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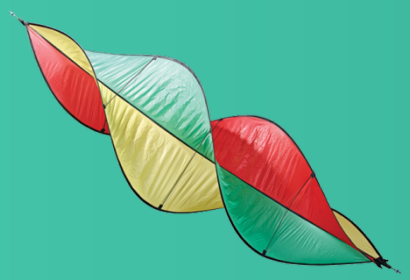

Heuristics: recent trends and real world applications

Dr. Tony Wauters
CODeS research group

Overview

- Recent trends
  - Hyperheuristics
  - Hybrid heuristics
    - Matheuristics
      - Example: Personnel Task scheduling
- Real world applications
  - Production scheduling in the food industry
  - Prisoner transportation
  - Lock scheduling
  - Personnel scheduling
  - Patient admission scheduling

Recent trends

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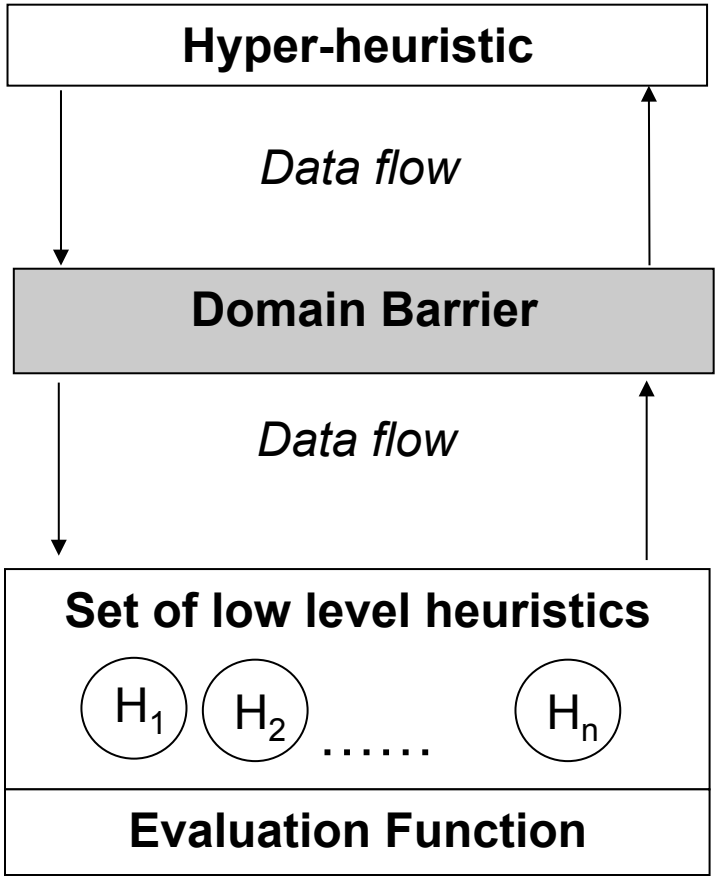
Hyperheuristics

- Simple Idea:
  - Heuristics to choose heuristics
  - Or heuristics to generate heuristics
- Operates on a search space of heuristics rather than directly on a search space of solutions

Hyperheuristics

- Metaheuristics tackle specific problems
- Are often tailored
- Can we increase the level of generality?
- Good enough, cheap enough, fast enough?

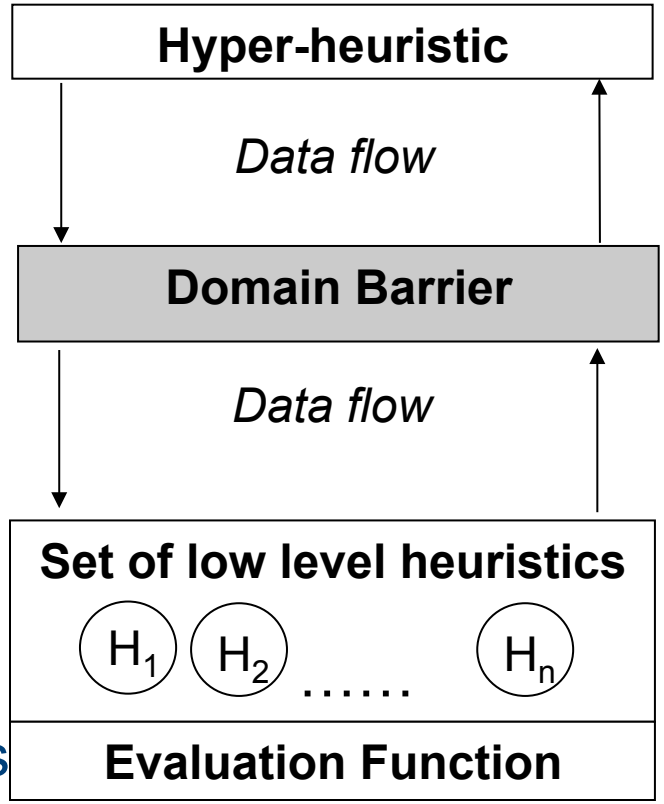


Hyperheuristics

- No ‘Superalgorithms’ (No Free Lunch)
- Intelligent schemes that function well on a number of problems
- Give the developer extra handles to bring in his domain expertise without requiring him to study the details of an advanced optimisation strategy

Hyperheuristic

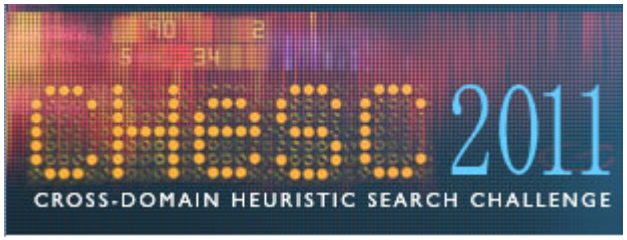
- Above the domain barrier
  - Genetic algorithm
  - Genetic programming
  - Tabusearch
  - Simple random
- Room for machine learning
  - Learn the weights of the low level heuristics
  - Relate them to solution space features (> expert?!)
  - Reduce the number of parameters
- Hyper heuristic competition



Cross-domain Heuristic Search Challenge

http://www.asap.cs.nott.ac.uk/external/chesc2011/

- 6 problem domains
  - Boolean satisfiability (MAX-SAT)
  - One dimensional bin packing
  - Permutation flow shop
  - Personnel scheduling
  - Traveling salesman problem (TSP)
  - Vehicle routing (VRP)



- Given for each problem domain:
  - low-level heuristics or operators (swaps, crossover, hillclimber,…)
  - Objective function to evaluate obtained solutions.
- Winner:
  - Best performing over all problem domains
  - Adaptive/learning hyperheuristic by Mustafa Misir(CODeS)

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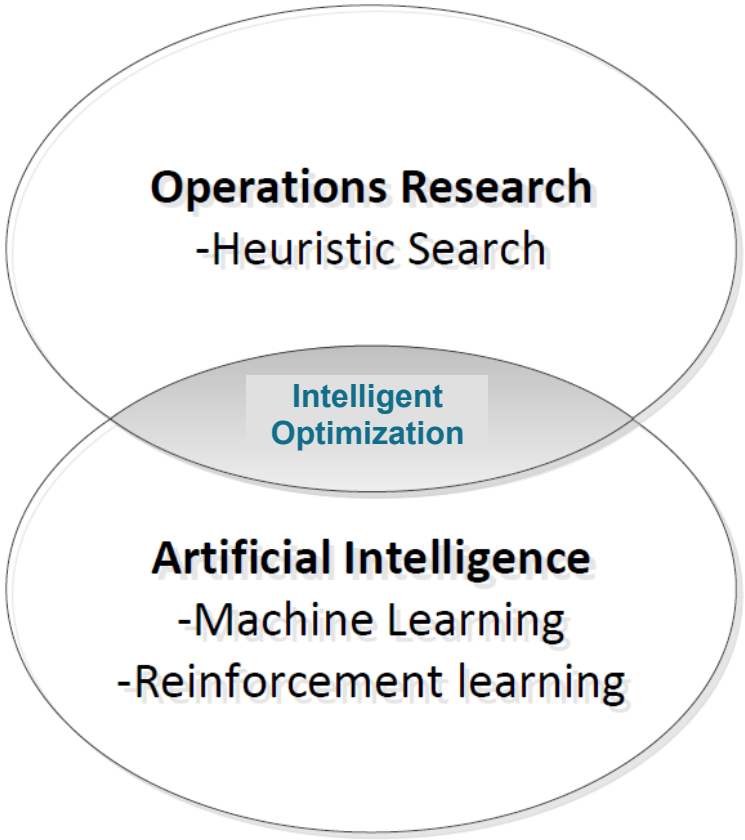
Hybrid heuristics

- Hybridizations of:
  - (meta)heuristics with (meta)heuristics
  - (meta)heuristics with exact methods (mathematical programming (MIP)) → **Matheuristics**
  - (meta)heuristics with constraint programming
  - (meta)heuristics with machine learning and data mining
    - Intelligent Optimization

Matheuristics

- Hybridization of **mathematical methods** and **heuristics**
  - Use MIP as metaheuristic neighbourhood
  - Use MIP to initialize metaheuristic
  - Use metaheuristic to initialize MIP
  - Exchange solution between MIP and metaheuristic
  - …

Reactive search – Intelligent optimization



BOOK:
ROBERTO BATTITI, MAURO BRUNATO AND FRANCO MASCIA, **Reactive Search and Intelligent Optimization**, *Operations Research/Computer Science Interfaces Series*, Vol. **45**, Springer, November 2008.

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Matheuristic example:

Hardness analysis and a new approach for the shift minimisation personnel task scheduling problem

Pieter Smet
Tony Wauters
Greet Vanden Berghe

CODeS, KAHO Sint-Lieven

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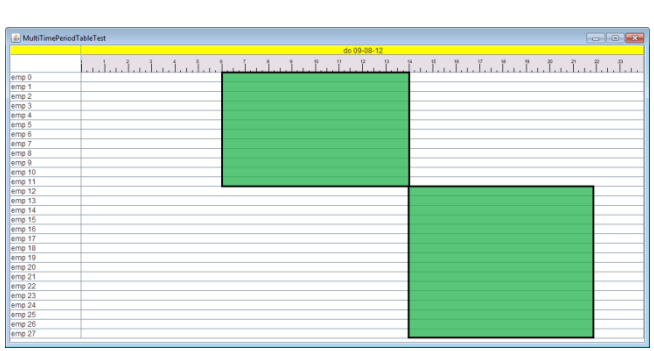
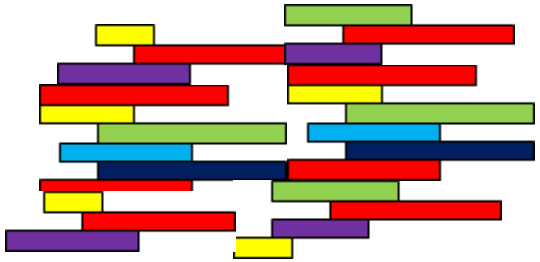
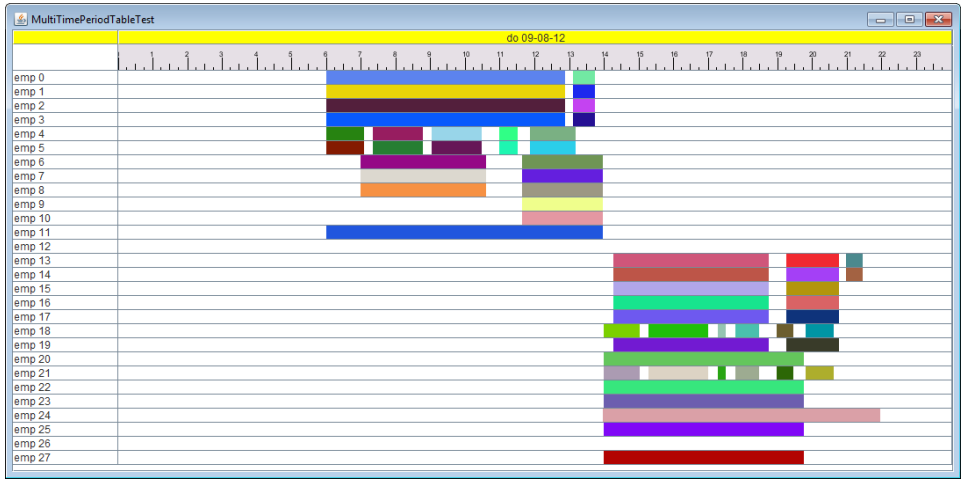
Introduction

- Assigning tasks to employees
  - Fixed working times of employees
  - Fixed start and end times of tasks







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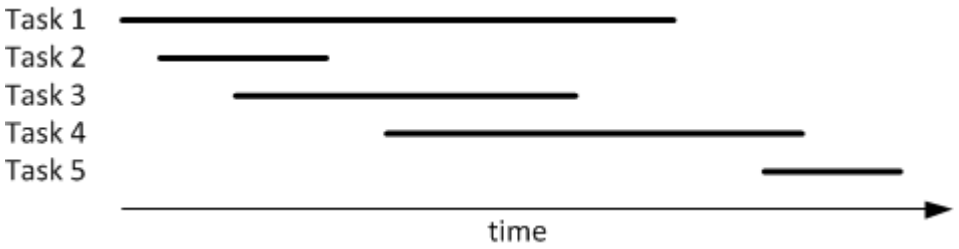
Problem description

- Shift minimisation personnel task scheduling problem (SMPTSP) (Krishnamoorthy et al, 2011)

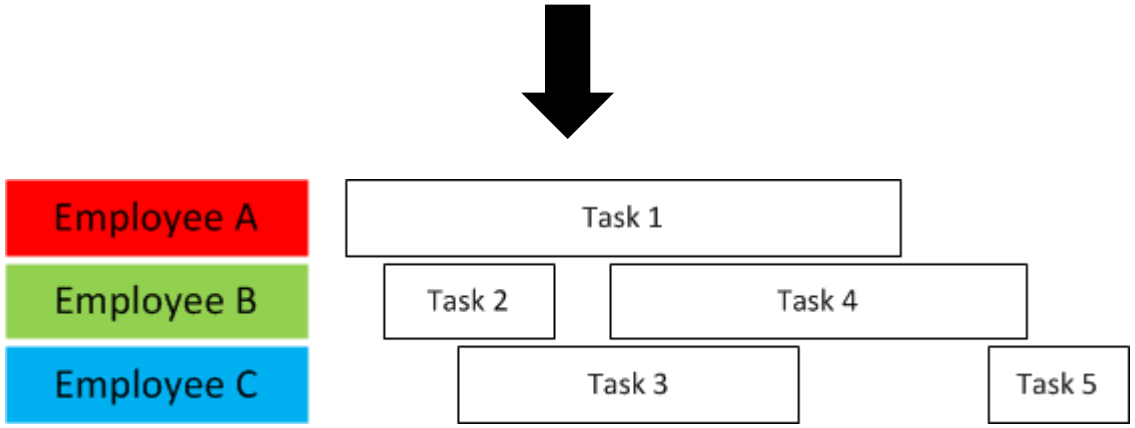
- Objective: minimise number of employees used

- Constraints
  - Each task is performed by one employee
  - No preemption
  - No overlapping tasks assigned to one employee
  - Respect qualification requirements
  - Assign only tasks to working employees

Example



	Task 1	Task2	Task 3	Task 4	Task 5
Employee A	x	x	x		
Employee B		x	x	x	
Employee C			x	x	x



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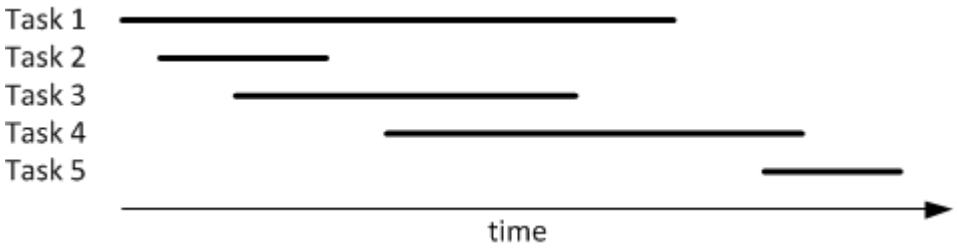
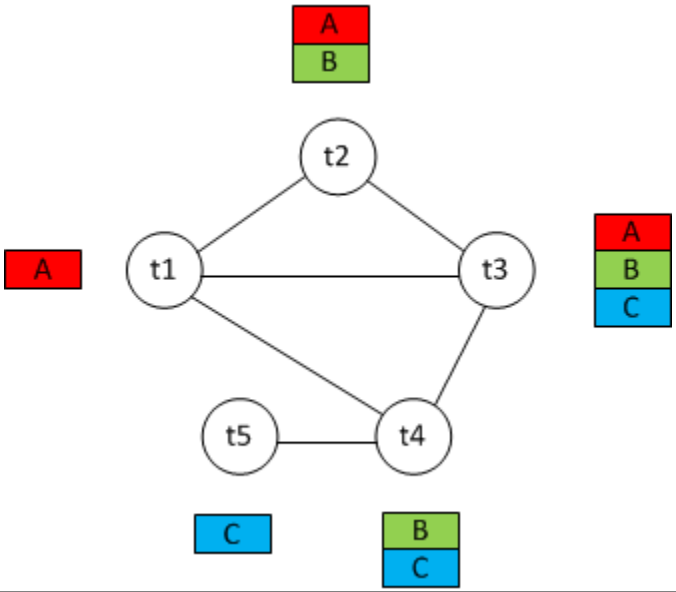
SMPTSP



	Task 1	Task2	Task 3	Task 4	Task 5
Employee A	x	x	x		
Employee B		x	x	x	
Employee C			x	x	x

As a list colouring problem



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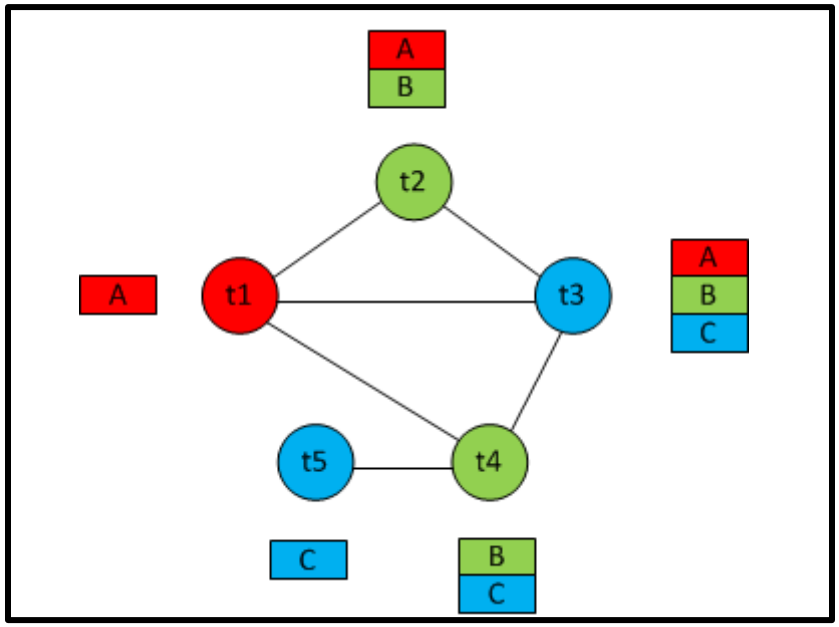
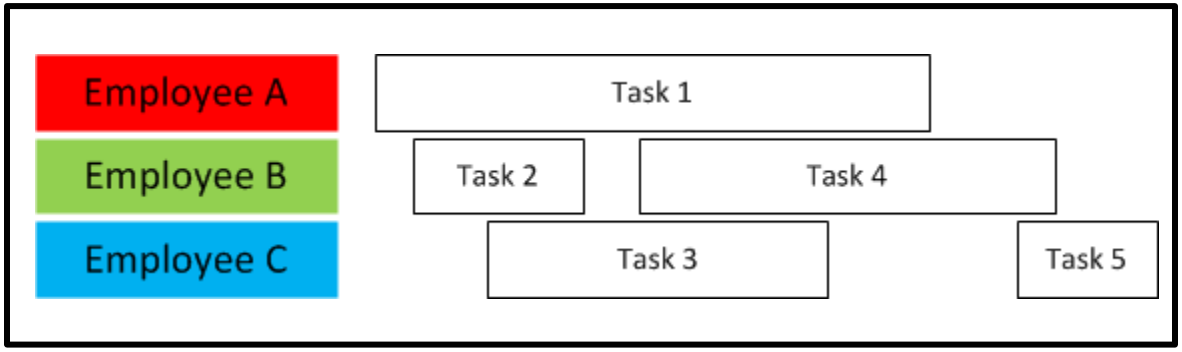
List colouring solution



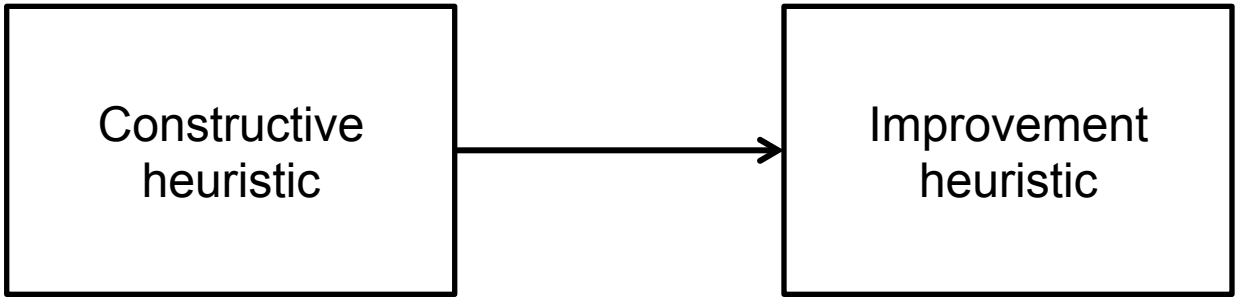
As a SMPTSP solution



State of the art

- Krishnamoorthy et al (2011)
  - Lagrangean relaxation based heuristic
- Smet and Vanden Berghe (2012)
  - Fix-and-optimize heuristic

Solution approach: overview



- Both phases use a general purpose MIP solver

MIP model

- Decision variables

$$x_{jw} = \begin{cases} 1 & \text{if task } j \in J \text{ is assigned to employee } w \in W \\ 0 & \text{otherwise} \end{cases}$$

$$y_w = \begin{cases} 1 & \text{if employee } w \in W \text{ is active, meaning that } w \text{ has at least one task} \\ 0 & \text{otherwise} \end{cases}$$

- MIP model

$$min \sum_{w \in W} y_w \tag{1}$$

$$s.t. \sum_{w \in P_j} x_{jw} = 1 \qquad \forall\ j \in J \tag{2}$$

$$\sum_{j \in K_t^w} x_{jw} \leq y_w \qquad \forall\ w \in W,\ K_t^w \in C^w \tag{3}$$

$$0 \leq y_w \leq 1 \qquad \forall\ w \in W \tag{4}$$

$$x_{jw} \in \{0, 1\} \qquad \forall\ j \in J,\ w \in W \tag{5}$$

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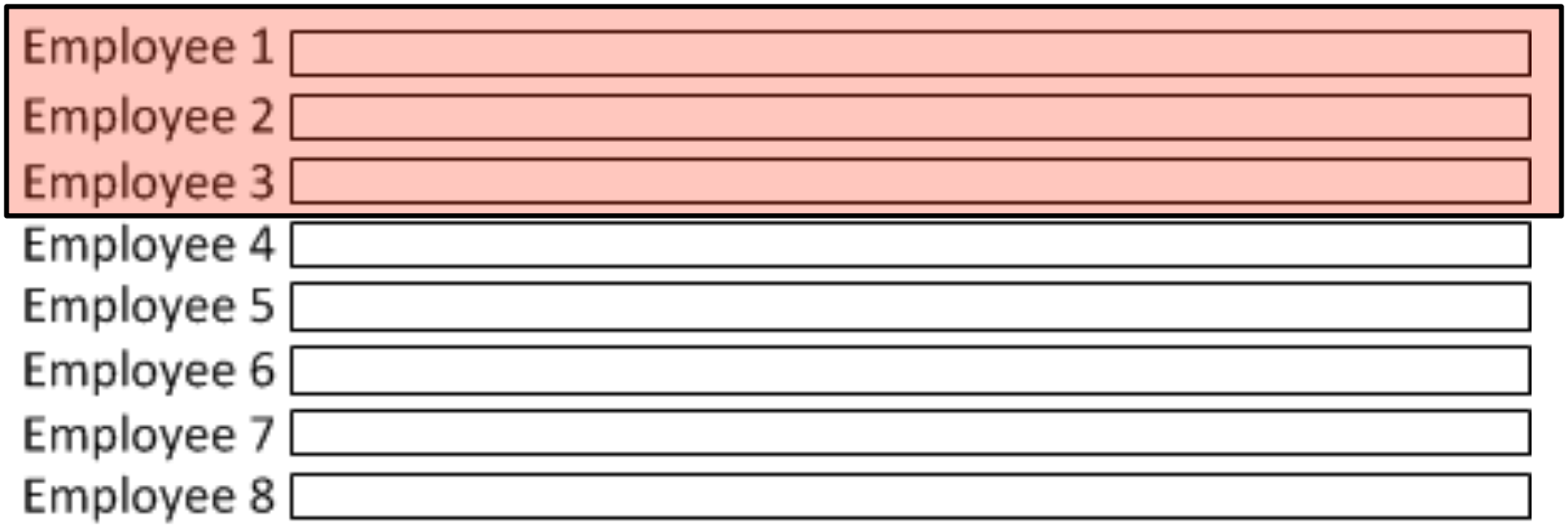
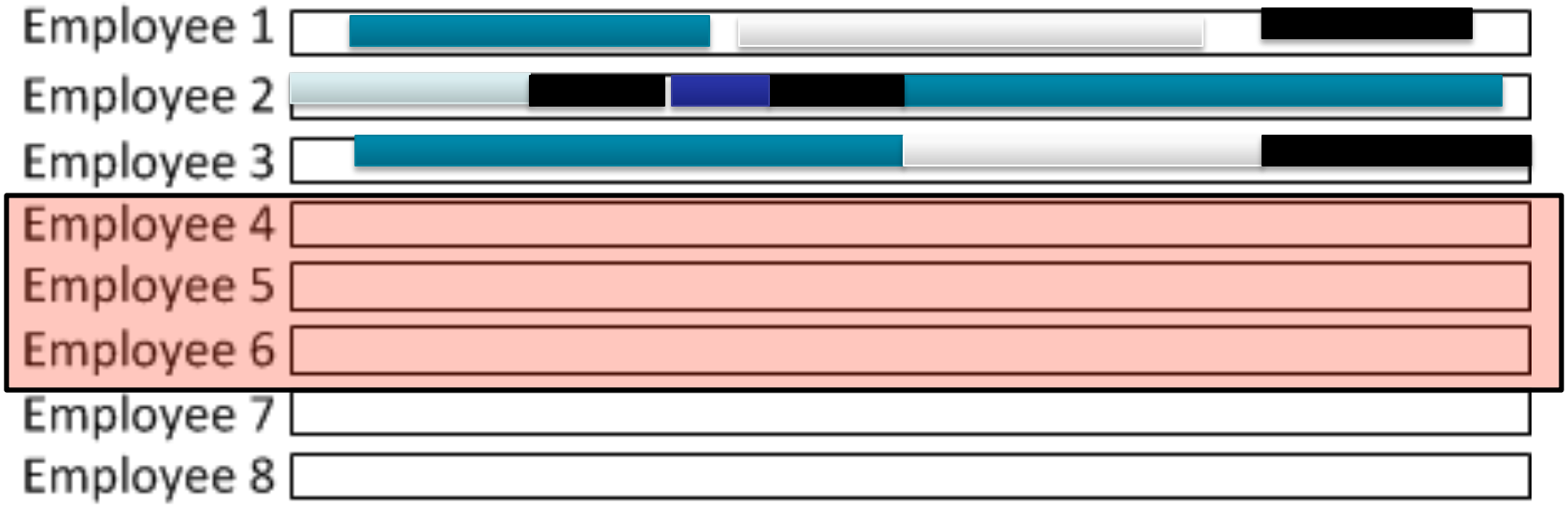
Solution approach: constructive heuristic

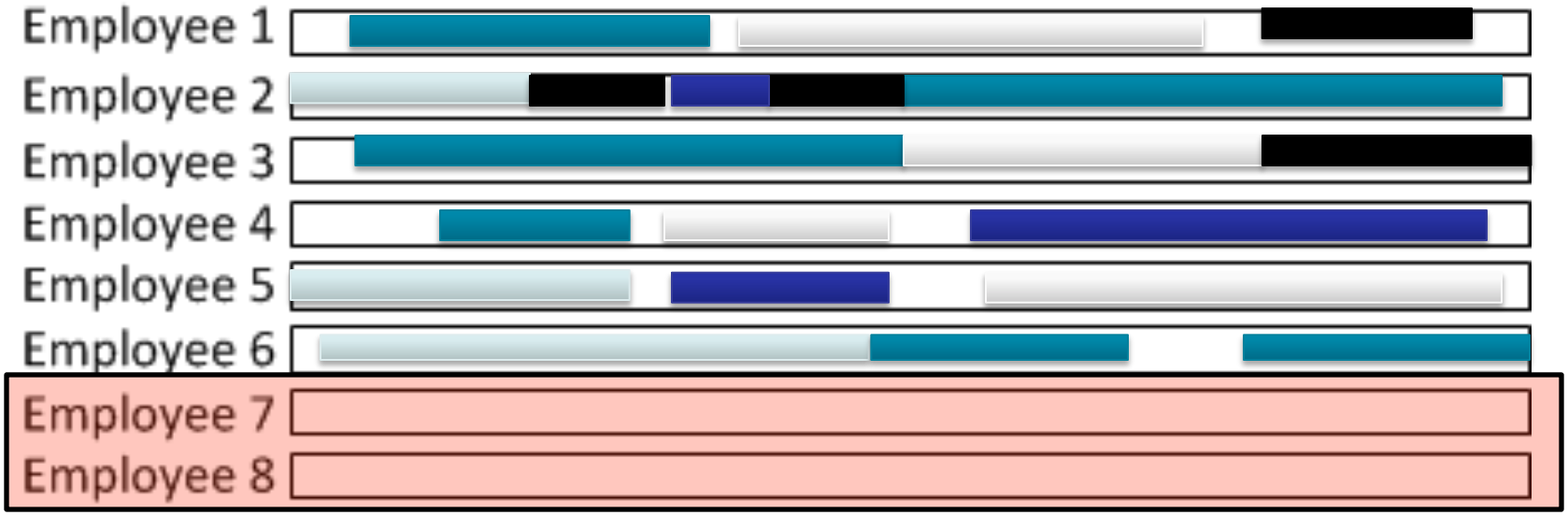
- Optimally assign tasks to a subset of *b* employees, maximizing the sum of assigned task durations



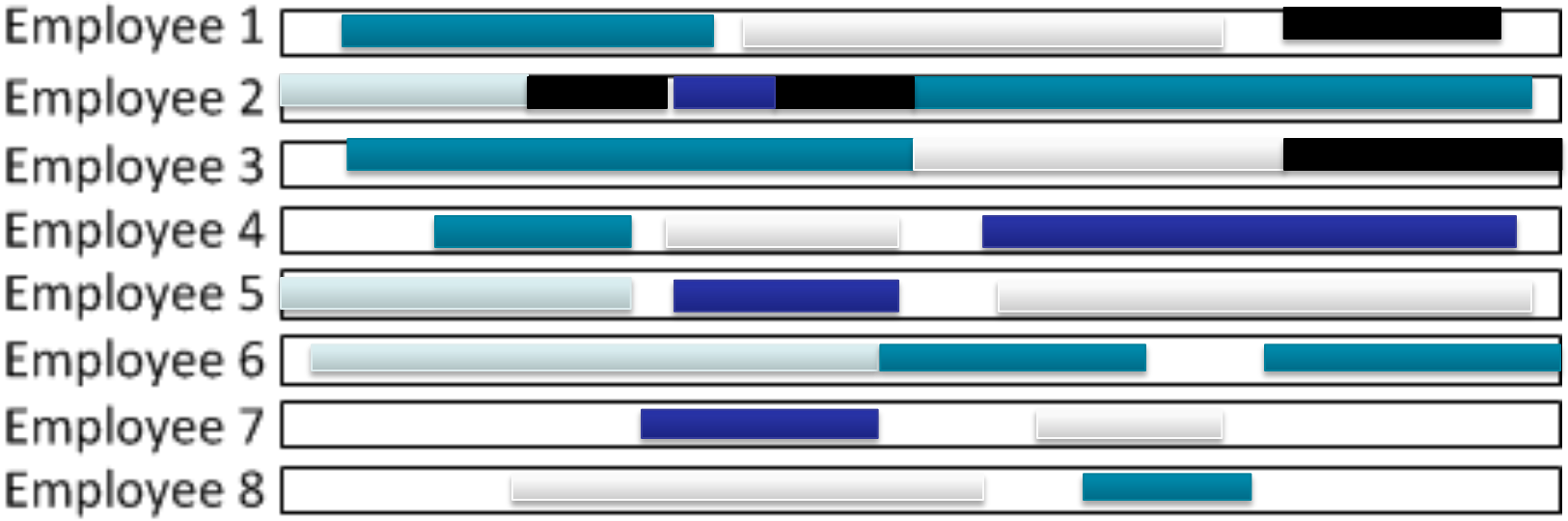
Solution approach: constructive heuristic



Solution approach: constructive heuristic



Solution approach: constructive heuristic



- It is possible that tasks remain unassigned

Solution approach: constructive heuristic

Algorithm 1 Constructive matheuristic (CMH)

Input: J := set of tasks
W := set of employees
b := number of employees in one subproblem
Output: a (partial) solution for the SMPTSP

while W is not empty **do**
  W' := sample and remove b employees from W
  optimally assign tasks $J' \subseteq J$ to W'
  remove assigned tasks J' from J
  if all tasks assigned **then**
    reoptimise W' using the original objective
  end if
end while
return employees with their assigned tasks

Fill employees' schedules by solving a subproblem iteratively untill all employees have tasks assigned.

Solve the subproblem by maximizing sum of assigned task durations.

Reoptimize last block for maximum efficiency.

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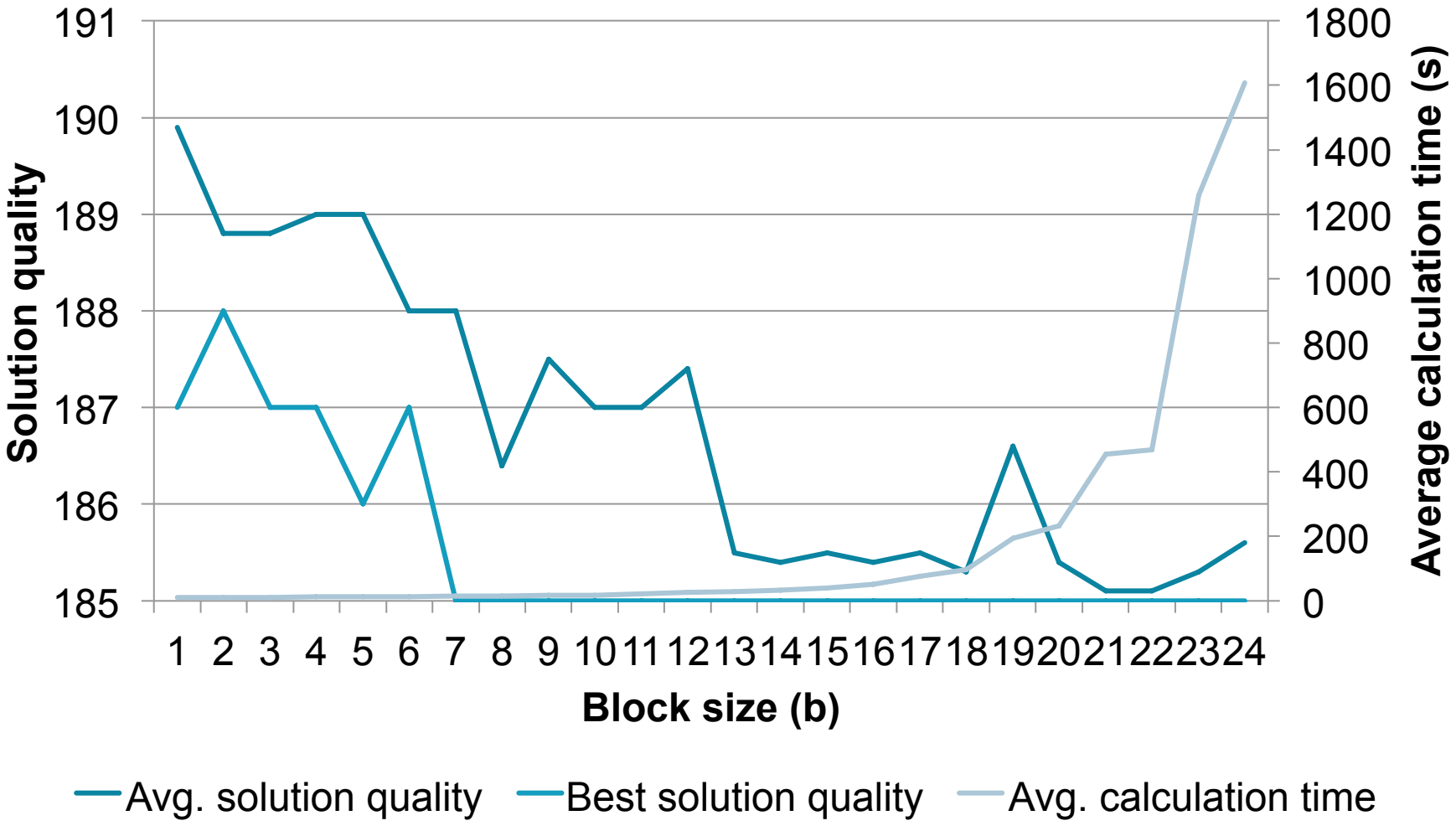
Solution approach: constructive heuristic



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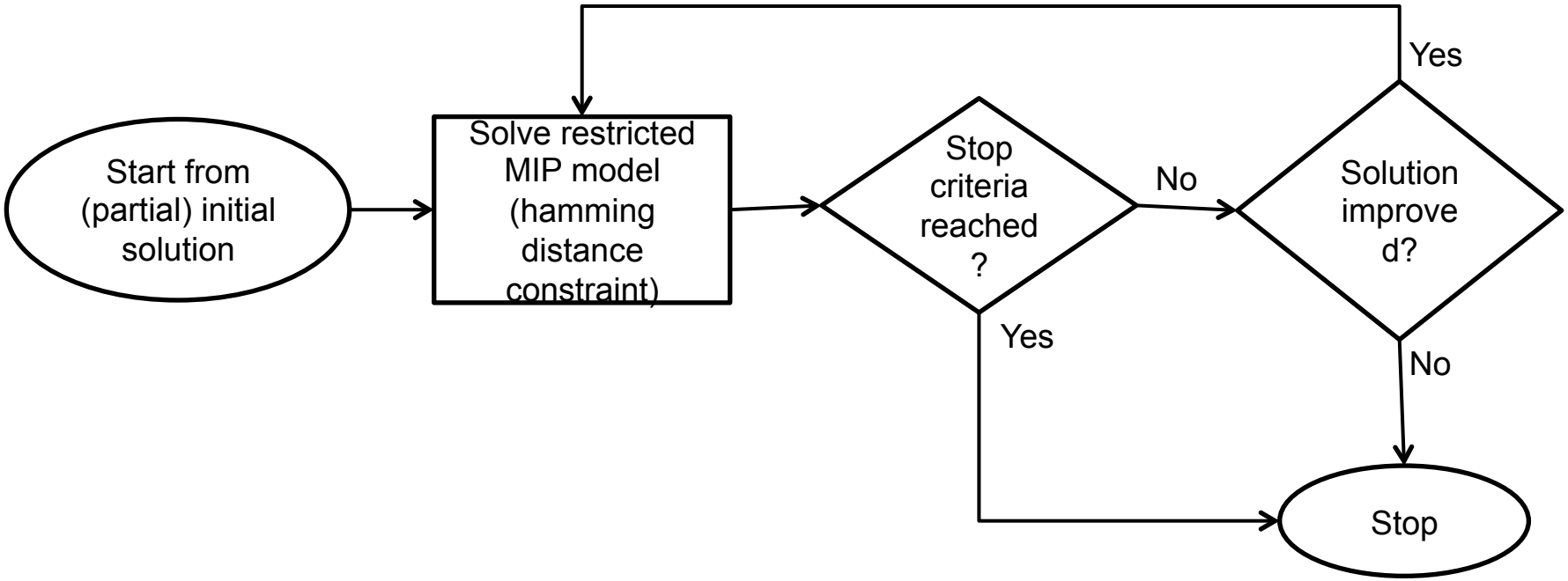
Solution approach: constructive heuristic

- Tested on 137 instances from literature
- # employees ranges from 23 to 245
- # tasks ranges from 40 to 2105

	CMH *b*=10	CMH *b*=15	First fit	Best fit
Number of optimal solutions	118	131	13	22
Average solution quality	123,00	122,6	125,40	125,80
Average calculation time (s)	12,26	44,93	0,02	0,11
Maximum calculation time (s)	169,30	889,80	0,21	1,10

Solution approach: improvement heuristic

- Iterative matheuristic procedure



Solution approach: improvement heuristic

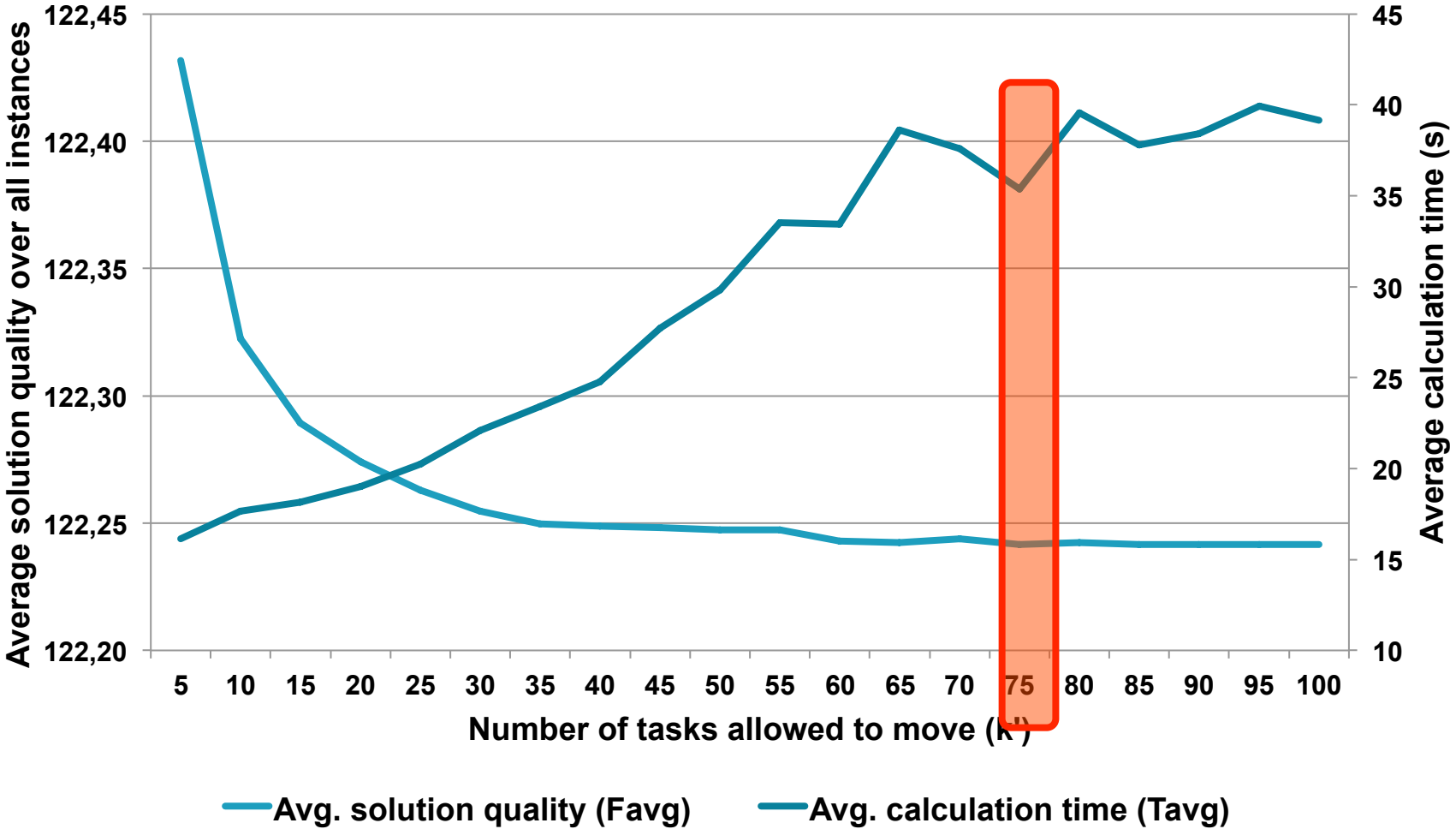
Algorithm 2 Hybrid improvement heuristic

Input: *F(x)* evaluation function value of *x*
LB := lower bound on the evaluation function value
x' := given reference solution
k' := maximum number of tasks to reassign
Output: a solution for the SMPTSP

improved := true
while *improved* **and** *F(x')* != *LB* **and** stop criterion not met **do**
x := *solverestricted(x', k')*
if *F(x)* < *F(x')* **then**
improved := true
else
improved := true
end if
end while
return *x'*

Starting from an initial solution, allow the MIP solver to only change a limited number of variables.

Solution approach: improvement heuristic



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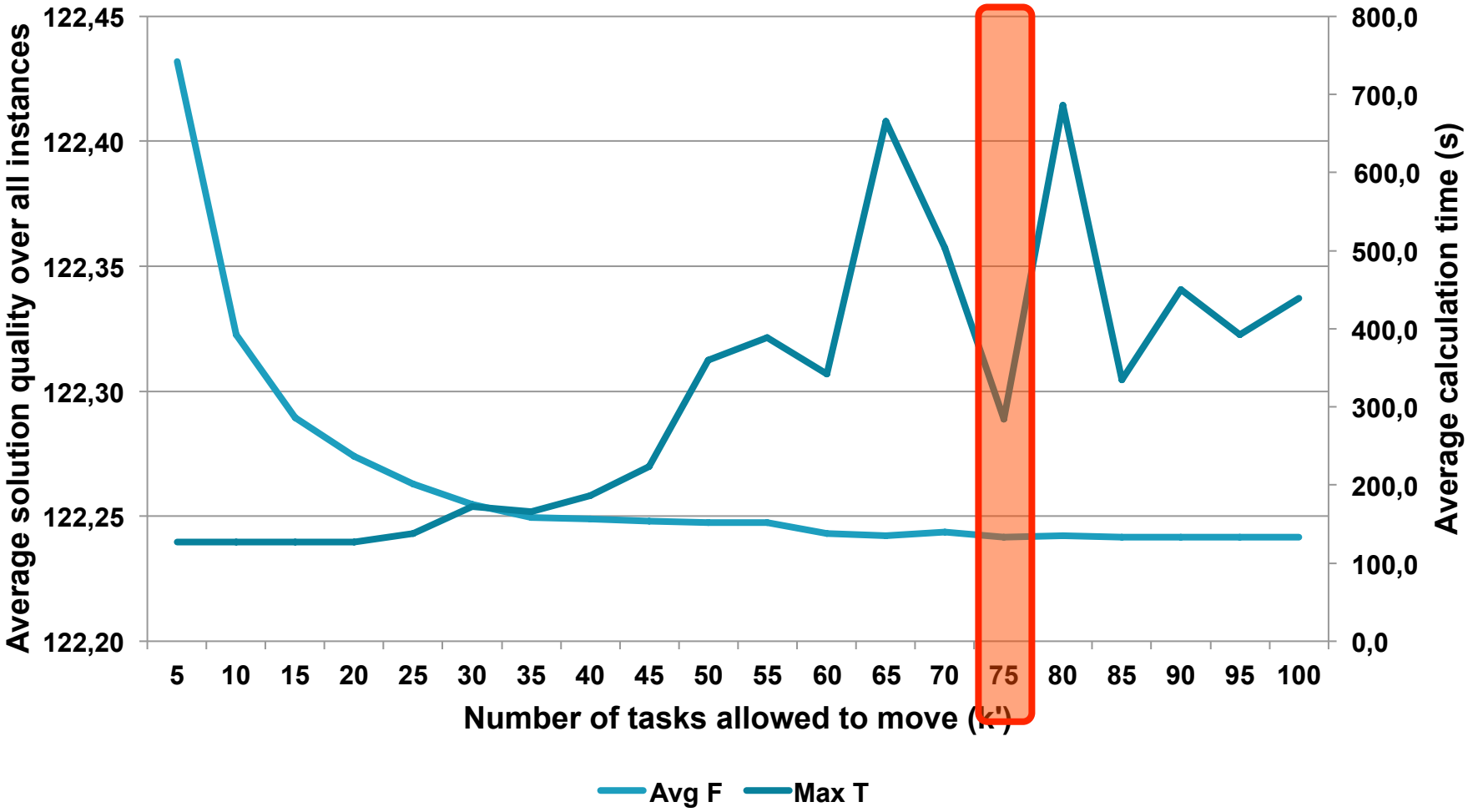
Solution approach: improvement heuristic



Solution approach: improvement heuristic

	KEB12	SV12	Our approach
Number of optimal solutions	67	81	137
Average solution quality	127,00	123,00	122,20
Average calculation time (s)	-	958,91	34,46
Maximum calculation time (s)	1800,00	1800,00	282,20

- On average 35 seconds required
- Maximum calculation time is less than 5 minutes

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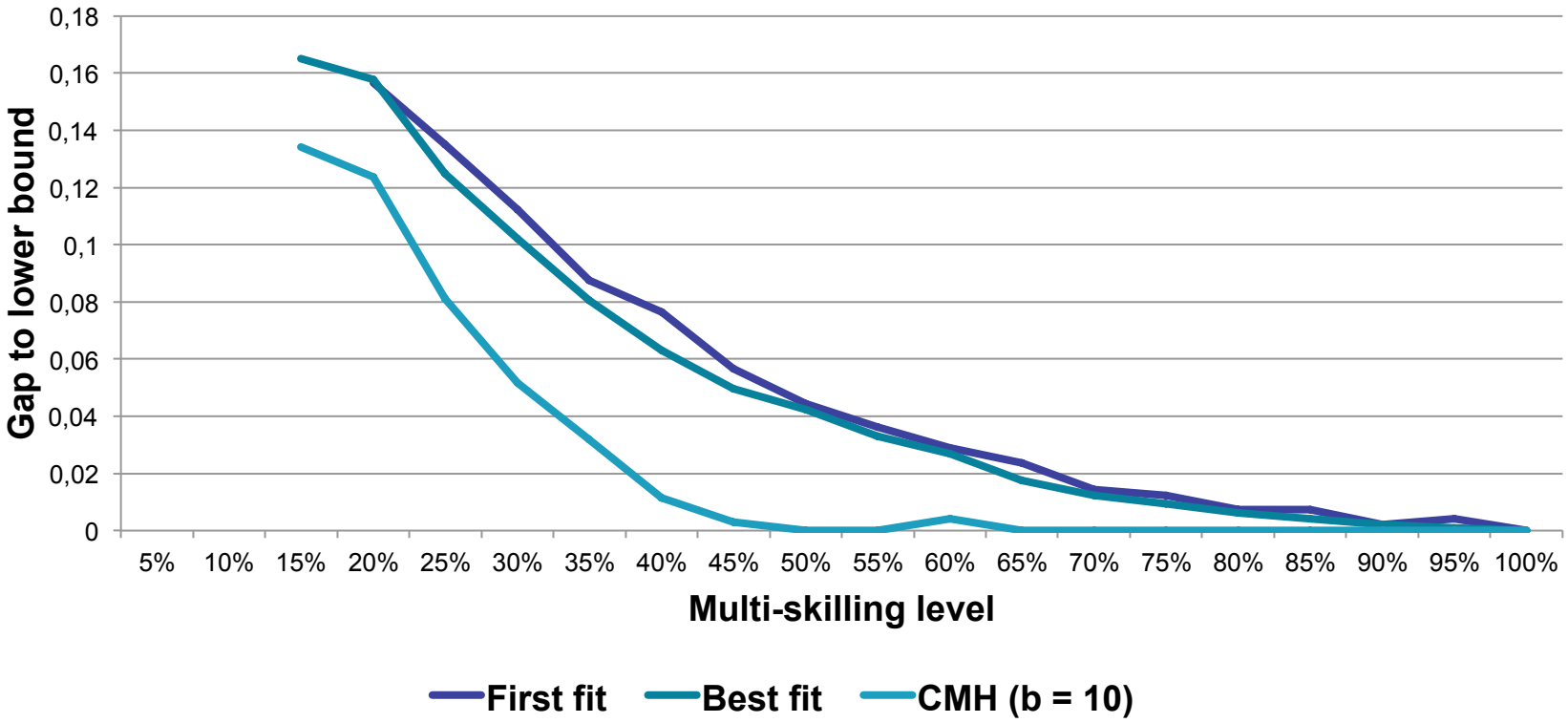
Empirial hardness

- Second longest calculation time (272 seconds) was on instance with
  - 70 shifts
  - 525 employees
- What makes this instance hard?

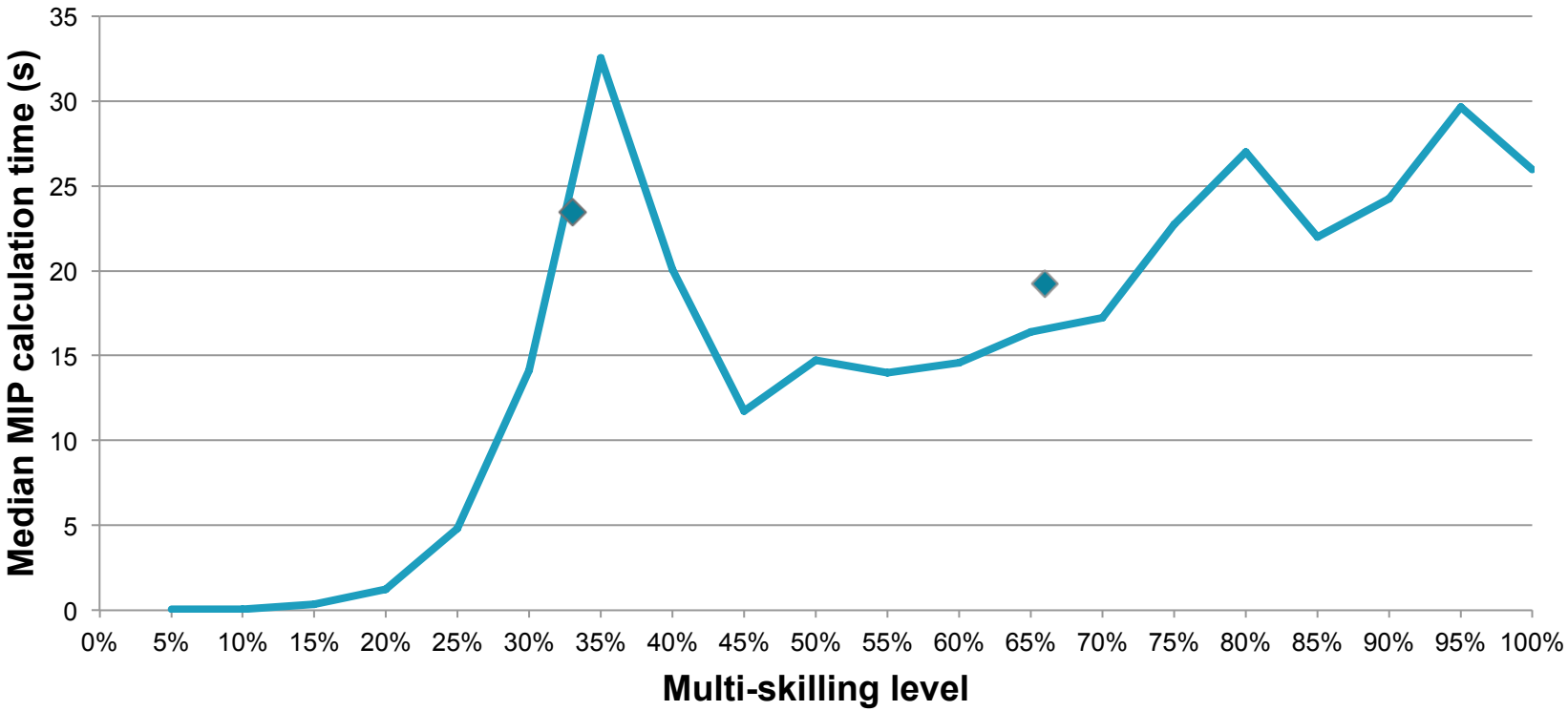
- Observe influence of problem properties on algorithmic performance
  - Multi-skilling level
  - Average task length

Empirial hardness: multi-skilling level



Behaviour of constructive heuristics

Empirial hardness: multi-skilling level



Behaviour of general purpose MIP solver

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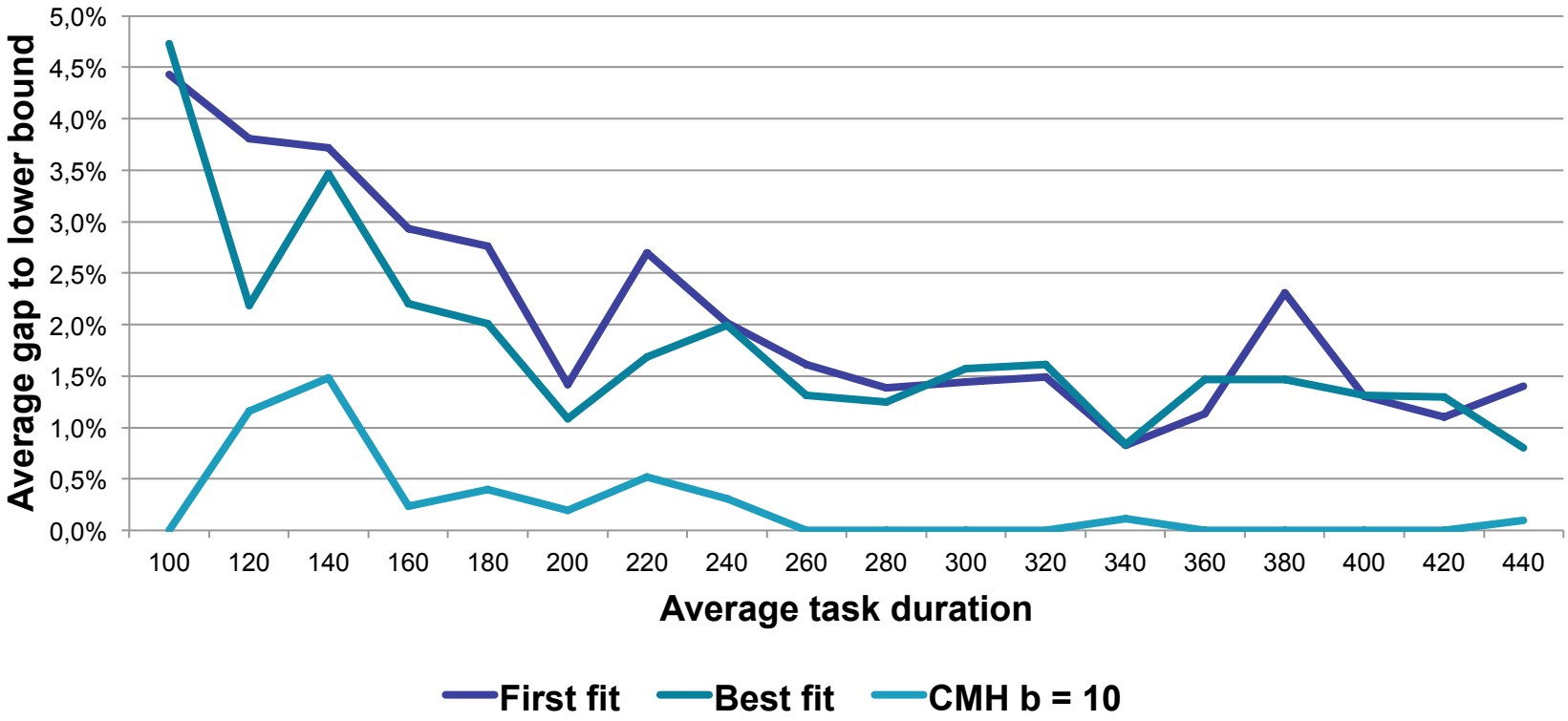
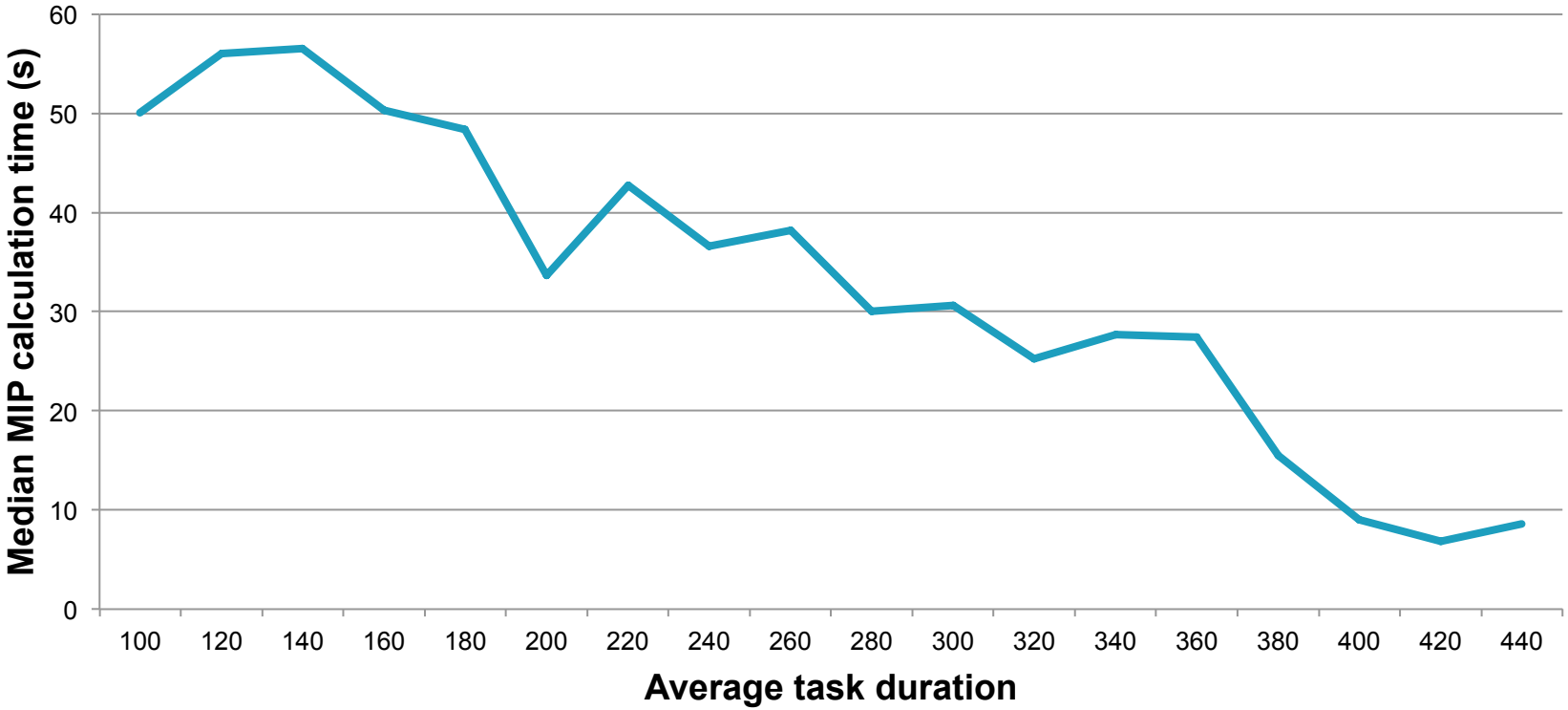
Empirial hardness: task length



Behaviour of constructive heuristics

Empirial hardness: task length



Behaviour of general purpose MIP solver

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Empirial hardness: new instances

- Based on results, harder instances were generated

Instance	LB	MIP solution
1_50_258_20	40	40
2_44_510_20	40	40
3_102_525_30	77	83
4_113_647_20	98	99
5_77_777_30	59	65
6_135_777_20	116	119
7_70_781_20	59	61
8_88_1022_20	79	80
9_125_1308_20	98	106
10_153_1577_20	116	153

- http://allserv.kahosl.be/~pieter/smptsp.html

Conclusions

- Illustration of potential of matheuristics
- Algorithm provides best known results for the SMPTSP
- Insight into what makes the SMPTSP hard for algorithms
- Methodology for generating harder instances

References

- Krishnamoorthy M, Ernst A, Baatar D (2011) Algorithms for large scale shift minimisation personnel task scheduling problems. European Journal of Operational Research
- Smet P, Vanden Berghe G, August 2012. A matheuristic approach to the shift minimisation personnel task scheduling problem. In: Proceedings of the International Conference on the Practice and Theory of Automated Timetabling (PATAT)

Real-world applications

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Real-world production scheduling in the food industry

BrightEye
Optimizing through insight.

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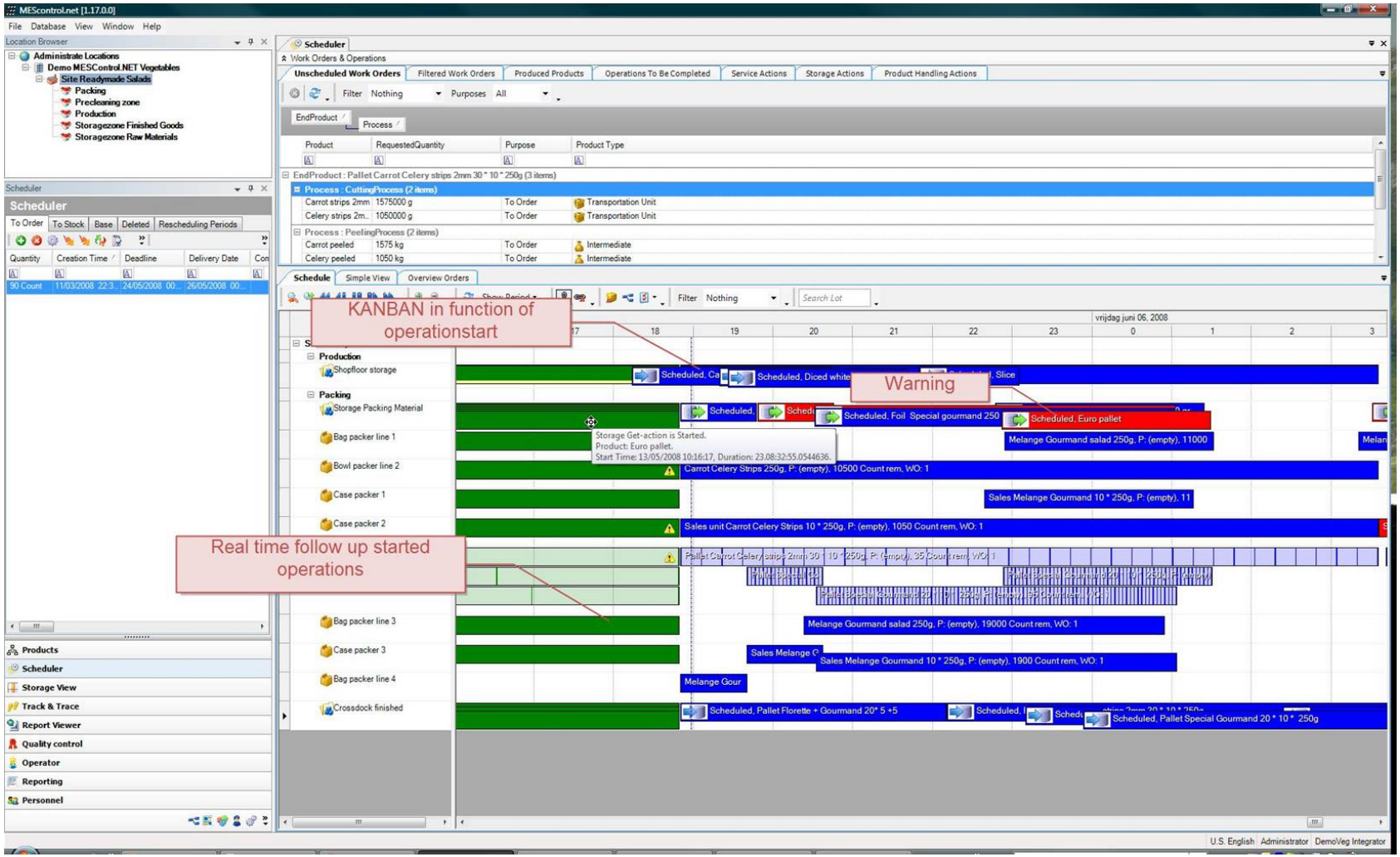
Real-world production scheduling in the food industry



Wauters, T., Verbeeck, K., Verstraete, P., Vanden Berghe, G., De Causmaecker, P. (2012). **Real-world production scheduling for the food industry: an integrated approach.** *Engineering Applications of Artificial Intelligence*, 25(2), 222-228.

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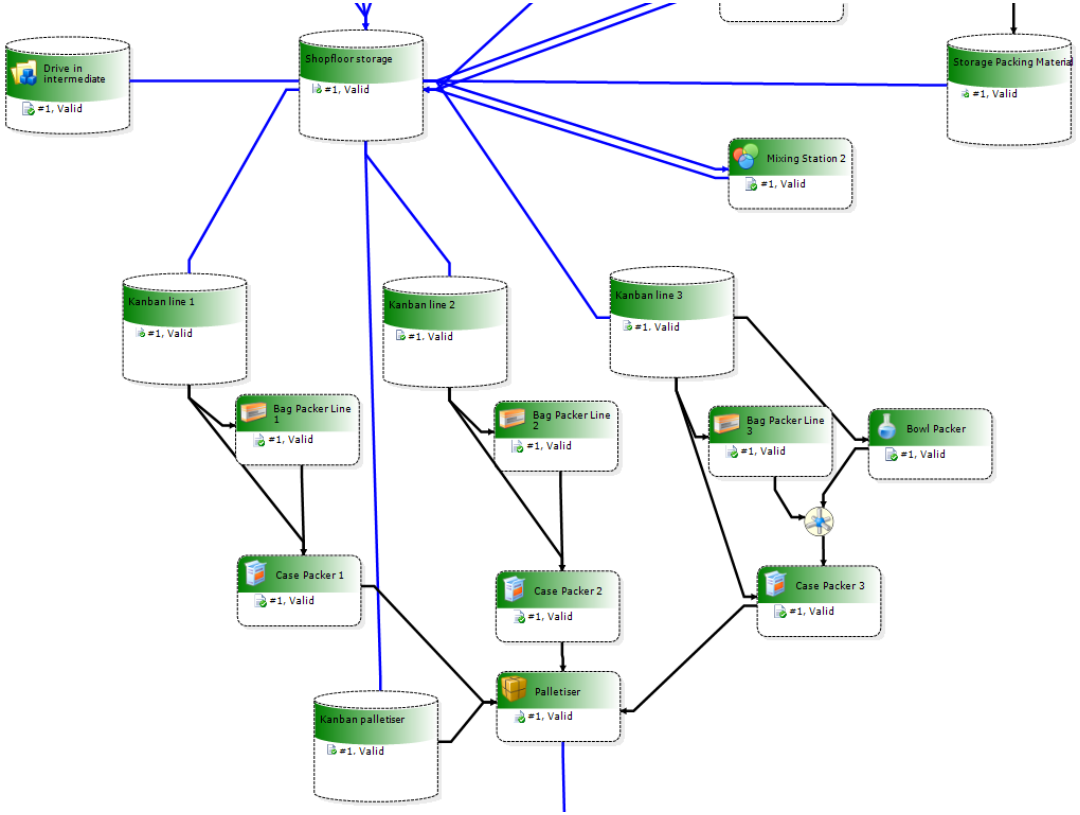
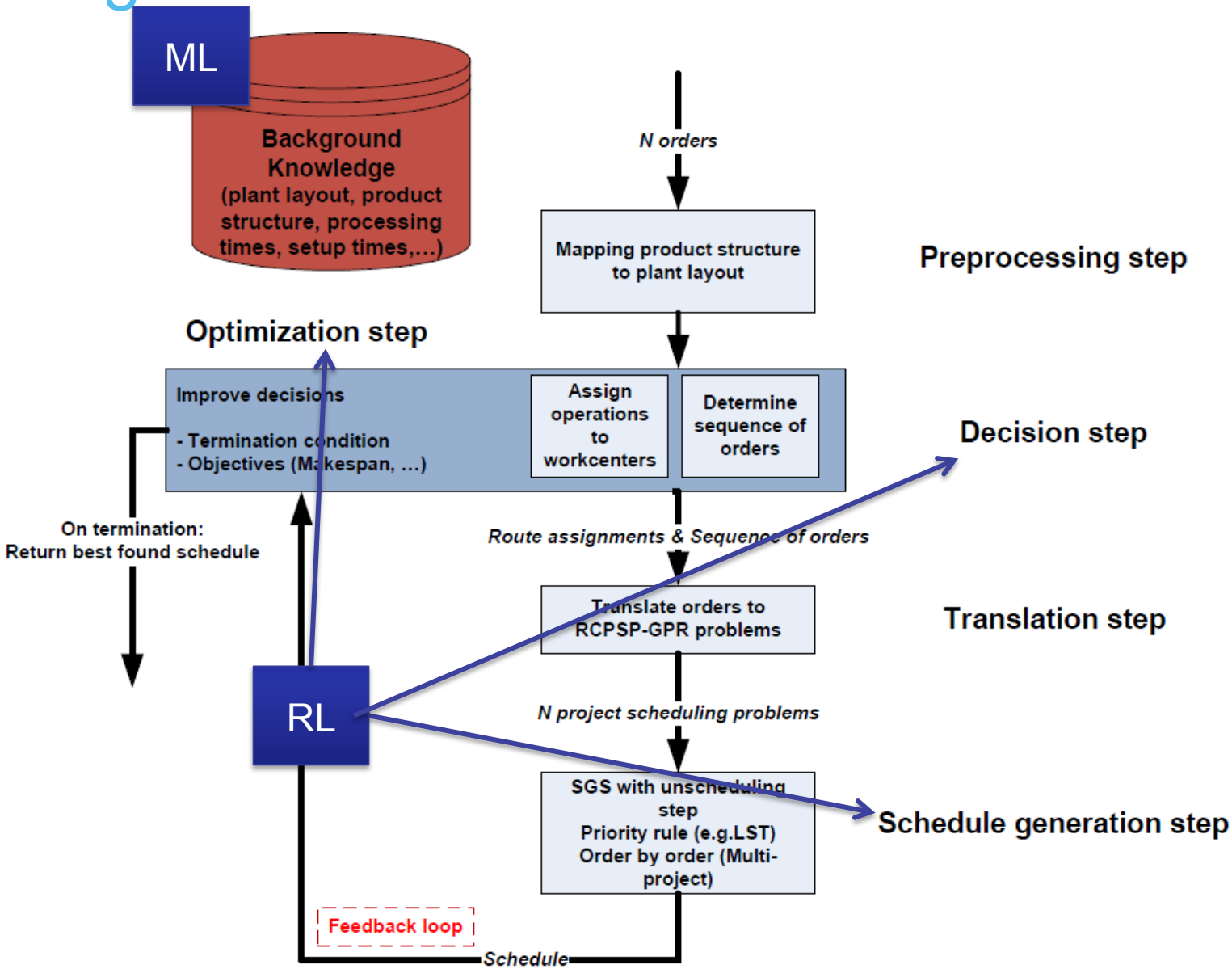
Plant Routing

- Mapping product structure to plant layout



Integrated approach high level overview



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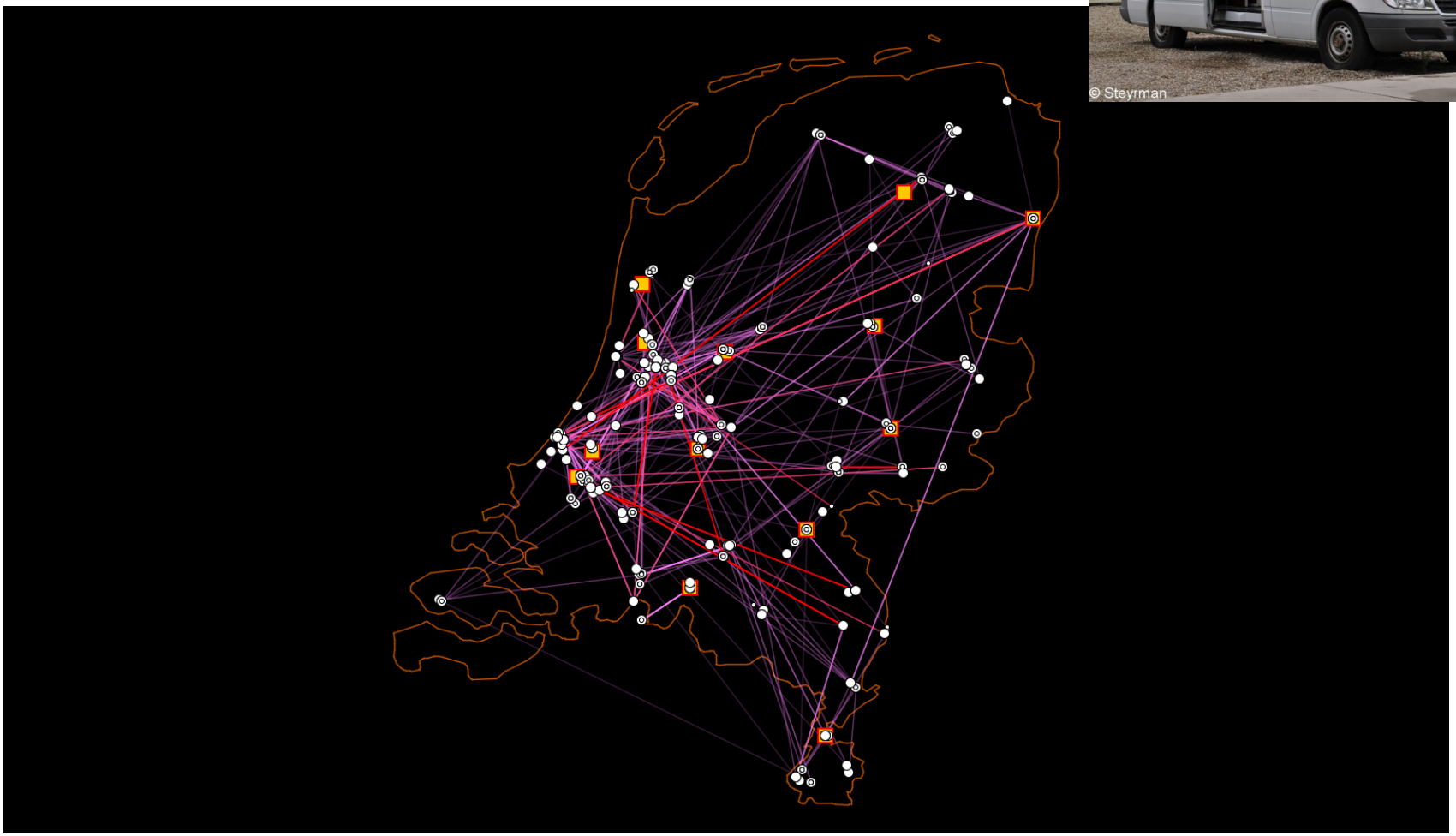
Prisoner transportation



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Prisoner transportation

- Case in the Netherlands (DV&O)
- Project with Mobius and Conundra
- Transportation service has about 300 employees
- Its head quarter receives 750 transportation requests per day on average
- **Problem:**
  - Pickup and delivery, Time windows
  - Heterogeneous fleet, multi-depot
  - Exclusionary constraints
    - Two prisoners cannot be transported in the same vehicle/compartment
  - Objective: minimize the total cost, which aggregates vehicle costs and personnel costs.

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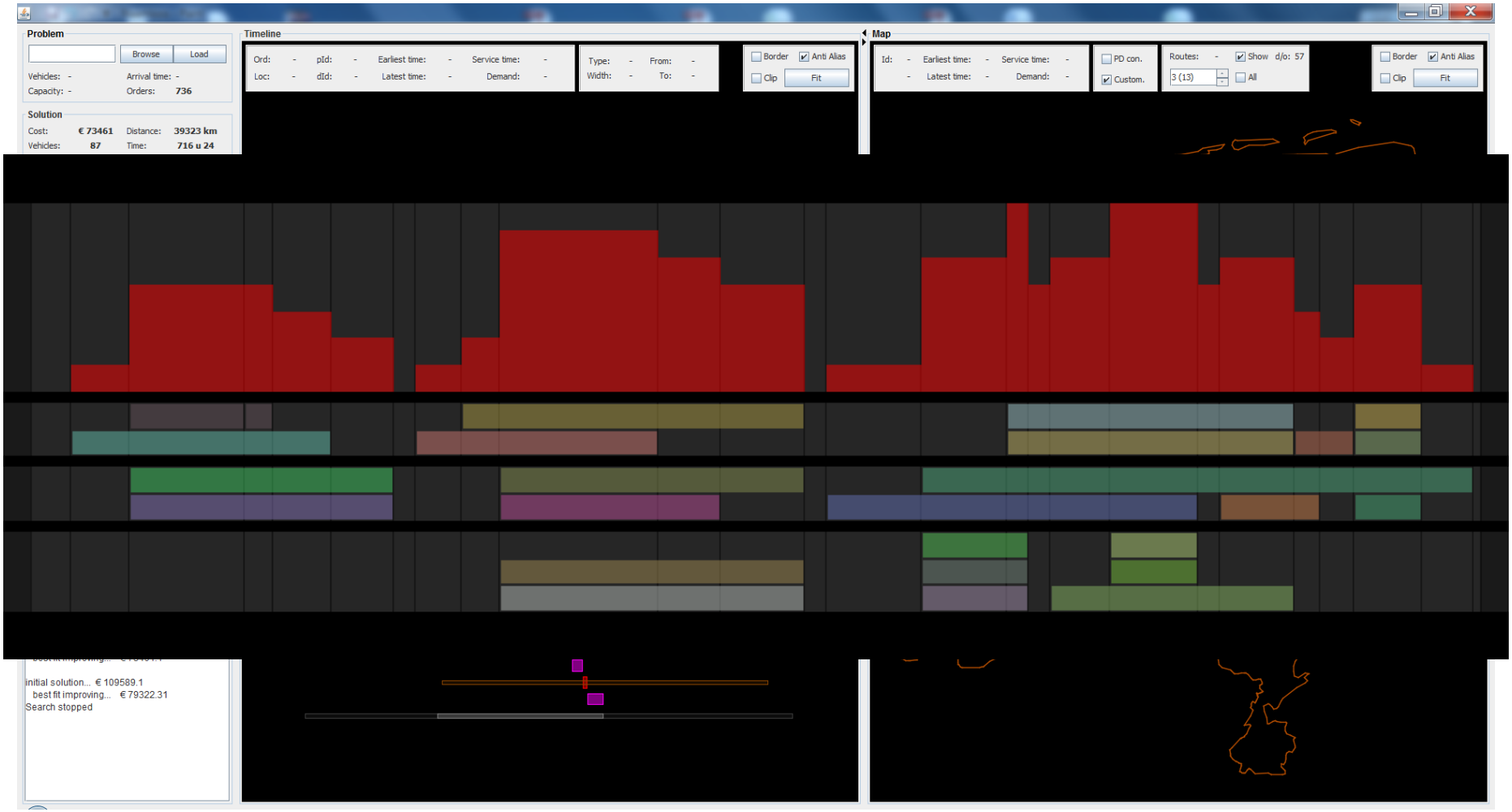
Prisoner transportation



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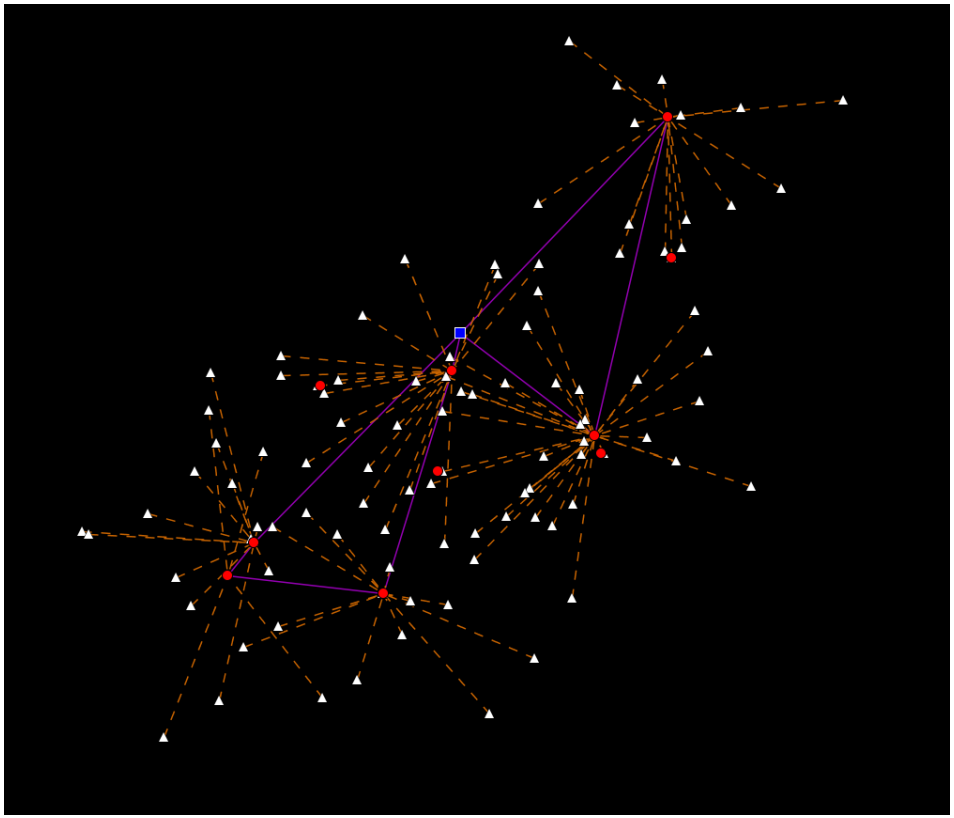
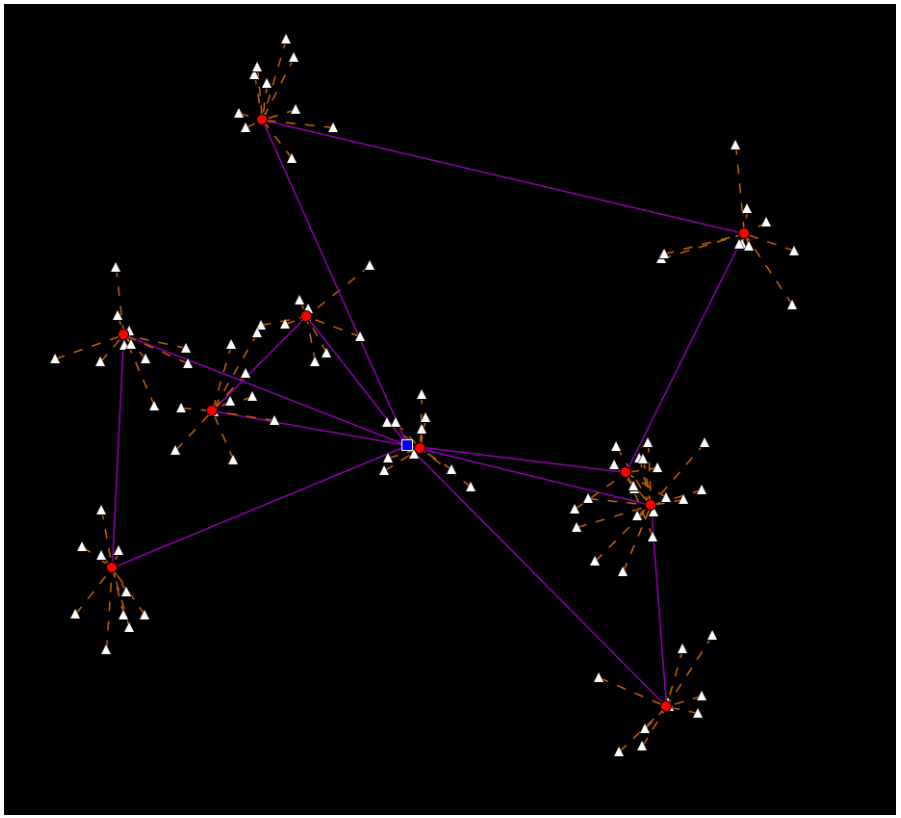
Prisoner transportation

- Approach:
  - Best fit heuristic
    - Add prisoner to “best” possible route
  - Efficient datastructure = very important!
    - Temporal data
    - Spatial data
  - Compartment assignment → graph colouring heuristic

School bus routing and ring-star





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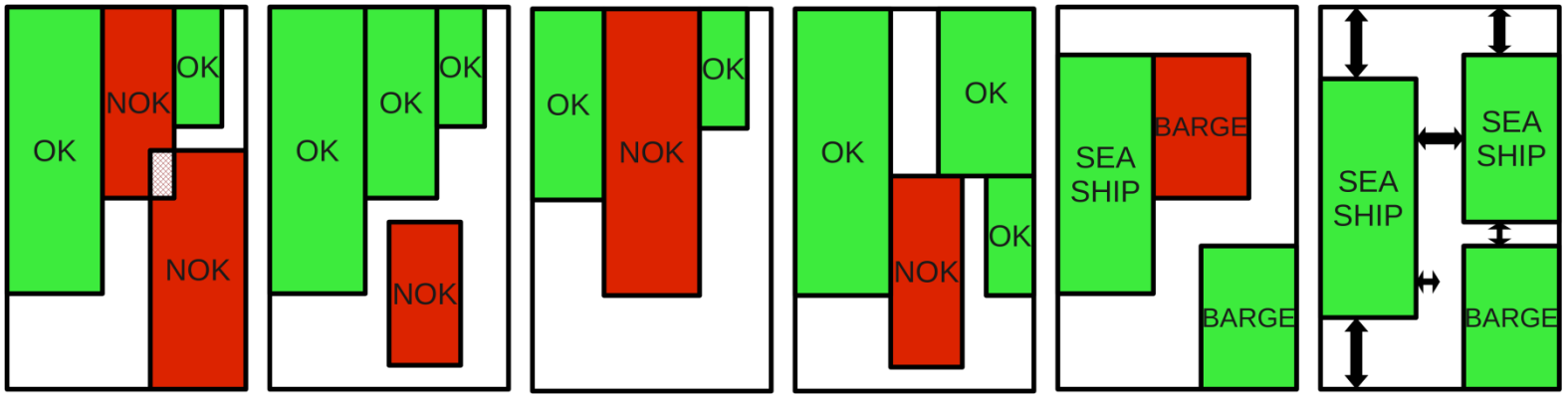
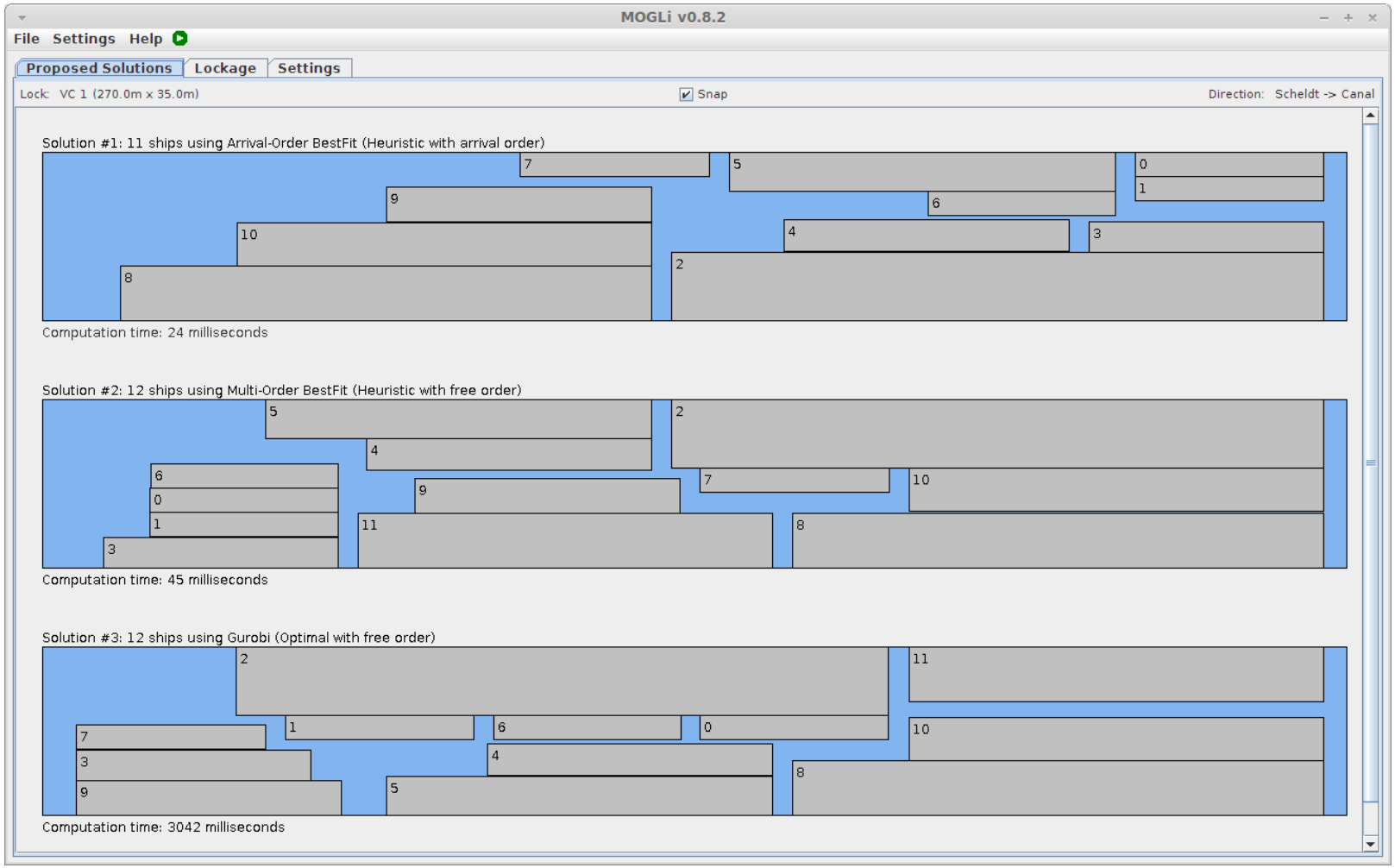
Lock scheduling

Placement constraints:



* Image by Jannes verstichel, CODeS KAHO Sint-Lieven

Lock Scheduling



Best-fit Heuristic
45ms

Optimal with MILP Solver
3042ms

* Image by Jannes verstichel, CODeS KAHO Sint-Lieven

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Barge scheduling

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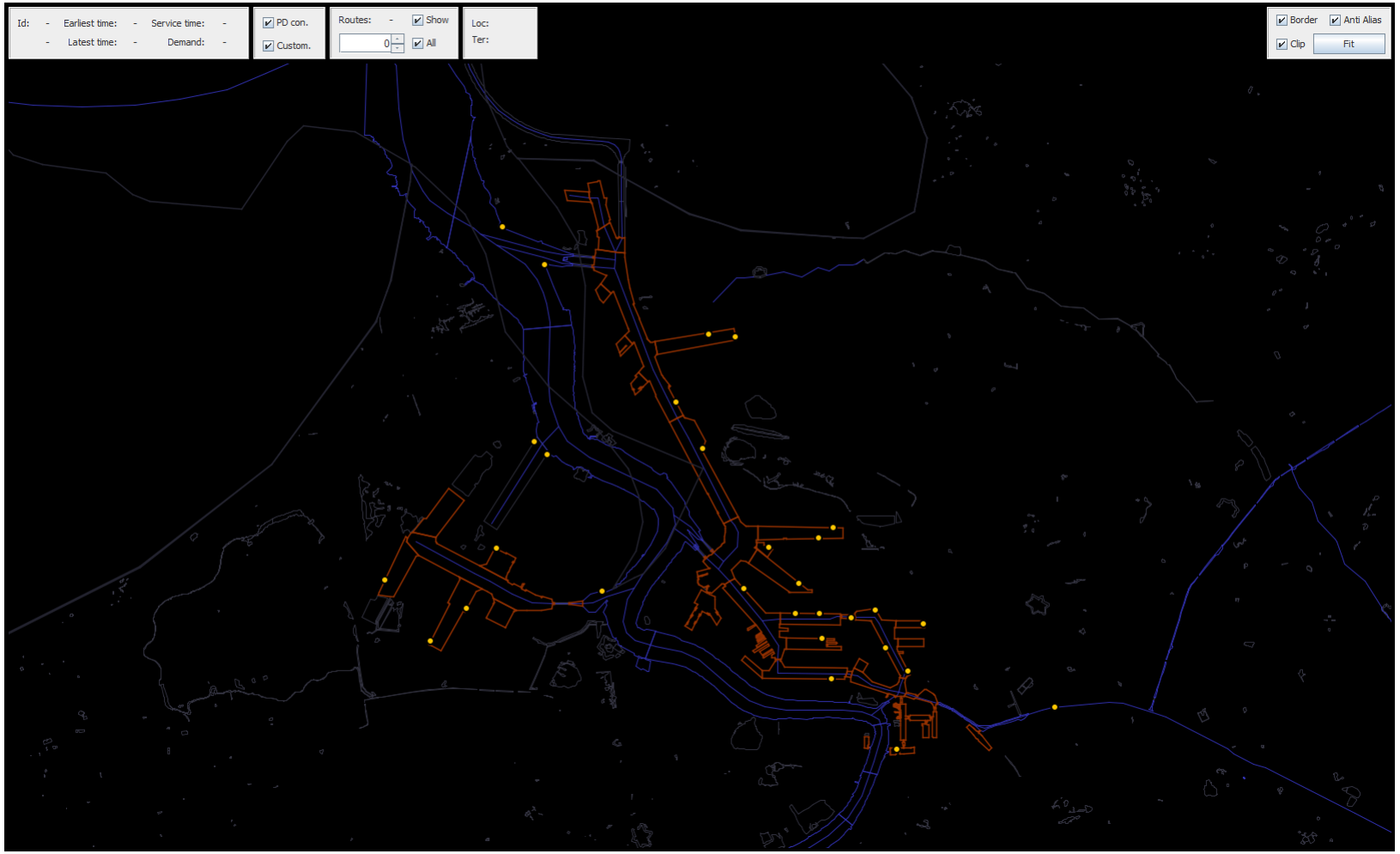
Barge scheduling



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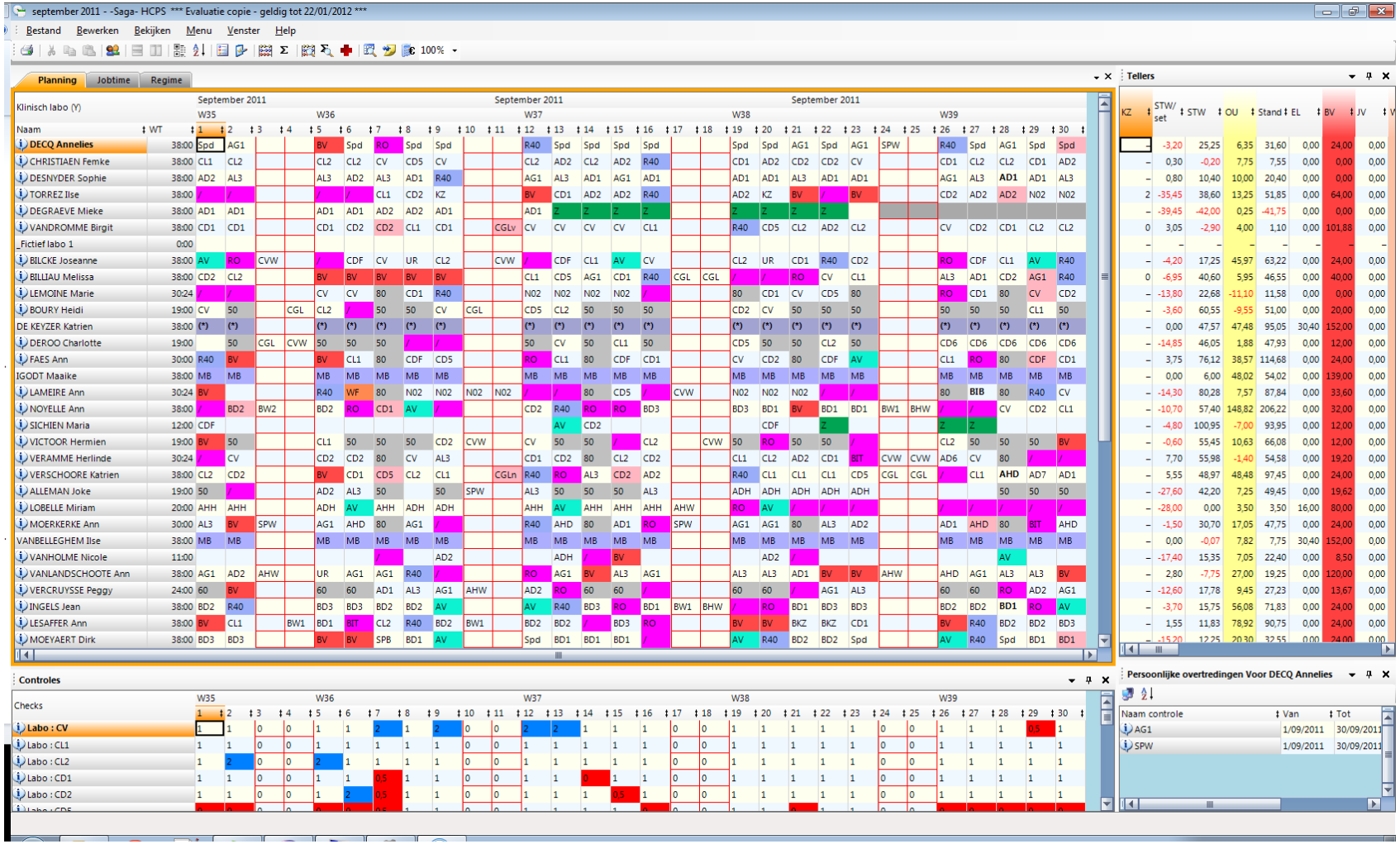
Personnel planning



* Image of SAGA HCPS

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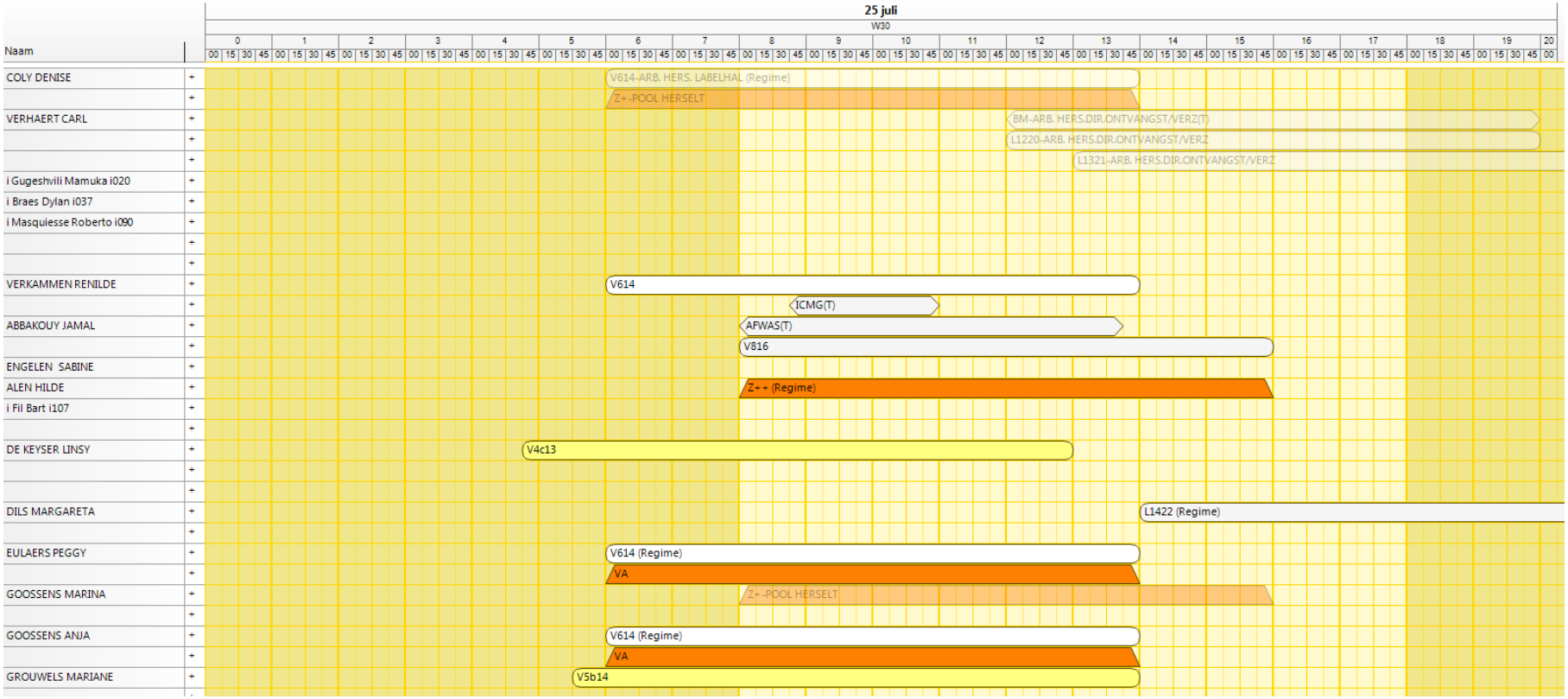
Personnel task planning



* Image of SAGA HCPS

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Patient admission scheduling

* Image by Wim Vancroonenburg (CODeS, KAHO Sint-Lieven)

Hospital
Multiple departments
Rooms with beds

Objective
Minimize constraint violations regarding to
- Age
- Gender
- Equipment
- Single/double room

→Weighted objective function

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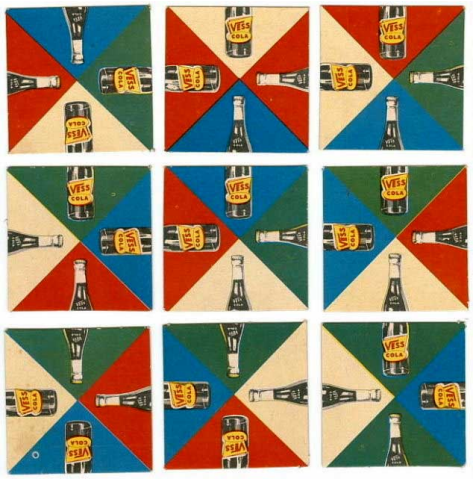
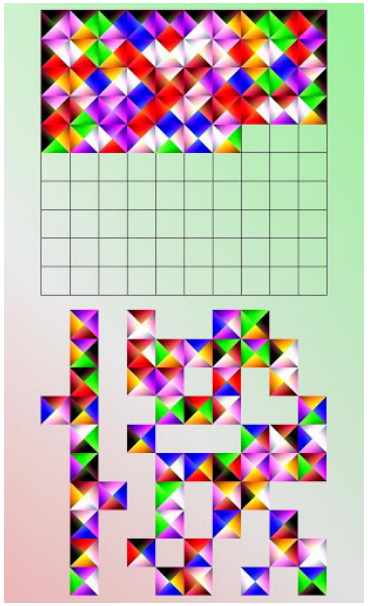
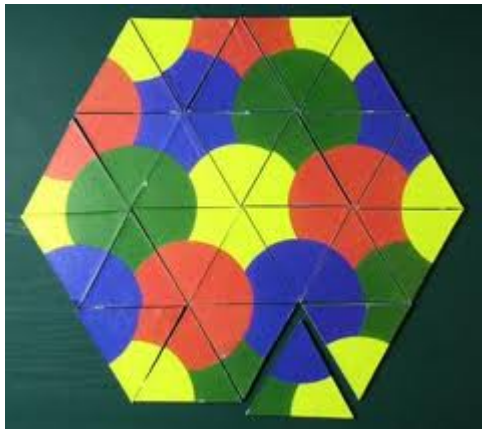
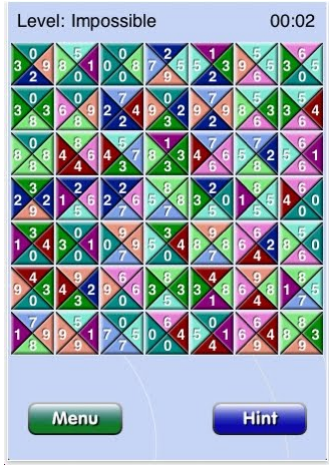
“Not so real-world” - Edge matching puzzles

- NP-complete









Edge matching puzzles

- Try it yourself – download the android app!
- Google play: https://play.google.com/store/apps/details?id=be.ka