Math Midterm Cheat Sheet

1 Week 1

1.1 Second Order Differential Equations (DE)

Formula	Description
$ay'' + by' + cy = 0$	Second order linear homogeneous DE
$r = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$	Formula to determine 'r'
$y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$	General solution if $r1$ and $r2$ two simple zeros of the characteristic eq
$y = (A + Bx)Ce^{r_1 x}$	General solution if $r1 = r2 = r$ is a double zero of the characteristic eq (multiplicity 2)
$y = e^{rx}(A\cos(\omega x) + B\sin(\omega x))$	General solution If $r1 = \alpha + i\omega andr2 = \alpha - i\omega(with\alpha, \omega \in \mathbb{R})$ are the complex zeros of the characteristic equation

1.2 Guidelines for finding a particular solution

- If f (t) is a polynomial in t, and r = 0 is not a solution of the characteristic polynomial, then try for yp(t) a polynomial in t of the same degree as f . If r = 0 is a solution of the characteristic equation, then try for yp(t) a polynomial of degree deg(f) + 1, if r = 0 is a zero of multiplicity 1, and a polynomial of degree deg(f) + 2 if r = 0 is a zero of multiplicity 2.
- If f (t) is of the form $f(t) = \alpha sin(\omega_0 t) + \beta cos(\omega_0 t)$, and $\pm i\omega_0$ are not the zeros of the characteristic equation, then try $yp(t) = Asin(\omega_0 t) + Bcos(\omega_0 t)$ as particular solution. If $\pm i\omega_0$ are the zeros of the characteristic equation, then try $yp(t) = Atsin(\omega_0 t) + Btcos(\omega_0 t)$ as particular solution
- If f (t) is of the form $f(t) = p(t) * e^{\alpha t}$, with p(t) a polynomial in t, and if α is not a zero of the characteristic equation, then try $yp(t) = q(t) * e^{\alpha t}$ as particular solution, with q(t) a polynomial in t of the same degree as p(t). If α is a zero of the characteristic polynomial of multiplicity k, then try $yp(t) = q(t) * t^k * e^{\alpha t}$ as particular solution, with q(t) a polynomial in t, with deg(q) = deg(p).

2 Week 2

2.1 Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$

Addition: $\vec{a} + \vec{b} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}$

Scaler/multiplication: $\alpha * \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \alpha * a_1 \\ \alpha * a_2 \end{bmatrix}$

2.2 Parametric description

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \lambda \begin{bmatrix} d \\ e \\ f \end{bmatrix} + \mu \begin{bmatrix} g \\ h \\ i \end{bmatrix}$$

The vector containing a, b *and* c is the **supporting vector** and the vectors containing d, e, f and g, h, i are the **directional vectors**.

Example:

Let the following points P(2,2, 0), Q(6, 0, 1) and R(3, 3, 1) be given. Find the parametric description of the plane containing P, Q and R.

Solution:

1. Determine supporting vector: $\vec{OP} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$

2. Determine directional vectors:

$\vec{PQ} = \vec{OQ} - \vec{OP} = \begin{bmatrix} 6-2 \\ 0-2 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$

$\vec{PR} = \vec{OR} - \vec{OP} = \begin{bmatrix} 3-2 \\ 3-2 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

3. Parametric description:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$