HW 7

Problem [1] A&M 1.2

(a) Within time 0 and t, the momenta of electrons are:

$$\vec{p}(0) \longrightarrow \vec{p}(t) = \vec{p}(0) + \vec{f}\, dt + O(t^2)$$

$\therefore$ Kinetic energies T are:

$$T(t) = \frac{|\vec{p}(t)|^2}{2m} = \frac{|\vec{p}(0)|^2}{2m} + \left(\vec{p}(0)\cdot\vec{f}\right)\frac{t}{m} + \frac{|\vec{f}|^2}{2m}t^2$$

Take average,

$$\langle T(t)\rangle = \langle T(0)\rangle + \langle \vec{p}(0)\rangle \cdot \vec{f}\,\frac{t}{m} + \frac{|\vec{f}|^2}{2m}t^2$$

$$\Rightarrow \langle T(t)\rangle - \langle T(0)\rangle = \underbrace{\langle \vec{p}(0)\rangle}_{=0} \cdot \vec{f}\,\frac{t}{m} + \frac{|\vec{f}|}{2m}t^2$$

$$= \frac{|\vec{f}|}{2m}t^2 = \frac{|e\vec{E}|^2 t^2}{2m} = \frac{e^2E^2t^2}{2m}$$

$\therefore$ Kinetic energy loss (on average) within t between two collisions is $\frac{(eEt)^2}{2m}$

(b) From (a), we know the average energy loss by time t is

$$\Delta T(t) = \frac{(eEt)^2}{2m}$$

And from 1.(b), we know the probability for collisions is:

$$e^{-\frac{t}{\tau}}\left(\frac{dt}{\tau}\right) = P(t)\,dt$$

$\therefore$ Total average energy loss for all time per collision ($t: 0 \to +\infty$) is

$\overline{\Delta T}\ (0 \to +\infty)$

$$= \int_0^\infty \Delta T(t)\, P(t)\, dt = \int_0^\infty \frac{(eEt)^2}{2m} e^{-\frac{t}{\tau}} \left(\frac{dt}{\tau}\right)$$

$$= \frac{(eE)^2}{2m}\tau^2 \int_0^\infty \left(\frac{t}{\tau}\right)^2 e^{-\left(\frac{t}{\tau}\right)} d\left(\frac{t}{\tau}\right)$$

$$= \frac{(eE)^2\tau^2}{2m} \cdot \Gamma(2) = \frac{(eE\tau)^2}{m} \quad (\because \Gamma(2) = 2! = 2)$$

$\therefore$ Power loss per time per volume is

$$\overline{P} = \overline{\Delta T} \cdot \frac{Ne^-}{V} \cdot \underline{\frac{1}{\tau}}$$

$\uparrow$

$\because$ Every τ, one collision happens

$$= \frac{(eE\tau)^2}{m} \cdot n \frac{1}{\tau} = \frac{ne^2\tau}{m} E^2$$

$$:= \sigma E^2 \quad \left(\sigma = \frac{ne^2\tau}{m} \text{ is conductivity}\right)$$

$\therefore$ Power loss in L and A is

$$\overline{P} \cdot AL = (\sigma AL) E^2 = \left(\frac{\sigma A}{L}\right)(EL)^2$$

$$= \left(\frac{1}{R}\right) V^2 = RI^2 \quad (\text{Ohm's law})$$

Problem [2]

(a) $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$

$\Rightarrow \tau = \frac{m}{\rho n e^2} = \frac{9.11\times10^{-31}\ kg}{1.8\times10^{-8}(\Omega\cdot m)\times(8.5\times10^{28}\ 1/m^3)\times(1.6\times10^{-19}\ C)^2}$

$= 2.3\times10^{-14}\ sec$

$= 23\ fs$

(b) From (A&M) (2.21 & 2.24), we have

$k_F = \sqrt[3]{3\pi^2 n}$

$V_F = \frac{\hbar}{m} k_F = \frac{\hbar}{m}\sqrt[3]{3\pi^2 n}$

$= \frac{6.63\times10^{-34}}{2\pi\times9.11\times10^{-31}} \times \sqrt[3]{3\pi^2\times8.5\times10^{28}}$ (SI)

$= 1.16\times10^{-4}\times1.36\times10^{10}$

$= 1.6\times10^{6}\ m/sec = 0.53\%\ c$ (speed of light)

(c) $l \equiv V_F\tau = 3.1\times10^{-8}\ m$

$= 31\ nm \gg a = 5\ \AA$

(Bohr radius)

Problem[3]

From (A&M) (1.12)

$$\frac{d\vec{p}(t)}{dt} + \frac{\vec{p}(t)}{\tau} = \vec{f}(t)$$

Since $\vec{f}(t) = e\,\vec{E}(t)$

$$= e\left(\vec{E}(\omega)e^{-i\omega t} + \vec{E}(\omega)^{*} e^{+i\omega t}\right)$$

We now can solve for the $e^{-i\omega t}$ component solution

$$\frac{dp}{dt} + \frac{1}{\tau}p = e\,E(\omega)e^{-i\omega t} \quad \left(\begin{array}{c}\text{Assume} \\ \vec{E} \parallel \vec{p}\end{array}\right)$$

Assume $p(t) \propto e^{-i\omega t}$, $p(t) = p(\omega)e^{-i\omega t}$

$$\left(-i\omega + \frac{1}{\tau}\right) p(\omega) = e\,E(\omega)$$

$$\Rightarrow j(\omega) = \frac{ne}{m}p(\omega) = \frac{ne^2}{m}\frac{E(\omega)}{-i\omega + \frac{1}{\tau}}$$

$$= \frac{ne^2\tau}{m}\frac{1}{-i\omega\tau + 1}E(\omega)$$

$$\therefore \sigma(\omega) = \frac{j(\omega)}{E(\omega)} = \frac{ne^2\tau}{m}\frac{1}{-i\omega\tau+1}$$

$$= \sigma_0 \frac{1 + i\omega\tau}{1 + (\omega\tau)^2} \quad \left(\sigma_0 := \frac{ne^2\tau}{m}\right)$$

Problem [4]

List of Resistivity

(a)Copper resistivity (=4.2K)

$$\rho_{Cu} = 6.7 \times 10^{-3} \mu\Omega.cm$$

Berman, R., & Macdonald, D. K. C. (1952). The thermal and electrical conductivity of copper at low temperatures. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, *211*(1104), 122–128. https://doi.org/10.1098/rspa.1952.0029

(b) Quartz resistivity (= room temp)

$$\rho_{Quartz} = 7.5 \times 10^{25} \mu\Omega.cm$$

https://www.thoughtco.com/table-of-electrical-resistivity-conductivity-608499

(c) $Ba_{0.75}La_{4.25}Cu_5O_5$ resistivity (=room temp)

$$\rho_{BaLaCO} = 2 \times 10^{4} \mu\Omega.cm$$

Bednorz, J. G., & M ller, K. A. (1986). Possible highT c superconductivity in the Ba?La?Cu?O system. *Zeitschrift f r Physik B Condensed Matter*, *64*(2), 189–193. https://doi.org/10.1007/BF01303701

(d) Distilled water resistivity (=room temp)

$$\rho_{water} = 1.8 \times 10^{13} \mu\Omega.cm$$

https://www.labmanager.com/resistivity-conductivity-measurement-of-purified-water-19691

Relaxation time for copper (4.2K)

(a)

$$\tau_{Cu} = \frac{m(= 9.11 \times 10^{-31} kg)}{\rho_{Cu} n(= 8.5 \times 10^{28} \frac{1}{m^3})e^2} = 6.2 \times 10^{-12} s = 6.2 ps$$

Density of conduction electrons

(b) Quartz

$$n_{Quartz} = \frac{m(= 9.11 \times 10^{-31} kg)}{\rho_{Quartz} \tau_{Quartz} (= 10^{-14} \mathrm{s}) e^2} = 4.7 \times 10^{3} \frac{1}{m^3}$$

(c) $Ba_{0.75}La_{4.25}Cu_5O_5$

$$n_{BaLaCO} = \frac{m(= 9.11 \times 10^{-31} kg)}{\rho_{BaLaCO}\tau_{BaLaCO}(= 10^{-14}\mathrm{s})e^2} = 1.8 \times 10^{25}\frac{1}{m^3}$$

(d) Distilled water (pH =7)

$$n_{water} = \frac{m(= 9.11 \times 10^{-31} kg)}{\rho_{water}\tau_{water}(= 10^{-14}\mathrm{s})e^2} = 2.0 \times 10^{16}\frac{1}{m^3}$$

The estimation for "conducting electrons" in distilled water is particularly wrong because the aqueous solutions conduct electricity through ions, not free electrons in the conduction band (liquids have no band structure at all). And the ion number density is $10^{-7}\frac{mol}{L} = 6 \times 10^{19}\frac{1}{m^3}$, which is much larger than the estimated "conduction electron number."