MSE893 - Advanced Kinematics for Robotic Systems

Robot for Airport Luggage Handling

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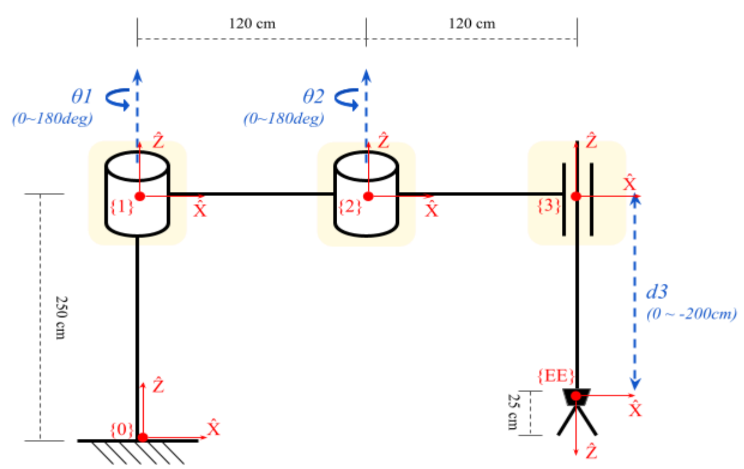
1. ABSTRACT

With the increasing labor shortage and heightened risk of injuries in airport ground handling, the need for automation has become crucial. The project focuses on designing a robot manipulator specifically for airport luggage handling. This report covers the results of project phase 2: inverse kinematic and workspace analysis. Through the analytic approach, we derive the inverse kinematic equations to convert a designated wrist position to corresponding joint values. We also utilize MATLAB to visualize the pose of the manipulator.

2. INVERSE KINEMATICS



Joint	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	250
2	0	120	θ_2	0
3	0	120	0	d_3
EE	180	0	0	0

Figure 1. Joint frames attached to the Robot's kinematic layout and corresponding DH Parameters

Following the manipulator's DH parameters shown in Figure 1, we derive the homogeneous transformation matrix in Project Report 1 as below:

$$
{}^0_3T = \begin{bmatrix} c_1 * c_2 - s_1 * s_2 & -c_1 * s_2 - s_1 * c_2 & 0 & 120 * c_1 + 120 * c_1 * c_2 - 120 * s_1 * s_2 \\ c_1 * s_2 + s_1 * c_2 & c_1 * c_2 - s_1 * s_2 & 0 & 120 * s_1 + 120 * c_1 * s_2 + 120 * c_2 * s_1 \\ 0 & 0 & 1 & d_3 + 250 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}
$$

The trigonometry expression in the matrix can be further simplified using the addition formula of sine and cosine, and we get the following simplified version of the transformation matrix:

$$ {}^0_3T = \begin{bmatrix} cos(\theta_1+\theta_2) & -sin(\theta_1+\theta_2) & 0 & 120*cos(\theta_1)+120*cos(\theta_1+\theta_2) \\ sin(\theta_1+\theta_2) & cos(\theta_1+\theta_2) & 0 & 120*sin(\theta_1)+120*sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & d_3+250 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

The desired position of the manipulator's wrist (x, y, z) is contributed by displacement components of the transformation matrix. Therefore, we get the following relationship between joint vector and position:

$$ 120cos(\theta_1) + 120cos(\theta_1+\theta_2) = x \tag{3} $$

$$ 120sin(\theta_1) + 120sin(\theta_1+\theta_2) = y \tag{4} $$

$$ D_3 + 250 = z \tag{5} $$

By squaring equations (3), (4) and adding them together, we get the following equation containing only one variable θ_2:

$$ 120^2 + 120^2 + 2\cdot 120^2 \cdot cos(\theta_2) = x^2 + y^2 \tag{6} $$

$$ cos(\theta_2) = \frac{x^2+y^2-120^2-120^2}{2\cdot 120\cdot 120} \tag{7} $$

Via the property of $sin(\theta_2) = \pm\sqrt{1 - cos(\theta_2)^2}$, we can find the value of θ_2 using inverse tangent:

$$ \theta_2 = atan2(sin(\theta_2), cos(\theta_2)) \tag{8} $$

After knowing the values of θ_2, the equation (3) and (4) can be rewritten as:

$$ x = k_1 * cos(\theta_1) - k_2 * sin(\theta_1) \tag{9} $$

$$ y = k_1 * sin(\theta_1) - k_2 * cos(\theta_1) \tag{10} $$

Where:

$$ k_1 = 120 + 120 * cos(\theta_2) \tag{11} $$

$$ k_2 = 120 * sin(\theta_2) \tag{12} $$

Then, we perform a change of variable by introducing two new variables r, ϕ where:

$$ r = \sqrt{{k_1}^2 + {k_2}^2} \tag{13} $$

$$ \phi = atan2(k_2, k_1) \tag{14} $$

$$ k_1 = r \cdot cos(\phi) \tag{15} $$

$$ k = r \cdot sin(\phi) \tag{16} $$

The above relationship between r, ϕ, k_1, k_2 can be visualized using a right triangle in Figure 2:

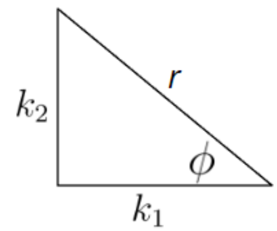


Figure 2. Visualization of change of variables: r, ϕ, k_1, k_2

Finally, the addition formula of sine and cosine can be applied to equations (3), and (4) to get the value of θ_1:

$$\frac{x}{r} = cos(\phi + \theta_1) \tag{17}$$

$$\frac{y}{r} = sin(\phi + \theta_1) \tag{18}$$

$$\phi + \theta_1 = atan2(y, x) \tag{19}$$

$$\theta_1 = atan2(y, x) - atan2(k_2, k_1) \tag{20}$$

Numerical Example

Let's say we want to move the wrist of the manipulator to the position $(134.98,\ 175.91,\ 150)$, in other words, $x = 134.98,\ y = 175.91,\ z = 150$.
For the value of d_3, it's straightforward to obtain its value as -100 via equation (5).

For joint values of θ_1 and θ_2 , by applying the position values to equation (7) (8), we get $\theta_2 = \pm 45^\circ$.
Next we apply both θ_2 values into equation (11) and (12) and get the corresponding values of k_1, k_2:

$$k_1 = 204.84$$
$$k_2 = \pm 84.85$$

Finally, by substituting numerical values of k_1, k_2 into equation (20), we get $\theta_1 = 30^\circ or\ 75^\circ$.
According to the designed joint range limit, both θ_1 values are valid. In practice, we would pick the joint values closest to the current pose.

3. WORKSPACE ANALYSIS

The first two revolute joints have joint limits in the range [0, 180°]; the third joint is a prismatic joint stretching between [0, -200]. To analyze the reachable workspace of the manipulator, we iterate through all possible joint vectors and visualize it using MATLAB. The reachable workspace in 3-dimensional space and X-Y plane are shown in Figure 3 and Figure 4 respectively.
We also observe that the line ($x = 0,\ y = 0,\ z = [50, 250]$) is the Dextrous workspace of the manipulator due to the equal arm length of links.

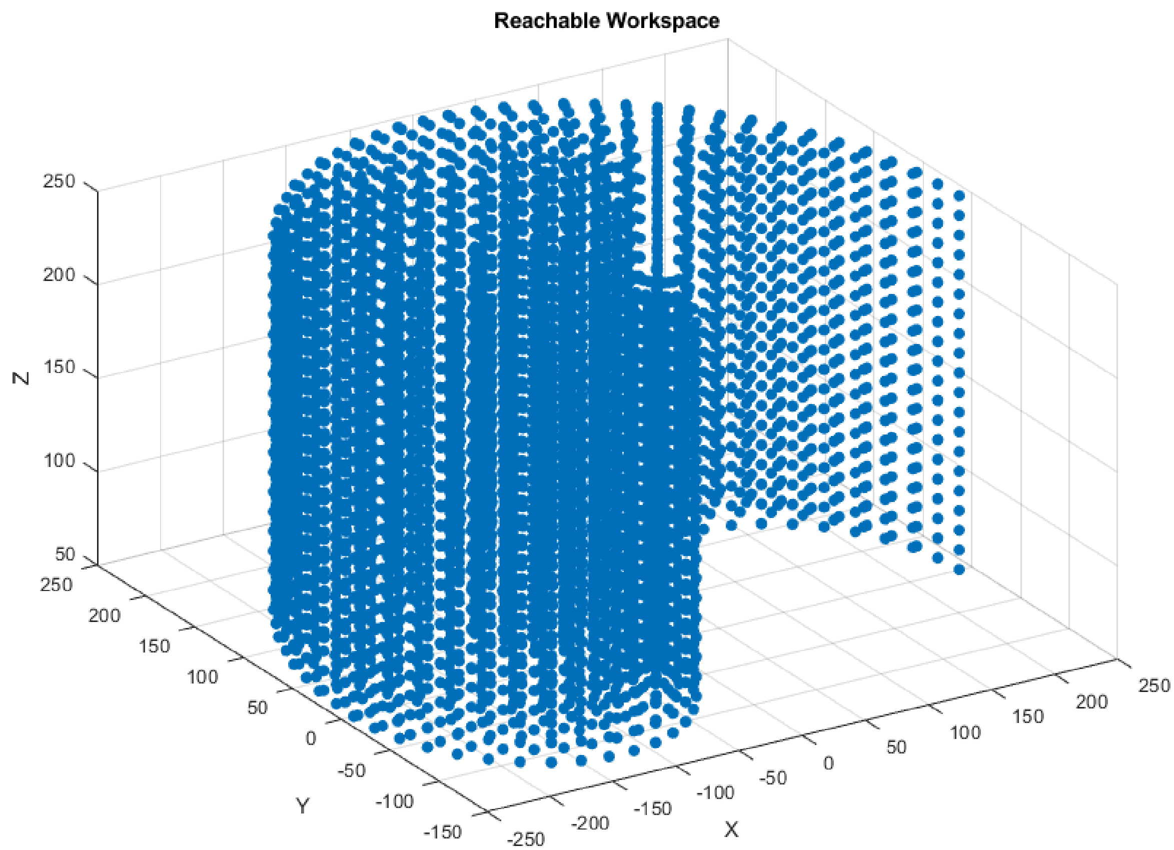


Figure 3. 3-dimensional visualization of manipulator's reachable workspace

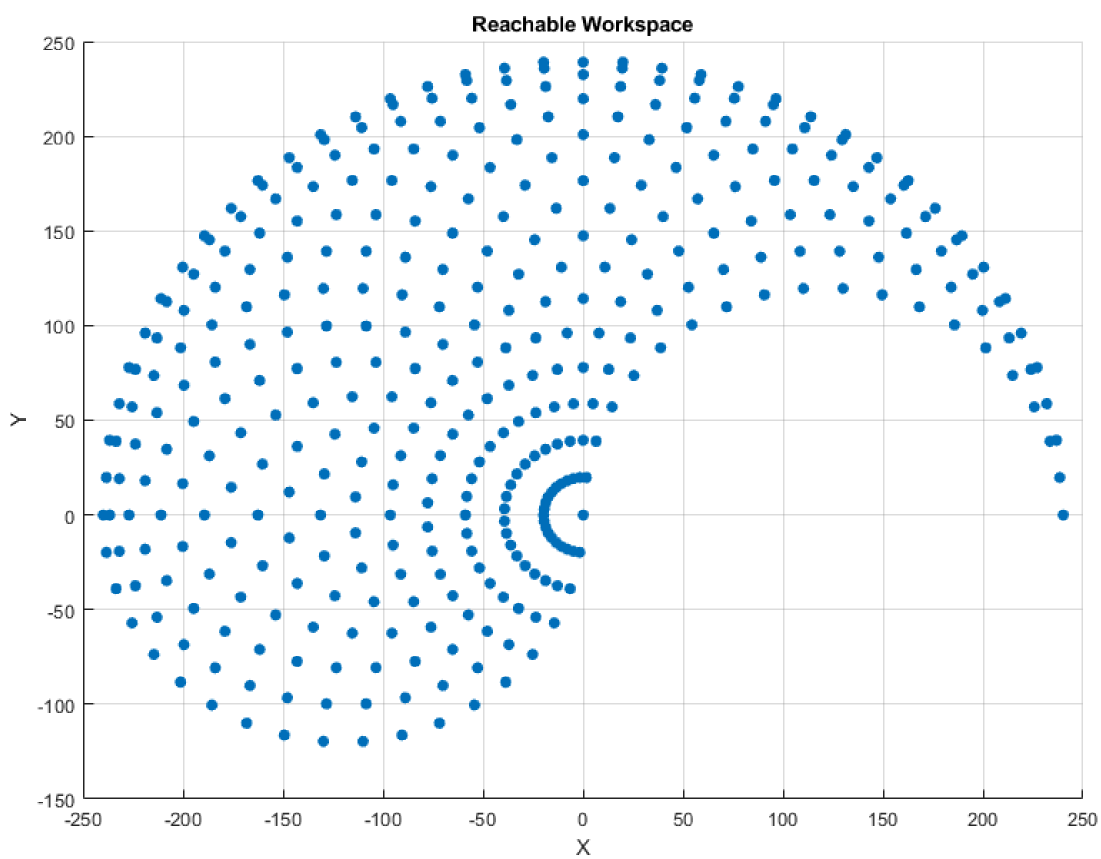


Figure 4. XY projection of manipulator's reachable workspace

4. KINEMATIC RECONSTRUCTION

All manipulator links are imported into MATLAB using the provided helper functions and project templates. The robot's zero position is shown in the following Figure 5. Note that the units on all axes are meters whereas the numbers shown in the previous D-H table are in centimeters. To demonstrate the execution of inverse kinematics, Figure 6 shows the corresponding joint settings to let the wrist of the manipulator reach the designated position. To validate it, please alter the values of variables X, Y, and Z in the submitted MATLAB script.

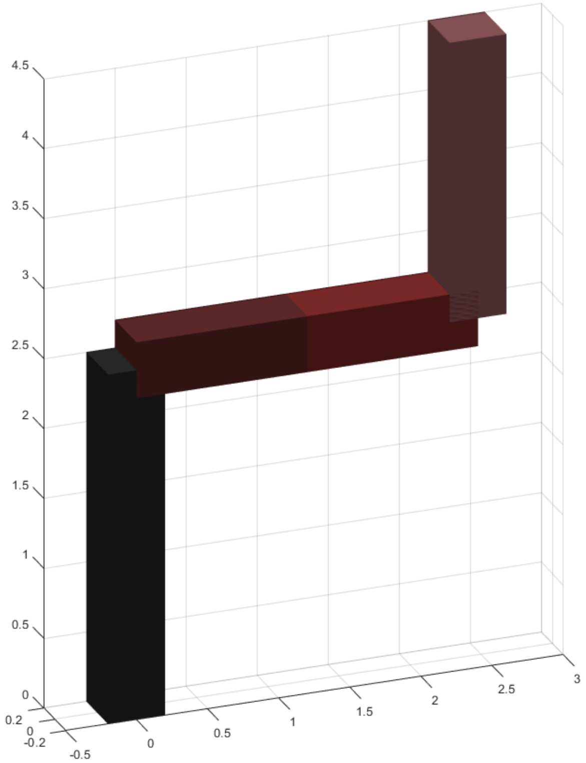


Figure 5. Robot visualization (zero position).

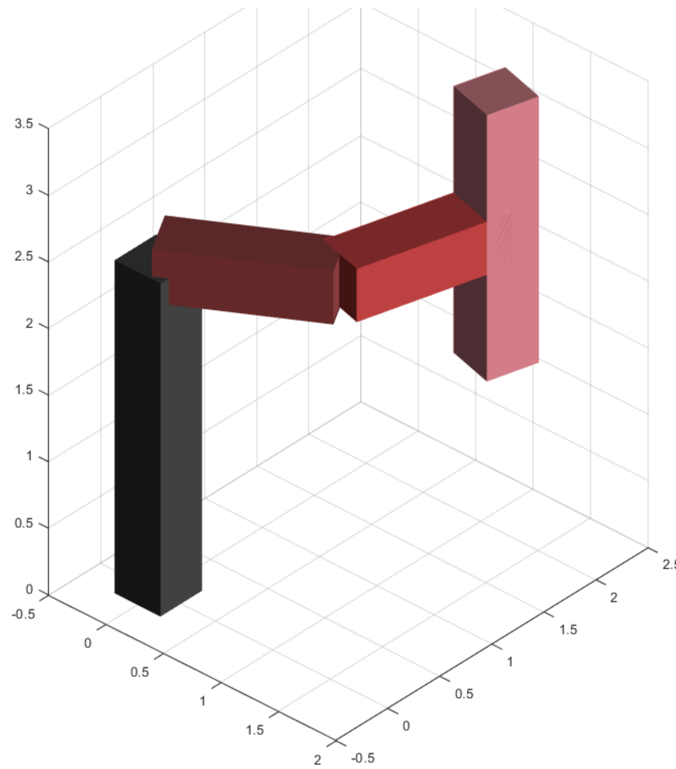


Figure 6. Result of inverse kinematic for position $(134.98,\ 175.91,\ 150)$

5. CONCLUSION

This report presents the findings from phase 2 of the project, which includes inverse kinematic and workspace analysis. Using an analytical approach, we derived the inverse kinematic equations necessary to convert a specified wrist position into corresponding joint values. Additionally, MATLAB was employed to visualize the manipulator's pose. The upcoming phase will focus on trajectory planning.

REFERENCE

[1] John J. Craig. "Introduction to robotics: Mechanics and control: John J. Craig." (1987): 263-264.