Exame Recurso 2007/2008

(I) $\vec{v}(t) = 4t^2\hat{i} - 3t^2\hat{j}$ (m/s)

a) $(t=0) \Rightarrow \vec{R} = 2\hat{i} + 2\hat{j}$ (m)

$\vec{R}(t) = \int v(t)dt = \left(\frac{4t^3}{3} + c\right)\hat{i} - \left(\frac{3t^3}{3} + c\right)\hat{j}$ (m)

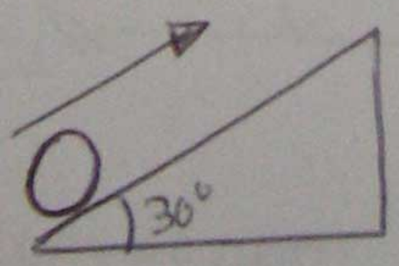
$\vec{R}(t) = \left(\frac{4t^3}{3} + 2\right)\hat{i} - \left(t^3 + 2\right)\hat{j}$ (m)

b) $\vec{a}(t) = 8t\hat{i} - 6t\hat{j}$ (m/s²)

c) $\cos\theta = \frac{v \cdot a}{\|v\|\|a\|} = \frac{(4t^2 - 3t^2)(8t - 6t)}{\sqrt{(4t^2)^2 + (3t^2)^2}\sqrt{(8t)^2 + (6t)^2}} = \frac{32t^3 + 18t^3}{\sqrt{16t^4 + 9t^4}\cdot\sqrt{64t^2 + 36t^2}} =$

$= \frac{50t^3}{\sqrt{25t^4}\cdot\sqrt{100t^2}} = \frac{50t^3}{5t^2 \cdot 10t} = \frac{50t^3}{50t^3} = 1$ //

(II) $R = 0{,}5\,m$
$M = 4\,Kg$
$\omega_i = 10\,rad/s$



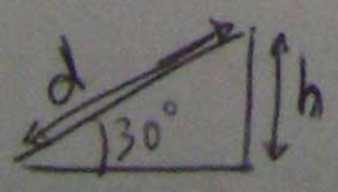
$I_{CM} = \frac{2}{5}MR^2 = \frac{2}{5}\cdot 4 \cdot 0{,}5^2 = \frac{2}{5}$

a) $E_c = \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times 5^2 = 50\,J$

$\boxed{V = \omega r} = 10 \times 0{,}5 = 5\,m/s$

$E_{C_{esf}} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{2}{5} \times 10^2 = 20\,J$

$E_{C_{Total}} = 50 + 20 = 70\,J$

b) 

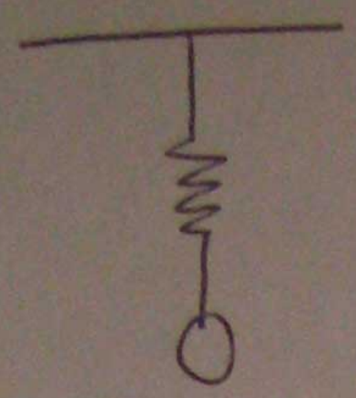
$\Delta\tau = 0$ $\quad E_{cf} - E_{pg} = E_{ci} - E_{pi}$ $\quad \sin(30) = \frac{h}{d} \Leftrightarrow \sin(30) = \frac{1{,}8}{d}$

$-E_{pg} = E_{ci}$

$-mgh = 70$

$h = \frac{70}{4 \times 9{,}8} = 1{,}8\,(m)$ $\quad \Leftrightarrow d = \frac{1{,}8}{\sin(20)} = 3{,}6$

III



$m = 2kg$

$A = 0,2\,m$

$F = 10N$

a) $\boxed{T = \frac{1}{f}}$ $\boxed{F = Kx}$ $\Leftrightarrow$ $K = \frac{F}{x} = \frac{10}{0,2} = 50\,(N/m)$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{50}{2}} = 5\,(rad/s)$$

$$\omega = 2\pi f \Leftrightarrow \omega = 2\pi . \frac{1}{T} \Leftrightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{5}\,s$$

b) $b = 2\,Kg$

$$x(t) = A\cos(\omega t + \phi)$$

$$x(t) = 0,2\cos(5t + \phi)$$

$$\frac{0,2}{0,2} = \cos(\phi) \Leftrightarrow \cos(\phi) = 1 \qquad \phi = 2\pi$$

$$x(t) = 0,2\cos(5t + 2\pi)$$

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c) $b = 2\,Kg$

$A = 0,2$

$$Em = \frac{1}{2} m A^2 \omega^2 . e^{\left(-\frac{b}{2m}\right)t}$$

$$Em = 1 \times 0,2^2 \times 5^2 = 1\,J$$

$$\frac{Em}{2} = 0,5\,J \qquad e^{\left(-\frac{2}{4}\right)t}$$

$$0,5 = \frac{1}{2} \times 2 \times 0,2^2 \times 5^2 \times e^{-\frac{1}{2}t}$$

$$0,5 = e^{-\frac{1}{2}t}$$

$$\ln(0,5) = -\frac{1}{2}t \Leftrightarrow t = 1,45\,s$$

1)

$A = 0{,}02\,m$

$B = 0{,}03\,m$

$\sigma = 5nC/m^2 = 5\times10^{-9}\,C/m^2$

a) $E = \frac{\sigma}{\varepsilon_0} = \frac{5\times10^{-9}}{9\times10^{-12}} = 5{,}55\times10^{2}$

b) $\Delta V = -\int_{0{,}02}^{0{,}03} \vec{E}\,.\,d\vec{s}$

VI

a) $\lambda = 20m$

b) $V = \frac{dx}{dt} = \frac{5}{2}\frac{m}{s} = 2{,}5\,(m/s)$

c) $\boxed{T = \frac{1}{f}}$ $T = \frac{\lambda}{V} = \frac{20}{2{,}5} = 8\,s$

d) $A = 2cm = 0{,}02m$

$x(Y,t) = A\,Sin\,(Kx - \omega t + \phi)$

$K = \frac{2\pi}{\lambda} = \frac{2\pi}{20} = \frac{\pi}{10}$

- $\omega = V.K = 2{,}5\times\frac{\pi}{10} = \frac{5\pi}{20} = \frac{\pi}{4}$
- $\omega = 2\pi f \Leftrightarrow \omega = 2\pi\cdot\frac{1}{8} \Leftrightarrow \omega = \frac{\pi}{4}$

$f = \frac{1}{T} = \frac{1}{8}$

$x(0{,}02,0) = 0{,}02\,Sin\left(\frac{\pi}{10}.0 - \frac{\pi}{4}.0 + \phi\right)$

$\frac{0{,}02}{0{,}02} = Sin\,(\phi)$

$Sin(\phi) = 1 \quad \phi = \frac{\pi}{2}$

$Y(x,T) = 0{,}02\left(\frac{\pi}{10}x - \frac{\pi}{4}t + \frac{\pi}{2}\right)$

Exame Recurso 2007/2008

① ~~$\vec{v}(t) = 4t^2\hat{i} - 3\hat{j}$~~

$\vec{v}(t) = 4t^2\hat{i} - 3t^2\hat{j}$ (m/s)

Em $(t=0) \Rightarrow \vec{r} = 2\hat{i} + 2\hat{j}$ (m)

a) $\vec{R}(t) = \int v\,dt = \left(\frac{4t^3}{3} + c_1\right)\hat{i} + \left(\frac{3t^2}{2} + c_2\right)\hat{j}$

$\vec{R}(t) = \left(\frac{4t^3}{3} + 2\right)\hat{i} + \left(\frac{3t^2}{2} + 2\right)\hat{j}$ (m)

b) $\vec{a}(t) = 8t\hat{i} - 6t\hat{j}$ m/s

c) $\cos\theta = \frac{v \cdot a}{\|v\| \cdot \|a\|} = \frac{(4t^2 - 3t^2)(8t - 6t)}{\sqrt{(4t^2)^2 + (3t^2)^2}\sqrt{(8t)^2 + (6t)^2}} = \frac{32t^3 + 18t^3}{\sqrt{16t^4 + 9t^4}\sqrt{64t^2 + 36t^2}}$

$= \frac{50t^3}{\sqrt{25t^4}\sqrt{100t^2}} = \frac{50t^3}{5t^2 \cdot 10} = \frac{50t^3}{50t^3} = 1$

Ou então

$|v| = \sqrt{(4t^2)^2 + (3t^2)^2} = \sqrt{16t^4 + 9t^4} = \sqrt{25t^4} = 5t^2$

$a_T = (5t^2)' = 10t$

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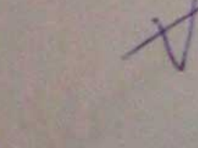
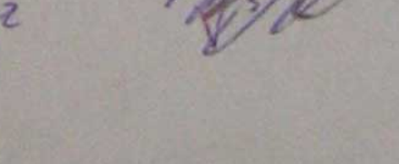
② 



$\omega_i = 10$ rad/s

$I_{CM} = \frac{2}{5}(MR^2)$

a) $E_C = \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times 5 = 50$ J

$V = \omega R = 10 \times 0,5 = 5$ m/s

$E_{CR} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{2}{5}(4 \times 0,5^2) \times 10^2 = \frac{1}{5}(0,25) \times 100 = 20$ J

$E_f = 50 + 20 = 70$ J

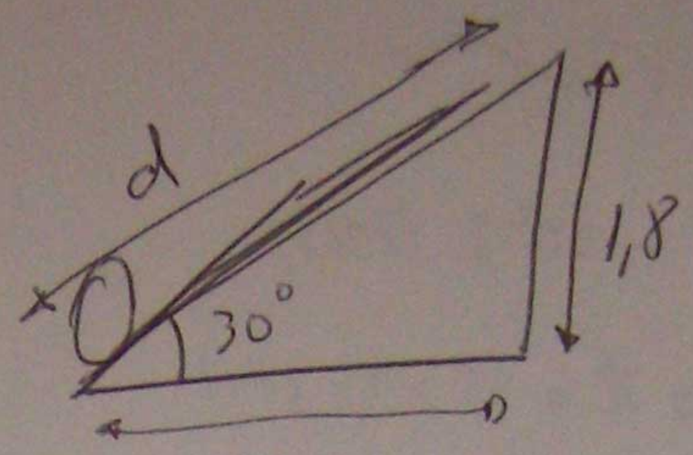
b) $\Delta\tau = 0$

$\epsilon_{cf} - \epsilon_{pg} = \epsilon_{ci} - \epsilon_{pi}$

$-\epsilon_{pg} = \epsilon_{ci}$

$-mgh = 70$

$h = \frac{70}{4 \times 9,8} = 1,8$

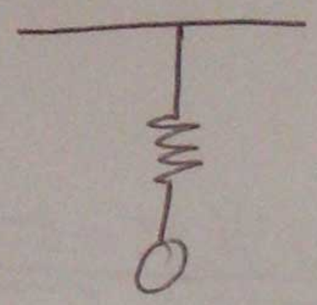


$\text{Sen } 30° = \frac{1,8}{d} \Leftrightarrow d = \frac{1,8}{\text{Sen } 30} = 3,6 \text{ m}$

(III) $m = 2,0 kg$

$F = 10 N$

$A = 0,2 m$



a) $F = Kx$

- $K = \frac{F}{x} = \frac{10}{0,2} = 50 \text{ N/m}$

- $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{50}{2}} = 5 \text{ rad/s}$

$T = \frac{1}{f} \Leftrightarrow f = \frac{\omega}{2\pi} \Leftrightarrow T = \frac{1}{\frac{\omega}{2\pi}} \Leftrightarrow T = \frac{1}{\frac{5}{2\pi}} = \frac{2\pi}{5} s$

$\omega = 2\pi f$

b) $x(t) = A\cos(\omega t + \phi)$

$x(t) = 0,2\cos(5t + \phi)$

$\frac{0,2}{0,2} = \cos(\phi)$

$\phi = \pi$

$x(t) = 0,2\cos(5t + \pi)$ //

c) $b = 2\,Kg/s$

$A = 0,2$

$$Em = \frac{1}{2} m A^2 \omega^2 \cdot e^{\left(-\frac{b}{2m}\right)\tau}$$

$$Em = 1 \cdot 0,2^2 \cdot 5^2 = 1\,J$$

$$\frac{Em}{2} = 0,5\,J$$

$$0,5 = \frac{1}{2} \cdot 2 \cdot 0,2^2 \cdot 5^2 \cdot e^{\left(-\frac{2}{4}\right)\tau}$$

$$0,5 = e^{-\frac{1}{2}\tau}$$

$$\ln(0,5) = -\frac{1}{2}\tau \qquad \tau = 1,4\,s$$

(IV) A e B a 2cm e 3cm $\quad A = 0,02m$ $\quad B = 0,03m$

$\theta = 5nC/m^2 = 5 \times 10^{-9}\,C/m^2$

~~a) $E = \frac{\theta}{\varepsilon_0} = \frac{5 \times 10^{-9}}{\varepsilon_0} = \frac{5,65 \times 10^{-9}}{9 \times 10^9} = 5,65 \times 10^{-22}\,N/C$~~

~~b) $\Delta V = -\int_A^B \vec{E}\, d\vec{s} = -5,65 \times 10^{24}$~~

a) $E = \frac{\theta}{\varepsilon_0} = \frac{5 \times 10^{-9}}{9 \times 10^{-12}} = 5,65 \times 10^{-22} \cdot N/C$

b) $\Delta V = -\int_A^B E \cdot ds = -5,65 \times 10^{-24}\,V$

(VI) $\tau = 0$ $\tau = 2$ $\lambda = v\tau$ $T = \frac{\lambda}{v}$

a) $x(y,t) = A\sin(kx - \omega t + \phi)$

$$30 = 0{,}02\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}\tau + \phi\right)$$

$$30 = 0{,}02\sin\left(\frac{2\pi}{T}(vx - \tau) + \phi\right)$$

b) $v = \frac{5}{2} = 2{,}5\ m/s$

c) $T = \frac{20}{2{,}5} = 8s$

$a = \lambda = 20$

d) $x(y,t) = A\sin(kx - \omega t + \phi)$

$$x(0{,}02, 0) = 0{,}02\sin\left(\frac{2\pi}{20}\cdot 0 - \frac{2\pi}{T}\cdot 0 + \phi\right)$$

$$0{,}02 = 0{,}02\,\mathrm{sen}(\phi)$$

$$\frac{\pi}{2} = \phi + 3\pi$$

$$\phi = -\frac{\pi}{2}$$

(I)

a) $\vec{R}(t) = \frac{4t^3}{3}\hat{i} - t^3\hat{j} + C$

$2\hat{i} + 2\hat{j} = C$

$\vec{R}(t) = \left(\frac{4t^3}{3} + 2\right)\hat{i} + (t^3 + 2)\hat{j}\ (m)$

b) $\vec{a}(t) = 8t\hat{i} - 6t\hat{j}$

(IV)

a) $\lambda = 20m$

b) $V = \frac{\lambda}{T}$ $\quad k = \frac{2\pi}{\lambda} \Leftrightarrow k = \frac{2\pi}{20} = \frac{\pi}{10}$

$\omega = v \cdot k = 0{,}25\pi \, rad/s$

$V = \frac{5}{2} m = 2{,}5 m/s$

c) $\boxed{T = \frac{1}{f}}$ $\quad \boxed{T = \frac{\lambda}{V}} = \frac{20}{2{,}5} = 8s$

d) $A = 2cm = 0{,}02m$

$Y(0,0) = 0 \Leftrightarrow 0 = 0{,}02 \, sen\left(\frac{\pi}{10}(0) - 0{,}25\pi(0) + \varphi\right)$

$\Leftrightarrow 0 = sen(\varphi)$

$\varphi = 0 + \pi k \quad , k \in \mathbb{Z}$

$Y(x,t) = 0{,}02 \, sen\left(\frac{\pi}{10} - 0{,}25\pi t\right)$

e) $Y_1(x,t) = 0{,}02 \, sen\left(\frac{\pi}{10}x - 0{,}25\pi t\right) (m)$

$Y_1(x,t) = 0{,}02 \, sen\left(\frac{\pi}{10}x + 0{,}25\pi t\right) (m)$

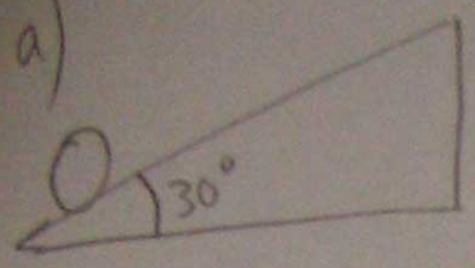
$Y_{total} = Y_1 + Y_2 = 0{,}04 \, sen\left(\frac{\pi}{10}x\right) \cos(0{,}25\pi t) (m)$

Y_{total} é 1 onda estacionária

f) $L = \frac{\lambda}{2} = 10m$

I

a)



$R = 0,5\,m$

$m = 4\,Kg$

$\omega = 10\,rad/s$

$I = \frac{2}{5}(MR^2) = \frac{2}{5} \cdot 4 \cdot 0,5^2 = 0,4$

$\boxed{\omega = \frac{V}{R}}$

$V = \omega . R = 10 \times 0,5 = 5\ m/s$

$E_c = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 4 \cdot 5^2 = 50$

$E_{c_{Rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2} \cdot 0,4 \cdot 10^2 = 20$

$E_{c_{Total}} = 70$

b) $\Delta E = 0$

$E_{pf} - E_{pg} = E_{ci} - E_{pi}$

$-E_{pg} = E_{ci}$

$-mgh = 0$

$\text{sen } 30 = \frac{1,8}{d}$

$d = 3,6$

$h = 1,8\,m$ //

VI

a) $y(x,T) = A \sin(\omega T + Kx + \varphi)$

$Kx - \omega T$

$A = 2$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$

a) $\lambda = vT = \frac{5}{2} \times 8 = 20$

b) $V = \frac{dx}{dT} = \frac{30-25}{2-0} = \frac{5}{2} = 2,5\ m/s$

c) $T = 8\,s$

d) $K = \frac{2\pi}{\lambda} = \frac{2\pi}{20} = 0,31 = \frac{\pi}{10}$

$\omega = \frac{\pi}{4}$

$y(x,T) = 2 \sin\left(\frac{\pi}{10} . x - \frac{\pi}{4}T + \varphi\right)$

$y(-2,2) = 2 \sin\left(\frac{\pi}{10} . -2 - \frac{\pi}{4}2 + \varphi\right)$

$y(-2,2) = 2 \sin\left(-\frac{\pi}{5} - \frac{\pi}{2} + \varphi\right)$

(2) (3)

$y(-2,2) = 2 \sin\left(\frac{-7\pi}{10} + \varphi\right)$

$-y(-2,2) = 2 \sin\left(\frac{-7\pi}{10} + \varphi\right)$

$2(x,T) = 2 \sin\left(\frac{\pi}{10} . 30 - \frac{\pi}{4}0 + \varphi\right)$

$\sin(3\pi + \varphi)$