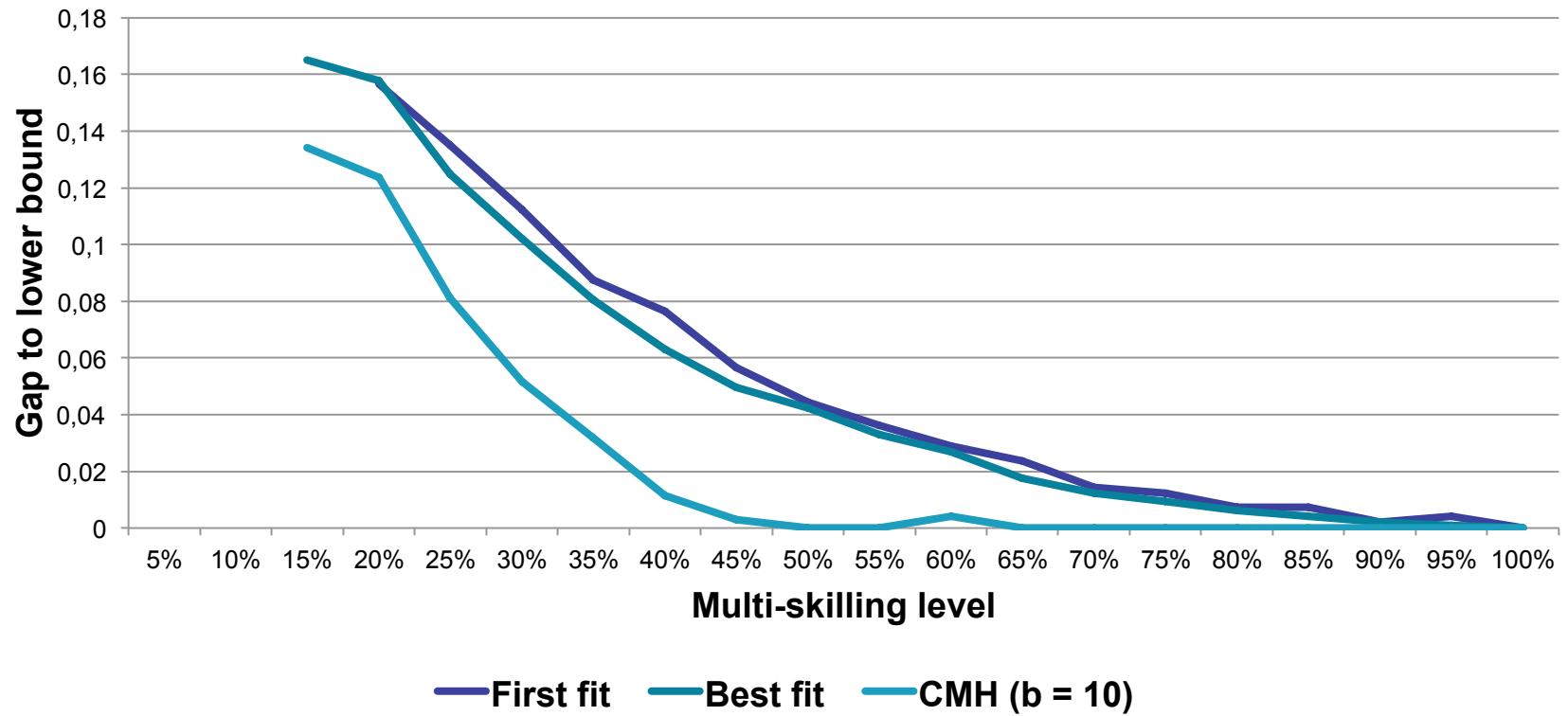
	KEB12	SV12	Our approach
Number of optimal solutions	67	81	137
Average solution quality	127,00	123,00	122,20
Average calculation time (s)	-	958,91	34,46
Maximum calculation time (s)	1800,00	1800,00	282,20

- On average 35 seconds required
- Maximum calculation time is less than 5 minutes

Empirical hardness

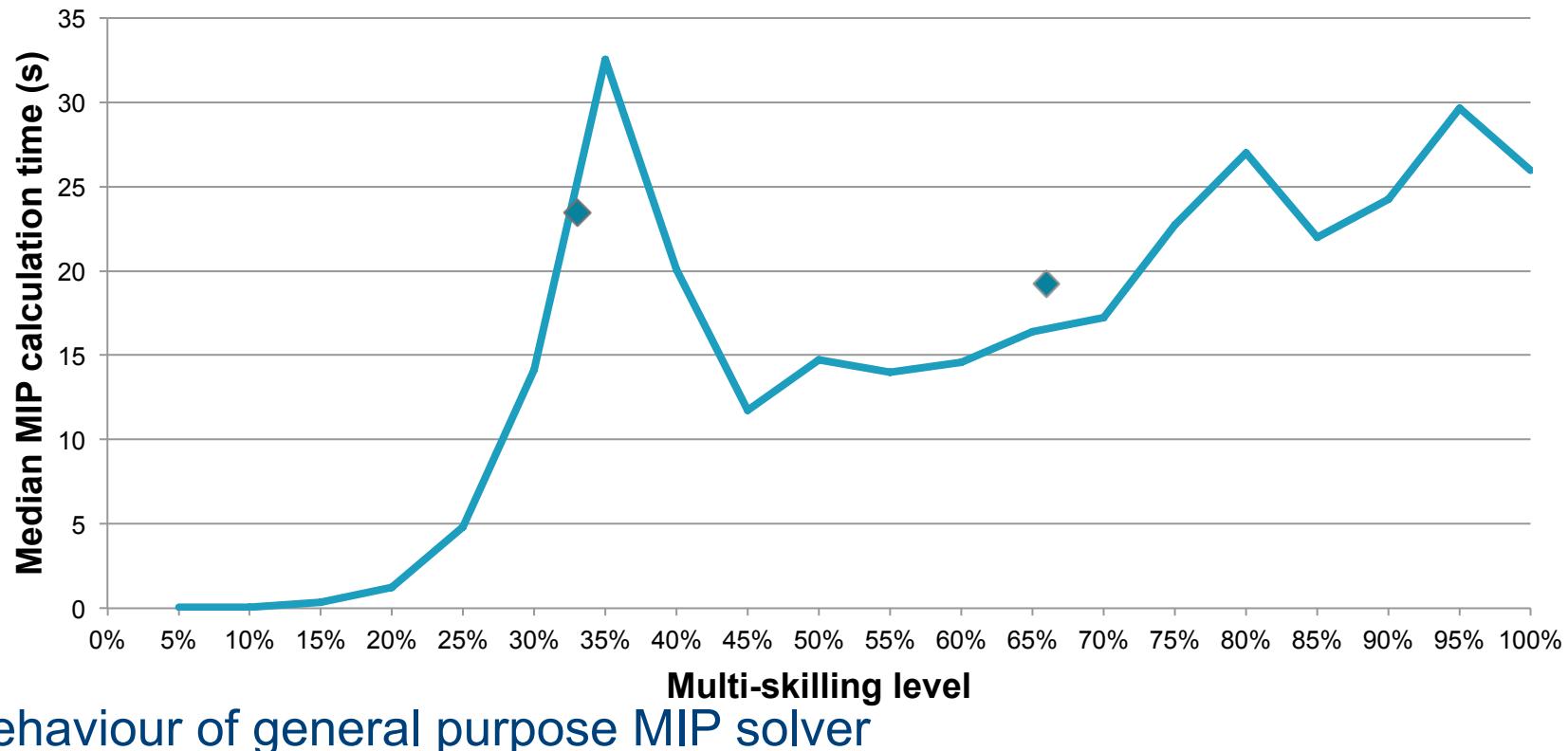
- Second longest calculation time (272 seconds) was on instance with
 - 70 shifts
 - 525 employees
- What makes this instance hard?
- Observe influence of problem properties on algorithmic performance
 - Multi-skilling level
 - Average task length

Empirical hardness: multi-skilling level

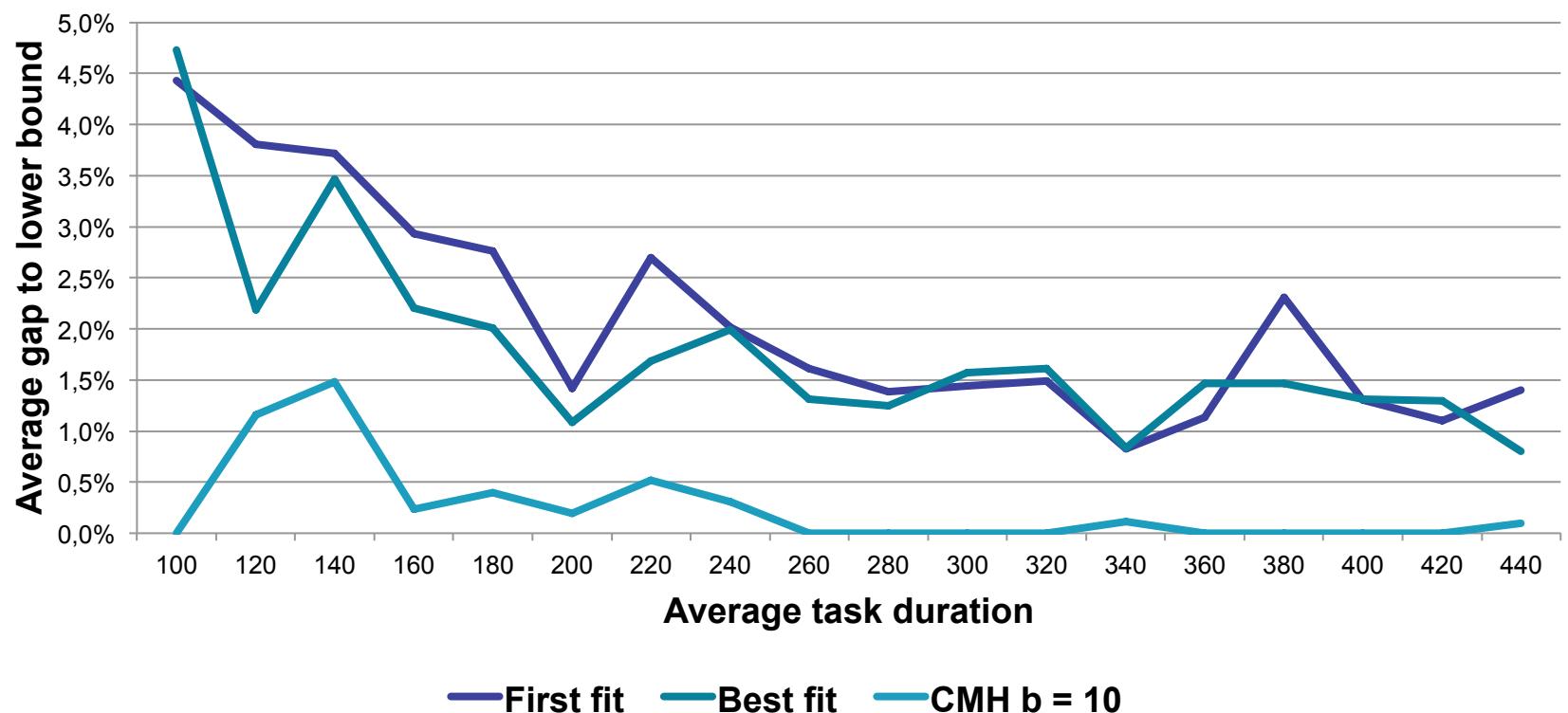


Behaviour of constructive heuristics

Empirical hardness: multi-skilling level

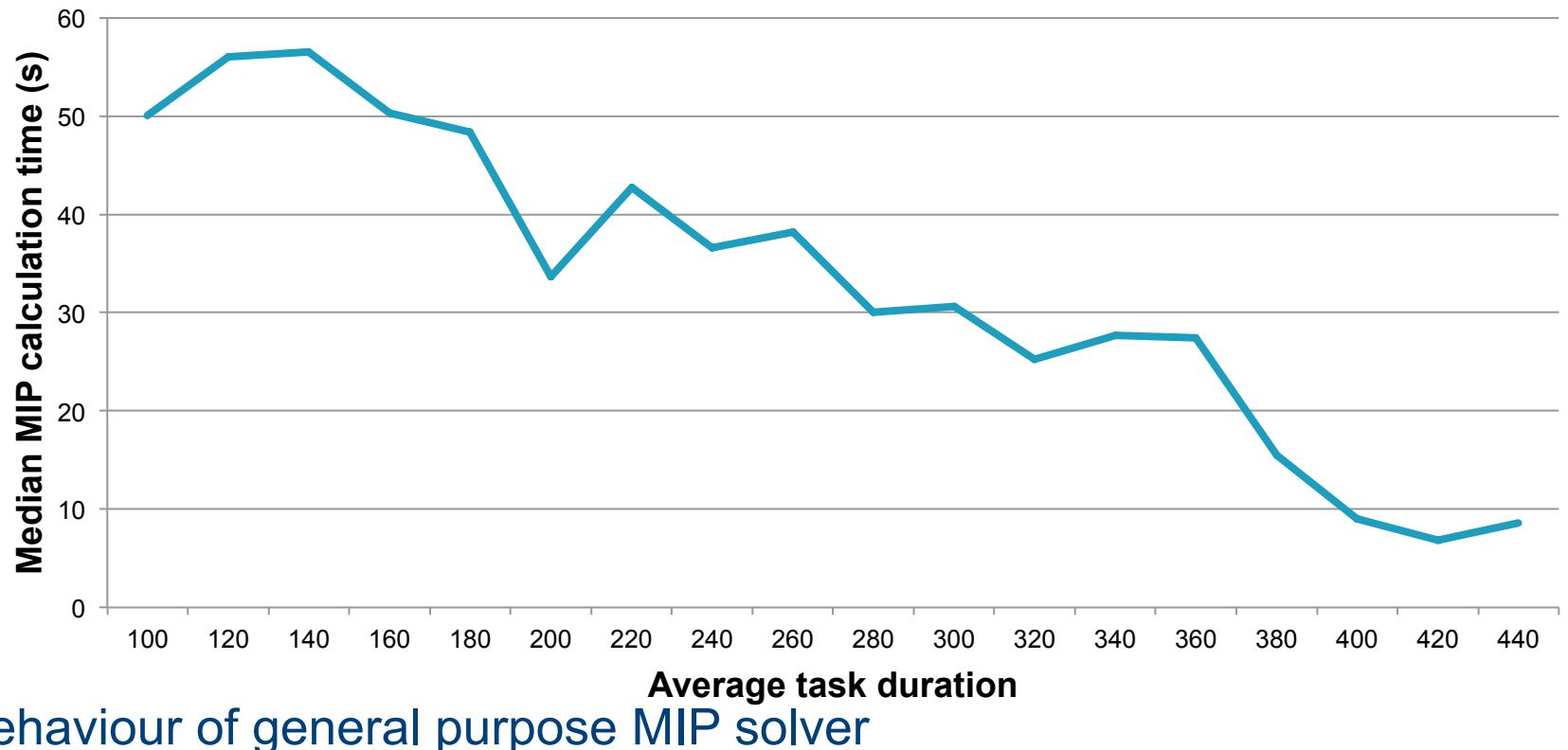


Empirical hardness: task length



Behaviour of constructive heuristics

Empirical hardness: task length



Empirical hardness: new instances

- Based on results, harder instances were generated

Instance	LB	MIP solution
1_50_258_20	40	40
2_44_510_20	40	40
3_102_525_30	77	83
4_113_647_20	98	99
5_77_777_30	59	65
6_135_777_20	116	119
7_70_781_20	59	61
8_88_1022_20	79	80
9_125_1308_20	98	106
10_153_1577_20	116	153

- <http://allserv.kahosl.be/~pieter/smptsp.html>

Conclusions

- Illustration of potential of matheuristics
- Algorithm provides best known results for the SMPTSP
- Insight into what makes the SMPTSP hard for algorithms
- Methodology for generating harder instances

References

- Krishnamoorthy M, Ernst A, Baatar D (2011) Algorithms for large scale shift minimisation personnel task scheduling problems. European Journal of Operational Research
- Smet P, Vanden Berghe G, August 2012. A matheuristic approach to the shift minimisation personnel task scheduling problem. In: Proceedings of the International Conference on the Practice and Theory of Automated Timetabling (PATAT)

Real-world applications

Real-world production scheduling in the food industry

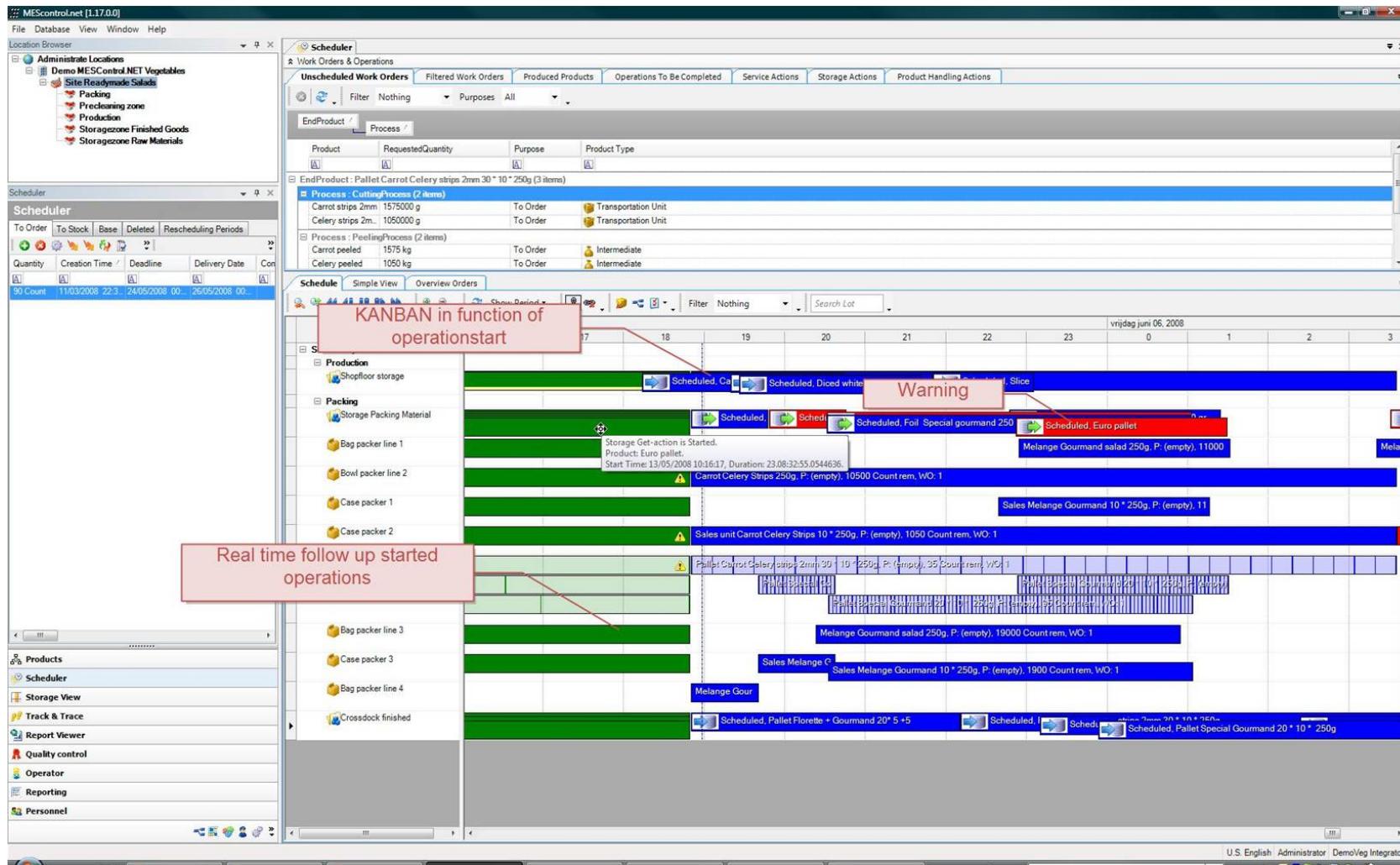


BrightEye
Optimizing through insight.

KU LEUVEN

 KAHO
SINT-LIEVEN

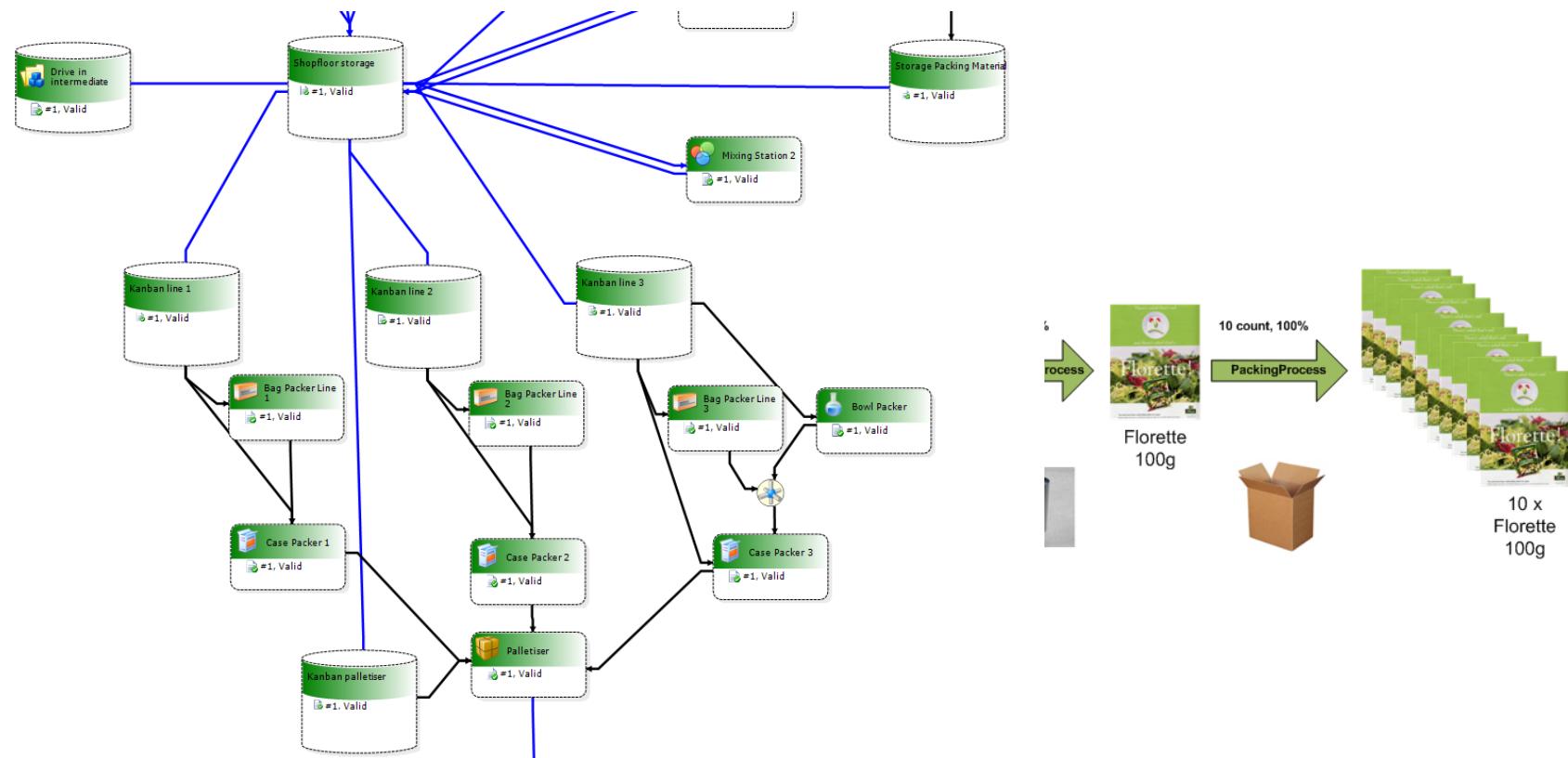
Real-world production scheduling in the food industry



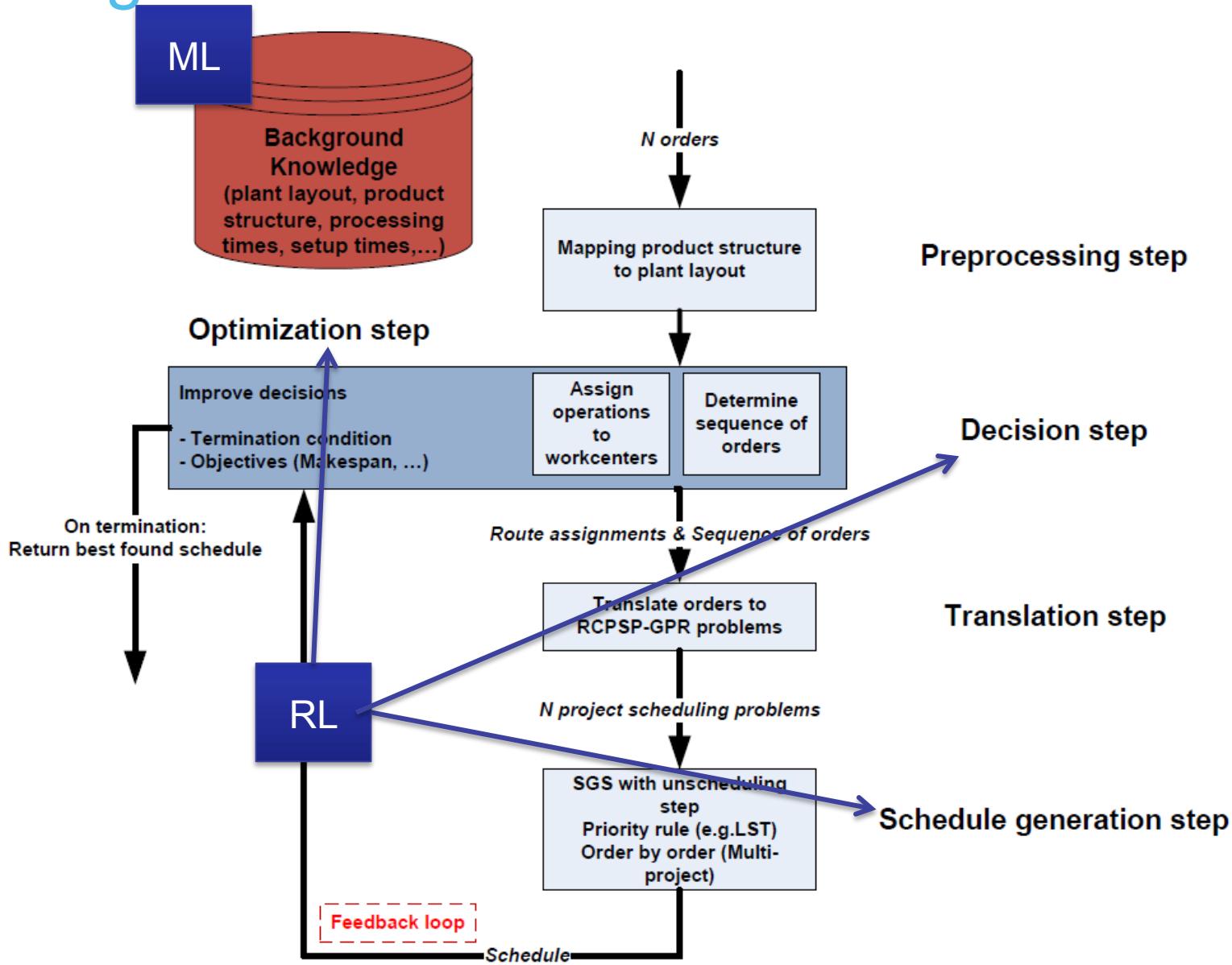
Wauters, T., Verbeeck, K., Verstraete, P., Vanden Berghe, G., De Causmaecker, P. (2012). **Real-world production scheduling for the food industry: an integrated approach.** *Engineering Applications of Artificial Intelligence*, 25(2), 222-228.

Plant Routing

- Mapping product structure to plant layout



Integrated approach high level overview



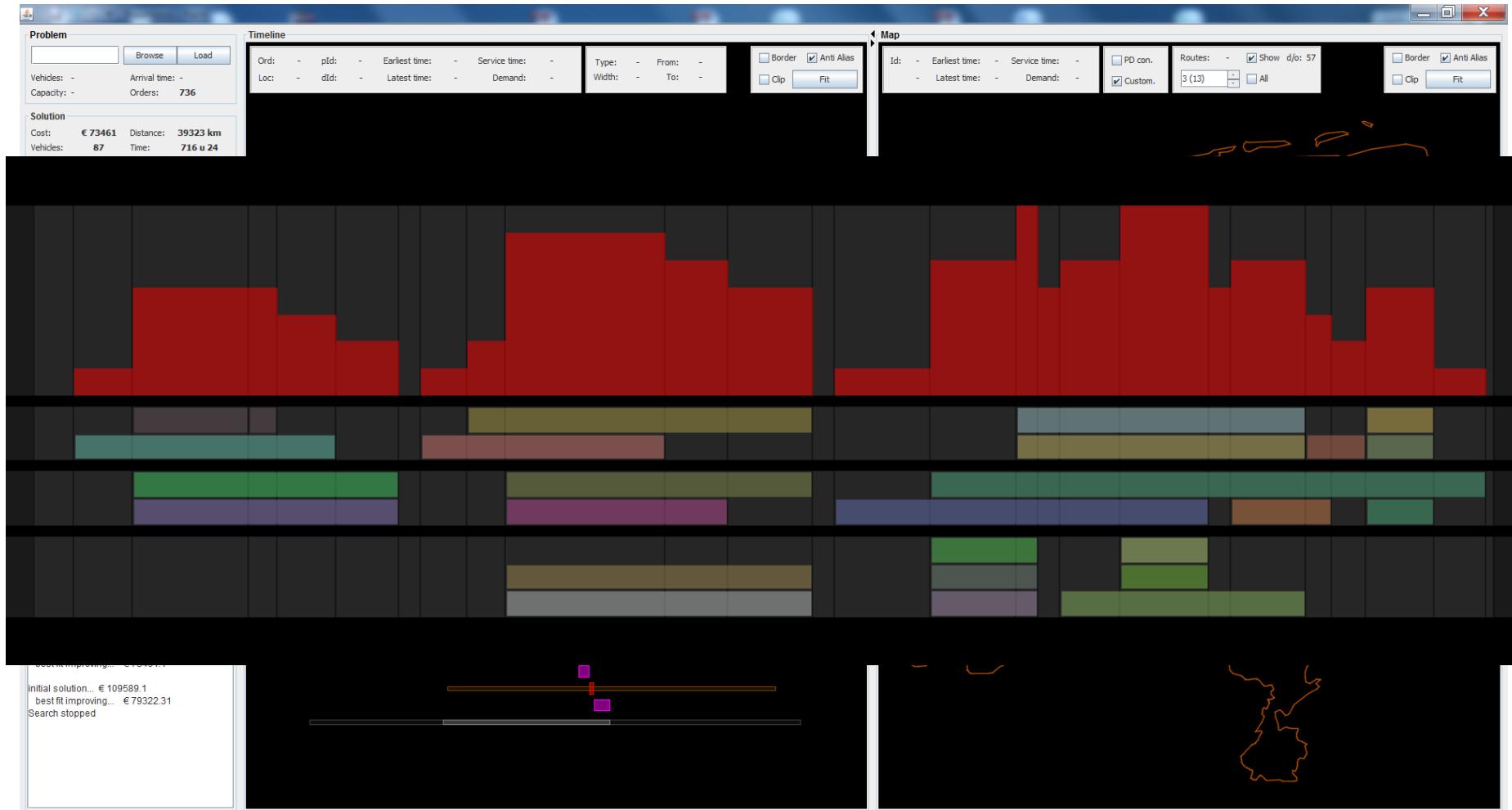
Prisoner transportation



Prisoner transportation

- Case in the Netherlands (DV&O)
- Project with Mobius and Conundra
- Transportation service has about 300 employees
- Its head quarter receives 750 transportation requests per day on average
- **Problem:**
 - Pickup and delivery, Time windows
 - Heterogeneous fleet, multi-depot
 - Exclusionary constraints
 - Two prisoners cannot be transported in the same vehicle/compartment
 - Objective: minimize the total cost, which aggregates vehicle costs and personnel costs.

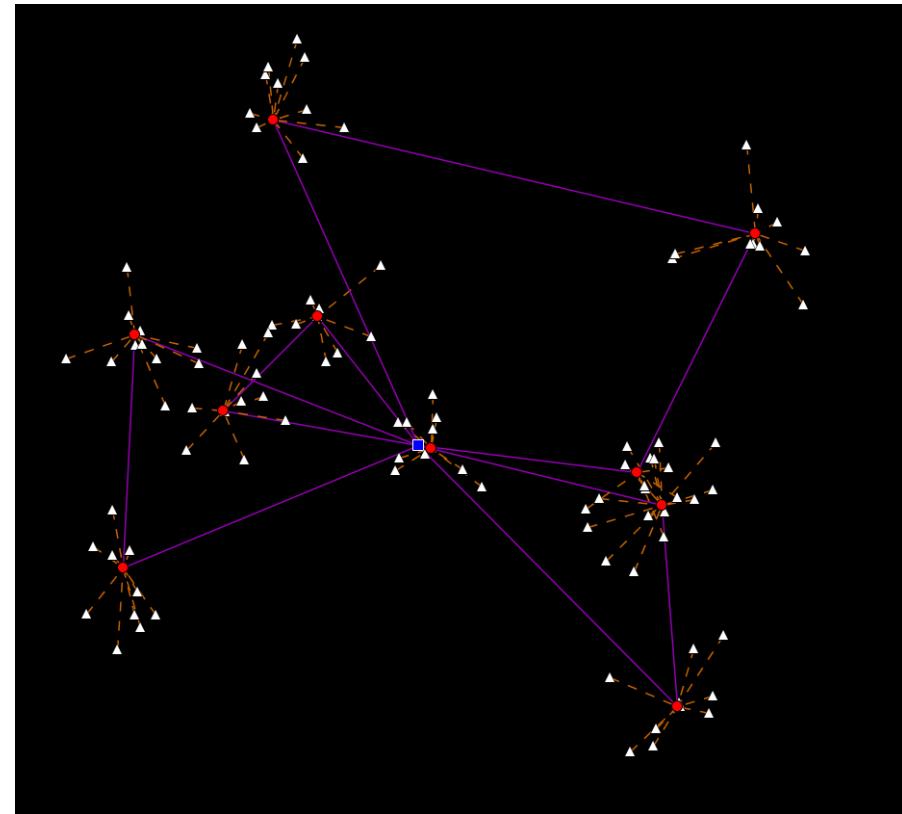
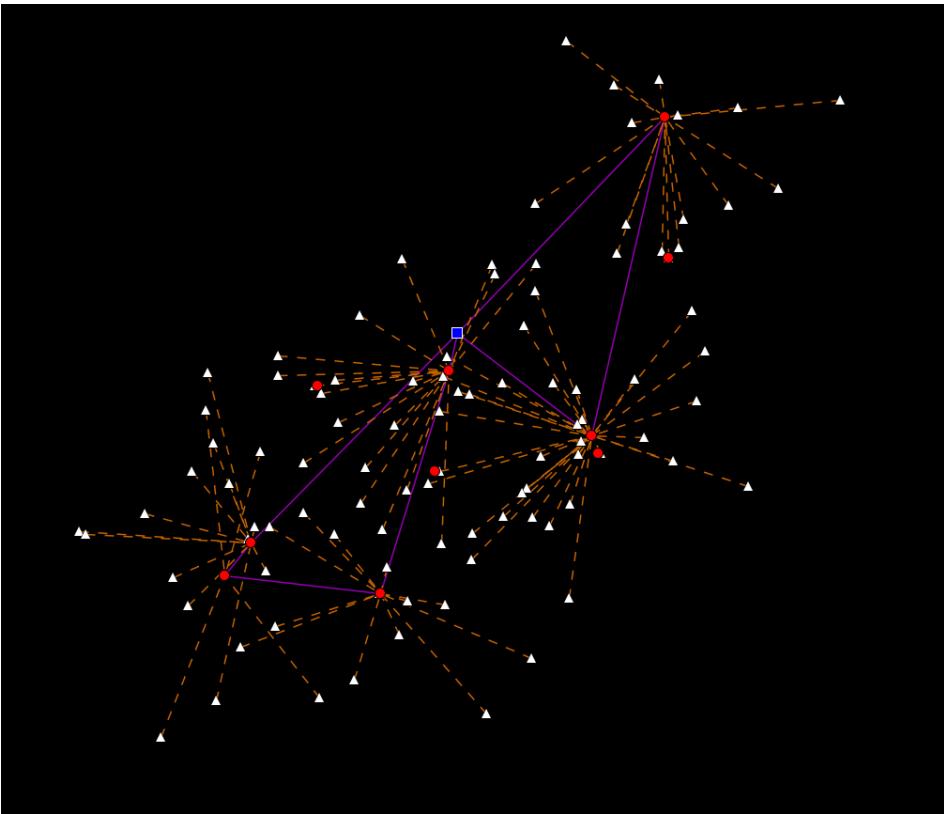
Prisoner transportation



Prisoner transportation

- Approach:
 - Best fit heuristic
 - Add prisoner to “best” possible route
 - Efficient datastructure = very important!
 - Temporal data
 - Spatial data
 - Compartment assignment → graph colouring heuristic

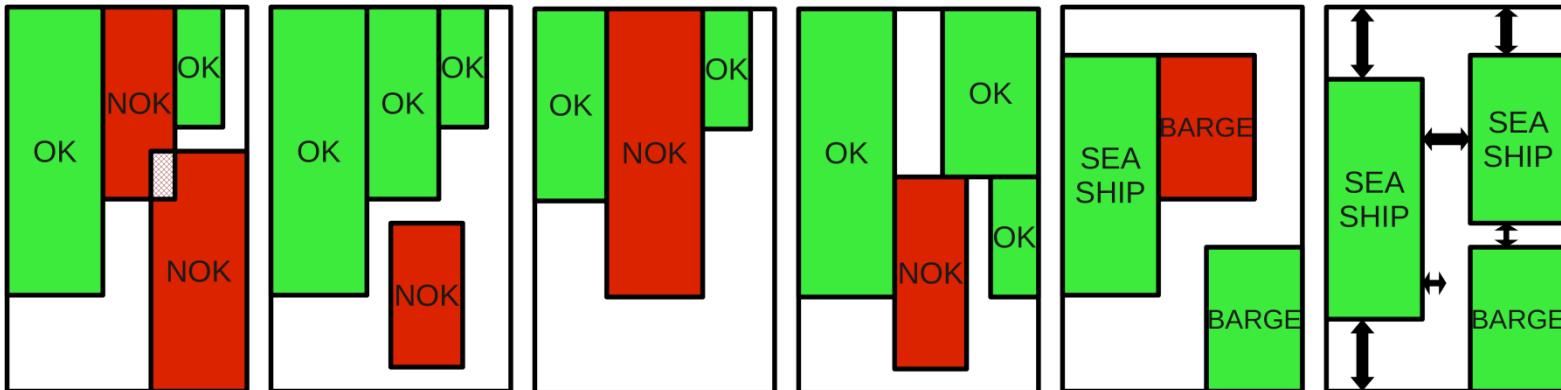
School bus routing and ring-star



Lock scheduling

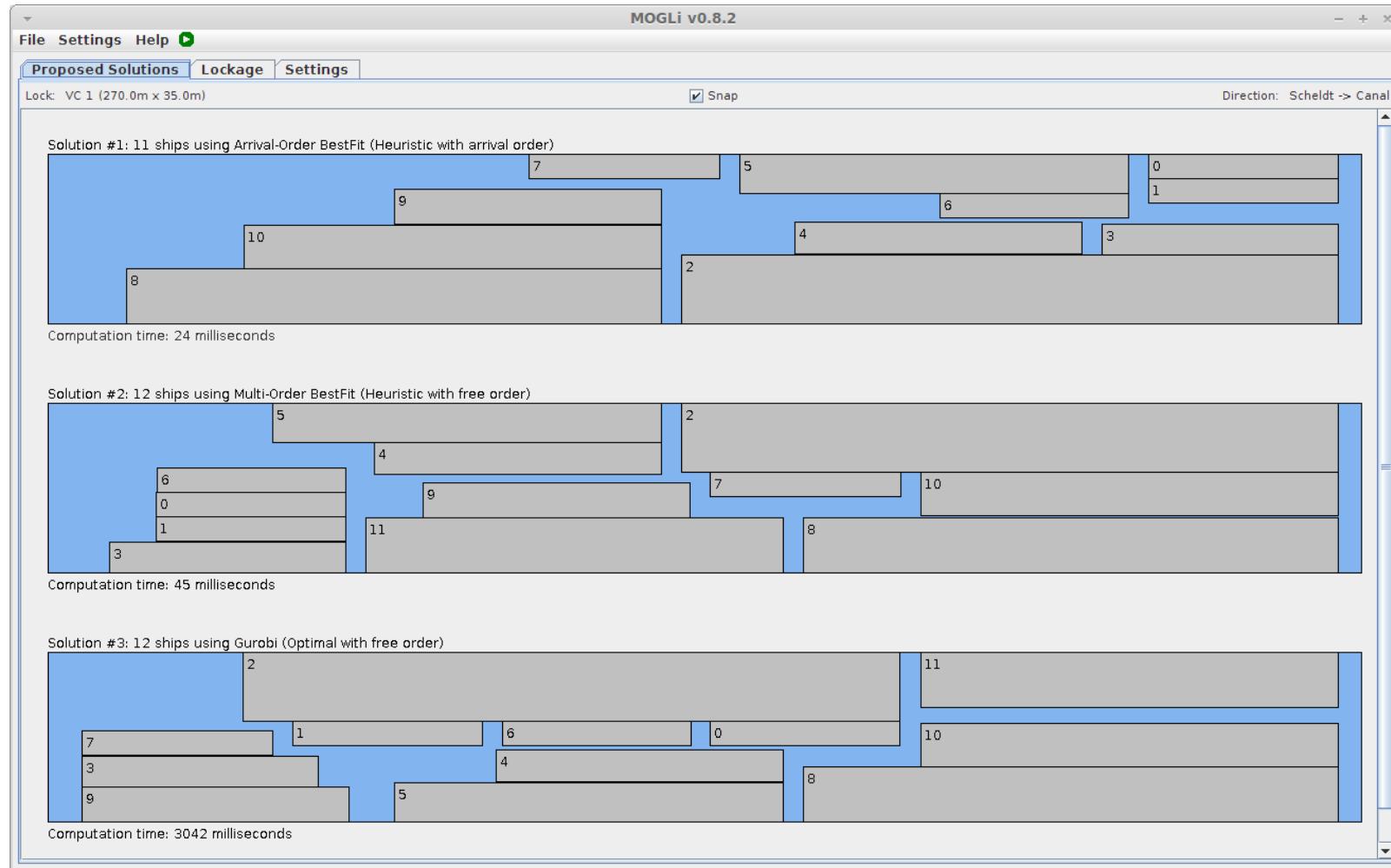


Placement constraints:



* Image by Jannes verstichel, CODES KAHO Sint-Lieven

Lock Scheduling



**Best-fit
Heuristic
45ms**

**Optimal
with MILP
Solver
3042ms**

* Image by Jannes verstichel, CODeS KAHO Sint-Lieven

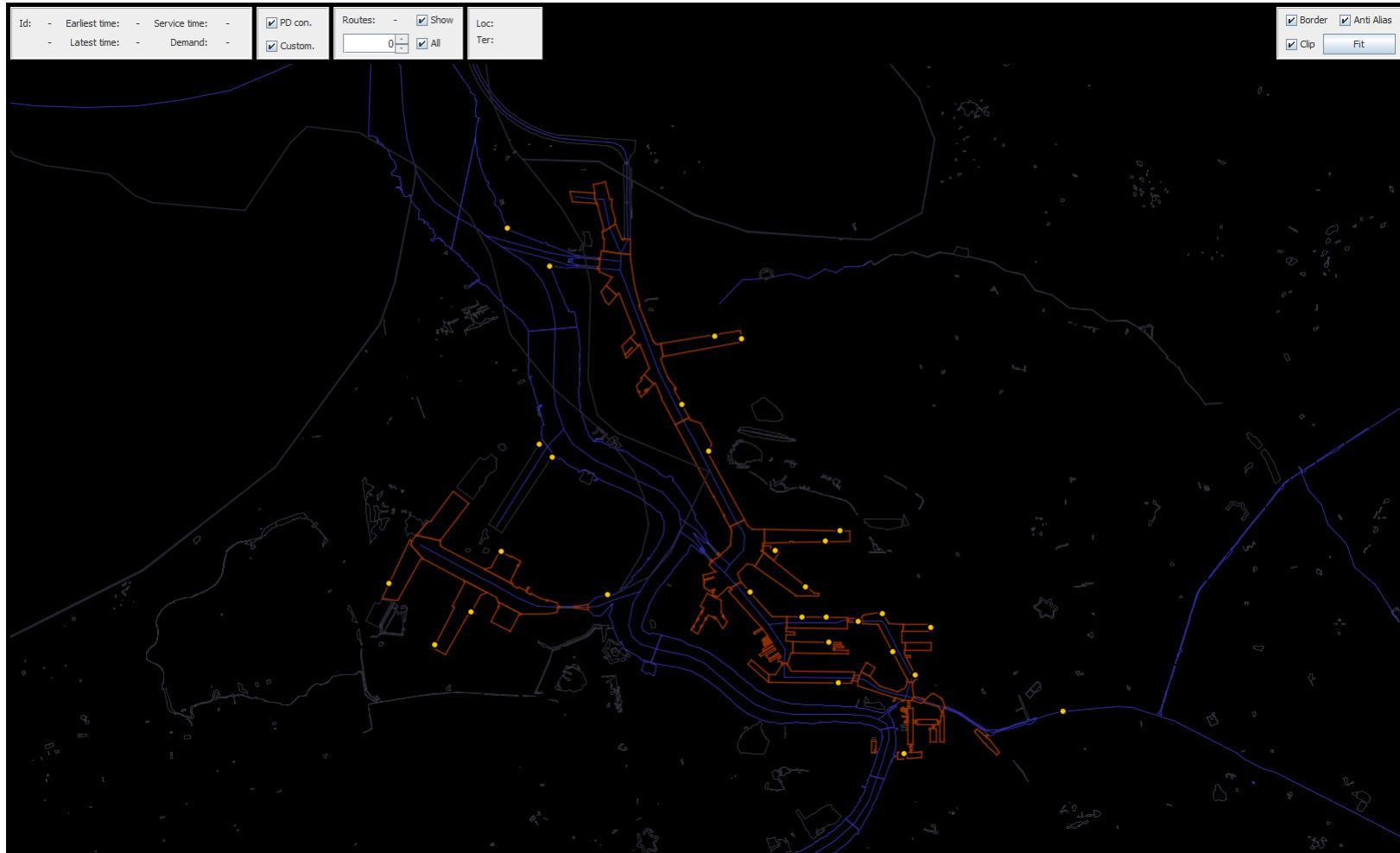
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SINT-LIEVEN

Barge scheduling



Barge scheduling



Personnel planning

september 2011 - Saga- HCPS *** Evaluatie copie - geldig tot 22/01/2012 ***

Bestand Bewerken Bekijken Menu Venster Help

Planning Jobtime Regime

Klinisch labo (Y)

Naam	WT	September 2011																											
		W35	W36	W37	W38																								
DECQ Annelies	38:00	Spd	AG1		BV	Spd	RO	Spd	Spd	R40	Spd	Spd	Spd	Spd	Spd	AG1	Spd	AG1	SPW	R40	Spd	AG1	Spd	Spd					
CHRISTIAEN Femke	38:00	CL1	CL2		CL2	CL2	CV	CDS	CV	CL2	AD2	CL2	AD2	R40		CD1	AD2	CD2	CD2	CV	CD1	CL2	CL2	CD1	AD2				
DENSYNDER Sophie	38:00	AD2	AL3		AL3	AD2	AL3	AD1	R40		AG1	AL3	AD1	AG1	AD1		AD1	AD1	AL3	AD1	AD1	AG1	AL3	AD1	AL3				
TORREZ Ilse	38:00	/	/		/	CL1	CD2	KZ		BV	CD1	AD2	AD2	R40		AD2	I2	BV	/	BV		CD2	AD2	AD2	N02	N02			
DEGRAEVE Mieke	38:00	AD1	AD1		AD1	AD1	AD2	AD2	AD1		AD1	Z	Z	Z	Z		Z	Z	Z	Z		CV	CD2	CD1	CL2	CL2			
VANDROMME Birgit	38:00	CD1	CD1		CD1	CD2	CD2	CL1	CD1	CGLn	CV	CV	CV	CV	CL1		R40	CD5	CL2	AD2	CL2								
Frichtelab 1	0:00																												
BILCKE Jossanne	38:00	AV	RO		CVW		CDF	CV	UR	CL2		CVW		CDF	CL1	AV	CV		CL2	UR	CD1	R40	CD2		RO	CDF	CL1	AV	R40
BILLIEU Melissa	38:00	CD2	CL2		BV	BV	BV	BV	BV		CL1	CDS	AG1	CD1	R40	CGL	CGL	/	RO	CV	CL1		AL3	AD1	CD2	AG1	R40		
LEMOINE Marie	30:24	/	/		CV	CV	80	CD1	R40		N02	N02	N02	N02	/		80	CD1	CV	CDS	80		RO	CD1	80	CV	C2D		
BOURY Heidi	19:00	CV	50		CGL	CL2	/	50	50	CV	CGL	CD5	CL2	50	50	50		CD2	CV	50	50	50	50	50	50	CL1	50		
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DEROO Charlotte	19:00	/	50	CGL	CVW	50	50	/	/		50	CV	50	CL1	50		CD5	50	50	CL2	50		CD6	CD6	CD6	CD6	CD6		
FAES Ann	30:00	R40	BV		BV	CL1	80	CDF	CD5		RO	CL1	80	CDF	CD1		CV	CD2	80	CDF	AV		CL1	RO	80	CDF	CD1		
IGODT Maaike	38:00	MB	MB		MB	MB	MB	MB	MB		MB	MB	MB	MB	MB		MB	MB	MB	MB	MB	MB	MB	MB	MB	MB			
LAMEIRE Ann	30:24	BV	50		R40	WF	80	N02	N02	N02	/	/	80	CDS	/	CVW	N02	N02	/	/	80	BIB	80	R40	CV				
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SICHEN Maria	12:00	CDF																											
VICTOR Hermien	19:00	BV	50		CL1	50	50	50	CD2	CVW	CV	50	50	/	CL2		CVW	50	RO	50	50	/	CL2	50	50	50	BV		
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LOBELLE Miriam	20:00	AHH	AHH		ADH	AV	AHH	ADH	ADH		AHH	AV	AHH	AHH	AHH		RO	AV	/	/	/	/	/	/	/	50	50	50	
MERKERKE Ann	30:00	AL3	BV	SPW	AG1	ADH	80	AG1	/		R40	ADH	80	AD1	RO	SPW	AG1	AG1	80	AL3	AD2		AD1	AHD	80	BIT	AHD	AHD	
VANBELLEGEM Ilse	38:00	MB	MB		MB	MB	MB	MB	MB		MB	MB	MB	MB	MB		MB	MB	MB	MB	MB	MB	MB	MB	MB	MB	MB	MB	
VANHOLME Nicole	11:00																												
VANDANSCHOOTE Ann	38:00	AG1	AD2		AHW	UR	AG1	AG1	R40	/		RO	AG1	BV	AL3	AG1		AL3	AL3	AD1	BV	BV	AHW	AHD	AG1	AL3	AL3	BV	
VERCRUYSE Peggy	24:00	60	BV		60	60	AD1	AL3	AG1	AHW	AD2	RO	60	60	/		60	60	/	AG1	AL3		60	60	RO	AD2	AG1		
INGELS Jean	38:00	BD2	R40		BD3	BD2	BD2	BD2	AV		AV	R40	BD2	RO	BD1	BW1	BHW	/	RO	BD1	BD3	BD3		BD2	BD2	BD1	RO	AV	
LESAFFER Ann	38:00	BV	CL1		BW1	BD1	BIT	CL2	R40	BD2	BW1	BD2	BD2	/	BD3	RO		BV	BV	RO	KZ	BD2	BD2	BD3	BD1	BD1			
MOEVAERT Dirk	38:00	BD3	BD3		BV	BV	SPD	BD1	AV		Spd	BD1	BD1	BD1	/		AV	R40	BD2	BD2	Spd	BD1	BD1						

Tellers

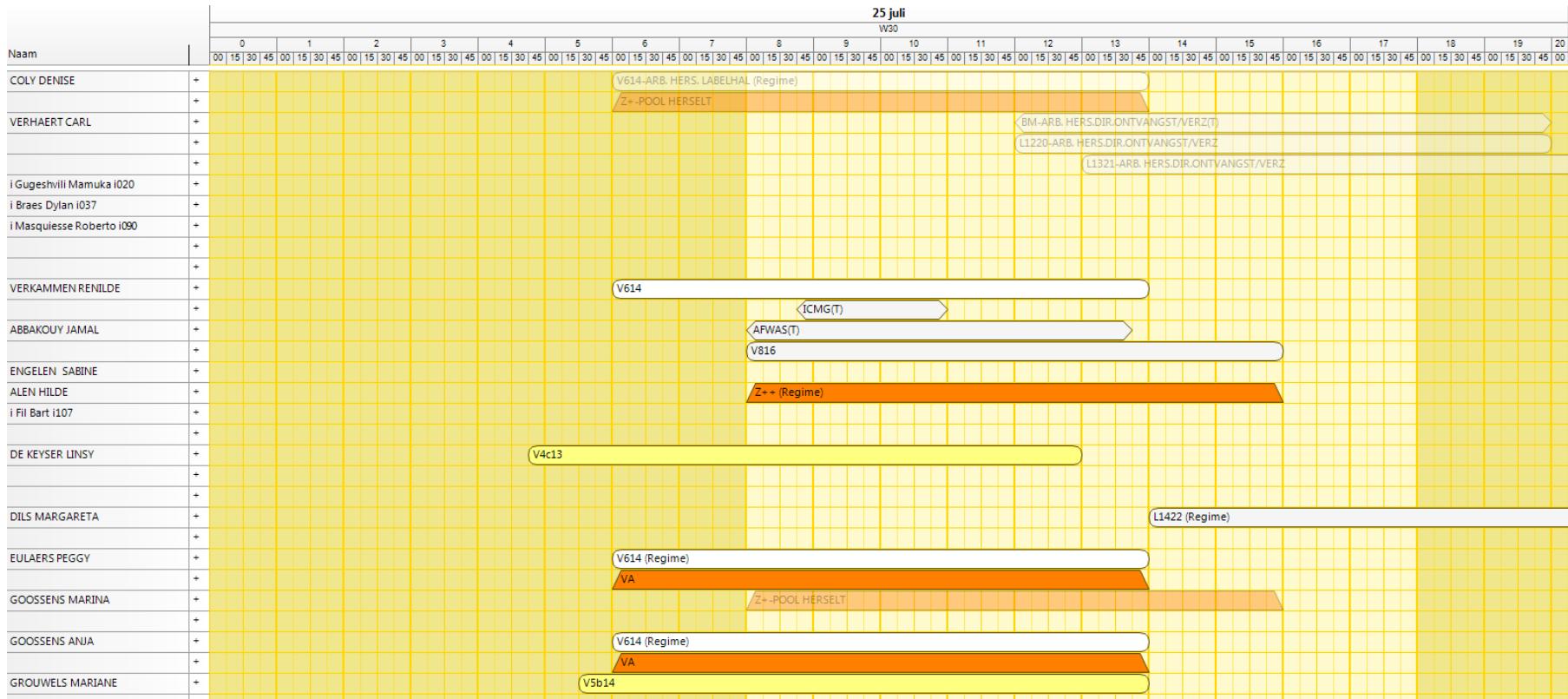
KZ	STW/ set	STW	OU	Stand	EL	BV	JV	V
-3,20	25,25	6,35	31,60	0,00	24,00	0,00		
0,30	-0,20	7,75	7,55	0,00	0,00	0,00		
0,80	10,40	10,00	20,40	0,00	0,00	0,00		
-35,45	38,60	13,25	51,85	0,00	64,00	0,00		
-39,45	-42,00	0,25	-41,75	0,00	0,00	0,00		
0,305	-2,90	4,00	1,10	0,00	101,88	0,00		
-	-	-	-	-	-	-		
-4,20	17,25	45,97	63,22	0,00	24,00	0,00		
-6,95	40,60	5,95	46,55	0,00	40,00	0,00		
-13,80	22,68	-11,10	11,58	0,00	0,00			
-3,60	60,55	-9,55	51,00	0,00	20,00			
0,00	47,77	47,49	95,05	30,40	152,00	0,00		
-14,85	46,05	1,88	47,93	0,00	12,00			
3,75	76,12	38,57	114,68	0,00	24,00			
0,00	6,00	48,02	54,02	0,00	139,00	0,00		
-14,30	80,28	7,57	87,84	0,00	33,60			
-10,70	57,40	148,82	206,22	0,00	32,00			
-4,80	100,95	-7,00	93,95	0,00	12,00			
-0,60	55,45	10,63	66,08	0,00	12,00			
7,70	55,98	-1,40	54,58	0,00	19,20			
5,55	48,97	48,48	97,45	0,00	24,00			
-27,60	42,20	7,25	49,45	0,00	19,62			
-28,00	0,00	3,50	3,50	16,00	80,00	0,00		
-1,50	30,70	17,05	47,75	0,00	24,00			
0,00	-0,07	7,82	7,75	30,40	152,00	0,00		
-17,40	15,35	7,05	22,40	0,00	8,50			
2,80	-7,75	27,00	19,25	0,00	120,00	0,00		
-12,60	17,78	9,45	27,23	0,00	13,67			
-3,70	15,75	56,08	71,83	0,00	24,00			
1,55	11,83	78,92	90,75	0,00	24,00			
-15,20	27,25	20,30	32,55	0,00	24,00			

Personlijke overtredingen Voor DECQ Annelies

Naam controle	Van	Tot
AG1	1/09/2011	30/09/2011
SPW	1/09/2011	30/09/2011

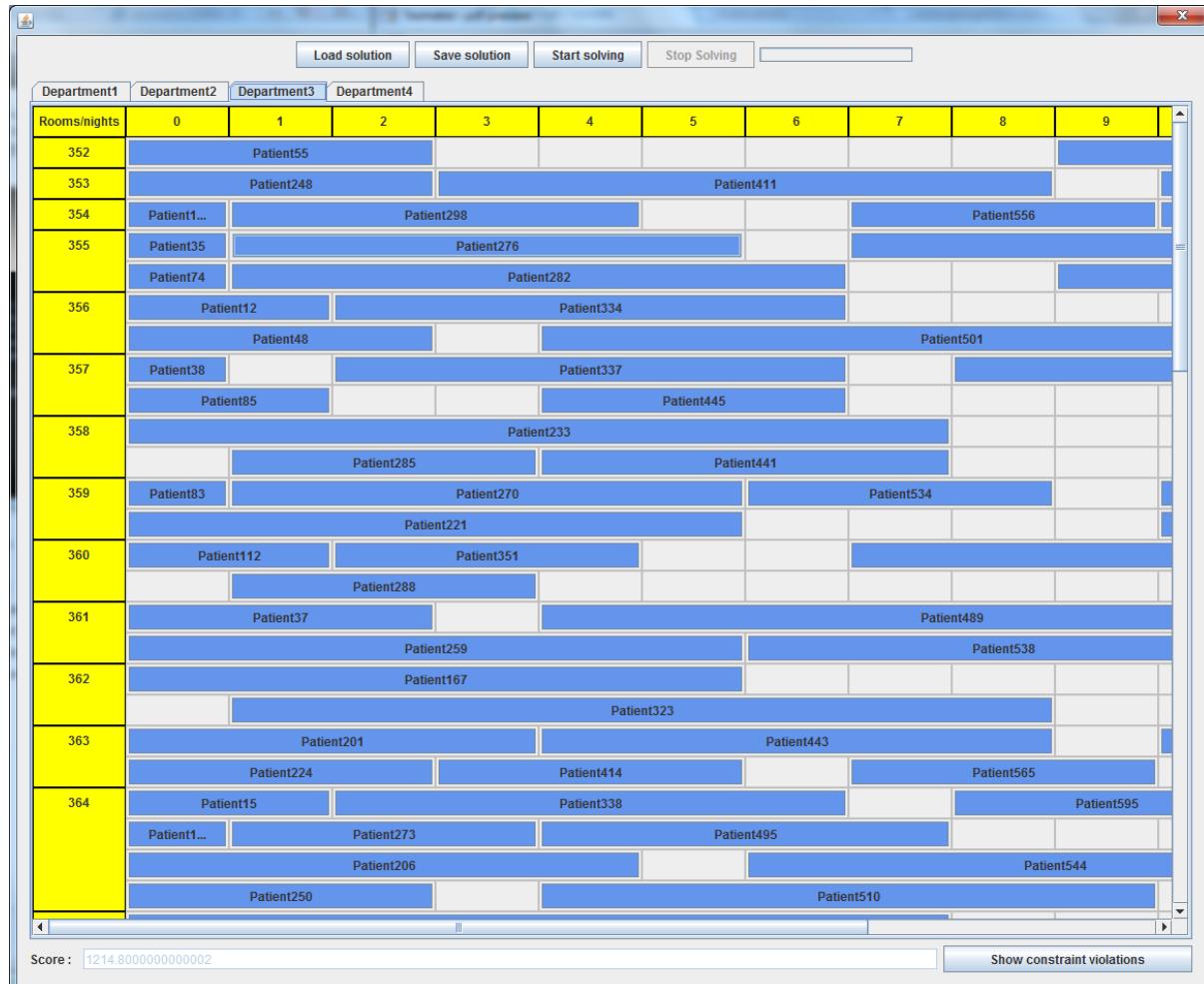
* Image of SAGA HCPS

Personnel task planning



* Image of SAGA HCPS

Patient admission scheduling



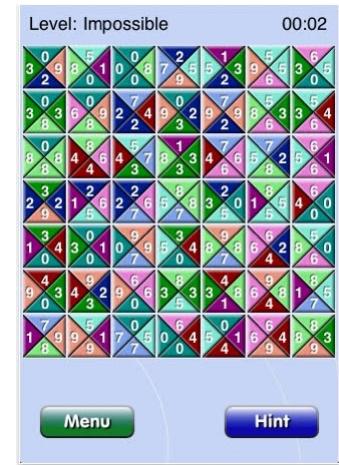
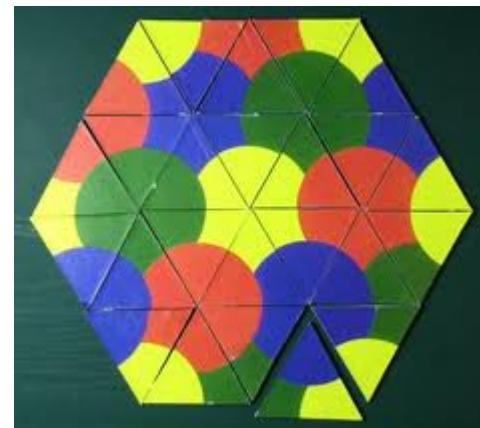
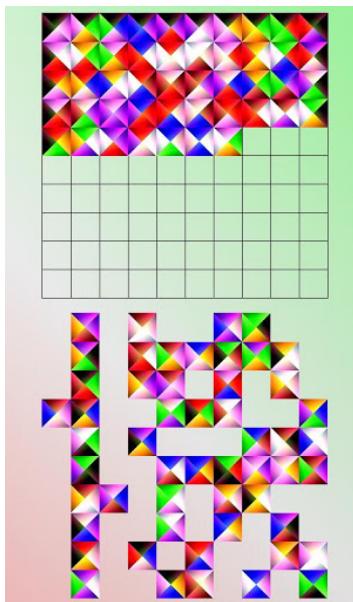
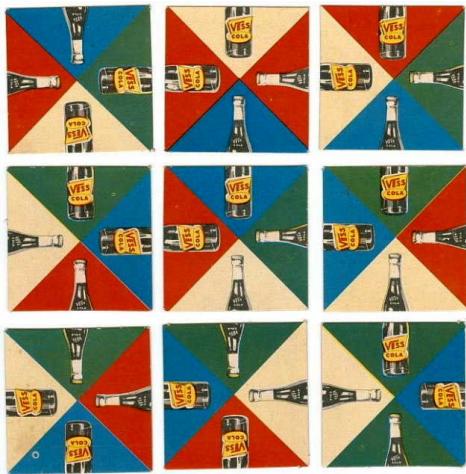
Hospital
Multiple departments
Rooms with beds

Objective
Minimize constraint violations regarding to
- Age
- Gender
- Equipment
- Single/double room
→ Weighted objective function

* Image by Wim Vancroonenburg (CDeS, KAHO Sint-Lieven)

“Not so real-world” - Edge matching puzzles

- NP-complete



Edge matching puzzles

- Try it yourself – download the android app!
- Google play:

[https://play.google.com/store/apps/details?
id=be.kahosintlieven.edgematching](https://play.google.com/store/apps/details?id=be.kahosintlieven.edgematching)

