We tried to make some plots for different values of A.

Here we set the parameters to be

We found that for three different values, x(t) behaves differently

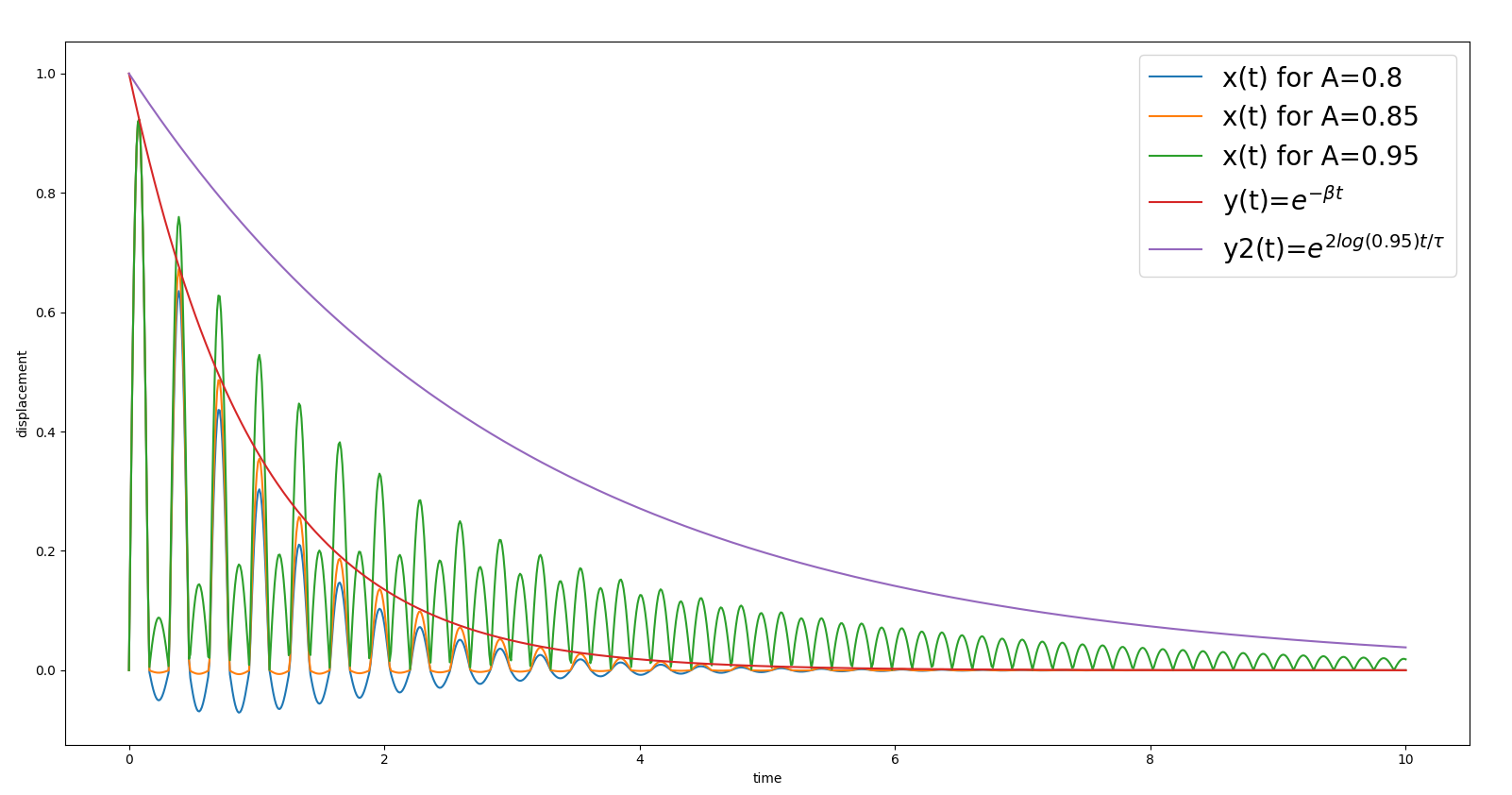


Fig. 2.1 Blue, orange, green lines are the x(t) for different A. Red line is a reference line for the damping rate , and the purple line is the reference line for , or equivalently .

We found that for A=0.95>0.85, for all t, and every half period , the peak height changes a lot, from big to small, and from small to big interchangeably and repetitively, but all the peaks are above 0.

For A=0.85, there is half a period that x(t) almost equals to 0 and then at the next half period, peaks appear again.

Thus the critical value is at around 0.85.

The reason for the critical value of A, , is because of the interference between the new response and the old response.

For each n,

And we can see easily that

at

Thus, if , the interference between the n-th and (n+1)-th response function will cause half a period to have 0 value, and at next half period, the new response function will again give a positive peak.

Thus the critical value of A is

in this case.

Moreover, it is also easy to see whether x(t) will be an exponential decrease or growth for .

We particularly look at the times where the peak may happen, i.e. when *.*

Where

And it has an iteration behavior

All we need to see is that whether converges or diverges as .

We can use mathematical induction rule to prove that will converge.

We assume that

For N=0,

And for N=1,

And if ,

Then

Since , , and since , .

Thus if ,

Thus we know by mathematical induction, for all N.

Thus must converge as and converge faster than

(see Fig. 2.1’s purple line).

(the peak heights) will decrease exponentially.

Thus, Kay is right.

By the way, the mathematical induction proof was done by Kay’s younger brother, who is a high school student, and knows nothing about the differential equations and Green functions.