ECE-6320 HWK 11

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Hespanha Problems1

16.2

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} u \tag{1}$$

$$y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x \tag{2}$$

a)

Checking the rank of Ω we see that it is 3 so the system is completely observable

b)

Now using a k matrix $\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$ and using this in the equation,

$$A + kc \tag{3}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} k_1 & 0 & k_1 \\ k_2 & 0 & k_2 \\ k_3 & 0 & k_3 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} 1+k_1 & 0 & k_1 \\ 1+k_2 & 1 & k_2 \\ -2+k_3 & 1 & 1+k_3 \end{bmatrix} \tag{6}$$

Now finding the characteristic equation,

$$s^3 + (-k_1 - k_3 - 3)s^2 + (4k_1 - k_2 + 2k_3 + 3)s - 4k_1 + k_2 - k_3 - 1 \tag{7}$$

Solving for for polynomial coefficients of 1, 3, 3, and 1 we get a k of,

$$k = \begin{bmatrix} -8 \\ -28 \\ 2 \end{bmatrix} \tag{8}$$

c)

The output feedback controller would have the form $u = f\hat{x}$ where the dynamics of $\hat{x}$ are,

$$\dot{\hat{x}} = A\hat{x} + bf\hat{x} + k(y - c\hat{x}) \tag{9}$$

24.3

Expressing the dynamics for the system with $u = -K\hat{x}$

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L(y - C\hat{x}) && (10)\\ &= A\hat{x} - BK\hat{x} + L(Cx - C\hat{x}) && (11)\\ &= (A - BK)\hat{x} + LC(x - \hat{x}) && (12) \end{aligned}$$

Now defining $e = x - \hat{x}$ and $\dot{e} = \dot{x} - \dot{\hat{x}}$,

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} && (13)\\ &= Ax - BK\hat{x} - ((A - BK)\hat{x} + LC(x - \hat{x}) && (14)\\ &= Ax - BK\hat{x} - (A\hat{x} - BK\hat{x} + LCe) && (15)\\ &= Ax - BK\hat{x} - A\hat{x} + BK\hat{x} - LCe && (16)\\ &= Ae - LCe && (17)\\ &= (A - LC)e && (18) \end{aligned}$$

Now solving for $\dot{x}$ in terms of e

$$\begin{aligned} \dot{x} &= Ax - BK\hat{x} && (19)\\ &= Ax - BK(x - e) && (20)\\ &= (A - BK)x + BKe && (21) \end{aligned}$$

Now expressing the aggregate dynamics,

$$\dot{z} = \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} (A - BK) & BK \\ 0 & (A - LC) \end{bmatrix} z \tag{22}$$

Since the system is a upper diagonal matrix we know that the system is asymptotically stable if (A-BK) is a stability matrix and (A-LC) is a stability matrix.

Output Feedback Control Design

$$\dot{x} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} u \tag{23}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} x \tag{24}$$

First checking the controllability and observability of the system.

For controllabillity we see gamma is not full rank so checking the controllability decomposition,

$$T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{25}$$

$$\bar{A} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{bmatrix} \tag{26}$$

$$\bar{b} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{27}$$

Thus showing the system is stabilizable. For observability we see that $rank(\Omega) = 3$ so the system is completely observable.

Now doing a change of state for the output $z = x - x_d$

$$\dot{z} = Ax + Bu - \dot{x_d} \tag{28}$$

$$\dot{z} = A(z + x_d) + Bu - \dot{x_d} \tag{29}$$

$$\dot{z} = Az + Bu + Ax_d - \dot{x_d} \tag{30}$$

Now splitting u into $u = -K\hat{z} + u_{ff}$

$$\dot{z} = Az - BK\hat{z} + Bu_{ff} + Ax_d - \dot{x_d} \tag{31}$$

Finding the feed forward term,

$$Bu_{ff} + Ax_d = 0 \tag{32}$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} u_{ff} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0 \tag{33}$$
$$u_{ff,2} + 3 = 0 \tag{34}$$
$$u_{ff,1} + 2 = 0 \tag{35}$$
$$u_{ff} = \begin{bmatrix} -2 \\ -3 \end{bmatrix} \tag{36}$$

Now finding k using lqr with the maximum values

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{9} \end{bmatrix} \tag{37}$$
$$R = \begin{bmatrix} \frac{1}{100} & 0 \\ 0 & 1 \end{bmatrix} \tag{38}$$
$$k = \begin{bmatrix} 0 & 1.347 & 4.2891 \\ 0 & 6.0 & 0.1347 \end{bmatrix} \tag{39}$$

So the full state feedback is

$$u = -k(\hat{x} - x_d) + \begin{bmatrix} -2 \\ -3 \end{bmatrix} \tag{40}$$

Checking the stability of this system we find eigenvalues of -3.183 ± 0.277 and -5 showing the controlled system is stable. Now defining the dynamics for $\hat{x}$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \tag{41}$$
$$\tag{42}$$

Solving for $e = x - \hat{x}$

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} && (43) \\ &= Ax - BKu - A\hat{x} + BKu - LC(x - \hat{x}) && (44) \\ &= Ae - LCe && (45) \\ &= \dot{e} = (A - LC)e && (46) \end{aligned}$$

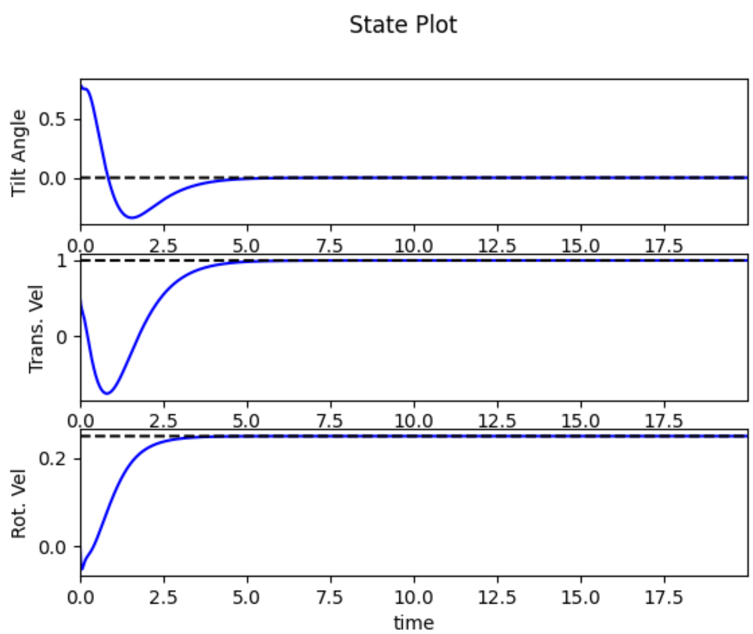
Using bryson's method I get an Luenberger observer of,

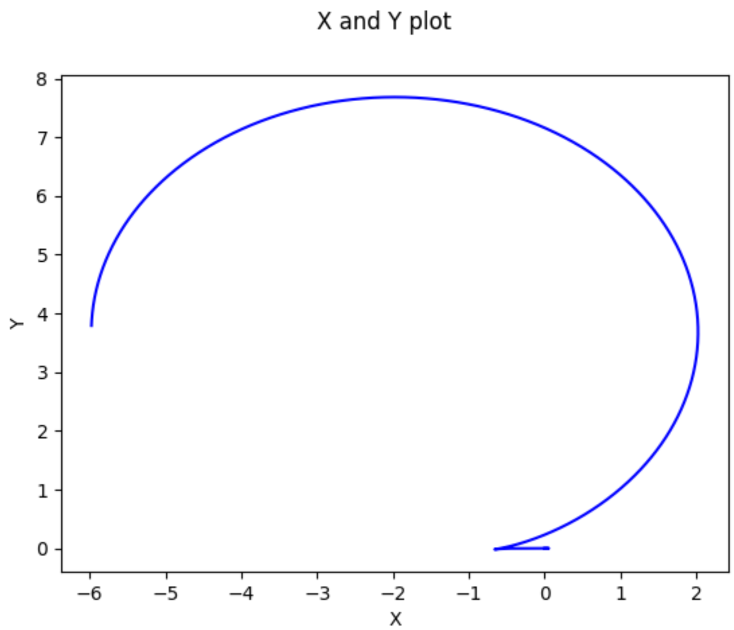
$$L = \begin{bmatrix} 0 & -.003998 \\ 39.59 & 0 \\ 17.11 & 0 \end{bmatrix} \tag{47}$$

The full controller is then

$$u = u_{ff} + -k(\hat{x} - x_d) \tag{48}$$
$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \tag{49}$$

1 Segway-Like Control




X and Y plot
Y
X
8
7
6
5
4
3
2
1
0
−6
−5
−4
−3
−2
−1
0
1
2