ODE Midterm 1 Oct. 19, 2023

Part A (65%): In-Class Exam

1. (10%) Solve $dy/dx=[2x - e^{x} \sin(y)]/[e^{x} \cos(y) + 1]$
2. (10%) Find a solution to the initial value problem $y'' + 4y' + 4y = 0$; $y(0) = 1$, $y'(0) = 3$.
3. (10%) Solve $y - x\, dx/dy=0$.
4. (15%) Consider the equation $(5x^2y + 6x^3y^2 + 4xy^2)dx + (2x^3 + 3x^4y + 3x^2y^2)dy = 0$. (a) Show that the equation is not exact. (b) Multiply the equation by $x^n y^m$ and determine values for n and m that make the resulting equation exact. (c) Use the solution of the resulting exact equation to solve the original equation.
5. (10%) Are the functions $f(x) = e^{x}$, $g(x) = xe^{x}$, and $h(x) = x^2 e^{x}$ linearly dependent on the real line? To find out, compute the Wronskian.
6. (10%) Consider the logic equation $u_{n+1} = \rho u_n(1 - u_n)$. Carry out the details in the linear stability analysis of the equilibrium solution $u_n = (\rho - 1)/\rho$.

Part B (35%): Take Home Exam. Due on Oct. 24 13PM. email to TA.

7. (12%) (a) (3%) Solve $dy/dx + y = xe^{x}y$, subject to the initial condition $y(0) =1$ by matlab command "dsolve". (b) (3%) Find the equilibrium point and determine their stability. (c) (3%) Plot the direction field and the solution trajectory. (d)(3%) Can $dy/dx=e^{y}/(xy)$ be solved by matlab command "dsolve"? Justify your answer.
8. (11%) (a) (3%) Use Euler's method by numerically solving $x'=x+t$, $x(0)=1$ by matlab. (b) (3%) Find its exact solution $x_{exact}(t)$. (c)(5%) Plot the relative error $[x(t) - x_{exact}(t)]/x_{exact}(t)$ for the step size h=0.1, 0.01, and 0.001 for t=0 to 10.
9. (12%) Consider the logic equation $u_{n+1} = \rho u_n(1 - u_n)$. (a)(6%) For $\rho = 3.2$, plot or calculate the solution of the logistic equation for several initial conditions, say, $u_0 = 0.2, 0.4, 0.6$, and 0.8. Observe that in each case the solution approaches a steady oscillation between the same two values. This illustrates that the long-term behavior of the solution is independent of the initial conditions. (b) (6%) Make similar calculations and verify that the nature of the solution for large n is independent of the initial condition for other values of ρ, such as 2.6, 2.8, and 3.4.