Knowing that the DM masses may vary in a large range. We want to see how much delayed time there is for a DM flux accelerated by SN neutrino shock to arrive the Earth, after the neutrino shock's arriving.
We then need to go back to our discussion about neutrino-DM colliding kinematics. In it, we know that the energy of DM particle after collision is

$$E_{lab,out,ave} = \frac{1+\frac{\beta^2}{2}}{1-\beta^2} m_{DM}$$

Where

$$\beta\ (E, m_{DM}) = \frac{\sqrt{E^2 - m_\nu^2}}{E + m_{DM}}$$

Thus

$$\frac{1+\frac{\beta^2}{2}}{1-\beta^2} \approx \frac{\frac{3}{2}}{\frac{2m_{DM}}{E}} = \frac{3E}{4m_{DM}}$$

When E>>$m_{DM}$
And the gamma value is $\overline{\gamma_{lab,out}} = \frac{3E}{4m_{DM}}$, and since $\overline{\gamma_{lab,out}} = \frac{1}{\sqrt{1-{\beta_{DM}}^2}}$
($\beta_{DM}$ is the velocity of DM particle in lab frame, unlike $\beta$ is its velocity in CM frame)

$$\beta_{DM} = \frac{1}{\sqrt{1-\overline{\gamma_{lab,out}}^{-2}}}$$

$$\approx 1 - \frac{1}{2}\overline{\gamma_{lab,out}}^{-2}$$

$$= 1 - \frac{8{m_{DM}}^2}{9E^2}$$

Because of the small mass of neutrino, we take its velocity to be exactly the same as light speed.

$$\beta_\nu = 1$$

And the delay time between Neutrino flux and DM flux is:

$$\Delta t = \frac{R}{\beta_{DM}} - \frac{R}{\beta_\nu}$$

$$\approx R\left(1 + \frac{8{m_{DM}}^2}{9E^2} - 1\right)$$

$$= \frac{8R}{9E^2}{m_{DM}}^2$$

Therefore we can see that the delay time is inversely proportional to the square of DM mass.

So consider a SN occurred in Milky Way, if DM mass is keV, the delay time of observing DM flux after observing neutrino flux will be several hours to several days. And when DM mass becomes larger, such as MeV or GeV, the delay time can be decades to centuries. Moreover, if we recall the DM flux formula, we shall notice the flux $\Phi_{DM} \propto \frac{1}{m_{DM}}$ . We conclude that the difficulty of detecting small amount of DM flux and the long delay time arise sharply as DM mass grows larger.

However, if we concern about the great amount of SNe happened in our Milky Way, we may detect a considerable amount of large-mass(say MeV or GeV) DM flux in our background signal, if we integrate all the DM fluxes of all SNe in our Milky Way.

Thus we shall resort to the formula of DM flux:

$$\Phi_{DM} = \frac{\rho_s \sigma_{\nu-DM} N_\nu}{4\pi t_{burst} m_{DM}} \frac{1}{\frac{r_{SN}}{r_s}\left(1+\frac{r_{SN}}{r_s}\right)^2} \frac{1}{R_i}$$

We can know how many DM particles per area were scattered up to the Earth through one explosion of a SN:

$$N_{DM,one\ SN} = \Phi_{DM} t_{burst} = \frac{\rho_s \sigma_{\nu-DM} N_\nu}{4\pi m_{DM}} \frac{1}{\frac{r_{SN}}{r_s}\left(1+\frac{r_{SN}}{r_s}\right)^2} \frac{1}{R_i}$$

And we integrate it on SNe in our galaxy:

$$N_{DM} = \int_{Milky\ Way} n_{SN} N_{DM,one\ SN}\, dV$$

$n_{SN}$ is the probability density $(1/cm\text{^}3 \cdot century)$ And its formula is by Scott et.al.

$$n_{SN} = Ae^{-R/R_d} e^{-|z|/H}$$

$R_d = 2.9kpc$ , $H = 95pc$, as is its ccSNe model

We take the DM-neutrino cross section to be 1e-40 cm^2, and DM mass is 10MeV

And $N_{DM} = 2.4e10\ 1/\ cm\text{^}2 \cdot century$