ECE-6320 HWK 10

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Hespanha Problems

15.4

$$\hat{G(s)} = \begin{bmatrix} \frac{s+1}{s} & \frac{1}{s+2} \end{bmatrix} \tag{1}$$

a)

First finding D we take the limit as s approaches infinity,

$$D = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{2}$$

Now we know the proper matrix is,

$$\hat{G_{sp}} = \hat{G(s)} - D \tag{3}$$
$$= \begin{bmatrix} \frac{s+1}{s} & \frac{1}{s+2} \end{bmatrix} - \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{4}$$
$$= \begin{bmatrix} \frac{s+1}{s} - \frac{s}{s} & \frac{1}{s+2} \end{bmatrix} \tag{5}$$
$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s+2} \end{bmatrix} \tag{6}$$

Now we know the least common denominator is $(s+2)s$,

$$G(\hat{s})_{sp} = \frac{\begin{bmatrix} s+2 & s \end{bmatrix}}{(s+2)s} \tag{7}$$

Now finding N's

$$N_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \tag{8}$$
$$N_2 = \begin{bmatrix} 2 & 0 \end{bmatrix} \tag{9}$$

Finding A, B, and C

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{10}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{11}$$

$$C = \begin{bmatrix} 1 & 1 & 2 & 0 \end{bmatrix} \tag{12}$$

b)

$$rank(\Gamma) = 4 \tag{13}$$

$$rank(\Omega) = 2 \tag{14}$$

From the controllability and observability matrices the system is C.C but not C.O

15.6

a)

Checking the controllable and observable matrices,

$$rank(\Gamma) = 1 \tag{15}$$

$$rank(\Omega) = 2 \tag{16}$$

Checking further, the system is not stabilizable. However it is completely Observable.

b)

The transfer function for the system is $\frac{1}{s+1}$

c)

The transfer function has a pole at -1 so it is BIBO Stable,

d)

The system is unstable because it has an eigenvalue of 1

15.7

a)

$$\dot{x} = \begin{bmatrix} x_2 \\ 1 - \frac{u^2}{x_1^2} \end{bmatrix} \tag{17}$$

b)

With $y_{eq} = 1$ we know $\dot{y} = 0$ so finding u

$$0 = 1 - \frac{u_{eq}^2}{1^2} \tag{18}$$

$$u_{eq}^2 = 1 \tag{19}$$

$$u_{eq} = \pm 1 \tag{20}$$

Yes because the control can be either positive or negative 1

c)

Computing the jacobians,

$$\frac{df}{dx} = \begin{bmatrix} 0 & 1 \\ 2\frac{u^2}{x_1^3} & 0 \end{bmatrix} \tag{21}$$

$$\frac{df}{du} = \begin{bmatrix} 0 \\ -2\frac{u}{x_1^2} \end{bmatrix} \tag{22}$$

Now finding the first point at y = 1, u = 1,

$$\dot{\delta x} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ -2 \end{bmatrix} \delta u \tag{23}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \delta x \tag{24}$$

And for the second y = 1 and u = -1

$$\dot{\delta x} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \delta u \tag{25}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \delta x \tag{26}$$

d)

Each system has the same eigenvalue of 1.4142 and -1.4142 causing both systems to be unstable.

e)

Evaluating the controllability and observability matrices,

$$rank(\Gamma) = 2 \tag{27}$$
$$rank(\Omega) = 2 \tag{28}$$

Thus both systems are C.C and C.O.

17.4

$$\dot{x} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{29}$$
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x \tag{30}$$

a)

Checking the controllability and observability matrices,

$$rank(\Gamma) = 2 \tag{31}$$
$$rank(\Omega) = 1 \tag{32}$$

Showing the system is C.C but not C.O.

b)

The transfer function for the matrix is $1/(s-2)$

c)

Finding the minimal realization,
I get

$$T = \begin{bmatrix} -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix} \tag{33}$$

$$\hat{A}_{11} = 2 \tag{34}$$
$$\hat{B}_1 = -0.7071 \tag{35}$$
$$\hat{C}_1 = -1.4142 \tag{36}$$

17.5

$$\dot{x} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \tag{37}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{38}$$

a)

Checking the controlability and observability,

$$rank(\Gamma) = 1 \tag{39}$$

$$rank(\Omega) = 2 \tag{40}$$

Showing the system is C.O and not C.C

b)

The transfer function for the matrix is $1/(s-2)$

c)

The minimal realization is ,

$$T = \begin{bmatrix} -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix} \tag{41}$$

$$\hat{A}_{11} = 2 \tag{42}$$

$$\hat{B}_1 = -1.414 \tag{43}$$

$$\hat{C}_1 = -0.7071 \tag{44}$$

Control Design

Linearize the System

Linearizing the system we get,

$$\dot{\delta z} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2.16615 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 72.5598 & 0 \end{bmatrix} \delta z + \begin{bmatrix} 0 & 0.029014 \\ -1.6687 & 0 \\ 0 & 0 \\ -24.1513 & 0 \end{bmatrix} \delta u \tag{45}$$

Controller

Designing the controller we do a change of state to w where $w = \delta z - x_d$. This then gives the following dynamics,

$$\dot{w} = Aw + B\delta u + Ax_d + \dot{x_d} \tag{46}$$

Now splitting δu into a control and feed forward term,

$$0 = Bu_{ff} + Ax_d + \dot{x_d} \tag{47}$$

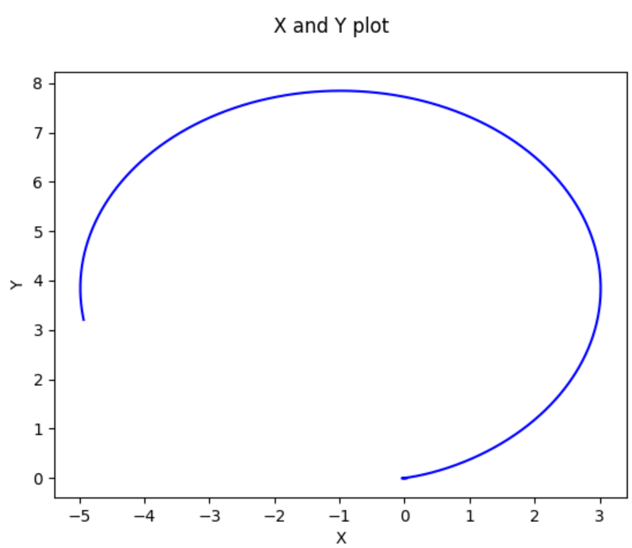
$$\hat{u} = -kw \tag{48}$$

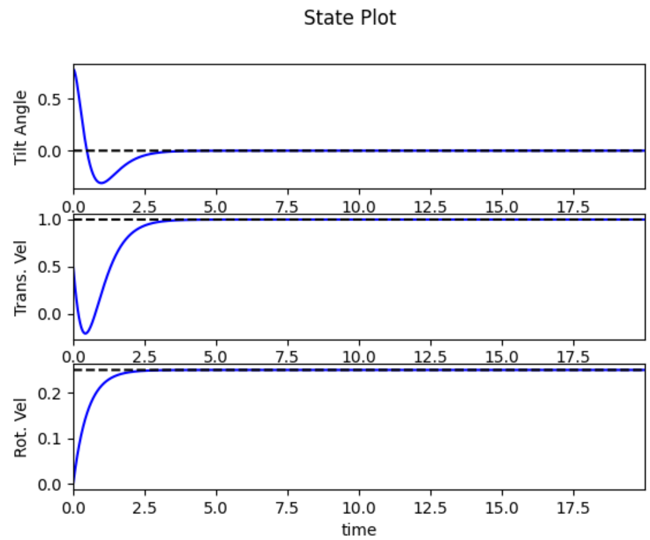
Now expressing the full control we get,

$$\delta u = -kw + u_{ff} \tag{49}$$

$$u = -kw + u_{ff} + u_{eq} \tag{50}$$

Plots





Extra Credit

From the previous problem we know that the control has the form,

$$\bar{a} = R_\epsilon^{-1}(-k(y - y_d) + \dot{y_d}) + \hat{w_\epsilon} \tag{51}$$

Where $y_d = \begin{bmatrix} q_d \\ \dot{q_d} \end{bmatrix}$. The model has the form,

$$\dot{y} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{52}$$

So substituting in the solution for the inner solution we get,

$$u = -k \left(\begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} - \begin{bmatrix} \sin(t) \\ t \\ \cos(t) \\ 1 \end{bmatrix} \right) + \begin{bmatrix} -\sin(t) \\ 0 \end{bmatrix} \tag{53}$$

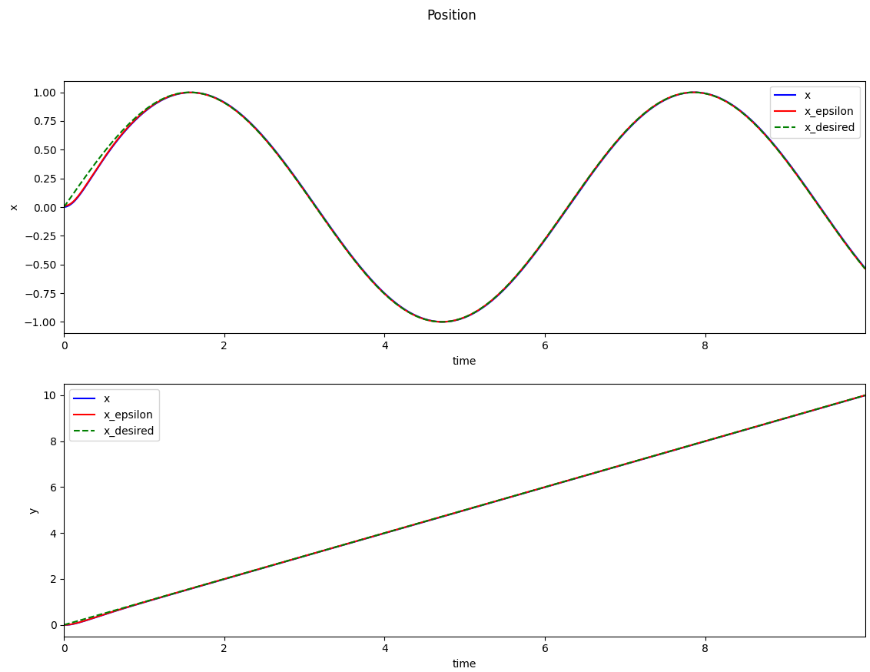
Using the place command with poles at -1, -2, -3, and -4,

$$k = \begin{bmatrix} 12 & 0 & 7 & 0 \\ 0 & 2 & 0 & 3 \end{bmatrix} \tag{54}$$

Now expressing the full control we get,

$$\begin{bmatrix} a \\ \alpha \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\epsilon \sin(\psi) \\ \sin(\psi) & \epsilon \cos(\psi) \end{bmatrix}^{-1} u - \begin{bmatrix} 0 & -\epsilon\omega \\ \frac{\omega}{\epsilon} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{55}$$

Simulation


Position
x
x_epsilon
x_desired
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
2
4
6
8
time
10
8
6
4
2
0
y