

Fall 2022

Practice Midterm Exam

MATH 3031

Department of Mathematics
Temple University

February 22, 2023

Name: _____

Instructor/Section: _____

This exam consists of 6 questions.
This exam will take 50 minutes to complete.
NO calculators.
Show all relevant work.
No work, no credit.
Good Luck!

Question	Max	Points
1	14	
2	12	
3	15	
4	12	
5	12	
6	20	
Total	20	

7pts 1. We have an urn with 3 red and 10 blue balls. We draw 5 balls, one by one, without replacement.

- (a) Find the probability that the colors we see in order are blue, blue, blue, red, blue.
- (b) Find the probability that our sample of balls were all blue.
 - (i) Using part (b), find the probability that the next two balls are red and blue (after the first 5 were blue).
- (c) Find the probability that our sample of 5 balls has at least 2 blue balls.

4pts **2.** If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, and $P(A \cap B) = \frac{1}{4}$,

(a) Find $P(A^C \cap B^C)$

(b) Find $P(A \cup B^C)$

8pts **3.** An urn contains 3 balls labeled 2, 3, and 4. We draw 2 balls one by one at random with replacement. Let X be the sum of the two numbers on the sample.

- (a) Find the possible values of X .
- (b) Find the probability mass function of X .
- (c) Let $Y = 3X - 21$. Find the probability mass function of Y .
- (d) Prove that X and Y are dependent for some $(i, j) \in \Omega^2$.

4pts 4. We have 3 urns. Urn I has 1 green and 2 red balls, urn II has 2 green and 3 red balls, and urn III has 3 green and 4 red balls.

- (a) We first choose an urn at random and then choose a ball randomly from the chosen urn. Find the probability that the ball is red.
- (b) Suppose we draw a random ball from urn I and transfer it to urn III. Then we choose a ball randomly from urn III. What is the probability that we draw a green ball from urn III.

4pts 5. There is a new test for a disease that occurs in about 0.01% of the population. It detects the disease 99% of the time. However it has a false positive rate of 10%.

- (a) What is the probability that the person's result is positive.
- (b) What is the probability that the person actually has the disease if they test positive.

4pts 6. Suppose $A, B, C \in \mathcal{F}$, s.t. A, B, C are mutually independent.

Set $P(A) = .1$, $P(B) = .2$, $P(C) = .3$.

- (a) $P(A \cap B \cap C^c)$
- (b) $P(A \cup C)$
- (c) $P((A \cup C) \cap B)$
- (d) Prove that $A \cup C$ is independent of B .