# Lesson Review

## *Learning Objectives*

Please list the learning objectives of this module that you have achieved:
I certified that I am able to:

- Recognise and translate atomic and compound propositions.
- Construct truth tables
- Apply the laws of Boolean algebra to show logical equivalence.

## *Learning Review*

Please complete the table below (refer to the attached Learning Process table).

| **Learning Objective** | **Concept**<br><br>What concept / key-word did you focus on? | **Step** | **Strategy**<br><br>What strategy did you apply? Why did you choose this? How did you apply it? Did it work well? How do you know? | **Resource**<br><br>What resource did you use? Why did you choose this? Did it work well? | **Reflection**<br><br>In hindsight, was this strategy and resource<br>• appropriate? Why?<br>• identify other options<br>• was this the best option? Why? | **Learning**<br><br>Generalise: what you learned that could be applied in the future in a different context |
|---|---|---|---|---|---|---|
| | | | | | | |
| Atomic and Compound Propositions | Recognise and translate atomic and compound propositions | Identify | Identify Concepts and make a list of resources needed | Unit Site Content | | |
| | | Making Sense | Read Text and Site Content, watch lecture videos, watch and follow external videos | Prescribed Text Book | | |
| | | | | Recorded Lectures | | |
| | | Making Meaning | Attempt practical questions, verify answers against online tools to identify any mistakes and try again | External Videos | | |
| | | | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| Truth Tables | Construct truth tables | Identify | Identify Concepts and make a list of resources needed | Unit Site content<br>Prescribed Text Book<br>Recorded Lectures<br>External Videos | | |
| | | Making Sense | Read Text and Site Content, watch lecture videos, watch and follow external videos | | | |
| | | Making Meaning | Attempt practical questions, verify answers against online tools to identify any mistakes and try again | | | |
| | | | | | | |
| Logical Equivalence | Apply the laws of Boolean algebra to show logical equivalence | Identify | Identify Concepts and make a list of resources needed | Unit Site content<br>Prescribed Text Book<br>Recorded Lectures<br>External Videos | | |
| | | Making Sense | Read Text and Site Content, watch lecture videos, watch and follow external videos | | | |
| | | Making Meaning | Attempt practical questions, verify answers against online tools to identify any mistakes and try again | | | |
| | | | | | | |

## *Learning Evidence*

Propositions & Truth tables

p it is cold
q it is ~~rainy~~ windy
r it is rainy
s it is sunny.

(a)

i, $p \wedge r$

p AND r

it is cold and it is rainy

ii, $q \vee \neg p$

q or not p

it is windy or is not raining

iii, $\neg p \rightarrow s$

NOT p if s

it is NOT cold if it is sunny.

iv, $\neg r \rightarrow (q \wedge \neg p)$

NOT r if (q AND NOT p)

it is not rainy if (windy & NOT cold)

v. $Q \Leftrightarrow \neg(S \vee R)$

Q if and only if not (S or R)

windy if and only if not (sunny or RAINY)

vi. $[[(P \vee Q) \wedge \neg P] \rightarrow Q$

((cold or windy) AND NOT cold) if and only if windy.

(6)

i, it is cold or sunny.

P or Q

$P \vee Q$

ii, neither cold nor windy

$\neg(P \vee Q)$

iii, cold or RAINY if not sunny.

$(\text{cold} \vee \text{RAINY}) \rightarrow \neg \text{sunny}$.

$(P \vee R) \rightarrow \neg S$

P v, NOT cold if & only if it is windy or not raining

$\neg cold \leftrightarrow (P \vee \neg R)$

$\neg P \leftrightarrow (P \vee \neg R)$

V, subjective not necessarily T or F

$2 \times 2^x$ x = # of VARS

Rows = VARS x 2

2 = 4
3 = 8
4 = 16

② Truth tables

⑨ $(P \wedge \neg Q) \vee Q$

Recall connectives.

AND

| PQ | P∧Q |
|---|---|
| FF | F |
| FT | F |
| TF | F |
| TT | T |

OR

| PQ | P∨Q |
|---|---|
| FF | F |
| FT | T |
| TF | T |
| TT | T |

NOT

| P | ¬P |
|---|---|
| F | T |
| T | F |

| P | Q | ¬Q | (P ∧ ¬Q) | (P ∧ ¬Q) ∨ Q |
|---|---|---|---|---|
| T | T | F | F | T |
| T | F | T | T | T |
| F | T | F | F | T |
| F | F | T | F | F |

✓

(6) $(P \wedge Q) \rightarrow P$

AND $\wedge$

| P | Q | $P \wedge Q$ |
|---|---|---|
| F | F | F |
| F | T | F |
| T | F | F |
| T | T | T |

if

| P | Q | $P \rightarrow Q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

| P | Q | $P \wedge Q$ | $(P \wedge Q) \rightarrow Q$ | |
|---|---|---|---|---|
| T | T | T | T | right for $(P \wedge Q \rightarrow Q)$ |
| T | F | F | T | |
| F | T | F | F | ? |
| F | F | F | T | |

| P | Q | $P \wedge Q$ | $P \wedge Q \rightarrow P$ | |
|---|---|---|---|---|
| T | T | T | T | |
| T | F | F | T | |
| F | T | F | T | $(P \wedge Q)$ left |
| F | F | F | T | Q right |

C.

$P \leftrightarrow \neg (Q \wedge R)$

| P | Q | R | $Q \wedge R$ | $\neg(Q \wedge R)$ | $P \leftrightarrow \neg(Q \wedge R)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | T | F | F | T | T |
| T | F | T | F | T | T |
| T | F | F | F | T | T |
| F | T | T | T | F | F |
| F | T | F | F | T | F |
| F | F | T | F | T | F |
| F | F | F | F | T | F |

AND

| P | Q | $P \wedge Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

NOT

| P | $\neg P$ |
|---|---|
| T | F |
| F | T |

$\leftrightarrow$

| P | Q | $P \leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

① $((P \vee Q) \wedge \neg P) \rightarrow Q$

AND

| P | Q | P∧Q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

OR

| P | Q | P∨Q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

NOT

| P | ¬P |
|---|---|
| T | F |
| F | T |

IF

| P | Q | P→Q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

| P | Q | ¬P | P∨Q | (P∨Q)∧¬P | ((P∨Q)∧¬P)→Q |
|---|---|---|---|---|---|
| T | T | F | T | F | T |
| T | F | F | T | F | T |
| F | T | T | T | T | F |
| F | F | T | F | F | T |

F
F
T
F

| P | Q | ¬P | P∨Q | (P∨Q)∧¬P | ((P∨Q)∧¬P)→Q |
|---|---|---|---|---|---|
| T | T | F | T | F | T |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | F | T | F | F | T |

$\neg\neg\neg(P \wedge (P \wedge \neg Q))$

$P \wedge (P \vee \neg(Q \vee \neg Q))$ — negation
$P$ AND $(P$ OR NOT $(TRUE))$ — negation
$P$ AND $(P$ OR $FALSE)$ — Identity
$P$ AND $P$ — Idempotent
$P$

$\neg(P \wedge Q) \wedge (P \vee \neg Q)$ — De Morgans
$(\neg P \wedge \neg Q) \wedge (P \vee \neg Q)$ — Commutative
$(\neg Q \wedge \neg P) \wedge (P \vee \neg Q)$ — not 455
$\neg Q \vee \neg P) \wedge (P \vee \neg Q)$ — not neg
$(\neg Q \cdot \vee \neg Q)$ — Idempotent
$\neg Q$

$\neg\neg(Q \vee P)$ $\quad 1 = T$ $\quad 0 = F$

| P | Q | $Q \vee P$ | $\neg(Q \vee P)$ | $\neg\neg(Q \vee P$ |
|---|---|---|---|---|
| T | T | 1 | 0 | 1 |
| T | F | 1 | 0 | 1 |
| F | T | 1 | 0 | 1 |
| F | F | 0 | 1 | 0 |

$$(P \vee (Q \wedge (Q \vee P))) \wedge \neg P$$

Double neg.

$$\neg\neg(P \vee (Q \wedge (Q \vee P))) \equiv P \vee Q$$

$\neg\neg Q \equiv A$ LAW

① $\equiv P \vee (Q \wedge (Q \vee P))$ Double neg.

$\equiv P \vee (\overline{\neg(\neg Q \vee F)} \wedge (Q \vee P))$ IDENTITY.

$\equiv P \vee (Q \vee (F \wedge P))$ DIST

$\equiv P \vee (Q \vee F)$ Dominance

$\equiv P \vee Q$ IDENTITY.

$[(p \vee q) \wedge \neg p)] \rightarrow q$

$A$ $\quad$ $B$

$q \rightarrow b \equiv \neg A \vee b$

$\neg A \vee B$

$\neg[(p \vee q) \wedge \neg p] \vee q$

$\neg[\underbrace{(p \vee q)}_{(B \vee C)} \wedge \underbrace{\neg p}_{A}] \vee q$

$p \wedge (q \vee r)$

$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge$

$\neg[(B \wedge A) \vee (C \wedge A)] \vee q$

$\neg[\underbrace{(p \wedge \neg p)}_{(A \wedge \neg A) = F} \vee (q \wedge \neg p)] \vee q$

negat

$p \wedge \neg p \equiv F$

$\neg[F \vee (q \wedge \neg p)] \vee q$

~~$\neg(F$~~

~~$\neg(q) \vee q$~~

$\neg(A) \vee q$

Identity.

$A \vee F \equiv A$

$\neg(q \wedge \neg p) \vee q$

$A \quad B \rightarrow$

$(\neg A \vee \neg B) \vee q$

De

$\neg(A \wedge B)$

$\neg q \vee$

## *Self-Assessment evidence*

# Propositional Logic

Click on a question number to see how your answers were marked and, where available, full solutions.

| Question Number | Score |
|---|---|
| 1 | 1 / 1 |
| 2 | 10 / 10 |
| **Total** | **11 / 11 (99%)** |

## Performance Summary

| | |
|---|---|
| **Exam Name:** | Propositional Logic |
| **Session ID:** | 15297330100 |
| **Exam Start:** | Thu Apr 16 2020 22:41:40 |
| **Exam Stop:** | Thu Apr 16 2020 22:45:54 |
| **Time Spent:** | 0:03:40 |

## Question 1

### Truth Table

#### Instructions

Please enter a truth table for the expression $q \wedge \neg(p \wedge q)$ .

#### Filling the table

- In the top row, enter boolean expressions. You can copy/paste from the following expression: q ∧ ¬(p ∧ q) or from the table below.
- In the remaining rows, enter boolean values (true or false). Please enter them as 1 (for true) and 0 (for false), for the system to check it.

## Entering expressions

You can enter expressions either by copy-pasting the expression below (or is sub-expressions), or typing the expression with English words - for example: "p or (q and not p)". The system will recognise and display the expression. If it does not display, then the system is unable to parse your input (make sure you check the brackets).

You can copy the sub-expressions from here: q ∧ ¬(p ∧ q)

Rows: 5 Columns: 5

| p | q | p∧q | ¬(p∧q) | q∧¬(p∧q) |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 |

*This feedback is based on your last submitted answer. Submit your changed answer to get updated feedback.*

Your answer is correct. You were awarded 1 mark.

You scored 1 mark for this part.

**Score: 1/1**

# Advice

# Question 2

Show that $p \vee \neg\neg(q \wedge \neg q) \equiv p$.

# Instructions to fill the gaps

- Apply **one and only one** law per step.

- Start from the left hand side, in the first gap, and finish with the right hand side. You can add additional steps, and any blank step at the end will signal the end of your work (so don't skip a line).
- The system will automatically interpret your entry and display a formula. *If no formula appears, it means that the system cannot parse your entry. Check the syntax.* Note that the displayed formula does not show all brackets, because it applies bodmas, which can be confusing at first. Unfortunately this is hardcoded and cannot be changed.
- You can enter formulas using either the operations ¬,∧,∨, or English words not, and, or. So, the input "p and (q or not p)" is the same as "p ∧ (q ∨ ¬ p)".
- At the bottom of this question you will find a table with the list of operations, and the keywords to use.

# Entering your solution

Please note that the system should accept any correct answers, but it is very strict about it: you can only apply one law per step, not more (note that this also includes the fact that you cannot apply the same law twice in one go - it has to be done in two steps: it is one law per step, not one type of law per step). The checker should tell you when it thinks that you entered more than one law.
It is not possible to reveal the solution: checking the solution automatically is easier than computing the solution.

You may copy-paste the expression in the first box: p ∨ ¬¬(q ∧ ¬q)

Select the number of steps you need: 4

| | | | |
|---|---|---|---|
| LHS | p ∨ ¬¬(q ∧ ¬q) | | |
| ≡ | | $p \vee \neg\neg (q \wedge \neg q)$ | |
| ≡ | p ∨ (q ∧ ¬q) | $p \vee q \wedge \neg q$ | Double Negation Law ✔ |
| ≡ | p ∨ false | $p \vee false$ | Negation Law ✔ |
| ≡ | p | $p$ | Identity Law ✔ |

You may enter the operations by copying from the following table. Either versions are fine.

| Operation/Value | True | False | negation | conjunction | inclusive disjunction | ex disj |
|---|---|---|---|---|---|---|
| ascii version | true | false | not | and | or | |
| symbol version | | | ¬ | ∧ | ∨ | |

*This feedback is based on your last submitted answer. Submit your changed answer to get updated feedback.*

The maximum you can score for this part is **10** marks. Your scores will be scaled down accordingly.

LHS

✔ Your answer is numerically correct. You were awarded **1.4285714286** marks.

Step 1

This part was marked using your answers to previous parts.

✔ Your answer is numerically correct.

Law 1

**Score: 10/10** ✔

# Advice