Lesson Review

Learning Objectives

Please list the learning objectives of this module that you have achieved:
I certified that I am able to:

- Find the elements of a sequence given in closed form or in recurrence form.
- Evaluate partial sums of sequences.
- Solve linear recurrence of order 1 and of order 2.

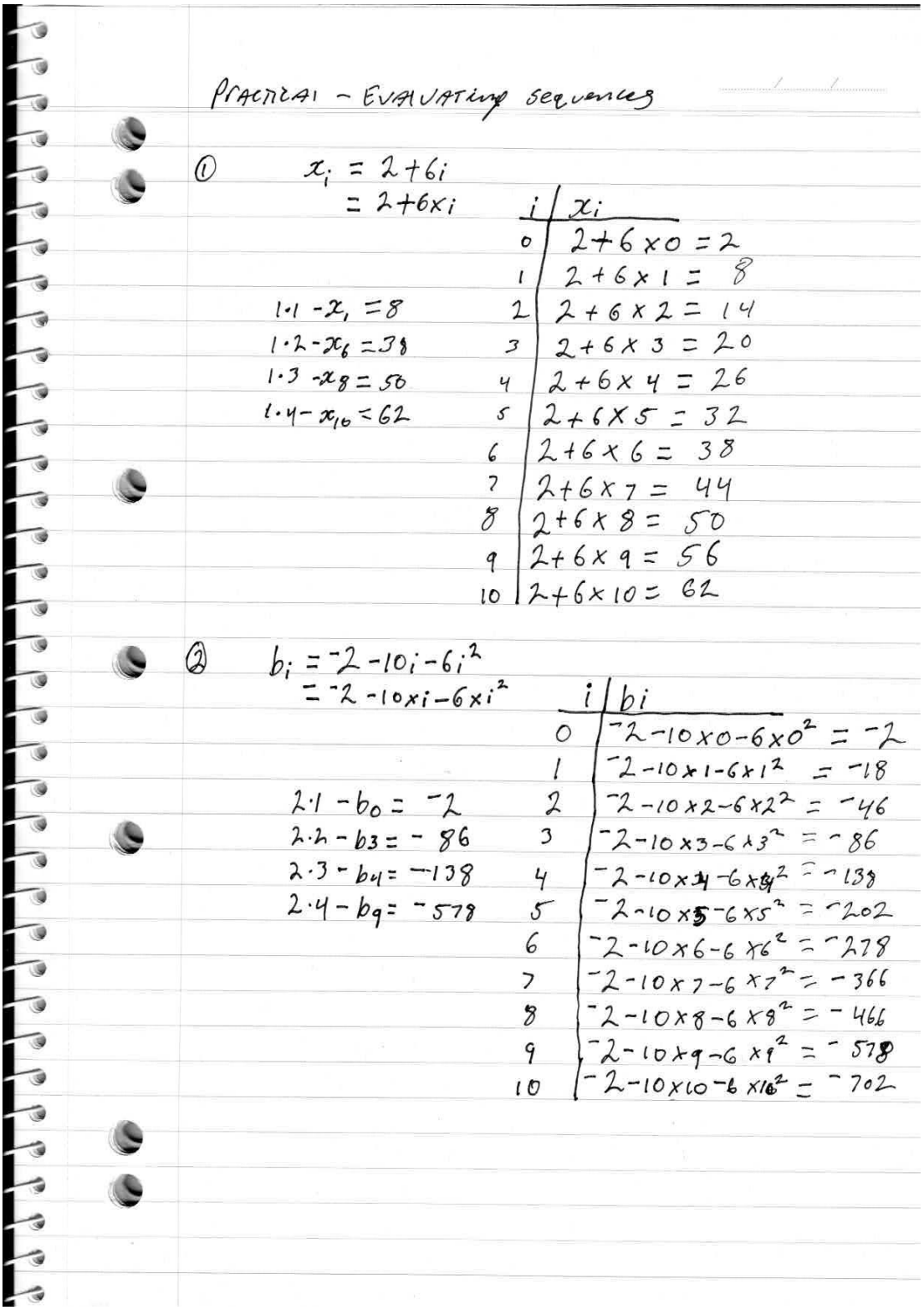
Learning Review

Please complete the table below (refer to the attached Learning Process table).

Learning Objective	**Concept** What concept / key-word did you focus on?	**Step**	**Strategy** What strategy did you apply? Why did you choose this? How did you apply it? Did it work well? How do you know?	**Resource** What resource did you use? Why did you choose this? Did it work well?	**Reflection** In hindsight, was this strategy and resource • appropriate? Why? • identify other options • was this the best option? Why?	**Learning** Generalise: what you learned that could be applied in the future in a different context
Recurrence	Find the elements of a sequence given in closed form or in recurrence form	Identify	Identify Concepts and make a list of resources needed	Unit Site Content		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos	Prescribed Text Book		
				Recorded Lectures		
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again	External Videos		

Sequences	Evaluate partial sums of sequences	Identify	Identify Concepts and make a list of resources needed	Unit Site content Prescribed Text Book Recorded Lectures External Videos		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos			
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again			
Recurrence	Solve linear recurrence of order 1 and of order 2	Identify	Identify Concepts and make a list of resources needed	Unit Site content Prescribed Text Book Recorded Lectures External Videos		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos			
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again			

Learning Evidence



PRACTICAL – EVALUATING sequences

① $x_i = 2 + 6i$

$= 2 + 6 \times i$

i	x_i
0	$2 + 6 \times 0 = 2$
1	$2 + 6 \times 1 = 8$
2	$2 + 6 \times 2 = 14$
3	$2 + 6 \times 3 = 20$
4	$2 + 6 \times 4 = 26$
5	$2 + 6 \times 5 = 32$
6	$2 + 6 \times 6 = 38$
7	$2 + 6 \times 7 = 44$
8	$2 + 6 \times 8 = 50$
9	$2 + 6 \times 9 = 56$
10	$2 + 6 \times 10 = 62$

1.1 – $x_1 = 8$

1.2 – $x_6 = 38$

1.3 – $x_8 = 50$

1.4 – $x_{10} = 62$

② $b_i = -2 - 10i - 6i^2$

$= -2 - 10 \times i - 6 \times i^2$

i	b_i
0	$-2 - 10 \times 0 - 6 \times 0^2 = -2$
1	$-2 - 10 \times 1 - 6 \times 1^2 = -18$
2	$-2 - 10 \times 2 - 6 \times 2^2 = -46$
3	$-2 - 10 \times 3 - 6 \times 3^2 = -86$
4	$-2 - 10 \times 4 - 6 \times 4^2 = -138$
5	$-2 - 10 \times 5 - 6 \times 5^2 = -202$
6	$-2 - 10 \times 6 - 6 \times 6^2 = -278$
7	$-2 - 10 \times 7 - 6 \times 7^2 = -366$
8	$-2 - 10 \times 8 - 6 \times 8^2 = -466$
9	$-2 - 10 \times 9 - 6 \times 9^2 = -578$
10	$-2 - 10 \times 10 - 6 \times 10^2 = -702$

2.1 – $b_0 = -2$

2.2 – $b_3 = -86$

2.3 – $b_4 = -138$

2.4 – $b_9 = -578$

③ $b_0 = 2$

$b_i = -3 + b_{i-1}$

$3 \cdot 1 - b_1 = -1$

$3 \cdot 2 - b_2 = -7$

$3 \cdot 3 - b_5 = -13$

$3 \cdot 4 - b_9 = -25$

i	b_i
0	2
1	$-3+2 = -1$
2	$-3+-1 = -4$
3	$-3+-4 = -7$
4	$-3+-7 = -10$
5	$-3+-10 = -13$
6	$-3+-13 = -16$
7	$-3+-16 = -19$
8	$-3+-19 = -22$
9	$-3+-22 = -25$
10	$-3+-22 = -28$

④ $a_0 = 1$

$a_i = 1 + 6a_{i-1} - 7a_{i-1}^2$

$4 \cdot 1 - a_3 = 0$

$4 \cdot 2 - a_4 = 1$

$4 \cdot 3 - a_5 = 0$

$4 \cdot 4 - a_{10} = 1$

i	b_i
0	1
1	$1+6\times1-7\times1^2 = 0$
2	$1+6\times0-7\times0^2 = 1$
3	$1+6\times1-7\times1^2 = 0$
4	$1+6\times0-7\times0^2 = 1$
5	$1+6\times1-7\times1^2 = 0$
6	$1+6\times0-7\times0^2 = 1$
7	$1+6\times1-7\times1^2 = 0$
8	$1+6\times0-7\times0^2 = 1$
9	$1+6\times1-7\times1^2 = 0$
10	$1+6\times0-7\times0^2 = 1$

Sums of sequences

(5) $a_i = 10 + 3i + 2i^2$

$= 10 + 3 \times i + 2 \times i^2$

$\sum_{i=0}^{6} a_i = 10 + 15 + 24 + 37 + 54 + 75 + 100$

$= 315$

$\sum_{i=0}^{6} q_i = 315$

i	a_i
0	$10 + 3 \times 0 + 2 \times 0^2 = 1$
1	$10 + 3 \times 1 + 2 \times 1^2 = 1$
2	$10 + 3 \times 2 + 2 \times 2^2 =$
3	$10 + 3 \times 3 + 2 \times 3^2 = 3$
4	$10 + 3 \times 4 + 2 \times 4^2 = 54$
5	$10 + 3 \times 5 + 2 \times 5^2 = 75$
6	$10 + 3 \times 6 + 2 \times 6^2 = 100$

(6) $s_i = -9 - 5i - 4i^2$

$= -9 - 5 \times i - 4 \times i^2$

$\sum_{i=2}^{5} s_i = -322$

i	s_i
0	$-9 - 5 \times 0 - 4 \times 0^2 = -9$
1	$-9 - 5 \times 1 - 4 \times 1^2 = -18$
2	$-9 - 5 \times 2 - 4 \times 2^2 = -35$
3	$-9 - 5 \times 3 - 4 \times 3^2 = -60$
4	$-9 - 5 \times 4 - 4 \times 4^2 = -93$
5	$-9 - 5 \times 5 - 4 \times 5^2 = -134$
6	$-9 - 5 \times 6 - 4 \times 6^2 = -183$
7	$-9 - 5 \times 7 - 4 \times 7^2 = -240$

(7) $b_0 = -5$

$b_i = -7 + 3b_{i-1}$

$\sum_{i=1}^{4} b_i = -1006$

i	b_i
0	-5
1	$-7 + 3 \times -5 = -22$
2	$-7 + 3 \times -22 = -73$
3	$-7 + 3 \times -73 = -226$
4	$-7 + 3 \times -226 = -685$

(8) $a_0 = -1$

$a_i = -5 a_{i-1}$

i	a_i
0	$-1 = 5$
1	$-5 \times a_0 = -25$
2	$-5 \times a_1 = +125$
3	$-5 \times a_2 = -625$
4	$-5 \times a_3 = 3125$
5	$-5 \times a_4 = -15625$

0) $5 \times 1 = 5 = 5^1$

1) $5 \times 5 = 25 = 5^2$

2) $5 \times 25 = 125 = 5^3$

3) $5 \times 125 = 625 = 5^4$

4) $5 \times 625 = 3125 = 5^5$

5) $5 \times 3125 = 15625 = 5^6$

$= a_i 5_i x_i (-5)^i$

$= -(-5)^i$

(9) $a_0 = -2$

$a_i = 3 a_{i-1}$

i	a_i
0	$-2 = -2$
1	$3 \times a_0 = -6$
2	$3 \times a_1 = -18$
3	$3 \times a_2 = -54$
4	$3 \times a_3 = -162$

0) $= -2$

1) $3 \times -2 = -6 = -2 \times 3^1$

2) $3 \times -6 = -18 = -2 \times 3^2$

3) $3 \times -18 = -54 = -2 \times 3^3$

4) $3 \times -54 = 162 = -2 \times 3^4$

$= a_i = 5_i \times 3^i$

$= -2 \times 3^i$

(10) $a_0 = 2$

$a_1 = -12$

$a_i = 3a_{i-1} + 18a_{i-2}$

2) $(3 \times -12 = -36) + (18 \times 2 = 36) = 0$

3) $(3 \times 0 = 0) + (18 \times -12 = -216) = -216$

4) $(3 \times -216 = -648) + (18 \times 0 = 0) = -648$

5) $(3 \times -648 = -1944) + (18 \times -216 = -3888) = -1944$

6) $(3 \times -1944 = -5832) + (18 \times -11664) = -5841$

7)

i	a_i
0	2
1	-12
2	$3 \times a_1 + 18 \times a_0 =$
3	$3 \times a_2 + 18 \times a_1 =$
4	$3 \times a_3 + 18 \times a_2 =$
5	$3 \times a_4 + 18 \times a_3 =$
6	$3 \times a_5 + 18 \times a_4 =$
7	$3 \times a_6 + 18 \times a_5 =$
8	$3 \times a_7 + 18 \times a_6 =$
9	$3 \times a_8 + 18 \times a_7 =$
10	$3 \times a_9 + 18 \times a_8 =$
11	$3 \times a_{10} + 18 \times a_9 =$
12	$3 \times a_{11} + 20 \times a_{10} =$
13	$3 \times a_{12} + 21 \times a_{11} =$
14	$3 \times a_{13} + 22 \times a_{12} =$
15	$3 \times a_{14} + 23 \times a_{13} =$
16	$3 \times a_{15} + 24 \times a_{14} =$
17	$3 \times a_{16} + 25 \times a_{15} =$
18	$3 \times a_{17} + 26 \times a_{16} =$

$= 51 \times (-6)^i + 52 \times 3^i$

$= a_i = 2 \times (-6)^i$

Self-Assessment evidence

Recurrence Relations

Click on a question number to see how your answers were marked and, where available, full solutions.

Question Number	Score			
Order 1				
1	1	/	1	Review
2	1	/	1	Review
3	1	/	1	Review
Order 2				
4	1	/	1	Review
5	1	/	1	Review
Total	**5**	**/**	**5 (100%)**	

Performance Summary

Exam Name: Recurrence Relations