Part A (65%): In-Class Exam.

1. (16%) Find the Fourier series for the periodic function f(t)=0, $-\pi < t \le 0$, and f(t)=t, $0 \le t < \pi$ with period 2π.

2. (20%) Find the Fourier series for $f(t) = t$, $-\pi < t < \pi$ first. Then, find the Fourier series for $g(t) = t^2$, $-\pi < t < \pi$ by integration of f(t).

3. (14%) Classify the critical point (0, 0) of each of the following linear systems X′=AX as either a stable node, an unstable node, or a saddle point. (a) $A = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}$; (b) $A = \begin{pmatrix} -10 & 6 \\ 15 & -19 \end{pmatrix}$; (c) $A = \begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix}$.

4. (15%) Solve $x' = \begin{pmatrix} 3 & 3 \\ 1 & 5 \end{pmatrix} x + \begin{pmatrix} 8 \\ 4e^{3t} \end{pmatrix}$.

Part B (35%): Take Home Exam. Due on Dec. 26 13PM. Upload your file to the course web in eeclass.

5. (20%) Approximate the function f(t) in problem 1 by graphing various partial sums of the Fourier series expansion in Matlab with the zero term plus n= 1 up to 6 terms for x in [-4, 4]. Also need to show theses error between the approximate function with the zero term plus n= 1 up to 6 terms and the original function. Show Gibbs phenomena in these figures.

6. (15%) Find the corresponding phase portrait of problem 3 near the critical point (0,0).