Math Midterm Cheat Sheet

Formulas

Second Order Differential Equations (DE)

Formula	Description
$ay'' + by' + cy = 0$	Second order linear homogeneous DE
$r = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$	Formula to determine 'r'
$y = C_1e^{r_1x} + C_2e^{r_2x}$	General solution if $r1$ and $r2$ two simple zeros of the characteristic eq
$y = (A + Bx)Ce^{r_1x}$	General solution if $r1 = r2 = r$ is a double zero of the characteristic eq (multiplicity 2)
$y = e^{rx}(A\cos(\omega x) + B\sin(\omega x))$	General solution If $r1 = \alpha + i\omega and r2 = \alpha - i\omega(with \alpha, \omega \in \mathbb{R})$ are the complex zeros of the characteristic equation

Guidelines for finding a particular solution

- If f (t) is a polynomial in t, and r = 0 is not a solution of the characteristic polynomial, then try for yp(t) a polynomial in t of the same degree as f . If r = 0 is a solution of the characteristic equation, then try for yp(t) a polynomial of degree deg(f) + 1, if r = 0 is a zero of multiplicity 1, and a polynomial of degree deg(f) + 2 if r = 0 is a zero of multiplicity 2.
- If f (t) is of the form $f(t) = \alpha sin(\omega_0 t) + \beta cos(\omega_0 t)$, and $\pm i\omega_0$ are not the zeros of the characteristic equation, then try $yp(t) = Asin(\omega_0 t) + Bcos(\omega_0 t)$ as particular solution. If $\pm i\omega_0$ are the zeros of the characteristic equation, then try $yp(t) = Atsin(\omega_0 t) + Btcos(\omega_0 t)$ as particular solution
- If f (t) is of the form $f(t) = p(t) * e^{\alpha t}$, with p(t) a polynomial in t, and if α is not a zero of the characteristic equation, then try $yp(t) = q(t) * e^{\alpha t}$ as particular solution, with q(t) a polynomial in t of the same degree as p(t). If α is a zero of the characteristic polynomial of multiplicity k, then try $yp(t) = q(t) * t^k * e^{\alpha t}$ as particular solution, with q(t) a polynomial in t, with deg(q) = deg(p).