To calculate the DM flux kicked by neutrinos emitted from SN, we must do a line of sight integration. From the calculation above we know that the DM particle nearly goes along the direction of the incident neutrino, and thus we may simply do the LOS integration along the direction from SN to the Earth.

We can obtain the DM scattering flux:

$$\Phi_{DM} = \int_{from\ SN\ to\ Earth} n_{DM} n_{\nu} \sigma_{\nu-DM} v\, dl$$

Here $n_{DM}\ and\ n_{\nu}$, are number density of DM and neutrino, $\sigma_{\nu-DM}$ is the cross section between DM and neutrino, which is about 1e-30~-40 cm^2. $v$ is relative velocity of neutrino, we may simply take it to be light speed. We adopt the NFW model for DM distribution, and neutrino number density is:

$$n_{\nu} = \frac{N_{\nu}}{4\pi R^2 c t_{burst}}$$

$N_{\nu}$ is the total number of neutrino in the shock, $t_{burst}$ is the shock duration, R is the distance between shock and SN.
And DM distribution is:

$$n_{DM} = \frac{\rho_s}{m_{DM}\frac{r}{r_s}(1+\frac{r}{r_s})^2}$$

$\rho_s$= 0.184 GeV/ cm3, $r_s$ = 24.42 kpc (M. Cirelli, G. Corcella, A. Hektor, G. Hutsi, M. Kadastik, et al., JCAP 1103, 051 (2011), 1012.4515.)
$m_{DM}$ is temporarily taken as 1keV.

We shall find that the integration shall decrease very soon as it is faraway from the SN emission source, since $n_{\nu} \propto \frac{1}{R^2}$ . Therefore we can consider the density of neutrino and DM particles near the SN only. And since $n_{\nu} \propto \frac{1}{R^2}$, the integration shall become divergent if we set the beginning radius $R_i$ to be 0. According to Janka et.al. , the neutrino shock is produced somewhere near R = 100km, we can begin the integral at $R_i = 100km$. We can now write out the approximation of our DM flux:

$$\Phi_{DM} = \int_{R_i}^{R_{from\ SN\ to\ Earth}} \frac{\rho_s}{m_{DM}\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2} \frac{N_{\nu}}{4\pi R^2 c t_{burst}} \sigma_{\nu-DM} c\, dR$$

$$= \frac{\rho_s \sigma_{\nu-DM} N_{\nu}}{4\pi t_{burst} m_{DM}} \int_{R_i}^{R_{from\ SN\ to\ Earth}} \frac{1}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2} \frac{1}{R^2}\, dR$$

$$\approx \frac{\rho_s \sigma_{\nu-DM} N_\nu}{4\pi t_{burst} m_{DM}} \frac{1}{\frac{r_{SN}}{r_s}\left(1+\frac{r_{SN}}{r_s}\right)^2} \frac{1}{R_i}$$

Where $r_{SN}$ is the distance from galactic center to the SN, and we take the approximation that $R_{from\ SN\ to\ Earth} \to \infty$

We shall refrain from discussing the actual value of DM flux from a SN with known position, because for now, we shall talk about some other interesting features.