Lesson Review

Learning Objectives

Please list the learning objectives of this module that you have achieved:
I certified that I am able to:

- Formulate complex statements using quantifiers and predicates.
- Nest and compound quantified statements.
- Prove or disprove quantified statements.

Learning Review

Please complete the table below (refer to the attached Learning Process table).

Learning Objective	**Concept** What concept / key-word did you focus on?	**Step**	**Strategy** What strategy did you apply? Why did you choose this? How did you apply it? Did it work well? How do you know?	**Resource** What resource did you use? Why did you choose this? Did it work well?	**Reflection** In hindsight, was this strategy and resource • appropriate? Why? • identify other options • was this the best option? Why?	**Learning** Generalise: what you learned that could be applied in the future in a different context
Quantifiers and Predicates	Formulate complex statements using quantifiers and predicates.	Identify	Identify Concepts and make a list of resources needed	Unit Site Content		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos	Prescribed Text Book		
				Recorded Lectures		
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again	External Videos		

Compound Statements	Nest and compound quantified statements.	Identify	Identify Concepts and make a list of resources needed	Unit Site content Prescribed Text Book Recorded Lectures External Videos		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos			
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again			
Compound Statements	Prove or disprove quantified statements.	Identify	Identify Concepts and make a list of resources needed	Unit Site content Prescribed Text Book Recorded Lectures External Videos		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos			
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again			

Learning Evidence

Predicate Logic

Quantifiers

notes

∀ = Universal Quantifier

- for all members of a Domain of Disclosure of (R, N, Z, Q)

∃ = Existential Quantifier

- there exists at least one in a Domain of Disclosure of (R, N, Z, Q)

Sets / Domains

∅ contains no elements
U universal (All elements)
N natural numbers (0, 1, 2, etc)
Z integers (−2, −1, 0, 1, 2 ..)

Q set of Rational numbers
R set of real numbers
P(A) power set
Any set (A) is the subset of A

∅ or 0 elements
contains no elements

U universal
All elements in a particular context or theory.

N natural numbers
"Those used for simple counting" eg, 0, 1, 2, 3

Z integers
similar to natural numbers but includes negative.

Q rational numbers
A number in the form of $\frac{P}{Q}$ is rational where P, Q are integers = fractions

R real numbers
includes rational, ~~irrational~~ irrational natural, integers = all numbers

P(A) Power set
set of all subsets of a set

NOTATION

{} (curly braces) to enclose a set

: "such as" $x : x > 2$ The set of all x such that x is greater than 2

$\in$ "element" $2 \in \{1,2,3\}$ 2 is an element of $\{1,2,3\}$

$\notin$ "not an element" $4 \notin \{1,2,3\}$ 4 is not an element of $\{1,2,3\}$

Lecture example

$Q(x)$ be the statement $x + 1 > 2x$ if Domain of is Real numbers

~~$Q(0) \equiv 0+1 > 2\times 0$~~

$Q(0) \equiv 0+1 > 2\times 0 \equiv \cancel{1}\ 1 > 0 \equiv$ True

$Q(1) \equiv 1+1 > 2\times 1 \equiv 2 > 2 \equiv$ FALSE

$\forall x\, Q(x) \equiv$ FALSE for all values for x $Q(1)$ disproves it · one example needed ·

$\exists x\, Q(x) \equiv$ True. "$Q(0)$ proves True – ~~for~~ Exists at least one.

$\exists x\, \neg Q(x) \equiv$ ~~FALSE~~ True. $\forall x\, Q(x) \equiv$ FALSE.

$\forall x\, \neg Q(x) \equiv$ ~~True~~ FALSE. $\exists x\, Q(x) \equiv$ True.

} negating Quantified Statements

$\exists x (x^2 = 2)$

Real numbers

$\exists x (x^2 = -1)$ F need general argument. Do not show examples for Existential Quantifiers as false.

$\forall x (x^2 + 2 \geq 1)$

Universal ~~is~~ True needs general Argument. Any non negative number + 2 will Always be greater From Than 1

Nested Quantifiers

$\forall x \exists y : xy = y$

$\forall x \quad \exists y : xy = y$

$\exists y \quad xy = y. \quad (x \times y = y)$

$$\forall x \exists y : xy = y$$

Start inside to outside. (right to left) after defining the quantifiers ($\forall x$ and $\exists y$)

quantifiers Predicate.

first ($\forall x$ - $\exists y : xy = y$) Third

($\exists y$ - $xy = y$) Second

One Quantifier at a time

* Tips

$\forall$ Universal True = general ~~Exam~~ Argument

$\forall$ Universal false = Example.

$\exists$ Existential True = Example.

$\exists$ Existential false = general argument.

Predicate Logic

①

ⓐ $P \vee \neg(P \vee Q) \equiv P \vee \neg Q$

LHS $P \vee \underbrace{\neg(P \vee Q)}$

$P \vee (\neg P \wedge \neg Q)$ Demorgans

$\underbrace{(P \vee \neg P)} \wedge (P \vee \neg Q)$ Dist

$T \wedge \underbrace{(P \vee \neg Q)}$ Logical ~~eq~~

$P \vee \neg Q \equiv$. IDENT

ⓑ $(P \wedge Q) \rightarrow (P \vee Q) \equiv T$

LHS $\equiv \underbrace{(P \wedge Q) \rightarrow (P \vee Q)}$

$\underbrace{\neg(P \wedge Q)} \vee (P \vee Q)$ EQUIV of $\rightarrow$

$\neg P \vee \neg Q \vee P \vee Q$ Demorgans

$\underbrace{(\neg P \vee P)} \vee \underbrace{(\neg Q \vee Q)}$ comm & assoc

$T \vee T \equiv T$ Logical

(C) $P \vee (P \wedge Q) \equiv P$

$LHS \equiv \underbrace{P \vee}(P \wedge Q)$

$\equiv (P \wedge T) \vee (P \wedge Q)$ IDENT

$\equiv P \wedge (T \vee Q)$ DIST

$\equiv P \wedge T$ DOM

$\equiv P$ RHS IDENT

(D) $(P \wedge Q) \rightarrow Q \equiv T$

$LHS = (P \wedge Q) \rightarrow Q$

$\underbrace{\neg (P \wedge Q)} \vee Q$ EQUIV of $\rightarrow$

$\neg P \vee \underbrace{\neg Q \vee Q}$ DIST

$\underbrace{\neg P \vee T}$ Log equiv

T Dom

(e) $p \land \neg(p \land q) \equiv p \land \neg q$

LHS $\equiv p \land \underbrace{\neg(p \land q)}$

$\equiv \underbrace{p \land (\neg p \lor} \neg q)$ De morgans

$\equiv \underbrace{(p \land \neg p)} \lor (p \land \neg q)$ DIST

$\equiv F \lor \underbrace{(p \land \neg q)}$ Log equiv

$\equiv p \land \neg q$ Ident

(2)

(A) $\underbrace{\forall x}_{\text{UNIVERSAL}} : x+2 > x$ = True

$x+2$ is always going to be greater than x

(B) $\underbrace{\exists x}_{\text{EXISTENSIAL}} : x = 3x$

(C) $\underbrace{\forall x}_{\text{UNIVERSAL}} : x = 3x$

(D) $\underbrace{\forall x \forall y}_{\text{UNIVERSAL}} : (x+y)^2 = x^2 + y^2$ = FALSE.

$$(x+y)^2 = x^2+y^2$$

$x = 1$ $(1+2)^2 = 9$

$y = 2$ $1^2+2^2 = 5$

$\therefore (1+2)^2 = 9 \neq 5 = 1^2+2^2.$

(E) $\exists y \, \forall x : (x+y)^2 = x^2+y^2$ = True.

EXISTENTIAL, UNIVERSAL

$x = 5$ $(x+y)^2 = 25$

$y = 0$ $x^2+y^2 = 25$

notes

1.5.1 Predicate logic 1.

Source YT MIT

$\forall$ Universal $\equiv$ For All

AND all must be true.

$\exists$ Existential $\equiv$ exists some.

or At least 1 must be true.

Existential Example.

$\varphi(y) ::= \exists x \mathrel{\dot{=}} x < y$

3 $\quad x < 3$ True if $x = 1$

1 $\quad x < 1$ True if $x = 0$.

0 $\quad x \neq 0$ False not true for any x in positive integer

Universal Example.

$R(y) ::= \forall x \mathrel{\dot{=}} x < y$.

1 $\quad x < 1$ False if $x = 5$

8 $\quad x < 8$ False if $x = 12$.

(E) $\forall x \; \exists y : 3x + y = 0$ True

Universal

Existential

$x = 3$

$y = -3$

$3x + y = 0 \therefore 3 + -3 = 0$

(F) $\exists y \; \forall x : \; 3x + y = 0$ FAlse.

$x = 3$

$y = -3$ As Above. True.

$x = 3$

$y = -4$ $3x + {}^{-4}y = -1$ FAlse.

Greater or equal ($\geq$)

(H) $\forall x : \; x^2 + 1 \geq 1$ True.

$x = 0$

$0^2 + 1 = 1$

~~(I) $\exists \forall x$~~

(I) $\exists x : \; x^2 = 7$ True.

$x = 2.64575$ (Approx)

$x^2 = 7$

Self-Assessment evidence

I quickly typed this up to make it easier to read, I did include the handwritten originals

Exercise: 3	**Prove or Disprove:** $\forall x \in N, \exists y \in N : x - y^3 = 0$		
Quantifiers	**Variables**	**Domain**	**Predicate**
$\forall$ - Universal	x	N	$P(x) \equiv \exists y \in N : x - y^3 = 0$
$\exists$ - Existential	y	N	$Q(x,y) \equiv x - y^3 = 0$
Statement is **FALSE**			
	X is Universally Quantified therefore to prove the statement false a counter argument is needed to show P(x) false, in this case x =4		
	Y is Existentially Quantified therefore a general argument is required to show the argument is false, in this case the cubed root of x is not a natural number		

Exercise: 5	**Prove or Disprove:** $\forall x \in N, \exists y \in N : (x + y)^2 = x^2 + 6x + y^2$		
Quantifiers	**Variables**	**Domain**	**Predicate**
$\forall$ - Universal	x	N	$P(x) \equiv \exists y \in N : (x + y)^2 = x^2 + 6x + y^2$
$\exists$ - Existential	y	N	$Q(x,y) \equiv (x + y)^2 = x^2 + 6x + y^2$
Statement is **TRUE**			
	Y is Existentially Quantified therefore an example is required X=12 Y=3 $(12 + 3)^2 = 255 \equiv (12 + 3)^2 = 12^2 + (6 \times 12) + 3^2 = 255$		

Exercise: 7	**Prove or Disprove:** $\exists y \in N, \exists x \in N : x - y = 0$		
Quantifiers	**Variables**	**Domain**	**Predicate**
$\exists$ - Existential	y	N	$P(y) \equiv \exists x \in N : x - y = 0$
$\exists$ - Existential	x	N	$Q(x,y) \equiv x - y = 0$
Statement is **TRUE**			
	Both variables are Existentially Quantified therefore to prove the statement is true an example is need X=0, Y=0 $X - y = 0 \equiv 0 - 0 = 0$		

Exercise: 9	**Prove or Disprove:** $\forall y \in N, \forall x \in N : x - y = 0$		
Quantifiers	**Variables**	**Domain**	**Predicate**
$\forall$ - Universal	y	N	$P(y) \equiv \forall x \in N : x - y = 0$
$\forall$ - Universal	x	N	$Q(x,y) \equiv x - y = 0$
Statement is **FALSE**			
	Both variables are Universally Quantified therefore to prove the statement is False an example is need X=0, Y=1 $X - y = 0 \neq 0 - 1 = -1$		

Work paper.

① $\forall x \in R, \exists y \in R: x - y^3 = 0$

QUANTIFIERS	VARS	DOMAIN	PREDICATE.
$\forall$ Universal	x	R	$\exists y \in R: x - y^3 = 0$
$\exists$ Existential	y	R	$Q(x,y) \equiv x - y^3 = 0$

for all x that exist in Real numbers

~~for~~ There exists at least one y in real numbers

$\forall x \in R \quad P(x) \equiv \exists y \in R \; x - y^3 = 0.$

$\exists y \in R \quad Q(x,y) \equiv x - y^3 = 0$

② There is at least one y in real numbers for any x

x is universally quantified therefore only a general statement is required $P(x)$ true for any x

y is existentially quantified. To prove true an example is required

$x = 8$
$y = 2 \quad x - y^3 \equiv 8 - 2^3 = 0$

→ self assessment answers given

$\sqrt[3]{x}$

Exersise 3

$\forall x \in N, \exists y \in N : x - y^3 = 0$

Quantifiers	Vars	Domain	Predicate
			$\exists y \in N : x - y^3 = 0$
$\forall$	x	N	
$\cancel{\exists}\ \exists$	y	N	$Q(x,y) \equiv x - y^3 = 0$

for all x that exist in x (natural numbers)

at least one y exist in natural numbers (x)

(?) at least one y exists in natural numbers for each x

x is universaly quantified to prove the statment false a counter example is needed $x=4$

y is existencly quantified to prove false a general argument is needed $\sqrt[3]{x}$ will not all ways result in a natural number

Counter example.

$x = 4$ not natural#

$y = \sqrt[3]{x}$

$x - y^3 \equiv 3 - 1.5874^3 = 0$

Exercise 4.5

$\forall x \in n, \exists y \in n: (x+y)^2 = x^2 + 6x + y^2$

$(x+y)^2 = (x^2) + (6 \times x) + (y^2)$

Quantifiers	Vars	Domain	Predicate
$\forall$	x	n	$P(x) = \exists y \in n: (x+y)2 = x2 + 6x + y2$
$\exists$	y	n	$(x+y)2 = x2 + 6x + y2$

for all x that exist in n (natural numbers) at least one y exist in n

(?) at least one y for all x in n

x is universally quantified. $P(x)$ is true for any x

y is is existentially quantified. Therefore one example is needed

Examples
$x = 12$
$y = 3$

$(12+3)^2 = 255$

$12^2 + (6 \times 12) + 3^2 = 255$

Exersise 7

$\exists y \in n, \exists x \in n : x - y = 0$

QUANT	VAR	DOMAIN	predicate
$\exists$	y	n	$P(y) \equiv \exists x \in n : x - y = 0$
$\exists$	x	n	$Q(x,y) \equiv x - y = 0$

~~for two~~
at least one in y
at least one in x

Both vars are existentially quantified. So we give an example of 0

$x = 0$

$x = 0$

$x - y = 0 \equiv 0 - 0 = 0$

Additionally there are infinite. if ~~x y are~~ $x = y$
this result will always be 0.

Exercise 9

Quantifiers	Vars	Domain	Predicate.
$\forall$	y	N	$P(y) \equiv \forall x \in N: x-y=0$
$\forall$	x	N	$Q(x,y) \equiv x-y=0$

Both variable are universaly Quantified
Therefore we need a counter example.
$y=1$
$x=0$

$x-y=0 \neq 0-1=-1$