Università della Svizzera italiana

Faculty of Informatics

Bound Information: analysis on the classical analog to Bound Entanglement

Student: Luca Dolfi **Advisor:** Prof. Stefan Wolf **Assistant:** Arne Hansen

Abstract

There is a correspondence between entanglement distillation in quantum mechanics and classical key agreement in information theory. In the same quantum-mechanical framework there are, furthermore, non-distillable, but entangled quantum states. So, considering the above analogy, does there exists some notion of bound information? As of today this remains an open question.

In this project we follow the intuition from bound entanglement, the related measures and their connections to concepts of classical key agreement, as well as related information-theoretical concepts, in order to further investigate this open question.
In the end we analyse a candidate probability distribution and formulate the question as an optimisation problem.

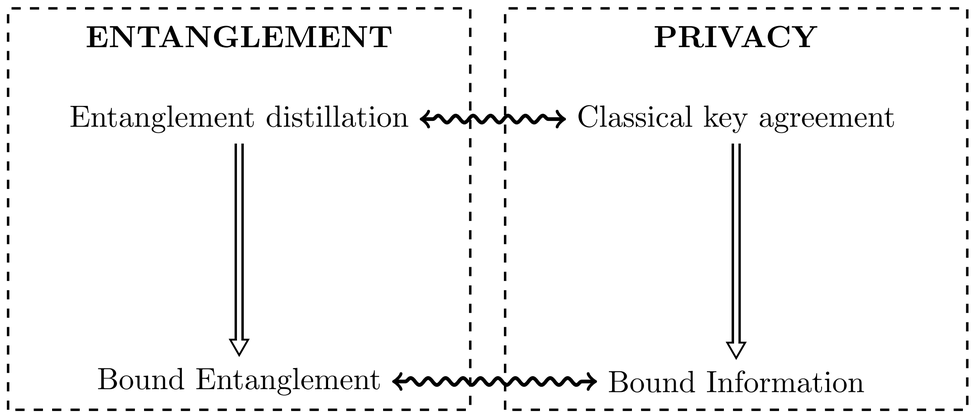
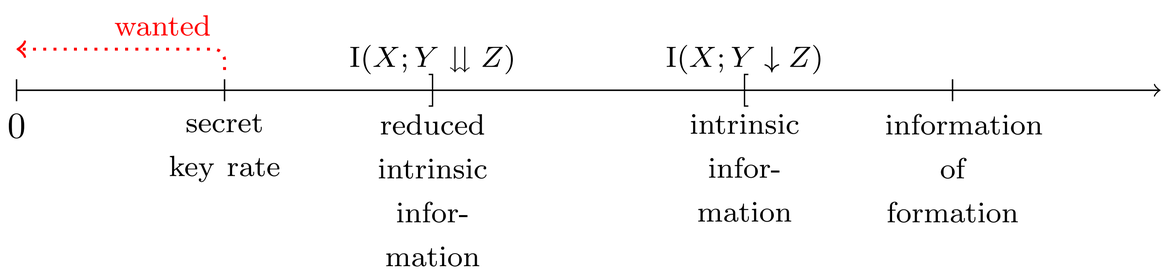


Fig. 1 *Some aspects of quantum mechanics can be mapped to classical information theory*



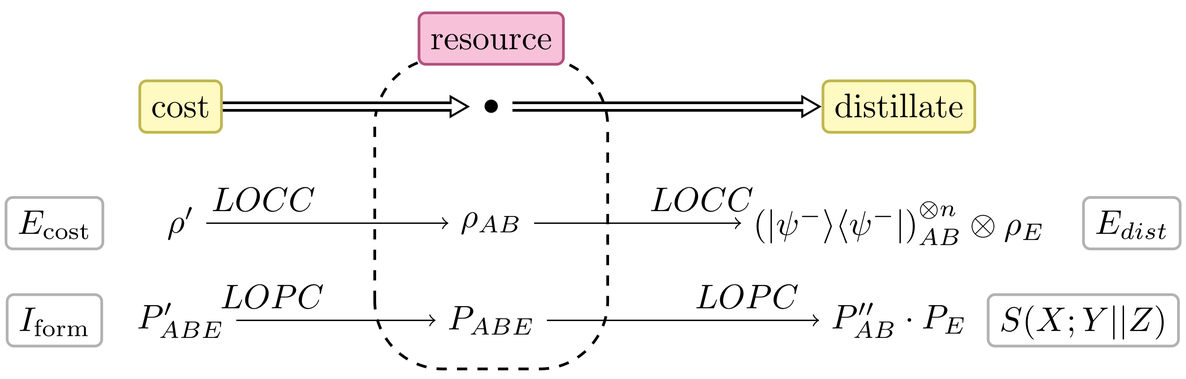


Fig. 2 *Entanglement distillation and CKA have similar concepts of resource, cost and distillate*

Entanglement distillation and CKA

To measure entanglement one might consider the maximal number of singlets that can be *distilled* from ρ by local operations and classical communication (LOCC). Bound entangled states are states that require a number of singlets for their preparation while they, in turn, do not allow to distill any singlets.

Similarly, in classical key agreement, there is the idea of the amount of perfectly secret bits required to synthesise a certain probability distribution by local operations and public communication (LOPC).

The **secret key rate** $S(X;Y\|Z)$ is defined as the maximal amount of correlated bits between Alice and Bob extractable from an arbitrarily large number of realisations of P_{XYZ}, through a protocol using LOPC, such that Eve has no information about them, i.e. she is factored out.

The **information of formation** $I_{\text{form}}(X;Y\,|\,Z)$ of X and Y given Z, for P_{XYZ}, express the rate at which bits of information known only to Alice and Bob are required to synthesise a distribution which is, in terms of privacy, at least as good as P_{XYZ} from Alice and bob's point of view, where the piece known to Eve, Z, is derived from the public communication of the protocol.

Bound Information

The counterpart to bound entanglement, is a kind of correlation which can not be used for generating a secret key.

> Is there a tripartite probability P_{XYZ}, corresponding to Alice and Bob wanting to establish a key unknown to Eve, that has non-zero key cost, while not allowing to distill any secret key (zero secret key rate but non-zero information of formation)?

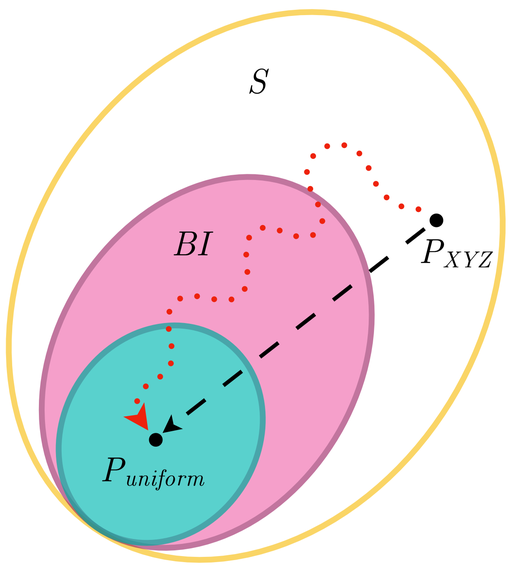


Fig. 4 *From the set* S *of distributions with extractable key we create a "path" towards distributions unserviceable for key agreement*

Fig. 5 *Graph showing results*

A candidate distribution

We implemented a numerical analysis of probability distributions in search for bound information.
Analogously to bound entanglement, we applied different noise functions (represented as "paths" in Fig. 4) to the distribution and measured for each steps the values of *reduced* and normal *intrinsic information*, as well as tests for *separability* of the translated quantum state.
We opted for a Monte Carlo method to calculate the infima of the information theoretical values.
The graph shows...