Lesson Review

Learning Objectives

Please list the learning objectives of this module that you have achieved:
I certified that I am able to:

- Analyse proofs.
- Construct simple proofs for mathematical statements.
- Select and explain appropriate methods of proofs.

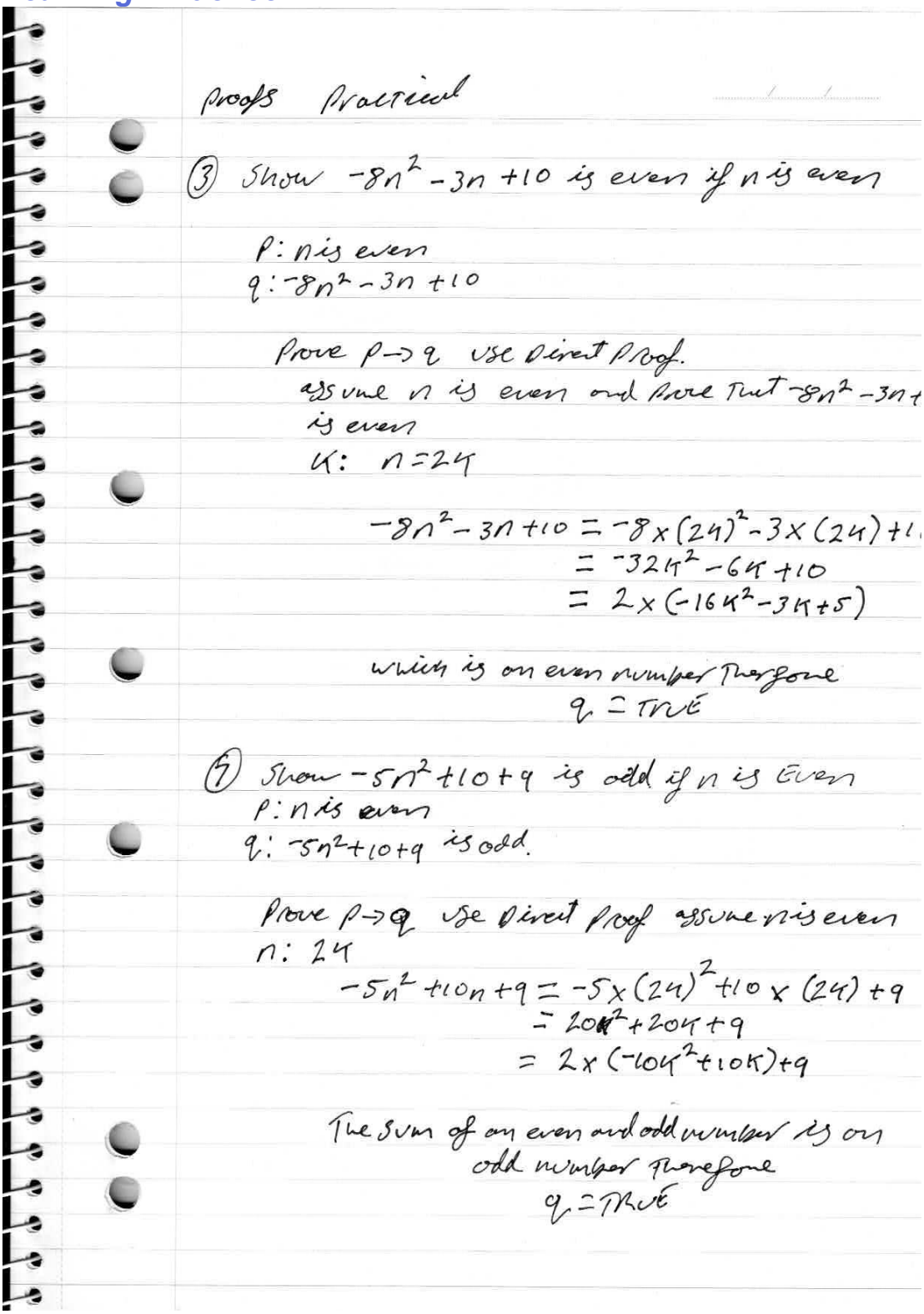
Learning Review

Please complete the table below (refer to the attached Learning Process table).

Learning Objective	**Concept** What concept / key-word did you focus on?	**Step**	**Strategy** What strategy did you apply? Why did you choose this? How did you apply it? Did it work well? How do you know?	**Resource** What resource did you use? Why did you choose this? Did it work well?	**Reflection** In hindsight, was this strategy and resource • appropriate? Why? • identify other options • was this the best option? Why?	**Learning** Generalise: what you learned that could be applied in the future in a different context
Proofs	Analyse proofs	Identify	Identify Concepts and make a list of resources needed	Unit Site Content		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos	Prescribed Text Book		
				Recorded Lectures		
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again	External Videos		

Proofs	Construct simple proofs for mathematical statements	Identify	Identify Concepts and make a list of resources needed	Unit Site content Prescribed Text Book Recorded Lectures External Videos		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos			
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again			
Proofs	Select and explain appropriate methods of proofs	Identify	Identify Concepts and make a list of resources needed	Unit Site content Prescribed Text Book Recorded Lectures External Videos		
		Making Sense	Read Text and Site Content, watch lecture videos, watch and follow external videos			
		Making Meaning	Attempt practical questions, verify answers against online tools to identify any mistakes and try again			

Learning Evidence



Proofs Practice

(3) Show $-8n^2 - 3n + 10$ is even if n is even

p: n is even
q: $-8n^2 - 3n + 10$

Prove $p \rightarrow q$ use Direct Proof.
assume n is even and prove that $-8n^2 - 3n +$
is even
k: $n = 2k$

$$-8n^2 - 3n + 10 = -8 \times (2k)^2 - 3 \times (2k) + 1$$
$$= -32k^2 - 6k + 10$$
$$= 2 \times (-16k^2 - 3k + 5)$$

which is an even number therefore
q = TRUE

(7) Show $-5n^2 + 10 + 9$ is odd if n is Even
p: n is even
q: $-5n^2 + 10 + 9$ is odd.

Prove $p \rightarrow q$ use Direct Proof assume n is even
n: $2k$

$$-5n^2 + 10n + 9 = -5 \times (2k)^2 + 10 \times (2k) + 9$$
$$= 20k^2 + 20k + 9$$
$$= 2 \times (-10k^2 + 10k) + 9$$

The sum of an even and odd number is an
odd number therefore
q = TRUE

(13) Show $+n-9$ is odd if n is even

P: n is even

q: $+n-9$ is odd

Prove $P \to q$ use Direct Proof

assume n is even and Prove $+n-9$ is odd

K: $n = 2y$

$$+n-9 = +(2y)-9$$
$$= +2y-9$$
$$= 2\times(+y)-9$$

the sum of an odd and even number is odd

Therefore q = TRUE

(14) $5n^2-3n-9$ is odd for any $n \in N$

n is even: κ: $n = 2y$

$$5n^2-3n-9 = 5\times(2y)^2-3\times(2y)-9$$
$$= 20y^2-6y-9$$
$$= 2\times(10y^2-3y)-9$$

the sum of an even & odd is odd.

n is odd : y: $n = 2y+1$

$$5n^2-3n-9 = 5\times(2y+1)^2-3\times(2y+1)-$$
$$= 5\times(4y+1)-6y-3-9$$
$$= 20y^2+20y+5-6y-12$$
$$= 20y^2+14y-7$$
$$= 2\times(10y^2+7y)-7$$

the sum of an even & odd is odd.

Self-Assessment evidence

Exercise 1

Show that $-9n^2 + 2n + 6$ is odd if n is odd	**TRUE**
P:	n is odd
Q:	$-9n^2 + 2n + 6$ is odd

P → Q - Direct proof: assume n is odd and prove that $-9n^2 + 2n + 6$

$-9n^2 + 2n + 6$	$= -9 \times (2k + 1)^2 + 2 \times (2k + 1) + 6$
	$= -9 \times (4k^2 + 4k + 1) + 4k + 2 + 6$
	$= -36k^2 - 36k - 9 + 4k + 8$
	$= -36k^2 - 32k - 1$
	$= 2 \times (-18k^2 - 16k) - 1$

The sum of an even and an odd number is always an odd number therefore Q is True

Exercise 3

Show that n is odd if $10n^2 - 5n - 6$ is odd	**TRUE**
P:	n is odd
Q:	$10n^2 - 5n - 6$ is odd

P ← Q – Indirect Proof – Prove ¬P → ¬Q

¬P n is not odd
¬Q: $10n^2$ - 5n - 6 is not odd

Assume n is not odd and prove that $10n^2$ - 5n - 6 is not odd
Since n is even, we can find an integer k such that n = 2k. Then

$10n^2$ - 5n - 6	$= 10 \times (2k)^2 - 5 \times (2k) - 6$
	$= 40k^2 - 10k - 6$
	$= 2 \times (20k^2 - 5k - 3)$

which is an even number. Therefore ¬Q is True.

Exercise 7

Show that n is even if $10n^2 + 7n - 6$ is even	**TRUE**
P:	n is even
Q:	$10n^2 + 7n - 6$ is even

P ← Q. We use an indirect proof - Prove ¬P ← ¬Q

¬P - is not even
¬Q - $10n^2 + 7n - 6$ is not even

Assume n is not even and prove $10\ n^2 + 7n - 6$ is not even
n is odd, k: n = 2k + 1

$10n^2 + 7n - 6$	$= 10 \times (2k + 1)^2 + 7 \times (2k + 1) - 6$
	$= 10 \times (4k2 + 4k + 1) + 14k + 7 - 6$
	$= 40k^2 + 40k + 10 + 14k + 1$
	$= 40k^2 + 54k + 11$
	$= 2 \times (20k^2 + 27k) + 11$

The sum of an even and an odd number is always an odd number therefore ¬Q is True