Exercise 1

Define an algorithm in Matlab based on dynamic programming for the following scheduling problem.

The problem and the algorithm are defined at page 23 of the file MTIA1516_P2_04_SchedulingPD.pdf

Verify that the solution, on a case study of 16 states on different examples of data at your choice, that has been obtained is optimal comparing with the solution obtained in a mathematical programming problem defined in Excel (or other spreadsheet tool with optimization module) or Lingo or Cplex.

Esercizio 2

Define in Matlab/Simulink the control specified below for the following system.

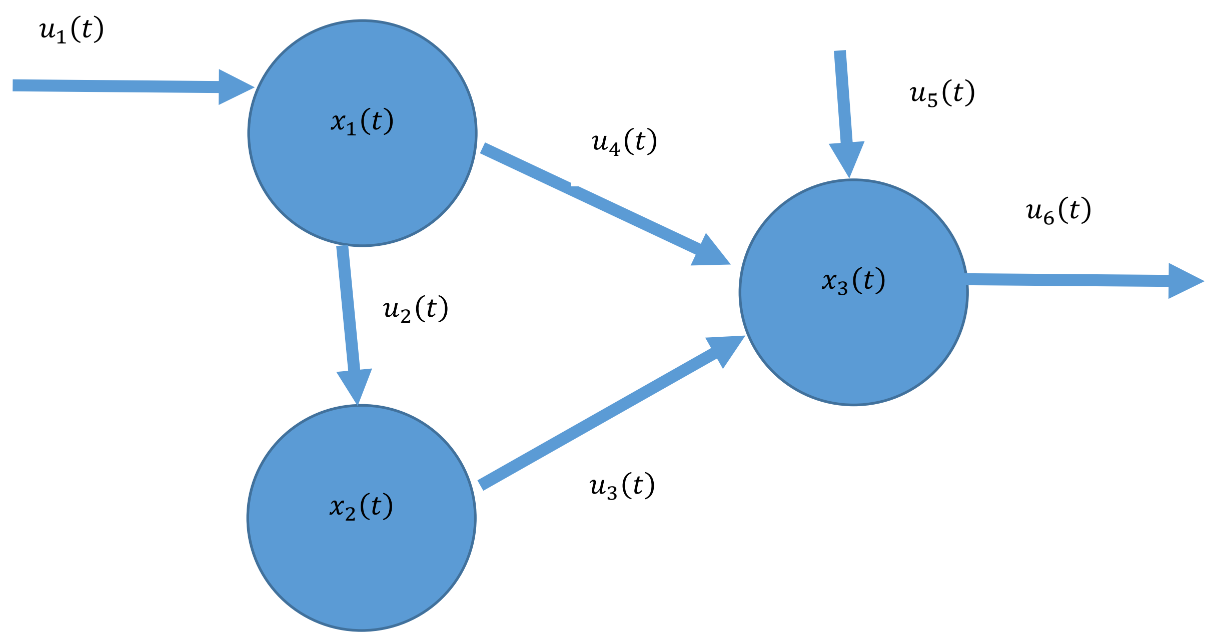
A production network is defined by the following state equation.

$$\underline{x}(t+1) = \underline{x}(t) + \alpha \underline{u}(t)$$

where:

- *x(t)* is the vector of the quantities of each output product (inventories) in each production center on the *t*-th day. It is defined as the deviation from a given steady state condition. Each production center is supposed to produce just one product
- *u(t)* is the vector of product flows in input (raw material) and in output (final product for that production center). Some products are imported/exported in/from the network. Other products or part of them are used within the same network.
- α represents a matrix of the rate of materials for each production center.

In your example, please take into account the production network drawn below, consisting of three production centers. For example, production center 1 has an inventory described by $x_1(t)$, i.e. the first component of the vector $\underline{x}(t)$. The production center 1 requires the raw material $u_1(t)$ which is transformed in the same day in the final product and stored in the warehouse (so increasing the inventory). Similarly $u_4(t)$ is the rate of the final product in output from production center 1 and entering production 3 as raw material for a new production process.



For the production network, the following equations hold:

$x_1(t+1) = x_1(t) + u_1(t) - u_2(t) - u_4(t)$
$x_2(t+1) = x_2(t) + u_2(t) - u_3(t)$
$x_3(t+1) = x_3(t) + u_4(t) + u_3(t) + u_5(t) - u_6(t)$

In addition, it is required that:
$u_3(t) = 4 * u_4(t)$
$u_4(t) = 2 * u_2(t)$
$u_5(t) \sim u_4(t) + u_3(t)$ (wished / desired, not necessarily equal)

1) Under the hypothesis of perfect observability of the state (i.e. the system output is the state), define the optimal control, in order to minimize the quadratic error of the state and of the control with respect to zero, on a time horizon of 7 days. Verify the solution with different parameters in the cost functions.
2) Under the hypothesis that it is not possible to measure $x_2(t)$, that the measure of the state is affected by a zero mean error with normal distribution (with known variance), and that the model itself is affected by a zero mean error with normal distribution (with known variance) define the optimal control.
   Suggestion. The control is LQG, which requires system identification by a Kalman filter. The system is so given by:

$x_1(t+1) = x_1(t) + u_1(t) - u_2(t) - u_4(t) + w_1(t)$
$x_2(t+1) = x_2(t) + u_2(t) - u_3(t) + w_2(t)$
$x_3(t+1) = x_3(t) + u_4(t) + u_3(t) + u_5(t) - u_6(t) + w_3(t)$
$u_3(t) = 4 * u_4(t)$
$u_4(t) = 2 * u_2(t)$
$u_5(t) \sim u_4(t) + u_3(t)$ (wished / desired, not necessarily equal)

Output
$y_1(t) = x_1(t) + v_1(t)$
$y_2(t) = -x_3(t) + v_2(t)$

3) Under the hypothesis of point 2, and relaxing the optimal control requirement, limit each control to be as an absolute value less than 1.
4) Apply a PID control to the system of point 2 (with no system identification) and compare such control with the optimal control computed in 2.
5) Under the hypothesis of point 1, identify an optimal control which minimize the quadratic value of the state and of the control to zero, but for the following value to be tracked so that $x_3(2) = 10$

Excercise 3.

Problem 1.2 [83] In a liquid-level control system for a storage tank, the valves connecting a reservoir and the tank are controlled by gear train driven by a D. C. motor and an electronic amplifier. The dynamics is described by a third order system

$$\begin{aligned}\dot{x}_1(t) &= -2x_1(t)\\ \dot{x}_2(t) &= x_3(t)\\ \dot{x}_3(t) &= -10x_3(t) + 9000u(t)\end{aligned}$$

where, $x_1(t) =$ is the height in the tank, $x_2(t) =$ is the angular position of the electric motor driving the valves controlling the liquid from reservoir to tank, $x_3(t) =$ the angular velocity of the motor, and $u(t) =$ is the input to electronic amplifier connected to the input of the motor. Formulate optimal control problem to keep the liquid level constant at a reference value and the system to act only if there is a change in the liquid level.

Use a discrete time control approach.