Aula 3: Funções hiperbólicas

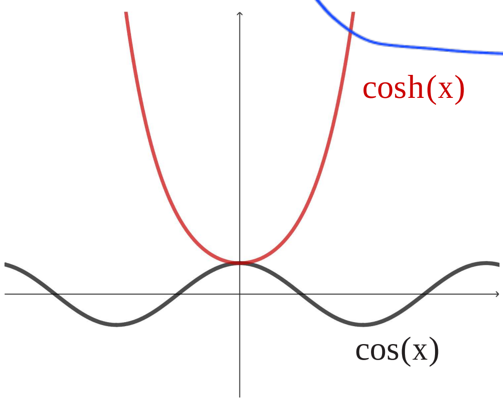
Fórmula de Euler: $\boxed{e^{ix} = \cos(x) + i\,\mathrm{sen}\,(x)}$ $\qquad (i^2 = -1)$

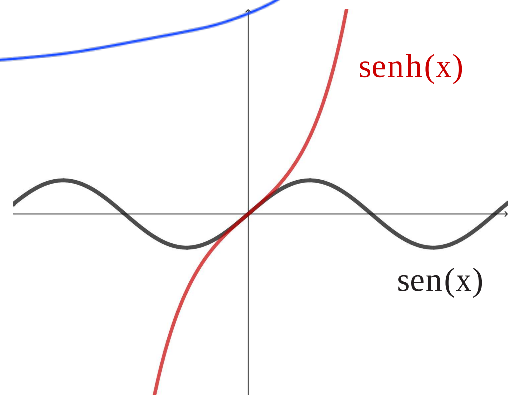
Daqui deduzimos:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}, \quad \mathrm{sen}\,(x) = \frac{e^{ix} - e^{-ix}}{2i} \tag{1}$$

Eliminando a i nas formulas anteriores obtemos o coseno e o seno hiperbólicos:

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \quad , \quad \mathrm{senh}\,(x) = \frac{e^{x} - e^{-x}}{2} \tag{2}$$





Aula 3: Fórmulas hiperbólicas e trigonométricas

$$\cosh(x) = \cos(ix)$$

$$\cos(x) = \cosh(ix)$$

$$\operatorname{senh}(x) = -i \operatorname{sen}(ix)$$

$$\operatorname{sen}(x) = -i \operatorname{senh}(ix)$$

$$e^{ix} = \cos(x) + i \operatorname{sen}(x)$$

$$e^{x} = e^{i(-ix)} = \cosh(x) + i \operatorname{senh}(x)$$

$$\cosh^2(x) - \operatorname{senh}^2(x) = 1$$

$$\cos^2(x) + \operatorname{sen}^2(x) = 1$$

$$\operatorname{tgh}(x) := \frac{\operatorname{senh}(x)}{\cosh(x)} = \frac{e^{2x}-1}{e^{2x}+1}$$

$$\operatorname{tg} x = \frac{\operatorname{sen}(x)}{\cos(x)}$$

$$\operatorname{cotgh}(x) := \frac{\cosh(x)}{\operatorname{senh}(x)} = \frac{e^{2x}+1}{e^{2x}-1}$$

$$\operatorname{cotg} x = \frac{\cos(x)}{\operatorname{sen}(x)}$$

$$\cosh(x+y) = \cosh(x)\cosh(y) + \operatorname{senh}(x)\operatorname{sen}(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \operatorname{sen}(x)\operatorname{sen}(y)$$

$$\cosh(x-y) = \cosh(x)\cosh(y) - \operatorname{senh}(x)\operatorname{sen}(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \operatorname{sen}(x)\operatorname{sen}(y)$$

$$\operatorname{senh}(x+y) = \operatorname{senh}(x)\cosh(y) + \cosh(x)\operatorname{senh}(y)$$

$$\operatorname{sen}(x+y) = \operatorname{sen}(x+y) = \operatorname{sen}(x)\cos(y) + \cos(x)\operatorname{sen}(y)$$

$$\operatorname{senh}(x-y) = \operatorname{senh}(x)\cosh(y) - \cosh(x)\operatorname{senh}(y)$$

$$\operatorname{sen}(x-y) = \operatorname{sen}(x-y) = \operatorname{sen}(x)\cos(y) - \cos(x)\operatorname{sen}(y)$$

$$\cosh(2x) = \cosh^2(x) + \operatorname{senh}^2(x)$$

$$\cos(2x) = \cos^2(x)\operatorname{sen}^2(x)$$

$$\operatorname{senh}(2x) = 2\operatorname{senh}(x)\cosh(x)$$

$$\operatorname{sen}(2x) = 2\operatorname{sen}(x)\cos(x)$$

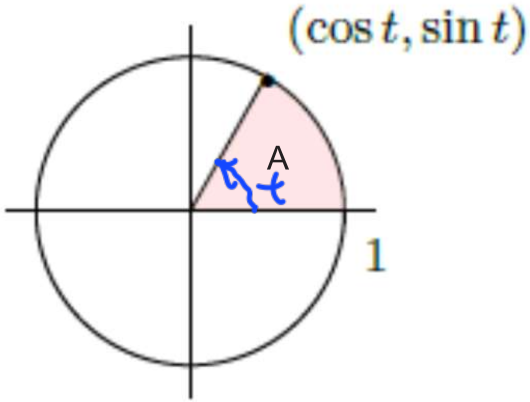
$$(\cosh(x))' = \operatorname{senh}(x)$$

$$(\cos(x))' = -\operatorname{sen}(x)$$

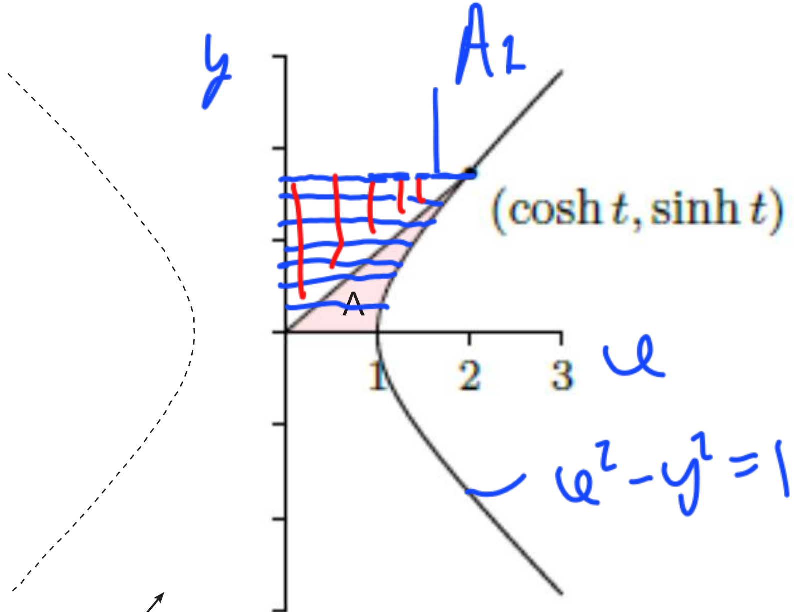
$$(\operatorname{senh}(x))' = \cosh(x)$$

$$(\operatorname{sen}(x))' = \cos(x)$$

Aula 3: Semelhanças



circunferência unitária



hipérbole unitária

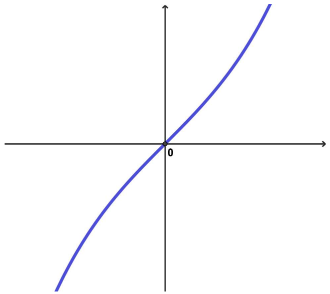
$$\pi \times 1^2 \times \frac{t}{2\pi} = \frac{t}{2}$$

$$A = \frac{t}{2}$$

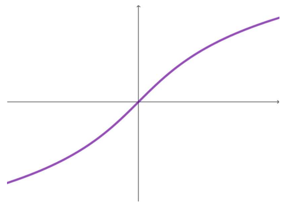
$$A_1 = \int_0^{\sinh t} \sqrt{1+y^2}\, dy$$

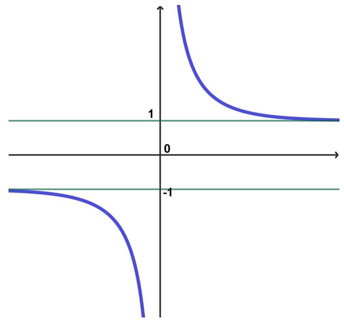
Aula 3: Funções hiperbólicas inversas

pf: parte de f ao subdominio (f restrita ao subdominio)

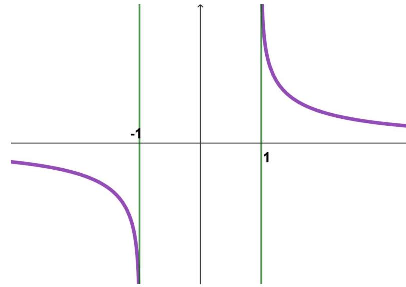
senh(x) 

inversa $\longrightarrow$ $\text{senh}^{-1}(x)=\ln(x+\sqrt{x^2+1})$



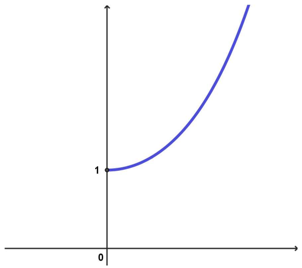
cotgh(x) 

inversa $\longrightarrow$ $\text{cotgh}^{-1}(x)=\frac{1}{2}\ln(\frac{x+1}{x-1})$

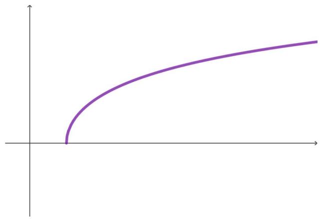


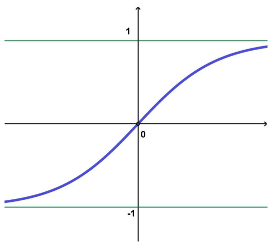
pcosh(x)

$x \geqslant 0$



inversa $\longrightarrow$ $\text{pcosh}^{-1}(x)=\ln(x+\sqrt{x^2-1})$



tgh(x) 

inversa $\longrightarrow$ $\text{tgh}^{-1}(x)=\frac{1}{2}\ln(\frac{1+x}{1-x})$