Spring 2023

Practice Midterm #1 Exam

MATH 3031

Department of Mathematics
Temple University

Februrary 22, 2023

Name: ______________________________

Instructor/Section: ______________________________

This exam consists of 6 questions.
This exam will take
1hr + 10min to complete.
A 4-function calculator may be used.
Show all relevant work.
No work, no credit.
Good Luck!

Question	Max	Points
1	**8**	
2	**4**	
3	**8**	
4	**4**	
5	**4**	
6	**8**	
Total	**36**	

$\boxed{\text{8pts}}$ **1.** We have an urn with 3 red and 10 blue balls. We draw 5 balls, one by one, without replacement.

(a) State Ω. Find the probability that the colors we see in order are blue, blue, blue, red, blue.

(b) Find the probability that our sample of balls were all blue.

(c) Consider the new sample space Ω', which is constructed after part (b), i.e. remove 5 blue balls. Suppose we changed our sample size to $k = 4$. State Ω'. Find the probability of the event $A = \{2 \text{ are red and } 2 \text{ are blue}\}$.

(d) Find the probability that the new sample A has at least 2 blue balls. ($\omega_i' \in \Omega'$)

$\boxed{\text{4pts}}$ **2.** If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, and $P(A \cap B) = \frac{1}{4}$,

(a) Find $P(A^C \cap B^C)$

(b) Find $P(A \cup B^C)$

$\boxed{\text{8pts}}$ **3.** An urn contains 3 balls labeled 2, 3, and 4. We draw 2 balls one by one at random with replacement. Let X be the sum of the two numbers of the sample.

(a) Find the possible values of X.

(b) Find the probability mass function of X.

(c) Let $Y = 3X - 21$. Find the probability mass function of Y.

(d) Prove that X and Y are dependent for some $(i, j) \in \Omega^2$.

$\boxed{\text{4pts}}$ **4.** We have 3 urns. Urn I has 1 green and 2 red balls, urn II has 2 green and 3 red balls, and urn III has 3 green and 4 red balls.

(a) We first choose an urn at ransom and then choose a ball randomly from the chosen urn. Find the probability that the ball is red.

(b) Suppose we draw a random ball from urn I and transfer it to urn III. Then we choose a ball randomly from urn III. What is the probability that we draw a green ball from urn III.

$\boxed{\text{4pts}}$ **5.** There is a new test for a disease that occurs in about 0.01% of the population. It detects the disease 99% of the time. However it has a false positive rate of 10%.

(a) What is the probability that the person's result is positive.

(b) What is the probability that the person actually has the disease if they test positive.

$\boxed{\text{8pts}}$ **6.** Suppose $A, B, C \in \mathcal{F},\ s.t.\ A, B, C$ are mutually independent.

Set $P(A) = .1,\ P(B) = .2,\ P(C) = .3.$

(a) $P(A \cap B \cap C^C)$

(b) $P(A \cup C)$

(c) $P((A \cup C) \cap B)$

(d) Prove that $A \cup C$ is independent of B.