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Bound Information: analysis on the classical analog to Bound Entanglement

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Abstract

One can show that there exists a correspondence between entanglement distillation in quantum mechanics and classical key agreement in information theory. In the same quantum-mechanical framework there are, furthermore, non-distillable, but entangled quantum states. So, considering the above analogy, does there exists some notion of bound information? As of today this remains an open question.

In the project we follow the intuition from bound entanglement, the related measures and their connections to concepts of classical key agreement, as well as related information-theoretical concepts, in order to further investigate this open question.

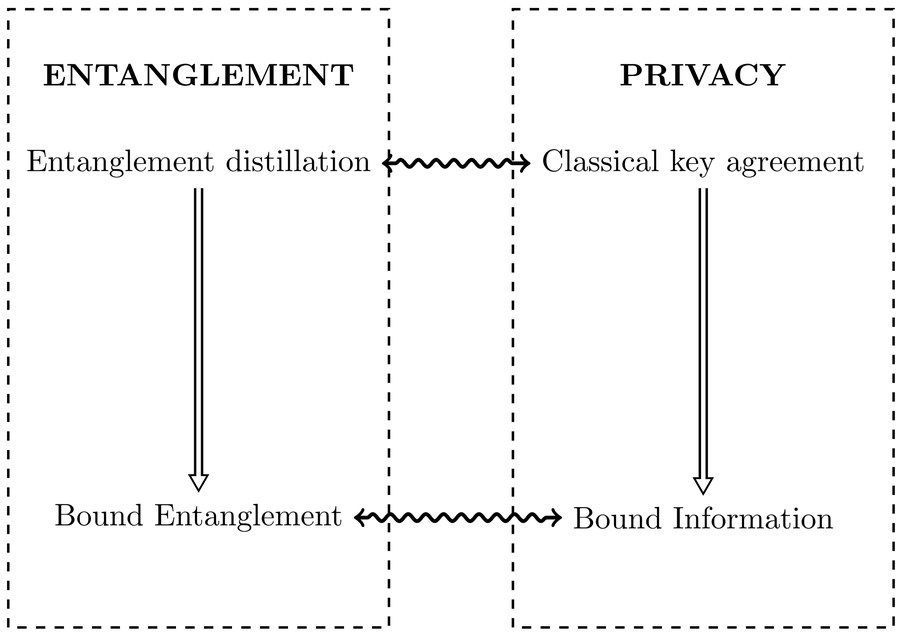


Fig. 1 *Some aspects of quantum mechanics can be mapped to classical information theory*

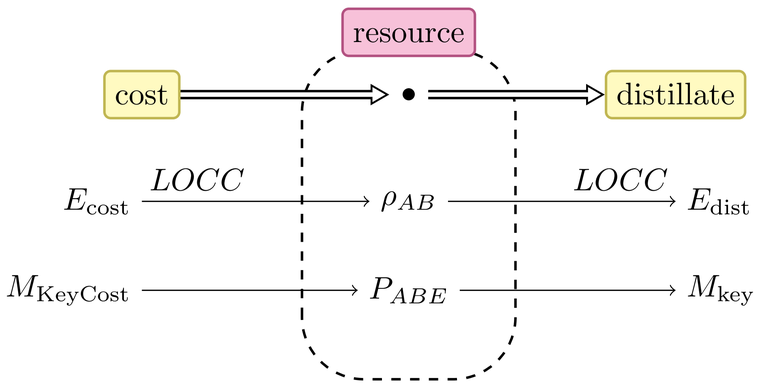


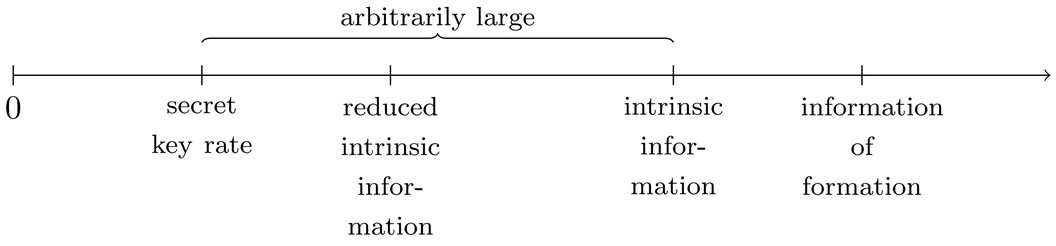
Fig. 2 *Entanglement distillation and CKA have similar concepts of resource, cost and distillate*

Entanglement distillation and CKA

To measure entanglement one might consider the maximal number of singlets that can be *distilled* from ρ by local operations and classical communication (LOCC). Bound entangled states are states that require a number of singlets for their preparation while they, in turn, do not allow to distill any singlets.

Similarly, in classical key agreement, there is the idea of the amount of perfectly secret bits required to synthesise a certain probability distribution by local operations and public communication (LOPC).

$S(X;Y\|\|Z)$	***Secret key rate***: the amount of extractable secret correlation from P_{XYZ}
$\mathrm{I}(X;Y\downarrow Z) := \inf_{Z\to\bar{Z}} \mathrm{I}(X;Y\|\bar{Z})$	***Intrinsic information***: the remaining correlated secrecy after Eve's best choice of a viewpoint
$\mathrm{I}(X;Y \downdownarrows Z) := \inf_{P_{U\|XYZ}} (\mathrm{I}(X;Y\downarrow ZU) + \mathrm{H}(U))$	***Reduced intrinsic information***: a stricter upper bound to secret key rate



Bound Information

The counterpart to bound entanglement, is a kind of correlation which can not be used for generating a secret key.

- Is there a tripartite probability P_{XYZ}, corresponding to Alice and Bob wanting to establish a key unknown to Eve, that has non-zero key cost, while not allowing to distill any secret key?

X / Y	0	1	2	3
0	1/8	1/8	a	a
1	1/8	1/8	a	a
2	a	a	1/4	0
3	a	a	0	1/4

$Z \equiv X + Y \pmod 2$ if $X, Y \in \{0,1\}$

$Z \equiv X \pmod 2$ if $X, Y \in \{2,3\}$

$Z = (X,Y)$ otherwise

Fig. 4 *Probability distribution (for a≥0, to be renormalised) for which S(X; Y \|\| Z) ≠ I(X; Y ↓ Z)*

Fig. 5 *Graph showing results*

A candidate distribution

We implemented a numerical analysis of the probability distribution given in Fig. 4 in search for bound information. This probability distribution was firstly proposed by Wolf and Renner in 2003.
Analogously to bound entanglement, we applied different noise functions to the distribution and measured for each steps the values of *reduced* and normal *intrinsic information*, as well as tests for *separability* of the translated quantum state.
We opted for a Monte Carlo method to calculate the infima of the information theoretical values.
The graph shows...