

# 字符串

## KMP

```
int len1, len2, nxt[MAXN], ans[MAXN];
char s1[MAXN], s2[MAXN]; //s1 文本串, s2 模式串

int main()
{
    scanf("%s%s", s1+1, s2+1);
    len1=strlen(s1+1), len2=strlen(s2+1);
    for(int i=2, p=0; i<=len2; i++)
    {
        while(p && s2[i]!=s2[p+1]) p=nxt[p];
        if(s2[i]==s2[p+1]) p++;
        nxt[i]=p;
    }
    for(int i=1, p=0; i<=len1; i++)
    {
        while(p && s1[i]!=s2[p+1]) p=nxt[p];
        if(s1[i]==s2[p+1]) p++;
        // p==len2 时, 存在一个匹配
    }
}
```

## AC自动机

```
int n, tot, tag[MAXN], fail[MAXN], t[MAXN][26], num[205];
char s2[205][MAXM], s1[MAXN]; //s1 文本串, s2 模式串

queue<int> q;

void insert(char s[], int id)
{
    int len=strlen(s), p=0;
    for(int i=0; i<len; i++)
    {
        int ch=s[i]-'a';
        if(!t[p][ch]) t[p][ch]=++tot;
        p=t[p][ch];
    }
    tag[p]=id;
}

void getfail()
{
    for(int i=0; i<26; i++)
        if(t[0][i]) fail[t[0][i]]=0, q.push(t[0][i]);
    while(!q.empty())
    {

```

```

    int x=q.front();
    q.pop();
    for(int i=0; i<26; i++)
    {
        if(t[x][i])
        {
            fail[t[x][i]]=t[fail[x]][i];
            q.push(t[x][i]);
        }
        else t[x][i]=t[fail[x]][i];
    }
}

void solve(char *s)
{
    int len=strlen(s), p=0;
    for(int i=0; i<len; i++)
    {
        int ch=s[i]-'a';
        p=t[p][ch];
        for(int j=p; j; j=fail[j])
            num[tag[j]]++;
    }
}

int main()
{
    scanf("%d", &n);
    for(int i=1; i<=n; i++)
    {
        scanf("%s", s2[i]);
        insert(s2[i], i);
    }
    getfail();
    scanf("%s", s1);
    solve(s1);
    //num[i] 为模式串 i 在文本串中出现次数
    return 0;
}

```

## 后缀自动机

```

int len, tot=1, last=1, b[MAXN], id[MAXN*2];
char s[MAXN];

struct State {int len, fa, val, nxt[26];} a[MAXN*2];

void insert(int ch)
{
    int cur=++tot, p=last;

```

```

a[cur].len=a[last].len+1;
a[cur].val=1;
for(; p && !a[p].nxt[ch]; p=a[p].fa) a[p].nxt[ch]=cur;
if(!p) a[cur].fa=1;
else
{
    int x=a[p].nxt[ch];
    if(a[p].len+1==a[x].len) a[cur].fa=x;
    else
    {
        int y=++tot;
        a[y]=a[x];
        a[y].len=a[p].len+1;
        for(; p && a[p].nxt[ch]==x; p=a[p].fa) a[p].nxt[ch]=y;
        a[x].fa=a[cur].fa=y;
    }
}
last=cur;
}

void topsort()
{
    for(int i=1; i<=tot; ++i) b[a[i].len]++;
    for(int i=1; i<=len; ++i) b[i]+=b[i-1];
    for(int i=1; i<=tot; ++i) id[b[a[i].len]--]=i;
    for(int i=tot; i>=2; --i)
    {
        int x=id[i];
        a[a[x].fa].val+=a[x].val;
    }
}

int main()
{
    scanf("%s", s+1);
    len=strlen(s+1);
    for(int i=1; i<=len; ++i) insert(s[i]-'a');
    //a[i].val 代表节点 i 对应字符串出现次数
    return 0;
}

```

# 数学

## 线性筛

```
int n, t, cnt, pri[MAXN], vis[MAXN];
int mu[MAXN], phi[MAXN]; //莫比乌斯函数和欧拉函数

void sieve(int lim)
{
    //phi[i]=mu[i]=1;
    for(int i=2; i<=lim; ++i) {
        if(!vis[i]) pri[++cnt]=i; // phi[i]=i-1, mu[i]=-1;
        for(int j=1; j<=cnt && pri[j]*i<=lim; ++j) {
            vis[i*pri[j]]=1;
            if(i%pri[j]==0) break;
            /*
            if(i%pri[j]==0) {
                mu[pri[j]*i]=0;
                phi[i*pri[j]]=pri[j]*phi[i];
                break;
            } else {
                mu[pri[j]*i]=-mu[i];
                phi[i*pri[j]]=(pri[j]-1)*phi[i];
            }
            */
        }
    }
}
```

## 组合数

```
int fac[MAXN], ifac[MAXN];

void init(int lim)
{
    ifac[0]=ifac[1]=fac[0]=fac[1]=1;
    for(int i=2; i<=lim; i++) ifac[i]=1LL*ifac[P%i]*(P-P/i)%P;
    for(int i=2; i<=lim; i++) {
        ifac[i]=1LL*ifac[i-1]*ifac[i]%P;
        fac[i]=1LL*fac[i-1]*i%P;
    }
}

int comb(int x, int y)
{
    if(y>x) return 0;
    return 1LL*fac[x]*ifac[y]%P*ifac[x-y]%P;
}
```

## 高斯消元

```
int n;
double a[MAXN][MAXN];

int main()
{
    scanf("%d", &n);
    for(int i=1; i<=n; ++i)
        for(int j=1; j<=n+1; ++j) scanf("%lf", &a[i][j]);
    int c=1, r=1;
    for(; c<=n; ++c) {
        int pos=0;
        for(int i=r; i<=n; ++i)
            if(a[i][c]!=0) pos=i;
        if(pos==0) continue;
        for(int i=c; i<=n+1; ++i) swap(a[r][i], a[pos][i]);
        for(int i=n+1; i>=c; --i) a[r][i]/=a[r][c];
        for(int i=1; i<=n; ++i) {
            if(r==i) continue;
            for(int j=n+1; j>=c; --j) a[i][j]-=a[i][c]*a[r][j];
        }
        r++;
    }
    if(r<=n) {
        printf("No Solution\n");
        return 0;
    }
    for(int i=1; i<=n; ++i) printf("%.21f\n", a[i][n+1]);
    return 0;
}
```

## 线性基

```
int n;
LL base[66];

void insert(LL *base, LL x)
{
    for(int i=60; i>=0; --i)
        if((x>>i)&1) {
            if(!base[i]) {
                base[i]=x;
                return;
            }
            x^=base[i];
        }
}

int main()
```

```

{
    scanf("%d", &n);
    for(int i=1; i<=n; ++i) {
        LL temp;
        scanf("%lld", &temp);
        insert(base, temp);
    }
    return 0;
}

```

## 质因数分解

```

const LL base[]={2, 325, 9375, 28178, 450775, 9780504, 1795265022};

int T;
LL n;
map<LL, int> mp;

LL ksm(LL x, LL y, LL p)
{
    LL num=1;
    while(y) {
        if(y&1) num=(I128)num*x%p;
        y>>=1, x=(I128)x*x%p;
    }
    return num;
}

bool MR(LL x)
{
    if(x<=2 || !(x%2)) return x==2;
    LL d=x-1, r=0;
    while(!(d%2)) d>>=1, r++;
    for(LL e: base) {
        LL v=ksm(e, d, x);
        if(v<=1 || v==x-1) continue;
        for(int i=0; i<r-1; ++i) {
            v=(I128)v*v%x;
            if(v==x-1 || v==1) break;
        }
        if(v!=x-1) return false;
    }
    return true;
}

LL PR(LL x)
{
    LL l=0, r=0, val=2, tmp;
    auto f=[x](LL y) {return ((I128)y*y+1)%x;};
    for(int i=0; ; ++i) {
        if(!(i%60) && __gcd(val, x)>1) break;

```

```

        if(l==r) l=rand()%(x-1)+1, r=f(l);
        if(tmp=(ll28)val*abs(r-l)%x) val=tmp;
        l=f(l), r=f(f(r));
    }
    return __gcd(val, x);
}

void find(LL x, int num)
{
    if(x<=1) return;
    if(MR(x)) {
        mp[x]+=num;
        return;
    }
    LL y=x, cnt=0;
    while(y==x) y=PR(x);
    while(x%y==0) x/=y, cnt++;
    find(x, num), find(y, cnt*num);
}

int main()
{
    srand(time(0));
    scanf("%d", &T);
    while(T--) {
        mp.clear();
        scanf("%lld", &n);
        find(n, 1); //指数形式分解质因数
        for(auto e: mp) printf("%lld %d\n", e.first, e.second);
    }
    return 0;
}

```

## BSGS

```

int a, b, x, p;

unordered_map<int, int> mp;

int ksm(int x, int y)
{
    int num=1;
    while(y) {
        if(y&1) num=1LL*num*x%p;
        x=1LL*x*x%p; y>>=1;
    }
    return num;
}

int bsgs(int a, int b)
{

```

```

    if(a%p==0) return (b==0)?1:-1;
    mp.clear();
    int siz=(int)(sqrt(p)+0.5), tmp=1LL*a*b%p;
    for(int i=1; i<=siz; ++i) {
        mp[tmp]=i;
        tmp=1LL*tmp*a%p;
    }
    int base=ksm(a, siz);
    tmp=1;
    for(int i=1; i<=siz; ++i) {
        tmp=1LL*tmp*base%p;
        if(mp[tmp]) return i*siz-mp[tmp];
    }
    return -1;
}

int main()
{
    scanf("%d%d%d", &a, &b, &p);
    x=bsgs(a, b); //a^x = b mod p
    if(x==-1) printf("No solution\n");
    else printf("%d\n", x);
}

```

## 快速傅里叶变换

```

int bit, len, len1, len2, pos[MAXN*4], ans[MAXN*4];
char s1[MAXN], s2[MAXN];

struct CP {
    double x, y;
    friend CP operator * (CP a, CP b) {
        return CP{a.x*b.x-a.y*b.y, a.x*b.y+a.y*b.x};
    }
    friend CP operator + (CP a, CP b) {
        return CP{a.x+b.x, a.y+b.y};
    }
    friend CP operator - (CP a, CP b) {
        return CP{a.x-b.x, a.y-b.y};
    }
} a[MAXN*4], b[MAXN*4];

void fft(CP a[], int n, int op)
{
    for(int i=0; i<n; ++i)
        if(i<pos[i]) swap(a[i], a[pos[i]]);
    for(int i=1; i<n; i<=<1) {
        CP wn{cos(PI/i), op*sin(PI/i)};
        for(int j=0; j<n; j+=(i<=<1)) {
            CP w{1, 0};
            for(int k=0; k<i; ++k, w=w*wn) {

```



```

        CP x=a[j+k], y=w*a[j+k+i];
        a[j+k]=x+y; a[j+k+i]=x-y;
    }
}
}
if(op== -1)
    for(int i=0; i<n; ++i) a[i].x/=n;
}

int main()
{
    scanf("%s%s", s1, s2);
    len1=strlen(s1);
    len2=strlen(s2);
    for(int i=0; i<len1; ++i) a[i].x=s1[len1-1-i]-'0';
    for(int i=0; i<len2; ++i) b[i].x=s2[len2-1-i]-'0';
    while((1<<bit)<len1+len2) bit++;
    for(int i=0; i<(1<<bit); ++i)
        pos[i]=(pos[i>>1]>>1)|((i&1)<<(bit-1));
    fft(a, 1<<bit, 1), fft(b, 1<<bit, 1);
    for(int i=0; i<(1<<bit); ++i) a[i]=a[i]*b[i];
    fft(a, 1<<bit, -1);
    for(int i=0; i<(1<<bit); ++i) {
        ans[i]+=(int)(a[i].x+0.5);
        ans[i+1]+=ans[i]/10;
        ans[i]%=10;
    }
    for(len=len1+len2; !ans[len] && len>=1; len--);
    for(int i=len; i>=0; i--) printf("%d", ans[i]);
    return 0;
}

```

## 多项式运算

```

namespace Polynomial {
    int bit, pos[MAXN*4];

    void init(int n)
    {
        while(1<<bit<n) bit++;
        for(int i=0; i<1<<bit; ++i)
            pos[i]=(pos[i>>1]|(i&1)<<bit)>>1;
    }

    void ntt(vi &a, int op)
    {
        int n=a.size();
        for(int i=0; i<n; ++i)
            if(pos[i]>i) swap(a[pos[i]], a[i]);
        for(int i=1; i<n; i<=<=1) {
            int wn=qpow(op>0?G:IG, (P-1)/(i<<1));

```

```

        for(int j=0; j<n; j+=(i<<1))
            for(int k=0, w=1; k<i; ++k, w=1LL*w*wn%P) {
                int x=a[j+k], y=1LL*a[i+j+k]*w%P;
                a[j+k]=add(x, y), a[i+j+k]=sub(x, y);
            }
    }
    if(op== -1) {
        int inv=qpow(n, P-2);
        for(int i=0; i<n; ++i) a[i]=1LL*a[i]*inv%P;
    }
}

vi mul(vi a, vi b)
{
    int n=a.size(), m=b.size();
    init(n+m-1);
    a.resize(1<<bit); b.resize(1<<bit);
    ntt(a, 1); ntt(b, 1);
    for(int i=0; i<1<<bit; ++i) a[i]=1LL*a[i]*b[i]%P;
    ntt(a, -1);
    a.resize(n+m-1);
    return a;
}

vi inv(vi a)
{
    int n=a.size(), m=(a.size()+1)>>1;
    if(n==1) return {qpow(a[0], P-2)};
    vi b=inv(vi(a.begin(), a.begin()+m));
    init(n<<1);
    a.resize(1<<bit); b.resize(1<<bit);
    ntt(a, 1); ntt(b, 1);
    for(int i=0; i<1<<bit; ++i)
        b[i]=1LL*(2+P-1LL*a[i]*b[i]%P)*b[i]%P;
    ntt(b, -1);
    b.resize(n);
    return b;
}

vi div(vi a, vi b)
{
    int n=a.size(), m=b.size();
    reverse(a.begin(), a.end());
    reverse(b.begin(), b.end());
    a.resize(n-m+1); b.resize(n-m+1);
    b=inv(b); a=mul(a, b);
    a.resize(n-m+1);
    reverse(a.begin(), a.end());
    return a;
}

vi mod(vi a, vi b)

```

```

{
    int n=a.size(), m=b.size();
    vi c=div(a, b);
    b=mul(b, c);
    a.resize(m-1);
    for(int i=0; i<m-1; ++i) a[i]=sub(a[i], b[i]);
    return a;
}

vi deriv(vi a)
{
    int n=a.size();
    for(int i=0; i<n-1; ++i) a[i]=1LL*a[i+1]*(i+1)%P;
    a.resize(n-1);
    return a;
}

vi integ(vi a, int c=0)
{
    int n=a.size();
    a.resize(n+1);
    vi inv(n+1);
    inv[0]=inv[1]=1;
    for(int i=2; i<=n; ++i) inv[i]=1LL*inv[P%i]*(P-P/i)%P;
    for(int i=n; i>=1; --i) a[i]=1LL*a[i-1]*inv[i]%P;
    a[0]=c;
    return a;
}

vi ln(vi a)
{
    int n=a.size();
    a=mul(inv(a), deriv(a));
    a.resize(n-1);
    return integ(a);
}

vi exp(vi a)
{
    int n=a.size(), m=(a.size()+1)>>1;
    if(n==1) return {1};
    vi b=Polynomial::exp(vi(a.begin(), a.begin()+m));
    b.resize(n);
    vi c=ln(b);
    init(n<<1);
    b.resize(1<<bit), c.resize(1<<bit);
    for(int i=0; i<n; ++i) c[i]=sub(a[i], c[i]);
    c[0]=add(c[0], 1);
    ntt(b, 1), ntt(c, 1);
    for(int i=0; i<1<<bit; ++i) b[i]=1LL*b[i]*c[i]%P;
    ntt(b, -1);
    b.resize(n);
}

```

```

        return b;
    }

vi pow(vi a, int k)
{
    int n=a.size();
    vi b=ln(a);
    for(int i=0; i<n; ++i) b[i]=1LL*b[i]*k%P;
    b=exp(b);
    return b;
}
}

```

## 杜教筛

```

int T, n, tot, vis[MAXN], pri[MAXN];
LL mu[MAXN], phi[MAXN];

unordered_map<int, LL> muf, phif;

void init()
{
    mu[1]=phi[1]=1;
    for(int i=2; i<=5e6; ++i)
    {
        if(!vis[i]) pri[++tot]=i, phi[i]=i-1, mu[i]=-1;
        for(int j=1; j<=tot && pri[j]*i<=5e6; ++j)
        {
            vis[pri[j]*i]=1;
            if(i%pri[j]==0)
            {
                phi[i*pri[j]]=phi[i]*pri[j];
                mu[i*pri[j]]=0;
                break;
            }
            phi[i*pri[j]]=phi[i]*(pri[j]-1);
            mu[i*pri[j]]=-mu[i];
        }
    }
    for(int i=1; i<=5e6; ++i) phi[i]+=phi[i-1], mu[i]+=mu[i-1];
}

LL mus(int x)
{
    if(x<=5e6) return mu[x];
    if(muf[x]) return muf[x];
    LL sum=1;
    for(int l=2, r; l<=x; l=r+1)
    {
        r=x/(x/l);
        sum-=1LL*(r-l+1)*mus(x/l);
    }
}

```

```

    }
    muf[x]=sum;
    return sum;
}

LL phis(int x)
{
    if(x<=5e6) return phi[x];
    if(phif[x]) return phif[x];
    LL sum=1LL*x*(x+1)/2;
    for(int l=2, r; l<=x; l=r+1)
    {
        r=x/(x/l);
        sum-=1LL*(r-l+1)*phis(x/l);
    }
    phif[x]=sum;
    return sum;
}

int main()
{
    init();
    scanf("%d", &T);
    while(T--)
    {
        scanf("%d", &n);
        printf("%lld %lld\n", phis(n), mus(n));
    }
    return 0;
}

```

## 拉格朗日插值

```

int T, n, k, ifac[MAXN];

int interp1(vi x, vi y, int k)
{
    int n=x.size(), ans=0;
    for(int i=0; i<n; ++i) {
        int up=y[i], down=1;
        for(int j=0; j<n; ++j) {
            if(i==j) continue;
            up=1LL*up*sub(k, x[j])%P;
            down=1LL*down*sub(x[i], x[j])%P;
        }
        ans=(ans+1LL*up*qpow(down))%P;
    }
    return ans;
}

int interp2(vi y, int k)

```

```

{
    int n=y.size(), ans=0;
    vi pre(n), suf(n);
    pre[0]=suf[n-1]=1;
    for(int i=0; i<n-1; ++i) pre[i+1]=1LL*pre[i]*sub(k, i)%P;
    for(int i=n-1; i>=1; --i) suf[i-1]=1LL*suf[i]*sub(k, i)%P;
    for(int i=0; i<n; ++i) {
        int up=1LL*y[i]*pre[i]%P*suf[i]%P;
        int down=1LL*ifac[i]*((n-i)&1?ifac[n-i-1]:P-ifac[n-i-1])%P;
        ans=(ans+1LL*up*down)%P;
    }
    return ans;
}

int main()
{
    init(2001);
    scanf("%d", &T);
    while(T--) {
        scanf("%d%d", &n, &k); //k>=0 && k<P
        vi x(n+1), y(n+1);
        for(int i=0; i<=n; ++i) scanf("%d%d", &x[i], &y[i]);
        printf("%d\n", interp1(x, y, k));
    }
    return 0;
}

```

## 线性递推

```

struct Mod {
    ll x;
    Mod(ll x = 0): x(x>=0?x%md:x%md+md) {}
    Mod operator+(Mod b) {ll y=x+b.x;return y<md ? y : y - md; }
    Mod operator-(Mod b) { return x - b.x + (x < b.x ? md : 0); }
    Mod operator * (Mod b) { return x * b.x % md; }
    void operator += (Mod b) { x += b.x; x < md ? : x -= md; }
    void operator *= (Mod b) { (x *= b.x) %= md; }
    void operator -= (Mod b) { x -= b.x; -x < 0 ? : x += md; }
};

int main() {
    constexpr ll root = 62, LIM = 1 << 17;
    auto mdPow = [&](Mod b, ll e) {
        Mod res = 1;
        for (; e; b *= b, e /= 2)
            if (e & 1) res *= b;
        return res;
    };
    auto mdInv = [&] {
        vc<ll> inv(LIM); inv[1] = 1;
        for (int i = 2; i < LIM; ++i)

```

```

    inv[i] = md - (md / i) * inv[md % i] % md;
    return inv;
};

auto iv = mdInv();
auto ntt = [&](vc<Mod> &a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vc<Mod> rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, ++s) {
        rt.resize(n);
        array<Mod, 2> z{1, mdPow(root, md >> s)};
        for (int i = k; i < k * 2; ++i)
            rt[i] = rt[i / 2] * z[i & 1];
    }
    vc<int> rev(n);
    for (int i = 0; i < n; ++i)
        rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    for (int i = 0; i < n; ++i)
        if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += k * 2) {
            auto it1 = &a[i], it2 = it1 + k;
            for (int j = 0; j < k; ++j, ++it1, ++it2) {
                Mod z = rt[j + k] * *it2;
                *it2 = *it1 - z, *it1 += z;
            }
        }
};

auto conv = [&](vc<Mod> a, vc<Mod> b) -> vc<Mod> {
    if (a.empty() || b.empty()) return {};
    int s = sz(a) + sz(b) - 1,
        n = 1 << (32 - __builtin_clz(s - 1));
    Mod inv = md - (md - 1) / n;
    vc<Mod> out(n);
    a.resize(n), b.resize(n);
    ntt(a), ntt(b);
    for (int i = 0; i < n; ++i)
        out[-i & (n - 1)] = a[i] * b[i] * inv;
    ntt(out);
    return {out.begin(), out.begin() + s};
};

auto invIter = [&](vc<Mod> &a, vc<Mod> &in, vc<Mod> &b) {
    int n = sz(in);
    vc<Mod> out(n);
    copy(a.begin(), a.begin() + min(sz(a), n), out.begin());
    auto conv = [&] {
        ntt(out);
        for (int i = 0; i < n; ++i) out[i] *= in[i];
        ntt(out), reverse(out.begin() + 1, out.end());
    };
    conv(), fill(out.begin(), out.begin() + sz(b), 0), conv();
    b.resize(n);
    Mod inv = md - (md - 1) / n; inv *= inv;
};

```

```

    for (int i = n / 2; i < n; ++i)
        b[i] = out[i].x ? inv * (md - out[i].x) : 0;
};

auto polyInv = [&](vc<Mod> a) -> vc<Mod> {
    if (a.empty()) return {};
    vc<Mod> b{mdPow(a[0], md - 2)};
    b.reserve(sz(a));
    while (sz(b) < sz(a)) {
        vc<Mod> in(sz(b) * 2);
        copy(all(b), in.begin()), ntt(in);
        invIter(a, in, b);
    }
    return {b.begin(), b.begin() + sz(a)};
};

auto polyMod = [&](vc<Mod> a, vc<Mod> b) -> vc<Mod> {
    if (sz(a) < sz(b)) return a;
    int n = sz(a) - sz(b) + 1;
    vc<Mod> da(a.rbegin(), a.rend()), db(b.rbegin(), b.rend());
    da.resize(n), db.resize(n);
    da = conv(da, polyInv(db));
    da.resize(n), reverse(all(da));
    auto c = conv(da, b);
    a.resize(sz(b) - 1);
    for (int i = 0; i < sz(a); ++i) a[i] -= c[i];
    return a;
};

auto deri = [&](vc<Mod> a) {
    for (int i = 1; i < sz(a); ++i) a[i - 1] = a[i] * i;
    a.pop_back();
    return a;
};

auto inte = [&](vc<Mod> a) {
    for (int i = sz(a) - 1; i >= 1; --i)
        a[i] = a[i - 1] * iv[i];
    a[0] = 0;
    return a;
};

auto polyLn = [&](vc<Mod> &a) -> vc<Mod> {
    if (a.empty()) return {};
    int n = 1 << (32 - __builtin_clz(2 * sz(a) - 2));
    Mod inv = md - (md - 1) / n;
    vc<Mod> b = polyInv(a), c = deri(a);
    b.resize(n), c.resize(n);
    ntt(b), ntt(c);
    for (int i = 0; i < n; ++i) b[i] = b[i] * c[i] * inv;
    ntt(b), reverse(b.begin() + 1, b.end());
    b = inte(b);
    return {b.begin(), b.begin() + sz(a)};
};

auto polyExp = [&](vc<Mod> &a) -> vc<Mod> {
    if (a.empty()) return {};
    vc<Mod> b{1}, ib{1};

```



```

b.reserve(sz(a)), ib.reserve(sz(a));
auto conv = [&](vc<Mod> &a, vc<Mod> &b) {
    ntt(a);
    for (int i = 0; i < sz(a); ++i) a[i] *= b[i];
    ntt(a), reverse(a.begin() + 1, a.end());
};
while (sz(b) < sz(a)) {
    int h = sz(b), n = h * 2;
    Mod inv = md - (md - 1) / n;
    vc<Mod> db(n), dib(n), A(deri(b)), B(n);
    copy(all(ib), dib.begin()), ntt(dib);
    copy(all(b), db.begin()), ntt(db);
    A.resize(n), conv(A, dib);
    for (int i = 0; i < n; ++i) B[i] = db[i] * dib[i];
    ntt(B), reverse(B.begin() + 1, B.end());
    fill(B.begin(), B.begin() + h, 0);
    vc<Mod> da(deri(vc<Mod>(a.begin(), a.begin() + h)));
    da.resize(n), ntt(da), conv(B, da);
    for (int i = min(n, sz(a)) - 1; i >= h; --i)
        A[i] = (A[i - 1] - B[i - 1] * inv) * iv[i] * inv - a[i];
    fill(A.begin(), A.begin() + h, 0), conv(A, db);
    b.resize(n);
    for (int i = h; i < n; ++i)
        b[i] = A[i].x ? inv * (md - A[i].x) : 0;
    if (sz(b) < sz(a)) invIter(b, dib, ib);
}
return {b.begin(), b.begin() + sz(a)};
};

auto polyPow = [&](vc<Mod> &a, ll k) {
    auto b = polyLn(a);
    for (Mod &e: b) e *= k;
    return polyExp(b);
};

cin.tie(0)->sync_with_stdio(0);
int m, k;
cin>>m>>k;
vc<Mod> a(k), b(k), c(k), f(k+1);
f[0]=1;
for(int i=1, tmp; i<=k; ++i) {cin>>tmp; f[i]=-tmp;}
for(int i=0, tmp; i<k; ++i) {cin>>tmp; c[i]=tmp;}
vc<Mod> g=polyInv(vc<Mod>(f.begin(), f.begin()+k-1));
reverse(all(f));
int s=2*k-1, n=1<<(32-__builtin_clz(s-1));
g.resize(n), f.resize(n);
ntt(g), ntt(f);

auto combine = [&](vc<Mod> a, vc<Mod> b) -> vc<Mod> {
    Mod inv = md - (md - 1) / n;
    vc<Mod> c(n), d(n);
    a=conv(a, b);
    copy(a.rbegin(), a.rbegin()+k-1, c.begin());

```

```

    ntt(c);
    for (int i=0; i<n; ++i) d[-i&(n-1)]=c[i]*g[i]*inv;
    ntt(d);
    copy(d.rend()-k+1, d.rend(), c.begin());
    fill(c.begin()+k-1, c.end(), 0);
    ntt(c);
    for (int i=0; i<n; ++i) d[-i&(n-1)]=c[i]*f[i]*inv;
    ntt(d);
    for(int i=0; i<k; ++i) a[i]-=d[i];
    return {a.begin(), a.begin()+k};
};

a[1]=1, b[0]=1;
for(; m; m>>=1) {
    if(m&1) b=combine(a, b);
    a=combine(a, a);
}
Mod ans;
for(int i=0; i<k; ++i) ans+=c[i]*b[i];
cout<<ans.x;
}

```

## 单位根反演

$$\omega_a^b = g^{(P-1) \cdot b/a}$$

$$[n|a] = \frac{1}{n} \sum_{k=0}^{n-1} \omega_n^{ak}$$

$$[a \equiv b \pmod n] = \frac{1}{n} \sum_{k=0}^{n-1} \omega_n^{ak} \omega_n^{-bk}$$

## 第 K 大反演

$$k^{th}max(s) = \sum_{T \subseteq S} (-1)^{|T|-k} \binom{|T|-1}{k-1} min(T)$$

## Burnside 引理

$l$  等价类个数

$G$  置换群

$c(p)$  置换  $p$  中不动点个数

$$l = \frac{1}{G} \sum_{p \in G} c(p)$$

# 数据结构

## 树状数组

```
int n, t[MAXN];

void add(int x, int y) {for(; x<=n; x+=(x&-x)) t[x]+=y;} //add(x, y) 位置 x 加 y

int sum(int x) {int y=0; for(; x; x-=(x&-x)) y+=t[x]; return y;} //sum(x) 1~x 区间和
```

## 线段树

```
int n, m, val[MAXN*4], tag[MAXN*4];

void up(int root)
{
    val[root]=val[ls]+val[rs];
}

void down(int root, int l, int r)
{
    if(!tag[root]) return;
    tag[ls]+=tag[root];
    tag[rs]+=tag[root];
    val[ls]+=tag[root]*(mid-l+1);
    val[rs]+=tag[root]*(r-mid);
    tag[root]=0;
}

void build(int root, int l, int r)
{
    if(l==r)
    {
        scanf("%d", &val[root]);
        return ;
    }
    build(ls, l, mid);
    build(rs, mid+1, r);
    up(root);
}

void add(int root, int l, int r, int x, int y, int k)
{
    if(x>r || y<l) return;
    if(l>=x && r<=y)
    {
        val[root]+=k*(r-l+1);
        tag[root]+=k;
        return;
    }
}
```

```

    down(root, l, r);
    add(ls, l, mid, x, y, k);
    add(rs, mid+1, r, x, y, k);
    up(root);
}

int query(int root, int l, int r, int x, int y)
{
    if(l>y || r<x) return 0;
    if(l>=x && r<=y) return val[root];
    down(root, l, r);
    return query(ls, l, mid, x, y)+query(rs, mid+1, r, x, y);
}

```

## 主席树

```

int n, m, tot, a[MAXN], root[MAXN];

struct Node {int ls, rs, val;} t[MAXN*40];

void update(int &rt1, int rt2, int l, int r, int x)
{
    rt1=++tot;
    t[rt1]=t[rt2], t[rt1].val++;
    if(l==r) return;
    if(x<=mid) update(t[rt1].ls, t[rt2].ls, l, mid, x);
    else update(t[rt1].rs, t[rt2].rs, mid+1, r, x);
}

int query(int rt1, int rt2, int l, int r, int k)
{
    if(l==r) return l;
    int temp=t[t[rt2].ls].val-t[t[rt1].ls].val;
    if(temp>=k) return query(t[rt1].ls, t[rt2].ls, l, mid, k);
    else return query(t[rt1].rs, t[rt2].rs, mid+1, r, k-temp);
}

int main()
{
    scanf("%d%d", &n, &m);
    for(int i=1; i<=n; ++i)
    {
        scanf("%d", &a[i]);
        update(root[i], root[i-1], 1, n, a[i]);
    }
    for(int i=1; i<=m; i++)
    {
        //query(root[x-1], root[y], 1, n, k) 区间 x~y 第k大
    }
}

```

## 莫队

```
int n, q, ans[MAXN], bol[MAXN];

struct Q {int l, r, id;} a[MAXN];

bool CMP(Q x, Q y)
{
    if(bol[x.l]==bol[y.l]) {
        if(bol[x.l]&1) return x.r<y.r;
        else return x.r>y.r;
    }
    return x.l<y.l;
}

int main()
{
    scanf("%d", &n);
    int l=1, r=0, siz=sqrt(n);
    for(int i=1; i<=n; ++i) bol[i]=(i-1)/siz+1;
    scanf("%d", &q);
    for(int i=1; i<=q; ++i) {
        scanf("%d%d", &a[i].l, &a[i].r);
        a[i].id=i;
    }
    sort(a+1, a+q+1, CMP);
    for(int i=1; i<=q; ++i) {
        while(r<a[i].r) {
            r++;
            //update(r, 1);
        }
        while(r>a[i].r) {
            //update(r, -1);
            r--;
        }
        while(l<a[i].l) {
            //update(l, -1);
            l++;
        }
        while(l>a[i].l) {
            l--;
            //update(l, 1);
        }
        //ans[a[i].id]=query();
    }
    for(int i=1; i<=q; ++i) printf("%d\n", ans[i]);
    return 0;
}
```

# 图论

## 最短路

```
int n, m, S, dis[MAXN], vis[MAXN];

struct Node
{
    int id, dis;
    bool friend operator < (Node x, Node y)
    {
        return x.dis>y.dis;
    }
};

vector<int> g1[MAXN], g2[MAXN];
priority_queue<Node> q;

void dijkstra()
{
    memset(dis, 0x3f, sizeof(dis));
    dis[S]=0;
    q.push(Node{S, 0});
    while(!q.empty())
    {
        int x=q.top().id; q.pop();
        if(vis[x]) continue;
        vis[x]=1;
        for(int i=0; i<g1[x].size(); ++i)
        {
            int to=g1[x][i];
            if(dis[to]>dis[x]+g2[x][i])
            {
                dis[to]=dis[x]+g2[x][i];
                q.push(Node{to, dis[to]});
            }
        }
    }
}

int main()
{
    scanf("%d%d%d", &n, &m, &S);
    for(int i=1; i<=m; ++i)
    {
        int x, y, z;
        scanf("%d%d%d", &x, &y, &z);
        g1[x].pb(y), g2[x].pb(z);
    }
    dijkstra();
    //dis[i] 为 s 到 i 的最短路距离
```

```

    return 0;
}

```

## 最小生成树

```

int n, m, ans, f[MAXN];

struct Edge {int x, y, dis;} edge[MAXM];

bool CMP(Edge x, Edge y)
{
    return x.dis<y.dis;
}

int find(int x)
{
    if(f[x]!=x) f[x]=find(f[x]);
    return f[x];
}

void kruskal()
{
    for(int i=1; i<=n; ++i) f[i]=i;
    sort(edge+1, edge+m+1, CMP);
    for(int i=1; i<=m; ++i)
    {
        int fx=find(edge[i].x), fy=find(edge[i].y);
        if(fx!=fy)
        {
            ans+=edge[i].dis;
            f[fx]=fy;
        }
    }
}

int main()
{
    scanf("%d%d", &n, &m); //n 个点 m 条边
    for(int i=1; i<=m; ++i)
        scanf("%d%d%d", &edge[i].x, &edge[i].y, &edge[i].dis);
    kruskal();
    //ans 为最小生成树边权之和
    return 0;
}

```

## 最近公共祖先

```

int n, m, s, dep[MAXN], f[MAXN][20];

vector<int> g[MAXN];

```

```

void dfs(int x, int fa)
{
    f[x][0]=fa, dep[x]=dep[fa]+1;
    for(int i=0; i<g[x].size(); ++i)
    {
        int to=g[x][i];
        if(to==fa) continue;
        dep[to]=dep[x]+1;
        dfs(to, x);
    }
}

int lca(int x, int y)
{
    if(dep[x]>dep[y]) swap(x, y);
    for(int i=19; i>=0; --i)
        if(dep[f[y][i]]>=dep[x]) y=f[y][i];
    if(x==y) return x;
    for(int i=19; i>=0; --i)
        if(f[x][i]!=f[y][i]) x=f[x][i], y=f[y][i];
    return f[x][0];
}

int main()
{
    scanf("%d%d", &n, &m);
    for(int i=1; i<n; ++i)
    {
        int x, y;
        scanf("%d%d", &x, &y);
        g[x].push_back(y);
        g[y].push_back(x);
    }
    dfs(1, 0);
    for(int i=1; i<=19; ++i)
        for(int j=1; j<=n; ++j) f[j][i]=f[f[j][i-1]][i-1];
    //lca(x,y) 为 x,y 的最近公共祖先
    return 0;
}

```

## 缩点

```

int n, m, cnt, ans, tot, dfn[MAXN], low[MAXN], ins[MAXN], id[MAXN];

queue<int> q;
stack<int> sta;
vector<int> g1[MAXN], g2[MAXN];

void tarjan(int x)
{

```



```

dfn[x]=low[x]=++cnt;
sta.push(x);
ins[x]=1;
for(int i=0; i<g1[x].size(); ++i)
{
    int to=g1[x][i];
    if(!dfn[to])
    {
        tarjan(to);
        low[x]=min(low[x], low[to]);
    }
    else if(ins[to])
        low[x]=min(low[x], dfn[to]);
}
if(low[x]==dfn[x])
{
    tot++;
    while(!sta.empty() && dfn[sta.top()]>=dfn[x])
    {
        int top=sta.top(); sta.pop();
        ins[top]=0;
        id[top]=tot;
    }
}
}

int main()
{
    scanf("%d%d", &n, &m);
    for(int i=1; i<=m; ++i)
    {
        int x, y;
        scanf("%d%d", &x, &y);
        g1[x].push_back(y);
    }
    for(int i=1; i<=n; ++i)
        if(!dfn[i]) tarjan(i);
    //id[x] 为 x 所在强连通分量的编号
    return 0;
}

```

## 割点

```

int n, m, cnt, ans, dfn[MAXN], low[MAXN], val[MAXN];
vector<int> g[MAXN];

void tarjan(int x)
{
    dfn[x]=low[x]=++cnt;
    for(int i=0; i<g[x].size(); ++i)
    {

```

```

        int to=g[x][i];
        if(!dfn[to])
        {
            tarjan(to);
            low[x]=min(low[x], low[to]);
            if(low[to]>=dfn[x]) val[x]++;
        }
        else low[x]=min(low[x], dfn[to]);
    }
}

int main()
{
    scanf("%d%d", &n, &m);
    for(int i=1; i<=m; ++i)
    {
        int x, y;
        scanf("%d%d", &x, &y);
        g[x].push_back(y);
        g[y].push_back(x);
    }
    for(int i=1; i<=n; ++i)
        if(!dfn[i])
        {
            ans++;
            tarjan(i);
            if(val[i]) val[i]--;
        }
    //val[i]>0 代表 i 为割点
    return 0;
}

```

## Dinic

```

int n, m, S, T, cnt=1, head[MAXN], dis[MAXN];
LL maxflow;

struct Edge {
    int next, to;
    LL flow;
} edge[MAXM*2];

queue<int> q;

inline void addedge(int from, int to, int flow)
{
    edge[++cnt].next=head[from];
    edge[cnt].to=to;
    edge[cnt].flow=flow;
    head[from]=cnt;
}

```

```

bool bfs()
{
    for(int i=1; i<=n; ++i) dis[i]=0;
    dis[S]=1;
    q.push(S);
    while(!q.empty())
    {
        int x=q.front(); q.pop();
        for(int i=head[x]; i; i=edge[i].next)
        {
            int to=edge[i].to;
            if(dis[to] || !edge[i].flow) continue;
            dis[to]=dis[x]+1;
            q.push(to);
        }
    }
    return dis[T]>0;
}

LL dfs(int x, LL flow)
{
    if(x==T) return flow;
    LL add=0;
    for(int i=head[x]; i && flow; i=edge[i].next)
    {
        int to=edge[i].to;
        if(dis[to]!=dis[x]+1 || !edge[i].flow) continue;
        LL f=dfs(to, min(edge[i].flow, flow));
        edge[i].flow-=f, edge[i^1].flow+=f;
        add+=f; flow-=f;
    }
    if(!add) dis[x]=0;
    return add;
}

int main()
{
    scanf("%d%d%d%d", &n, &m, &S, &T);
    for(int i=1; i<=m; i++)
    {
        int u, v, w;
        scanf("%d%d%d", &u, &v, &w);
        addedge(u, v, w);
        addedge(v, u, 0);
    }
    while(bfs()) maxflow+=dfs(S, INF);
    printf("%lld\n", maxflow);
}

```

```

int n, m, S, T, cnt=1, head[MAXN];
int maxflow, mincost, vis[MAXN], dis[MAXN];

struct Edge {int next, to, flow, cost;} edge[MAXM*2];
struct Pre {int id, from;} pre[MAXN];

queue<int> q;

void addedge (int from, int to, int flow, int cost)
{
    edge[++cnt].next=head[from];
    edge[cnt].cost=cost;
    edge[cnt].flow=flow;
    edge[cnt].to=to;
    head[from]=cnt;
}

bool spfa()
{
    for(int i=0; i<=n; ++i) vis[i]=0, dis[i]=INF;
    vis[S]=1; dis[S]=0;
    q.push(S);
    while(!q.empty())
    {
        int x=q.front(); q.pop();
        vis[x]=0;
        for(int i=head[x]; i; i=edge[i].next)
        {
            int to=edge[i].to;
            if(dis[to]>dis[x]+edge[i].cost && edge[i].flow)
            {
                dis[to]=dis[x]+edge[i].cost;
                pre[to].from=x, pre[to].id=i;
                if(!vis[to])
                {
                    q.push(to);
                    vis[to]=1;
                }
            }
        }
    }
    return dis[T]<dis[0];
}

int main()
{
    scanf("%d%d%d%d", &n, &m, &S, &T);
    for(int i=1; i<=m; i++)
    {
        int u, v, w, f;
        scanf("%d%d%d%d", &u, &v, &w, &f);
    }
}

```

```

        addedge(u, v, w, f);
        addedge(v, u, 0, -f);
    }
    maxflow=0, mincost=0;
    while(spfa())
    {
        int flow=INF;
        for(int i=T; i!=S; i=pre[i].from) flow=min(flow, edge[pre[i].id].flow);
        for(int i=T; i!=S; i=pre[i].from)
        {
            edge[pre[i].id].flow-=flow;
            edge[pre[i].id^1].flow+=flow;
        }
        maxflow+=flow;
        mincost+=dis[T]*flow;
    }
    printf("%d %d\n", maxflow, mincost);
    return 0;
}

```

# 集合运算

## 枚举子集

```
int n;

int main()
{
    scanf("%d", &n);
    // 预处理
    for(int sta=1; sta<(1<<n); ++sta)
        for(int sub=sta; sub; sub=(sub-1)&sta)
        {
            // sub 为 sta 的子集
        }
    return 0;
}
```

## SOS DP

```
int n, f[MAXN], g[MAXN];

int main()
{
    scanf("%d", &n);
    for(int i=0; i<1<<n; ++i) scanf("%d%d", &f[i], &g[i]);
    for(int i=0; i<n; ++i)
        for(int sta=0; sta<1<<n; ++sta) {
            if((sta>>i)&1) f[sta]+=f[sta^(1<<i)]; //子集和
            if(!((sta>>i)&1)) g[sta]+=g[sta^(1<<i)]; //母集和
        }
    return 0;
}
```

## 快速沃尔什变换

```
void or_fwt(vi &a, int op)
{
    for(int n=sz(a), step=1; step<n; step*=2)
        for(int i=0; i<n; i+=2*step) for(int j=i; j<i+step; ++j) {
            int &u=a[j], &v=a[j+step];
            tie(u, v)=op>0?MP(add(u, v), u):MP(v, sub(u, v));
        }
}

void and_fwt(vi &a, int op)
{
    for(int n=sz(a), step=1; step<n; step*=2)
        for(int i=0; i<n; i+=2*step) for(int j=i; j<i+step; ++j) {
```

```

        int &u=a[j], &v=a[j+step];
        tie(u, v)=op>0?MP(v, add(u, v)):MP(sub(v, u), u);
    }
}

void xor_fwt(vi &a, int op)
{
    for(int n=sz(a), step=1; step<n; step*=2)
        for(int i=0; i<n; i+=2*step) for(int j=i; j<i+step; ++j) {
            int &u=a[j], &v=a[j+step];
            tie(u, v)=MP(add(u, v), sub(u, v));
        }
    if(op<0) {
        int inv=qpow(sz(a));
        for(int i=0; i<sz(a); ++i) a[i]=1LL*a[i]*inv%P;
    }
}

vi or_conv(vi a, vi b)
{
    or_fwt(a, 1), or_fwt(b, 1);
    for(int i=0; i<sz(a); ++i) a[i]=1LL*a[i]*b[i]%P;
    or_fwt(a, -1);
    return a;
}

vi and_conv(vi a, vi b)
{
    and_fwt(a, 1), and_fwt(b, 1);
    for(int i=0; i<sz(a); ++i) a[i]=1LL*a[i]*b[i]%P;
    and_fwt(a, -1);
    return a;
}

vi xor_conv(vi a, vi b)
{
    xor_fwt(a, 1), xor_fwt(b, 1);
    for(int i=0; i<sz(a); ++i) a[i]=1LL*a[i]*b[i]%P;
    xor_fwt(a, -1);
    return a;
}

```

## 子集卷积

### WC2018 州区划分

```

int n, m, p, vis[22], w[22], v[MAXN], iv[MAXN];
vi vec[22], f[22], g[22];

int dfs(int x, int sta)
{

```

```

int cnt=1, deg=0;
vis[x]=1;
for(int to: vec[x])
    if((sta>>to)&1) {
        deg++;
        if(vis[to]) continue;
        int tmp=dfs(to, sta);
        if(tmp==-1) return -1;
        else cnt+=tmp;
    }
return deg%2==0?cnt:-1;
}

int main()
{
    scanf("%d%d%d", &n, &m, &p);
    for(int i=1; i<=m; ++i) {
        int x, y;
        scanf("%d%d", &x, &y);
        vec[x-1].PB(y-1);
        vec[y-1].PB(x-1);
    }
    for(int i=0; i<n; ++i) scanf("%d", &w[i]);
    for(int i=0; i<=n; ++i) f[i].resize(1<<n), g[i].resize(1<<n);
    for(int sta=1; sta<1<<n; ++sta) {
        memset(vis, 0, sizeof(vis));
        int cnt=0;
        for(int i=0; i<n; ++i)
            if((sta>>i)&1) cnt++, v[sta]+=w[i];
        if(!p) v[sta]=1;
        else if(p==2) v[sta]=1LL*v[sta]*v[sta]%P;
        if(dfs(__builtin_ctz(sta), sta)==cnt) g[cnt][sta]=0;
        else g[cnt][sta]=v[sta];
        iv[sta]=qpow(v[sta]);
    }
    for(int i=0; i<=n; ++i) fwt(g[i], 0);
    for(int i=0; i<=n; ++i) {
        if(i==0) {
            f[0][0]=1;
            fwt(f[0], 0);
            for(int sta=0; sta<1<<n; ++sta) f[2][sta]=1LL*f[0][sta]*g[2][sta]%P;
            fwt(f[2], 1);
        } else {
            fwt(f[i], 1);
            for(int sta=0; sta<1<<n; ++sta) {
                //printf("%d %d %d\n", i, sta, f[i][sta]);
                if(__builtin_popcount(sta)==i) f[i][sta]=1LL*f[i][sta]*iv[sta]%P;
                else f[i][sta]=0;
            }
            fwt(f[i], 0);
        }
        for(int j=0; j<=n-i; ++j)

```



```
        for(int sta=0; sta<1<<n; ++sta)
            f[i+j][sta]=(f[i+j][sta]+1LL*f[i][sta]*g[j][sta])%P;

    }
    fwt(f[n], 1);
    printf("%d\n", f[n][(1<<n)-1]);
    return 0;
}
```

# 其他

## 离散化

```
int n, cnt, a[MAXN], b[MAXN], temp[MAXN*2], suba[MAXN], subb[MAXN];

int main()
{
    scanf("%d", &n);
    for(int i=1; i<=n; ++i)
    {
        scanf("%d%d", &a[i], &b[i]);
        temp[i*2-1]=a[i], temp[2*i]=b[i];
    }
    sort(temp+1, temp+2*n+1);
    cnt=unique(temp+1, temp+2*n+1)-temp-1;
    for(int i=1; i<=n; ++i)
    {
        suba[i]=lower_bound(temp+1, temp+cnt+1, a[i])-temp;
        subb[i]=lower_bound(temp+1, temp+cnt+1, b[i])-temp;
    }
    // suba subb 离散化后数组
    return 0;
}
```

## 大数运算

```
string add(string a, string b)//只限两个非负整数相加
{
    string ans;
    int na[MAXL]={0}, nb[MAXL]={0};
    int la=a.size(), lb=b.size();
    for(int i=0; i<la; i++) na[la-1-i]=a[i]-'0';
    for(int i=0; i<lb; i++) nb[lb-1-i]=b[i]-'0';
    int lmax=la>lb?la:lb;
    for(int i=0; i<lmax; i++) na[i]+=nb[i], na[i+1]+=na[i]/10, na[i]%10;
    if(na[lmax]) lmax++;
    for(int i=lmax-1; i>=0; i--) ans+=na[i]+'0';
    return ans;
}

string sub(string a, string b)//只限大的非负整数减小的非负整数
{
    string ans;
    int na[MAXL]={0}, nb[MAXL]={0};
    int la=a.size(), lb=b.size();
    for(int i=0; i<la; i++) na[la-1-i]=a[i]-'0';
    for(int i=0; i<lb; i++) nb[lb-1-i]=b[i]-'0';
    int lmax=la>lb?la:lb;
    for(int i=0; i<lmax; i++)
```

```

{
    na[i] -= nb[i];
    if(na[i] < 0) na[i] += 10, na[i+1]--;
}
while(!na[--lmax] && lmax > 0) lmax++;
for(int i = lmax - 1; i >= 0; i--) ans += na[i] + '0';
return ans;
}

string mul(string a, string b) //高精度乘法 a, b 均为非负整数
{
    string ans;
    int na[MAXL] = {0}, nb[MAXL] = {0}, nc[MAXL] = {0}, La = a.size(), Lb = b.size(); //na存储被乘数, nb存储乘数, nc存储积
    for(int i = La - 1; i >= 0; i--) na[La - i] = a[i] - '0'; //将字符串表示的大整数数组转成i整数数组表示的大整数数

    for(int i = Lb - 1; i >= 0; i--) nb[Lb - i] = b[i] - '0';
    for(int i = 1; i <= La; i++)
        for(int j = 1; j <= Lb; j++)
            nc[i + j - 1] += na[i] * nb[j]; //a的第i位乘以b的第j位为积的第i+j-1位 (先不考虑进位)
    for(int i = 1; i <= La + Lb; i++)
        nc[i + 1] += nc[i] / 10, nc[i] %= 10; //统一处理进位
    if(nc[La + Lb]) ans += nc[La + Lb] + '0'; //判断第i+j位上的数字是不是0
    for(int i = La + Lb - 1; i >= 1; i--)
        ans += nc[i] + '0'; //将整数数组转成字符串
    return ans;
}

string div(string a, int b) //高精度a除以单精度b
{
    string r, ans;
    int d = 0;
    for(int i = 0; i < a.size(); i++)
    {
        r += (d * 10 + a[i] - '0') / b + '0'; //求出商
        d = (d * 10 + (a[i] - '0')) % b; //求出余数
    }
    int p = 0;
    for(int i = 0; i < r.size(); i++)
        if(r[i] != '0') {p = i; break;}
    return r.substr(p);
}

```