COM 5140 Error-Correcting Codes — Chi-chao Chao
Spring 2025

Project No. 2
Due 5:00pm, June 17, 2025

You are expected to produce a computer program capable of decoding the $(63, 42)$ Reed-Solomon code over $\mathrm{GF}(2^6)$, which was used in Cinema Digital Sound. This code consists of all vectors $(C_0, \ldots, C_{62})$, with each $C_i \in \mathrm{GF}(2^6)$, such that

$$\sum_{i=0}^{62} C_i \alpha^{ij} = 0, \qquad \text{for } j = 1, 2, \ldots, 21$$

where α is a primitive element in $\mathrm{GF}(2^6)$ satisfying $\alpha^6 + \alpha + 1 = 0$. Assume that the first 21 characters $C_0, \ldots, C_{20}$ are the parity-check characters, and the last 42 characters $C_{21}, \ldots, C_{62}$ are the information characters.

The deliverable will consist of three parts:

- **Part I, Demonstration.** At the time of demonstration, we will test your program by giving it several garbled codewords of the form $(R_0, \ldots, R_{62})$, differing from a codeword by t_0 erasures and t_1 errors. If $t_0 + 2t_1 \leq 21$, your program should find the codeword; but if $t_0 + 2t_1 > 21$, your program should output an appropriate failure message.

  The elements of $\mathrm{GF}(2^6)$ will be encoded as integers in the range 0 to 63, with integer 0 corresponding to [000000], 1 corresponding to [000001], 2 corresponding to [000010], ..., and 63 corresponding to [111111]. (Here $[a_5 a_4 a_3 a_2 a_1 a_0]$ corresponds to the element $a_5\alpha^5 + a_4\alpha^4 + a_3\alpha^3 + a_2\alpha^2 + a_1\alpha + a_0$ in $\mathrm{GF}(2^6)$.) An erasure will be represented by a $*$ sign. The input to the program will be a file consisting of several garbled codewords in the following format:

  ```
  45 63 * 0 2 ··· 15   (the first garbled codeword)
  ...
  13 0 5 * * ··· 0   (the last garbled codeword)
  ```

- **Part II, Report.** When the project is due (and after the demonstration is passed), you need to hand in a *report* (in a hard copy), which should include, among other things, description of your project, discussions, etc. Your computer program *with comments* should be attached at the end of the report.

- **Part III, Program file.** You also need to submit, before the deadline, your program file. Please put all of your programs into a single file with your registration number and `proj2` as the file name, say, `113064501_proj2.c` or `113064501_proj2.cpp`. (If, after all kinds of attempts, you are still unable to put all of your programs in a single file, please compress your files into a single `zip` or `rar` file and use your registration number along with `proj2` as the file name, say, `113064501_porj2.zip` or `113064501_proj2.rar`.) Upload your file to the *eeclass* system.

The coefficients of the generator polynomial for the (63, 42) RS code:

```
g[ 0]=58    g[11]=43
g[ 1]=62    g[12]=44
g[ 2]=59    g[13]=27
g[ 3]= 7    g[14]= 7
g[ 4]=35    g[15]=53
g[ 5]=58    g[16]=39
g[ 6]=63    g[17]=62
g[ 7]=47    g[18]=52
g[ 8]=51    g[19]=41
g[ 9]= 6    g[20]=44
g[10]=33    g[21]= 1
```

The values of the polynomial:

alpha^ 0 = 1	g(alpha^ 0) = 34	alpha^32 = 9	g(alpha^32) = 22
alpha^ 1 = 2	g(alpha^ 1) = 0	alpha^33 = 18	g(alpha^33) = 20
alpha^ 2 = 4	g(alpha^ 2) = 0	alpha^34 = 36	g(alpha^34) = 28
alpha^ 3 = 8	g(alpha^ 3) = 0	alpha^35 = 11	g(alpha^35) = 17
alpha^ 4 = 16	g(alpha^ 4) = 0	alpha^36 = 22	g(alpha^36) = 13
alpha^ 5 = 32	g(alpha^ 5) = 0	alpha^37 = 44	g(alpha^37) = 19
alpha^ 6 = 3	g(alpha^ 6) = 0	alpha^38 = 27	g(alpha^38) = 14
alpha^ 7 = 6	g(alpha^ 7) = 0	alpha^39 = 54	g(alpha^39) = 56
alpha^ 8 = 12	g(alpha^ 8) = 0	alpha^40 = 47	g(alpha^40) = 25
alpha^ 9 = 24	g(alpha^ 9) = 0	alpha^41 = 29	g(alpha^41) = 45
alpha^10 = 48	g(alpha^10) = 0	alpha^42 = 58	g(alpha^42) = 10
alpha^11 = 35	g(alpha^11) = 0	alpha^43 = 55	g(alpha^43) = 46
alpha^12 = 5	g(alpha^12) = 0	alpha^44 = 45	g(alpha^44) = 45
alpha^13 = 10	g(alpha^13) = 0	alpha^45 = 25	g(alpha^45) = 8
alpha^14 = 20	g(alpha^14) = 0	alpha^46 = 50	g(alpha^46) = 12
alpha^15 = 40	g(alpha^15) = 0	alpha^47 = 39	g(alpha^47) = 14
alpha^16 = 19	g(alpha^16) = 0	alpha^48 = 13	g(alpha^48) = 44
alpha^17 = 38	g(alpha^17) = 0	alpha^49 = 26	g(alpha^49) = 14
alpha^18 = 15	g(alpha^18) = 0	alpha^50 = 52	g(alpha^50) = 17
alpha^19 = 30	g(alpha^19) = 0	alpha^51 = 43	g(alpha^51) = 26
alpha^20 = 60	g(alpha^20) = 0	alpha^52 = 21	g(alpha^52) = 31
alpha^21 = 59	g(alpha^21) = 0	alpha^53 = 42	g(alpha^53) = 22
alpha^22 = 53	g(alpha^22) = 16	alpha^54 = 23	g(alpha^54) = 59
alpha^23 = 41	g(alpha^23) = 13	alpha^55 = 46	g(alpha^55) = 29
alpha^24 = 17	g(alpha^24) = 43	alpha^56 = 31	g(alpha^56) = 52
alpha^25 = 34	g(alpha^25) = 41	alpha^57 = 62	g(alpha^57) = 31
alpha^26 = 7	g(alpha^26) = 48	alpha^58 = 63	g(alpha^58) = 57
alpha^27 = 14	g(alpha^27) = 15	alpha^59 = 61	g(alpha^59) = 48
alpha^28 = 28	g(alpha^28) = 11	alpha^60 = 57	g(alpha^60) = 45
alpha^29 = 56	g(alpha^29) = 52	alpha^61 = 49	g(alpha^61) = 51
alpha^30 = 51	g(alpha^30) = 33	alpha^62 = 33	g(alpha^62) = 13
alpha^31 = 37	g(alpha^31) = 1		