

Lesson Review

Learning Objectives

Please list the learning objectives of this module that you have achieved:

I certified that I am able to:

- Apply common set operations.
- Express sets using enumeration or set builder notation, and go back and forth between the two representations.
- Identify common properties of functions.

Learning Review

Please complete the table below (refer to the attached Learning Process table).

| Learning Objective | Concept | Step | Strategy | Resource | Reflection | Learning |
|--------------------|---|----------------|--|---|--|---|
| | What concept / key-word did you focus on? | | What strategy did you apply? Why did you choose this? How did you apply it? Did it work well? How do you know? | What resource did you use? Why did you choose this? Did it work well? | In hindsight, was this strategy and resource <ul style="list-style-type: none"> • appropriate? Why? • identify other options • was this the best option? Why? | Generalise: what you learned that could be applied in the future in a different context |
| Set Operations | Apply common set operations | Identify | Identify Concepts and make a list of resources needed | Unit Site Content | | |
| | | Making Sense | Read Text and Site Content, watch lecture videos, watch and follow external videos | Prescribed Text Book | | |
| | | | | Recorded Lectures | | |
| | | Making Meaning | Attempt practical questions, verify answers against online tools to identify any mistakes and try again | External Videos | | |

| | | | | | | |
|-------------------------------------|---|----------------|---|---|--|--|
| enumeration or set builder notation | Express sets using enumeration or set builder notation, and go back and forth between the two representations | Identify | Identify Concepts and make a list of resources needed | Unit Site content Prescribed Text Book Recorded Lectures External Videos | | |
| | | Making Sense | Read Text and Site Content, watch lecture videos, watch and follow external videos | | | |
| | | Making Meaning | Attempt practical questions, verify answers against online tools to identify any mistakes and try again | | | |
| | | | | | | |
| properties of functions | Identify common properties of functions | Identify | Identify Concepts and make a list of resources needed | Unit Site content Prescribed Text Book Recorded Lectures External Videos | | |
| | | Making Sense | Read Text and Site Content, watch lecture videos, watch and follow external videos | | | |
| | | Making Meaning | Attempt practical questions, verify answers against online tools to identify any mistakes and try again | | | |

Learning Evidence

Sets and functions

Sets

Set is a collection of objects

A set is indicated by curly brackets $\{ \}$

\in is an element of.

\notin is not an element of.

Upper case denotes sets

Lower case denotes elements of sets

Subset

$A \subset B$ A is a subset of B if every element of A is in B

$B \supset A$ B is a superset of A

Q.

$\{1, 2, 3\} \subset \mathbb{N}$ $\{1, 2, 3\}$ is a subset of NATURAL no

$\mathbb{N} \supset \{1, 2, 3\}$ NATURAL numbers is a superset of the elements $\{1, 2, 3\}$

Set operation definition

\cup = UNION

The union of A and B is the set of elements in A or B (or both) denoted by $A \cup B$

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

\cap = INTERSECTION

The intersection of A and B is the set of elements in both A and B denoted by $A \cap B$

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

\bar{A} or A' or A^c = complement.

The complement of A is the set of all elements in the universal set U but not in A denoted by \bar{A} or A' or A^c

$$\bar{A} = \{x : x \in U \text{ and } x \notin A\}$$

$A \setminus B$ or $A - B$ = Set Difference

The set difference is the set of all elements in A excluding those in B denoted by $A \setminus B$ or $A - B$

$$A \setminus B = \{x : x \in A \text{ and } x \notin B\} = A \cap B^c$$

$A \oplus B$ or $A \Delta B$ = Symmetric Difference

Set operations example 1

Suppose.

(Universal set)

$$U = \{x \in \mathbb{N} : 1 \leq x \leq 9\}$$

$$x = \forall A$$

$$e = \exists B$$

$$A = \{1, 2, 3\} \quad (\text{Set A})$$

$$\mathbb{N} = \text{NATURAL NUMBERS}$$

$$B = \{2, 3, 4, 5\} \quad (\text{Set B})$$

$$A \cup B = \{1, 2, 3, 4, 5\}$$

↳ The union of A and B

Join

$$A \cap B = \{2, 3\}$$

↳ The intersection of A and B is in BOTH

$$\bar{A} = \{4, 5, 6, 7, 8, 9\}$$

↳ complement includes all elements in universal set that are NOT in A

$$\bar{B} = \{1, 6, 7, 8, 9\}$$

↳ complement includes all elements in universal set that are NOT in B

$$A \setminus B = \{1\}$$

↳ set difference, elements in A that are not in B

$$B \setminus A = \{4, 5\}$$

↳ set difference, elements in B that are not in A

$$A \oplus B = \{1, 4, 5\}$$

↳ Symmetric Difference elements in either set but not in the other

~~Set operations example 2:~~

Set operations

$$x \in A \cup B \iff x \in A \vee x \in B$$

$$x \in A \vee x \in B$$

$$x \in A \cap B$$

$$x \in A \wedge x \in B$$

$$x \in A^c$$

$$\neg (x \in A)$$

PRACTICAL 4 - SETS & FUNCTIONS

$$U = \{x \in \mathbb{Z} : 1 \leq x \leq 10\} \\ = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{x \in U : x \text{ is even}\} \\ = \{2, 4, 6, 8, 10\}$$

$$B = \{x \in U : x \leq 4\} \\ = \{1, 2, 3, 4\}$$

$$C = \{x \in U : 3 \leq x < 9\} \\ = \{3, 4, 5, 6, 7, 8\}$$

①

$$A = \{2, 4, 6, 8, 10\}$$

$$B \cap A = \{1, 3\}$$

$$B = \{1, 2, 3, 4\}$$

$$A \oplus B = \{1, 3, 6, 8, 10\}$$

$$C = \{3, 4, 5, 6, 7, 8\}$$

$$A \cap B \cap C = \{4\}$$

$$A' = \{1, 3, 5, 7, 9\}$$

$$C' = \{1, 2, 9, 10\}$$

$$A \cup B = \{1, 2, 3, 4, 6, 8, 10\}$$

$$C \setminus A = \{3, 5, 7\}$$

$$B \cap C = \{3, 4\}$$

$$(C \setminus A) \cap B = \{4\}$$

$$A \setminus B = \{6, 8, 10\}$$

②

④ $2 \in A \cap B = \text{True}$

⑤ $\{1\} \in A'$

⑦ $\{4\} \subseteq A \cap B = \text{True}$

⑧ $1 \leq 0$

} False in both worlds

③

① $(A \cap B) \cup (A' \cup B)' = A$

LHS $(A \cap B) \cup (A' \cup B)'$

$= (A \cap B) \cup (A'' \cap B')$

De Morgan's

$= (A \cap B) \cup (A \cap B')$

Double.

$= A \cap (B \cup B')$

Distrib

$= A \cap U$

Set

$= A = \text{RHS}$

QED

~~②~~ ~~③~~ ~~④~~ ~~⑤~~

⑥ $B' - (B \cap A') = B'$

LHS $= B' - (B \cap A')$

$= B' \cap (B \cap A')'$

Def of -

$= B' \cap (B' \cup A'')$

De Morgan's

$= B' \cap (B' \cup A)$

Double

$= B' \cup \emptyset \cap (B' \cup A)$

Ident

$= B' \cup (\emptyset \cap A)$

Distrib

$= B' \cup \emptyset$

Dominance

$= B' = \text{RHS}$

Ident

(C) ~~LAB~~ $A - (A - B) = A \cap B$

$$\begin{aligned}
 \text{LHS} &= A - (A - B) \\
 &= A - (A \cap B') && \text{Def of } - \\
 &= A \cap (A \cap B')' && \text{Def of } - \\
 &= A \cap (A' \cup B'') && \text{De Morgan's} \\
 &= A \cap (A' \cup B) && \text{Double} \\
 &= (A \cap A') \cup (A \cap B) && \text{Dist} \\
 &= \emptyset \cup (A \cap B) && \text{Set} \\
 &= A \cap B = \text{RHS} && \text{Ident}
 \end{aligned}$$

Self-Assessment evidence

Sets and Functions

Click on a question number to see how your answers were marked and, where available, full solutions.

| Question Number | Score |
|-----------------|--------------|
| Sets | |
| 1 | 3 / 3 |
| Functions | |
| 2 | 1 / 1 |
| 3 | 1 / 1 |
| 4 | 1 / 1 |
| Total | 6 / 6 (100%) |

Performance Summary

| | |
|-------------|--------------------------|
| Exam Name: | Sets and Functions |
| Session ID: | 1569397211 |
| Exam Start: | Wed Apr 29 2020 17:49:30 |
| Exam Stop: | Wed Apr 29 2020 17:57:06 |
| Time Spent: | 0:07:36 |

Question 1

Compute the following set operations, given the sets:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

$$A = \{4, 9, 12, 15, 16, 17, 18, 19, 20\}$$

$$B = \{x \in U : x \text{ is a multiple of } 4\}$$

$$C = \{x \in U : x > 15\}$$