An overview of research on multi-objective optimization techniques

chen ziang

E-mail: 3269770140@qq.com

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Abstract In the research and application technology of many disciplines, it is necessary to optimize multiple conflicting objectives, so the theory of multi-objective optimization is very important. This paper summarizes the progress in the field of multi-objective optimization in recent ten years, and introduces several main algorithms and their advantages and disadvantages.

Keywords muti-objective optimization, algorithm

1. Introduction

Optimization has a vital role in daily life, with numerous individuals applying optimization methods in their everyday tasks. In the realm of computing, optimization entails enhancing system or application performance while minimizing resource usage. Among various optimization challenges, multi-objective optimization holds exceptional significance, especially in engineering applications.

Multi-objective optimization technology primarily addresses scenarios where several conflicting objectives must be balanced concurrently. This is particularly pertinent in engineering and design, where trade-offs between cost, efficiency, and performance are commonplace. For instance, in automotive engineering, optimizing for fuel efficiency, engine power, and emissions simultaneously is a multi-objective problem.

The complexity of multi-objective optimization lies in the fact that improving one objective often leads to the detriment of another. Therefore, the goal is not to find a single optimal solution but rather a set of 'Pareto optimal' solutions. Each solution in this set is non-dominated, meaning that no other solution is better in all objectives. The concept of Pareto optimality is central to multi-objective optimization, providing a framework to understand and compare different solutions.

Additionally, the advancement of computational methods has revolutionized the field of multi-objective optimization. Techniques such as genetic algorithms, swarm intelligence, and other evolutionary algorithms have proven particularly effective in solving complex multi-objective problems. These methods simulate natural processes and are capable of exploring a vast solution space to find a diverse set of Pareto optimal solutions.

However, despite significant progress, multi-objective optimization continues to present unique challenges. These include handling high-dimensional objectives, ensuring diversity among solutions, and managing computational complexity. The ongoing research in this field is focused on developing more efficient algorithms, enhancing solution quality, and applying these techniques to an ever-wider range of practical problems.

In summary, multi-objective optimization is a critical and dynamic field with significant practical implications. Its applications span various sectors, including engineering, economics, logistics, and environmental management, reflecting its versatility and importance in tackling complex decision-making scenarios.

2. The basic concept of multi-objective optimization

A multi-objective technique is used to optimize two or more competing goals at the same time by considering the constraints. Multi-objective problems are defined by an objective function vector F(x) that is minimized or maximized in terms of decision variables vector X. For every solution x in the decision variable space, there is a point in the objective function space. Function f ： X → Y measures the efficiency of a given solution by assigning the objective function vector (y1 , y2 , ... , yk) in objective space Y. Also, there can be some inequality (g (x)) and equality (h (x)) constraints for some multi-objective problems .

3. Multi-objective optimization algorithms

This part will introduce the main algorithms in multi-objective optimization and the new algorithms published in the last three years.

3.1 Multi-objective Evolutionary Algorithms (MOEAs)

3.1.1 NSGA-II

NSGA-II is a multi-objective optimization method based on genetic algorithm, which was proposed by Deb et al in 2002. Its core idea is to introduce the concepts of non-dominant ordering and crowding degree under the framework of genetic algorithm to effectively deal with multi-objective problems. The main steps include: (1) initialization and population generation: Starting from random solutions, the initial population is generated. (2) non dominated sorting: according to the solution of the control relationship of population stratification. Each layer contains solutions that are not dominated by each other, and each solution has a different level of non-domination. (3) the crowded degree calculation: in each of the non control layer, calculating the congestion of solution, in order to keep the diversity of solution set. (4) selection, crossover and mutation, choose according to non dominated sorting and crowded degree solution for genetic operation, produce new species. (5) elite reserved strategy: by combining parent and offspring populations, and using non dominated sorting and crowded degree compared to form the next generation of population.

Time complexity of the NSGA - II mainly depends on non dominated sorting and crowded degree calculation. For a population of size N, the time complexity of non-dominated sorting is approximately O(MN^2), where M is the number of objective functions. The time complexity of the congestion calculation is O(MNlog N). Therefore, overall, the time complexity of NSGA-II is mainly affected by the population size and the number of targets.

The NSGA - II has a efficient non dominated sorting, maintain good diversity and widely used; However, for large-scale problems or cases with a large number of targets, the computational cost of NSGA-II is higher, and NSGA-II may not be as powerful as some other optimization techniques in terms of local search. In general, the NSGA - II is a within the field of multi-objective optimization algorithm, especially suitable for the goal conflict and need to maintain the diversity of the solution set problem significantly. However, it may encounter some challenges in dealing with high peacekeeping on a large scale.

3.1.2 SPEA2

SPEA2 is an algorithm for multi-objective optimization proposed by Zitzler et al in 2001 as an improved version of its predecessor SPEA. The main principle of SPEA2 consists of the following steps: (1) Population and external archive: SPEA2 maintains a fixed size population and an external archive. The population is used to generate new candidate solutions, while the external archive is used to store solutions that approximate the Pareto frontier. (2) Fitness assignment: SPEA2 assigns a fitness value to each individual in the population and archive based on the number of solutions the individual dominates and which other individuals dominate. (3) Density estimation: In order to maintain diversity, SPEA2 calculates a density estimate for each solution, which is usually based on the distance from other solutions. (4) Selection and genetic manipulation: fitness and density estimation are used to select individuals for crossover and mutation manipulation to generate new populations. (5) Environment selection: The best individuals are selected from the current population and external archives to form a new archive for next generation iteration.

SPEA2 time complexity is mainly affected by population size and number of the objective function. The time complexity of fitness assignment is O(TN^2), where T is the target number and N is the total size of the population plus archive. Density estimates typically have a time complexity of O(N^2). As a result, SPEA2 can become computationally intensive for large problems.

SPEA2 has the advantages of: (1) to improve the diversity of keeping: through the density estimation mechanism, SPEA2 can effectively maintain the diversity of solution set. (2) Powerful fitness assignment mechanism: combining the number of solutions dominated by an individual and the degree of being dominated, the algorithm's ability to find Pareto frontier solutions is improved. (3) External archive: Maintaining an external archive helps to save and update solutions close to the Pareto frontier. SPEA2 shortcomings are: (1) the high computational complexity, answer to the question of the contains a large number of targets and the decision variables of SPEA2 calculation cost is higher. (2) Parameter setting: Archive size and other parameters need to be carefully selected, which may require multiple tests and adjustments. (3) Limitation of density estimation: Although density estimation contributes to diversity, it may cause the algorithm to fall into local optimality in some cases. Overall, SPEA2 multi-objective optimization algorithm is a powerful, especially suitable for the need to the distribution of highly diverse and effective fitness complex problems. However, due to its high computational complexity, it may not be suitable for problems of very large scale or high dimensions.

3.1.3 S-Metric Selection Evolutionary Multi-Objective Algorithm(SMS-EMOA)

S-Metric Selection Evolutionary Multi-Objective Algorithm is an evolutionary algorithm for multi-objective optimization, which mainly guides the search process by using overvolume index. SMS - EMOA core principles include the following key parts: (1) based on the volume of choice: algorithm using ultra volume index (S) - Metric to measure a set of solution to cover the quality of the target space. The hypervolume metric measures the volume of the region defined between the solution set and the reference point. (2) Update: population in every generation, the algorithm generates a new solution, and then to add new explanation to the current population, forming a temporary population. (3) the environmental choice: choose the next generation of populations from the temporary population. This selection process is based on hypervolume contribution - removing the least contributing individual and retaining the most contributing individual. (4) the genetic operation: adopt standard genetic operation such as crossover and mutation to generate new candidate solutions. SMS - EMOA time complexity is mainly influenced by super volume calculation. The computation of hypervolume is a NP-hard problem, and its time complexity increases sharply with the increase of the number of targets. In practice, for higher-dimensional target Spaces, hypervolume computing can be a major bottleneck for algorithms. SMS - EMOA has the advantages of: (1) the evolution of the clear guidance: by using super volume index, SMS - EMOA multi-objective optimization provides a clear guidance to help better space exploration target. (2) balance the exploration and utilization: based on the choice of the largest contribution to the volume of the individual, the SMS - EMOA balance the exploration and utilization of the solution space. (3) effective Pareto frontier approach: SMS - EMOA excels at finding the Pareto frontier, especially under the condition of the target number is less. SMS - EMOA shortcomings are: (1) the high computational cost, volume of high computational cost limits the SMS - EMOA application in higher dimensional target problem. (2) Parameter selection: Appropriate parameter Settings (such as the selection of reference points) are required, which may need to be adjusted based on specific problems. (3) scalability problem: with a large number of target problem, SMS - EMOA may be less effective, because the super volume calculation in the high-dimensional space become very complex and time-consuming. Overall, SMS - EMOA is a multi-objective optimization algorithm performed well, especially in the target quantity is small. However, it faces challenges in dealing with high-dimensional target problems, mainly due to the high time complexity of hypervolume computation.

3.1.4 Multi-Objective Evolutionary Algorithm based on Decomposition(MOEA/D)

MOEA/D is a multi-objective optimization algorithm proposed by Qingfu Zhang and Hui Li in 2007. Its main principles are as follows: (1) Decomposition based method: The core idea of MOEA/D is to decompose a multi-objective optimization problem into several sub-problems, and optimize these sub-problems at the same time. Each subproblem is usually a single objective optimization problem. (2) the neighborhood structure: algorithms maintain a neighborhood structure, used to guide the search process. Each subproblem interacts only with other subproblems in its neighborhood. (3) Cooperative evolution: the solution of each subproblem not only affects itself, but may also have an impact on other subproblems in the neighborhood. In this way, the solutions of the whole population gradually approach the Pareto frontier through cooperative evolution. The time complexity of MOEA/D mainly depends on the number of subproblems and each sub-problem optimization process. In general, this time complexity is low because each subproblem is a single-objective optimization and the interaction in the algorithm mainly occurs in the local neighborhood. However, when the number of sub-problems is very large, the overall computational cost can still become quite high. MOEA/D has the advantages of (1) efficient problem decomposition: the multi-objective problem is decomposed into more child can simplify the complexity of the problem, make the algorithm more efficient. (2) good extensibility, relative to other multi-objective optimization algorithm, the MOEA/D in dealing with high dimensional target has a better performance. (3) the flexibility and adaptability, can choose different decomposition method according to different problem and neighborhood structure. MOEA/D shortcomings are: (1) the decomposition strategy choice: decomposition strategy choice has a significant influence on the algorithm performance, but in practice may be difficult to determine the optimal strategy. (2) the mutual influence between the subproblems: although local neighborhood structure can improve the efficiency, but also may lead to some problem of the solution is not comprehensive, affect the quality of the overall solution set. (3) parameter Settings: need reasonable set the neighborhood size and other parameters, which may need to adjust and optimize for the specific problem. In general, the MOEA/D is a kind of efficient, flexible and adaptable multi-objective optimization algorithm, especially suitable for complicated problem with multiple targets. However, it requires careful selection and adjustment of decomposition strategies and parameters in practical applications.

3.2 Swarm Based Algorithms

3.2.1 Multi objective Ant Colony Optimization (MOACO)

MOACO (Multi-Objective Ant Colony Optimization) is a multi-objective optimization algorithm based on the ACO principle. Ant colony optimization is an algorithm that mimics the foraging behavior of ants in nature and is used to solve optimization problems. In MOACO, this principle is extended to the context of multiple objectives. Ants release pheromones as they search for routes, and other ants choose routes based on the concentration of pheromones. In multi-objective optimization, ant behavior is used to explore solutions for each objective simultaneously. Unlike traditional ACOs, MOACO considers multiple targets when pheromone updates. Each target has its own pheromone matrix, and the ants' routing and pheromone renewal mechanisms take all targets into account simultaneously. MOACO attempts to find a set of solutions that are non-dominant in the Pareto sense, i.e. no other solution is superior on all goals. MOACO time complexity depends on the number of ants, the number of iterations, and the scale of the problem. In each iteration, each ant builds a solution, and then all solutions are non-dominated sorted and pheromone updated. Therefore, the time complexity may increase significantly as the ant population and the size of the problem increase. In the multi-target case, the computational cost is further increased due to pheromone updates involving multiple targets. Good MOACO exploring ability, strong adaptability, the diversity of solution set can be found. But his high computational cost, parameter adjustment and complex, may be trapped in local optimum. Overall, promising MOACO is a multi-objective optimization algorithm, especially in needs strong exploration ability and the diversity of solution set. However, its efficiency and performance are highly dependent on the parameter Settings and the specific characteristics of the problem.

3.2.2 MOPSO

MOPSO (Multi-Objective Particle Swarm Optimization) is an algorithm for solving multi-objective optimization problems, based on the traditional particle swarm optimization (PSO) algorithm. This algorithm is very effective when dealing with problems that require optimization of multiple objective functions at the same time, especially in cases where there is a potential conflict between these objectives. MOPSO simulates the collective behavior of flocks of birds or schools of fish, in which each particle represents a potential solution; Each particle updates its position in the search space according to its own experience (individual optimal) and group experience (global optimal or neighborhood optimal); In the multi-objective version, the algorithm maintains an external archive to store the currently found non-dominated solutions. The motion of the particle is affected by the solution in this archive. Time complexity of MOPSO depends on the number of particles, the number of iterations and the dimensions of the problem.

With each iteration, each particle needs to change its position and speed according to the update rules. Although MOPSO is more efficient than some other multi-objective optimization algorithms, the computational costs can still be significant when faced with large-scale problems or high-dimensional targets. MOPSO advantages are: (1) is simple and easy to realize: MOPSO simple structure, easy to understand and implement. (2) a good global search ability, can effectively explore the solution space, find a wide range of solutions. (3) new space: in dealing with continuous search space is especially effective. MOPSO shortcomings are: (1) challenge: maintaining diversity in multi-objective optimization, to maintain the diversity of solution is a challenge, MOPSO may require additional mechanism to maintain the diversity. (2) parameter adjustment: the performance of the algorithm to a large extent depends on the parameter Settings, such as particle velocity, inertia weight, etc. (3) may be trapped in local optimum, although MOPSO in global search performance is good, but in some cases may be trapped in local optimal solution. Overall, MOPSO is a powerful, widely used in the multi-objective optimization problem tools, applicable to various types of application scenarios, especially those involving the continuous variable problem. However, in order to improve its performance on specific problems, it may be necessary to fine-tune and optimize its parameters.

3.2.3 MOMSA

In the multi-objective optimization algorithm (MSA), three groups of moths (trailblazers, prospectors, and bystanders) and a light source are considered. Pioneers can use the "first in, last out" principle to find the best position in the optimized space and guide the movement of the main group. Prospectors tend to move in random spiral paths near light sources that are marked by trailblazers. Onlookers move directly towards the globally optimal solution (moonlight), which was discovered by the prospector moth. Thus, potential solutions in MSA are represented by the light source position, whose quality is viewed as the brightness intensity of the light source.

MSA algorithm through the initialization, reconnaissance and lateral positioning of three phases. At the beginning of the flight, the position of each moth (initial solution) is randomly determined by a randomization function (initialization phase). Then, the type of each moth in the population is selected based on the fitness value (objective function). The best moths were seen as pioneers (light sources), and the best and worst groups were seen as prospectors and bystanders, respectively. During exploration, moths may become concentrated in certain parts of the response space, leading to local optimality and reducing the mass of some moth populations. To prevent premature convergence and improve the diversity of solutions, a portion of the moth population is needed to explore areas with fewer populations. Trailblazer is responsible for that role. As a result, they update their position by interacting and cross-operating with each other and being able to fly long distances (known as levy variants) and prevent them from staying locally optimal (the reconnaissance phase). The path of the moth towards the light source can be described by the conical logarithmic spiral shape. Accordingly, a set of paths located on the surface of the cone, with fixed central angles, can describe the moth's flight path toward the light source. The moths with the highest brightness intensity were selected as prospectors. Each iteration should reduce the number of prospector (horizontal status section).

In the process of optimization of MSA, by reducing the number of explorers, the number of bystanders increases, which leads to faster convergence to the global solution. The increased rate of convergence is actually due to celestial navigation. The least luminous bystander moths can fly directly towards the best solution (the moon). Thus, in order to control recent movement, this step of the MSA algorithm is designed in such a way that bystanders are forced to search more effectively by focusing on the important points of the prospector. To this end, a bystander is divided into two parts, respectively by gauss walking and learning mechanism. In MSA, the type of each moth alternates. Therefore, every prospector (brighter than the light source) who provides a better solution is promoted to pioneer. At the end of each step, new light sources and lunar light sources are offered as possible solutions.

Overall, mimic the behavior of moth group of MSA algorithm, through the three roles (Portland, prospectors, and bystanders) and the interactive light source for multi-objective optimization. The algorithm consists of three stages: initialization, reconnaissance and lateral positioning, and uses the position and brightness intensity of moths to find the optimal solution. The focus of the algorithm is to maintain diversity, prevent premature convergence, and accelerate the convergence of the global optimal solution by changing the type of moth and using different search mechanisms.

4. Conclusion

After reviewing the significant developments and innovations in the field of multi-objective optimization over the past decade, it is clear how the field continues to evolve and mature. Research during this period not only enhanced our understanding of traditional algorithms such as NSGA-II, SPEA2, and MOEA/D, but also saw the rise of new algorithms that showed significant improvements in processing complexity, computational efficiency, and diversity of solutions. These advances demonstrate the dynamic and innovative capabilities of multi-objective optimization as a research field, while also underscoring its importance in solving increasingly complex problems in the real world.

In addition, the research of this period also reveals the multi-objective optimization of key intersection between theory and practice, especially in the applicability of the algorithm, extensibility and processing ability of high and large scale problems. With the increasing availability of computing resources and continuous innovation in algorithm design, we expect that multi-objective optimization will continue to play an important role in a variety of engineering, scientific, and business applications. Future research may focus on further improving the efficiency of the algorithm, optimizing the parameter tuning process, and exploring more complex and dynamic multi-objective problems. In summary, the future of multi-objective optimization is full of challenges, but it is also full of opportunities, indicating that this field will continue to make significant advances in technology and applications.

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