Mathematical Modeling and Simulation Notebook

Dean Quach

Course Info

Mathematical Modeling and Simulation

Janurary 17, 2023 to May 9, 2023
TR — 10:00 am - 12:20 pm — Wachman 10
Prof. Mrs. Rujeko Chinomona
CRN 48064

3 credits $\Longrightarrow$ 6 hours of homework

Text

No course textbook, however all readings are on the Canvas page under "Topic Schedule".

Other Notes

Homework due $\approx$ weekly on friday, through Github. Generally, MATLAB excercises.

No exams, but eventually, everyother week there will be a check-in for the project.

Final project presentation 1 week before end of semester.

Project files/report due 1 week after semester ends.

Office Hours: Wachman 512

TR 1:30-3:30pm

Spring 2023
Math 2121
Section 001

Contents

Chapter 1

Git/Github and Intro to Agent Based Modeling

For the Git/Github Setup, its simple enough, go download:

1. Make an account here
2. Notepad++
3. Git
4. Github Desktop

in that order. The main part being "Notepad++" before "Git". This is so that when you configure the installation of "Git", you can choose "Notepad++" as the default terminal, instead of "Vim" editor (I believe). Also we will be using MATLAB in this course.

1.1 Types of Statistics

- Deterministic
- Stochastic
  - which is Randomness

both of which you will see in various models.

The focus of this class will be

1.2 Agent Based Modeling (or ABM)

Specifically, Complex adaptive systems

We'll look at individual agents, which have

- Emergent behavior
- macro scale behavior > micro scale behavior

Definition. *Computational study of processes modeled as dynamic systems of interacting agents*

- Key idea: identifying individuals/agents

Things to look for can include

- Agents: organisms, humans, businesses, institutions
- Unique: different size location, resource reserves, history, etc.
- Interacting locally: agents usually interact with neighbors and not all agents in geographic space or network.
- Autonomous:
- Adaptive:

Key things to find (what are ABMs composed of)

- Number of agents
- Set of decision making for each agentSet of learning rules for each agent
- A space in which the agents can move/operate and an environment in which they can interact
  - (It's kind of like game design)

1.3 ABM Simulation

1. Setup environment and agents
2. Run for each time step
3. (Do Something)

E.g.

- Agents: people at temple

- Characteristic:
  - masks or not
  - age/height/weight/etc.
- Rules: (what to keep track of)
  - Simplify to within one room
  - Track mask or not
- Movement (gas model for example)
- Probability of collisions, getting infected, etc.
  - Think stochastic video games,
    movie simulation/cgi/large wars and ragdolls

1.4 ABM Properties and Methodology

The general properties of an ABM deals with

1. Individuals/Agents
2. Act according to self interest
3. Can interact with each other
4. Can interact with the environment

We'll look at economics, social studies, biology, etc. as a broad overview.

For example, for Epidemiology

- Disease modeling (and spreading through out a population
- e.g. Covid-19, HIV, SARS, Flu, Measles, Chickenpox

So for modeling, what would you need to consider?
(2 main ways)

1.4.1 Way 1: For an ABM

Represent individuals in population (which may be humans, different animals... or be spiecific to human like old/young/ healthy/etc.) Simulate what happens when individuals meet.

E.g. - Characteristics of disease

- How does the disease spread?
  - High/Low Probability of infecting individuals (old/young or healthy/unhealthy)
- Time
  - recovery speed
  - period of infection
  - period of exposure before you get infected

Attributes of agents

- Age
- Gender
- Immunocomprimised?

- Location
  - Related: Healthy vs. Unhealthy

Now that we have recognized/observed these properties, we need to

- Define an environment
- Define a set of rules, by which agents move/interact. The interaction can be
  - agent - agent
  - agent - environment
- Create Code
- Run Simulation
  - Discrete steps (a.k.a. time steps)
- Analyze Results

Side note: ABM's can get large too quickly with
$O(f(x)) \to \infty$. So before you start your modeling task, you need to identify the key questions your model needs to answer.

1.4.2 Way 2: Quickly (we won't talk about this for now)

This method by groups "loses resolution",

- Group your population (e.g. SIR model)
  - Suseptible $= s$
  - Infected $= i$

  - recovered = r

And this leads to a differential equation model, for an SIR model with variables... you'll analyize/model with

$$\frac{\partial S}{\partial t},\ \frac{\partial i}{\partial t},\ \text{and}\ \frac{\partial r}{\partial t}.$$

1.5 About Github

Clone Repository - a.k.a. ”download”
Make a MATLAB folder on desktop/USB drive for organizing files. e.g. MATH2121Sp23

- MATH2121
  - HW1
  - HW2
  - ...

Refer to the “github” cheat sheet and “gittools” pdf for things you need to know. Namely,

- Clone
- Commit
- Fetch
- Push
- Pull

Chapter 2

Elaborating on Stochasticity/Randomness

Recall we're looking at "Mathematical Modeling" and to be honest, the following equation shall be implemented as much to heart as you possibly can.

$$Model = Simplification\ of\ reality$$

Part of this includes choosing the correct type of statistical measurement method.

2.1 Deterministic vs. Stochastic

Preface: It is not specifically one or the other, there can be a little bit of both in eachother.

2.1.1 "Deterministic"

...generally predicts what will happen.

- $F = ma$
- Physical laws modeled by differential equations
- Initial conditions leads to same output everytime.

2.1.2 "Stochasticity"

...are generally random events where different results can be obtained from the same initial condition.

— Why introduce stochasticity in a model?

Usually because of some human/living being indecisiveness := Natural variability.

E.g.

- modeling crowd behavior
- model consumer behavior
- model behavior on stock market

Use stochasticity to assign initial conditions,
and you can Model variable outcome processes.

Random numbers in models, where
you pick random # distribution.

The main point is that Stochasticity/Randomness has

- Natural Variability
- Stochastic Variable to model complex processes

- Initialize model
- Monte Carlo Simulation (Run it many times)
  - Use a histogram to visualize different frequences

2.2 Continuous vs. Discrete Distributions

For a random variable, which one to use and when?
For continuous distributions

- normal distribution
- uniform distribution
- log-normal distribution

Chapter 3

Random Walks

There are many takes/definitions on what a random walk is.

- Process by which randomly moving objects wander away from their starting points.
- Stochastic Process for determining the probable location of a point subject to random motions, givin the probabilities of moving some distance in some direction.

3.1 1D Random Walks

You can imagine a number line, where

- you start at 0
- you move ± 1 steps (right or left/up and down), where we can set a probability measure
  - assume we set $P(\pm 1) = .5$
- Per discrete step in time, we do an action.

- One of the main questions: Where would you end up (probability)

3.2 2D Random Walks

This would similarly have two dimensions, which implies a whole plane of freedom.

Random walks can represent:

1. Path of molecules in a gas/molecules in a gas/liquid (Brownian Motion)
2. Path of an animal looking for food
3. Short term flucuating Stock
4. Diffusion

Questions:

1. What is the expected position of the particle after n steps?
2. How far does the particle travel after n steps?

Chapter 4

Statistical Measurements

Definition. *Mean*

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Definition. *Variance*[1]

If looking at the entire population

$$var(x) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

If looking at a sample of popultaion

$$var(x) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Definition. *Standard Deviation*

$$\sigma = \sqrt{var(x)} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

[1](Note: Matlab will default to $\frac{1}{n-1}$ (basel's correction) i.e. the entire population)

Chapter 5

Simulated Annealing

...metallurgy...

Instead of picking best move, pick a random move.

Set probability of accempting a bad move, P will go down as you take more steps.

If the temp is

- High $\implies$ more likely to accept bad move
- Low $\implies$ more likely to reject a bad move.

5.1 Pseudocode

Let $x = x_0$, and $f(x) = f(x_0)$.

Let n be total # of steps in random walk.

```
for j=1:n
then, Set temperature

Pick a random move,
x_{mean}=x+random variable
```

Bibliography