ODE Midterm 2 Nov. 30, 2023

Part A (70%): In-Class Exam

1. (15%) Solve $y''-y'=t$, $y(0)=1$, $y'(0)=1$ by Laplace Transform.
2. (15%) Solve $y''+y=2t$, $y(0.25\pi)=1$, $y'(0.25\pi)=2-\sqrt{2}$ by Laplace Transform.
3. (15%) Solve $y''+5y'+6y=f(t)$; $y(0)=y'(0)=0$, with $f(t)=-2$ for $0 \le t < 3$ and $f(t)=0$ for $t \ge 3$.
4. (10%) Solve for f(t) in the integral equation $f(t)=2t^2+\int_0^t f(t-\tau)e^{-\tau}d\tau$.
5. (15%) Solve $y''+2y'+2y=\delta(t-3)$; $y(0)=y'(0)=0$, where $\delta(.)$ is the impulse function.

Part B (30%): Take Home Exam. Due on Dec. 5 13PM. Upload to eeclass.

6. (18%) The Laguerre differential equation is $xy''+(1-x)y'+\lambda y=0$.
   a. Show that $x=0$ is a regular singular point. (4%)
   b. Determine the indicial equation, its roots, and the recurrence relation. (4%)
   c. Find one solution (for $x>0$). Show that if $\lambda=m$, a positive integer, this solution reduces to a polynomial. When properly normalized, this polynomial is known as the Laguerre polynomial, $L_m(x)$. (6%)
   d. Verify your answer by using the command "laguerrel" in matlab for m=1, 2, 3, 4. (4%)
7. (12%) Find two solutions (not multiples of each other) of the Bessel equation of order

$$3/2 \quad x^2y''+xy'+(x^2-9/4)y=0,\ x>0.$$