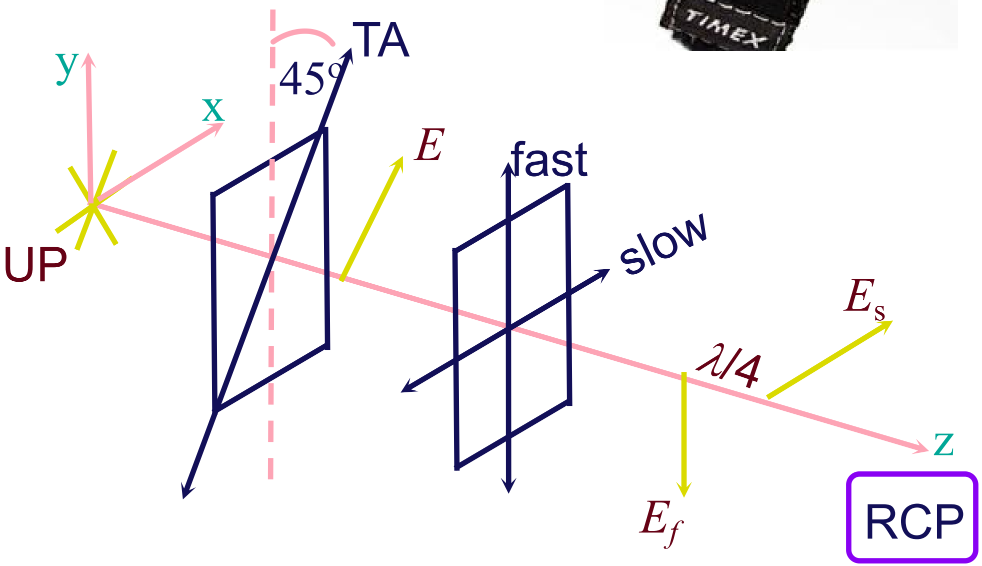
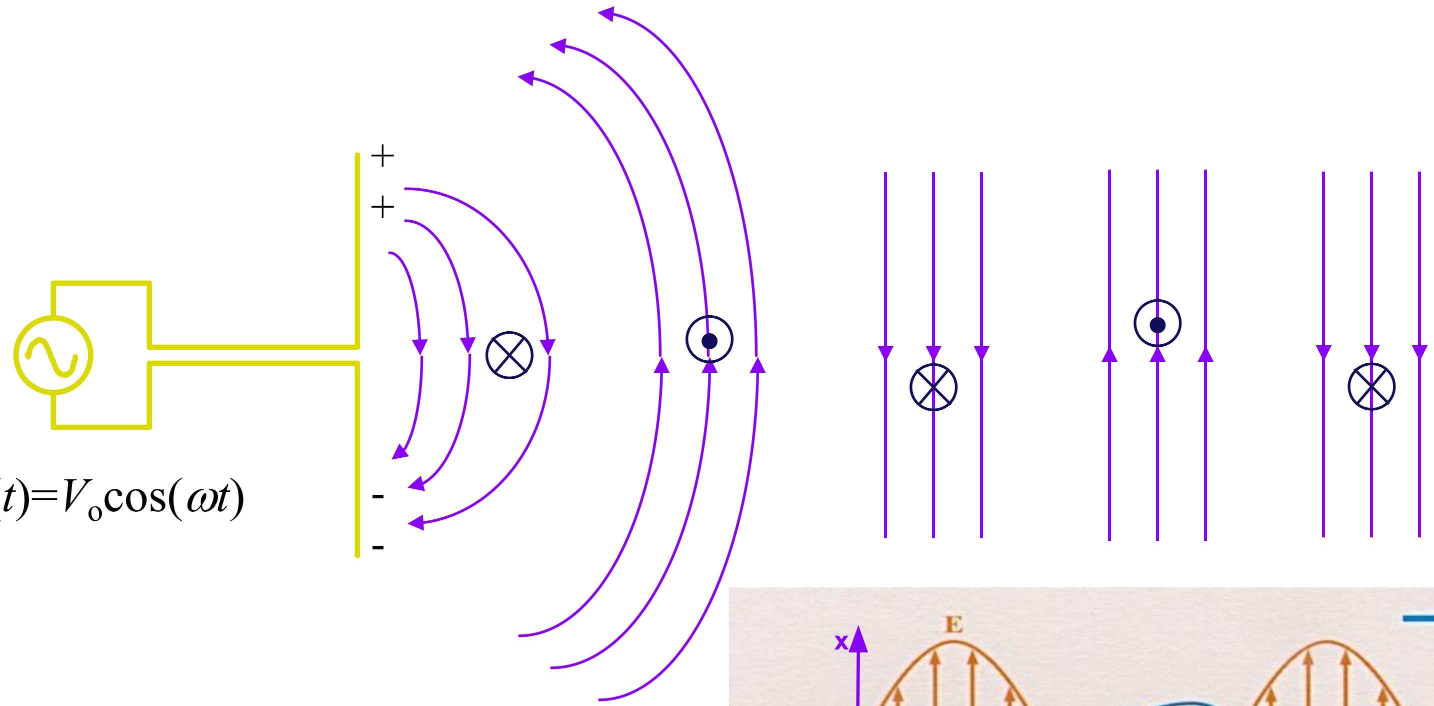
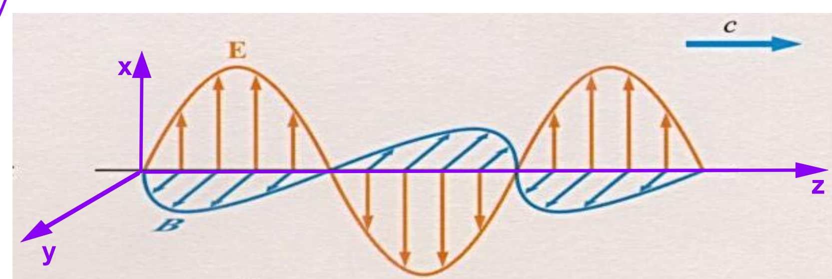
Chapter 44
Wave Optics(3)
Polarization (偏振)



Radiation from oscillating dipole

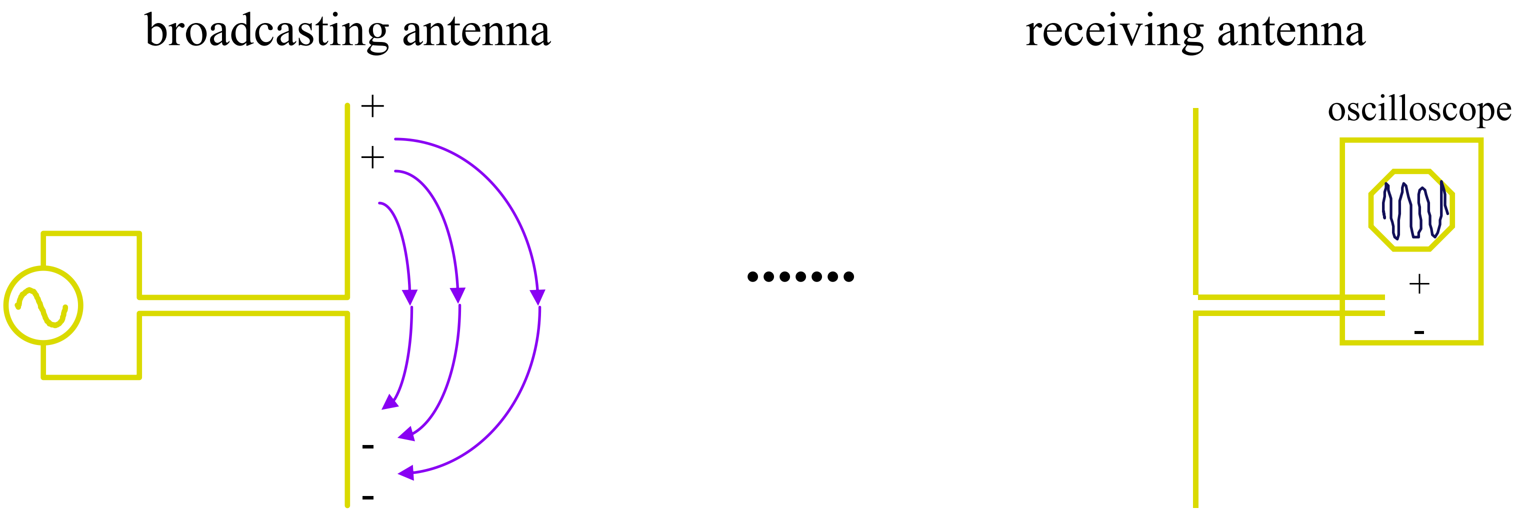


- $V(t)=V_o\cos(\omega t)$



- **Wave propagates outward at speed of light**

Demonstrate transmitting and receiving of E-M radiation

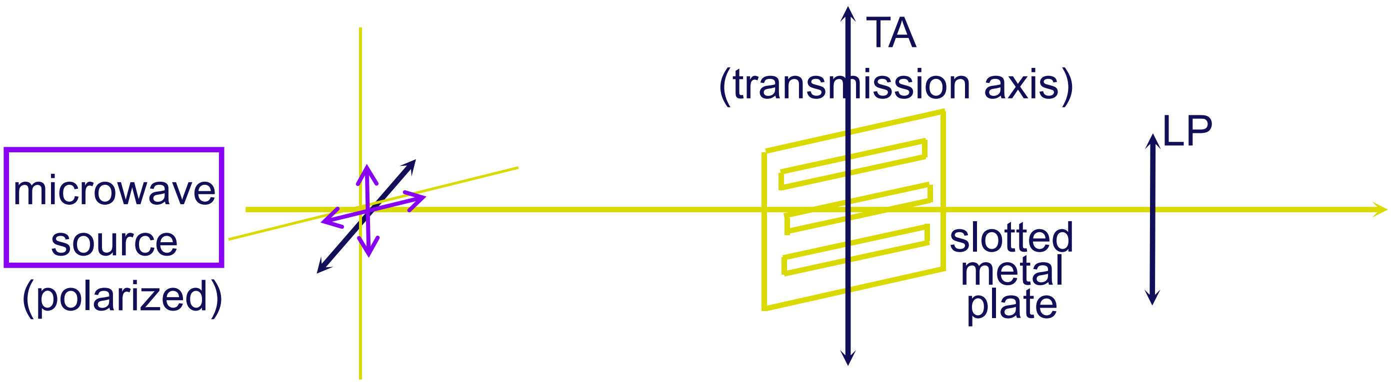


Broadcasting antenna transmits e-m wave at 200 MHz
- linearly polarized

Receiving antenna also tuned to 200 MHz
- also linearly polarized
- signal displayed on fast oscilloscope screen
- demonstrate polarization sensitivity

44-1 Linear Polarization (线偏振)

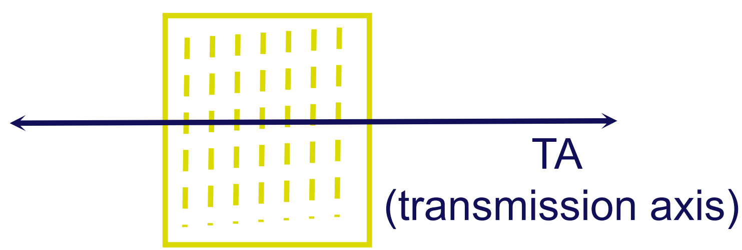
- **How else can we produce linearly polarized (LP) e-m waves?**
  - **Absorp./reflect. of vector component of wave perp to "polarizer"**



The E-field component parallel to the slots is absorbed and/or reflected.
The E-field component perpendicular to the slots is transmitted.

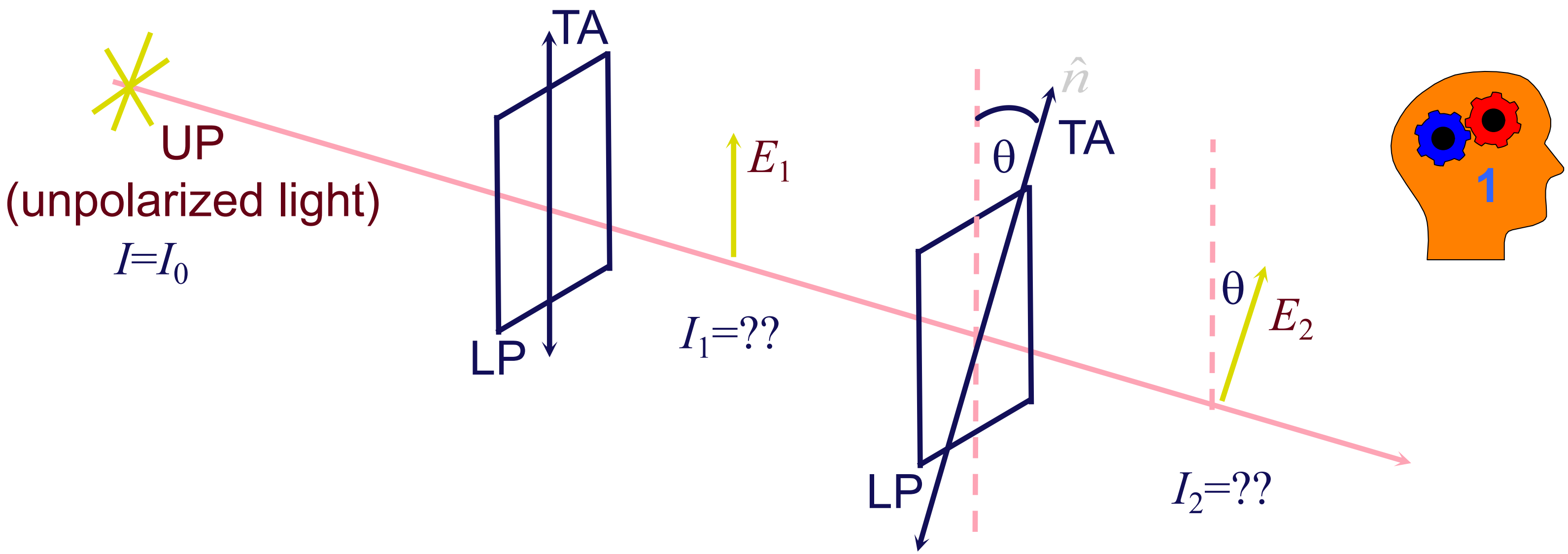
Polaroid （偏振片）(sunglasses)

Long molecules absorb E-field parallel to molecule.



- **Absorption produces LP e-m waves but in so doing it also reduces intensity of the wave (波的强度下降). How much??**

LP Intensity Reduction



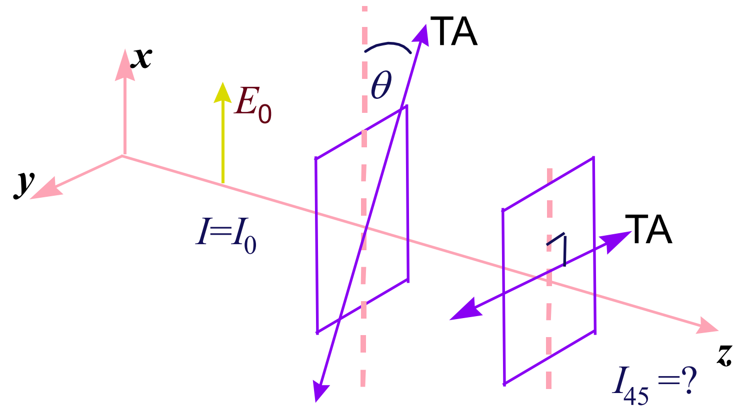
- **This set of two linear polarizers produces LP light. What is the final intensity?**
  - **First LP transmits 1/2 of the unpolarized light:** $\mathbf{I_1 = 1/2\ I_0}$
  - **Second LP projects out the $\boldsymbol{E}$-field component parallel to the TA:**

$$\vec{E}_2 = (\vec{E}_1 \bullet \hat{n})\hat{n} \qquad E_2 = E_1\cos\theta \qquad I \propto E^2 \qquad \boxed{I_2 = I_1\cos^2\theta}$$

This result is called the **Law of Malus (马隆定律)** (for LP light incident on LP)

Chapter 44, ACT 1

- **Light of intensity I_0, polarized along the x direction is incident on a set of 2 linear polarizers as shown.**

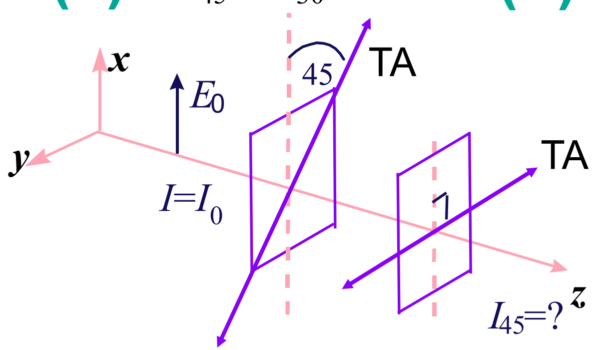


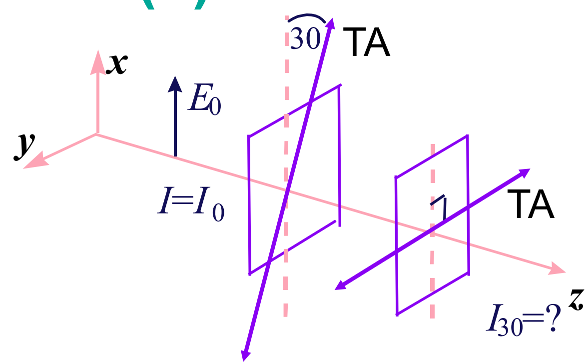
1A – **Assuming θ = 45°, what is I_{45}, the intensity at the exit of the 2 polarizers, in terms of I_0?**

(a) $I_{45} = \frac{1}{2} I_0$ **(b)** $I_{45} = \frac{1}{4} I_0$ **(c)** $I_{45} = 0$

1B • **What is the relation between I_{45} and I_{30}, the final intensities in the situation above when the angle $\theta = 45°$ and $30°$, respectively?**

(a) $I_{45} < I_{30}$ **(b)** $I_{45} = I_{30}$ **(c)** $I_{45} > I_{30}$





Chapter 44, ACT 1

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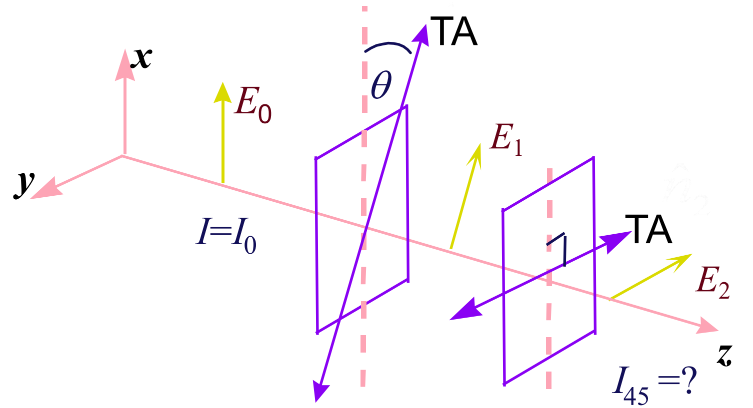

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(a) $I_{45} = \frac{1}{2} I_0$ (b) $\boxed{I_{45} = \frac{1}{4} I_0}$ (c) $I_{45} = 0$

- We proceed through each polarizer in turn.
  - The electric field following the first polarizer is: $\vec{E}_1 = \left(\vec{E}_0 \bullet \hat{n}_1\right)\hat{n}_1$
  - Therefore, $\vec{E}_1 = E_0 \cos(45^\circ)\hat{n}_1 = \frac{E_0}{\sqrt{2}}\hat{n}_1$
  - The electric field following the second polarizer is: $\vec{E}_2 = \left(\vec{E}_1 \bullet \hat{n}_2\right)\hat{n}_2$
  - Therefore, $\vec{E}_2 = E_1 \cos(45^\circ)\hat{n}_2 = \frac{E_1}{\sqrt{2}}\hat{n}_2$

Putting together, $\vec{E}_2 = \frac{E_1}{\sqrt{2}}\hat{n}_2 = \frac{E_0}{2}$ ⇒ $\frac{I_2}{I_0} = \frac{E_2^2}{E_0^2}$ ⇒ $\boxed{I_2 = \frac{1}{4} I_0}$

Chapter 44, ACT 1

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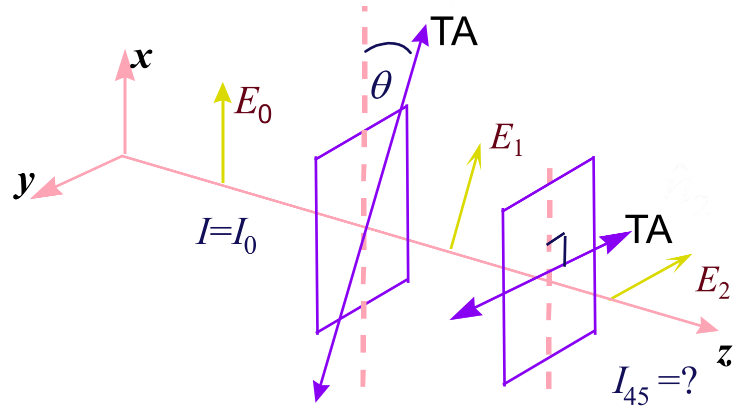

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(a) $I_{45} < I_{30}$ (b) $I_{45} = I_{30}$ (c) $\boxed{I_{45} > I_{30}}$

$$E_1 = E_0 \cos\theta$$

$$E_2 = E_1 \cos(\frac{\pi}{2} - \theta) = E_0 \cos\theta \cdot \cos(\frac{\pi}{2} - \theta)$$

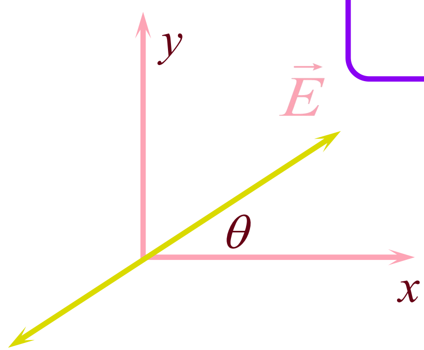
$$\therefore I_2 = I_0 \sin^2\theta \cos^2\theta$$

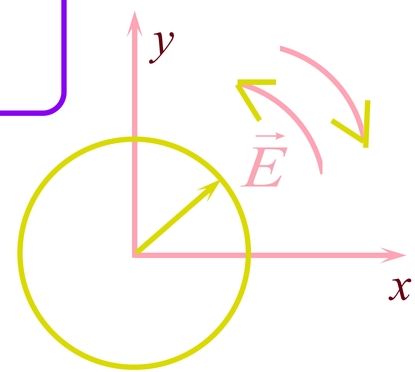
$$I_{45} = \frac{1}{4} I_0, \quad I_{30} = \frac{3}{16} I_0, \qquad \therefore I_{45} > I_{30}$$

44-2 Other Polarization States (其它偏振态)?

- **Are there polarizations other than linear?**
  - **Sure!!**
  - **The general harmonic solution for a plane wave traveling in the +z-direction is:**

$$E_x = E_{x0}\sin(kz - \omega t + \phi_x)$$
$$E_y = E_{y0}\sin(kz - \omega t + \phi_y)$$





Linear Polarization (线偏振)

$$\phi \equiv \phi_x - \phi_y = 0$$

$$\frac{E_{yo}}{E_{xo}} = \tan\theta$$

Circular Polarization (园偏振)

$$\phi \equiv \phi_x - \phi_y = \pm\frac{\pi}{2}$$

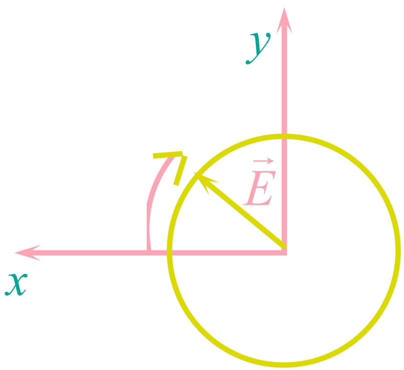
$$E_{yo} = E_{xo}$$

Circular Polarization (园偏振光)

(E_{0x} and E_{0y} are $\pm 90^\circ$ out of phase.)

- Right(右)-handed (RCP)

– Electric field spirals CW in space (at fixed time)



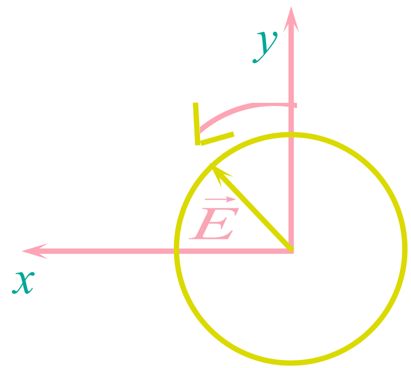
RCP = CW

$$E_x = E_0 \sin(kz - \omega t + \frac{\pi}{2})$$

$$E_y = E_0 \sin(kz - \omega t)$$

- Left(左)-handed (LCP)

– Electric field spirals CCW in space (at fixed time)



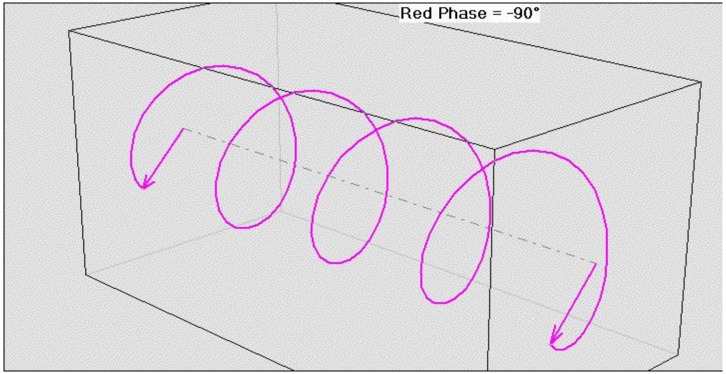
LCP = CCW

$$E_x = E_0 \sin(kz - \omega t - \frac{\pi}{2})$$

$$E_y = E_0 \sin(kz - \omega t)$$

Visualization

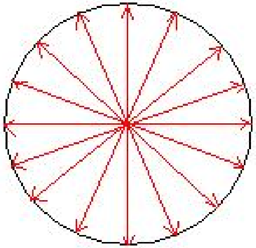
- **Why do we call this circular polarization?**

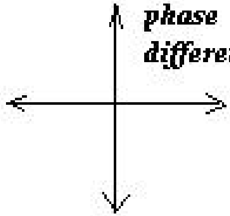


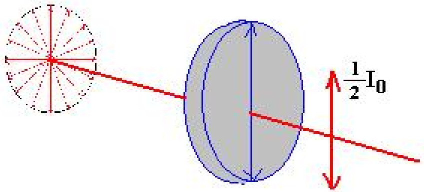
Note: If you shine circularly polarized light onto an absorber, it will in principle start to rotate $\rightarrow$ *conservation of angular momentum*!

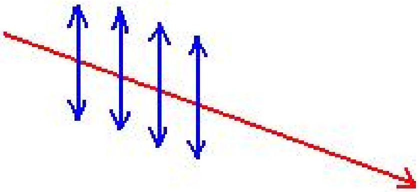
There are 5 kinds Polarization States

- **Unpolarized Light (无偏振光, 自然光)**









偏振度：$P = \dfrac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$

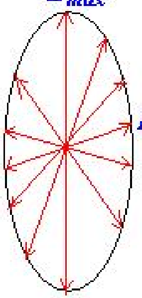
Linearly Polarized Light:

$$I_{\min} = 0, \ P = 1$$

Unpolarized light:

$$I_{\max} = I_{\min}, \quad P = 0$$

- **Linearly Polarized light(线偏振光)**
- **Partial Polarized light (部分偏振光)**



- **Circular Polarized light(圆偏振光)**

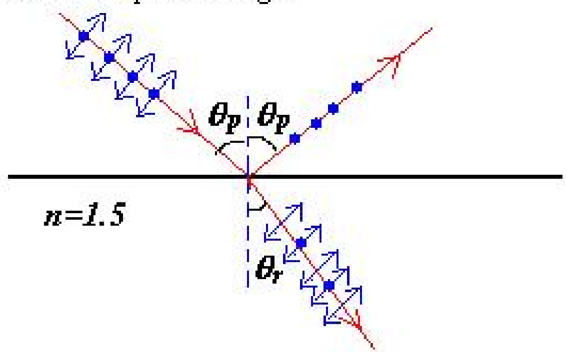
$$E_x = E_0 \sin(kz - \omega t \pm \frac{\pi}{2})$$
$$E_y = E_0 \sin(kz - \omega t)$$

- **Ellipse Polarized light(椭圆偏振光)**

$$E_x = E_1 \sin(kz - \omega t + \delta)$$
$$E_y = E_2 \sin(kz - \omega t)$$

and $E_1 \neq E_2$, or $\delta \neq \pm\dfrac{\pi}{2}$

44-3 Polarization by Reflection
(反射产生的偏振)



For glass or other dielectric materials, there is a particular angle of incidence.

Brewster's angle:
(布儒斯特角)

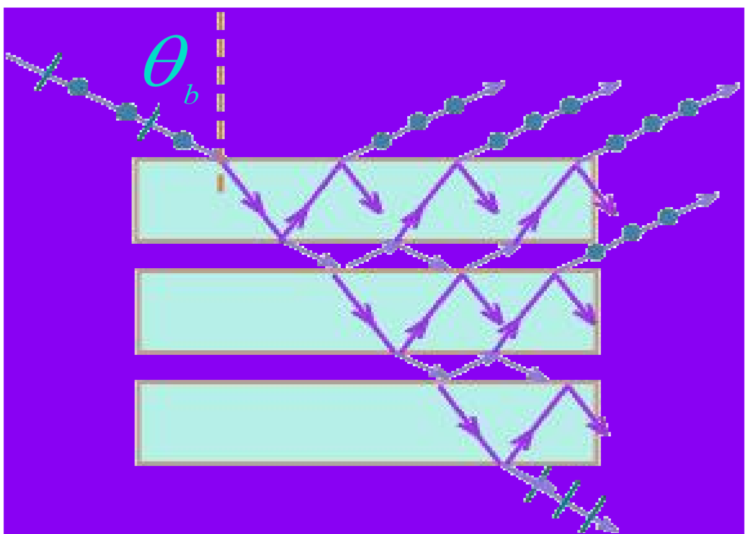
$$\begin{cases}\theta_p + \theta_r = 90^o \\ n_1 \sin\theta_p = n_2 \sin\theta_r\end{cases}$$

$$n_1 \sin\theta_p = n_2 \sin(90^0 - \theta_p) = n_2 \cos\theta_p$$

$$\therefore tg\theta_p = \frac{n_2}{n_1} \quad \text{----------Brewster's Law}$$

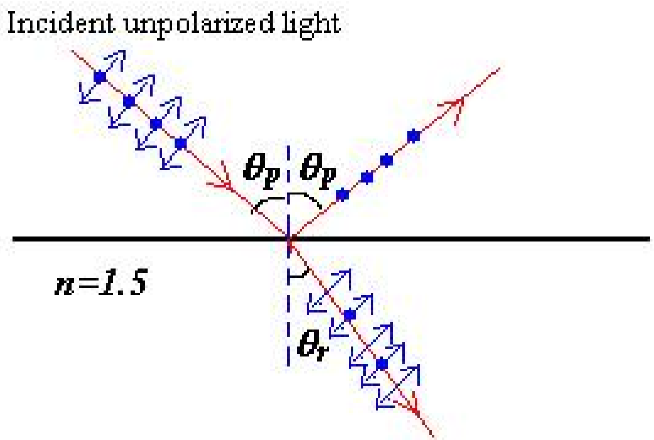
In general: $n_1 = 1, \quad \Rightarrow \quad tg\theta_p = n$

if $n = 1.5, \quad \theta_p = 56.3^o$



Stack of glass plates

Notes: From Maxwell's Equations



Brewster's angle;

Phase change on Reflection

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\bullet\vec{r}-\omega t)}$$

$$\vec{H} = \vec{H}_0 e^{i(\vec{k}\bullet\vec{r}-\omega t)}$$

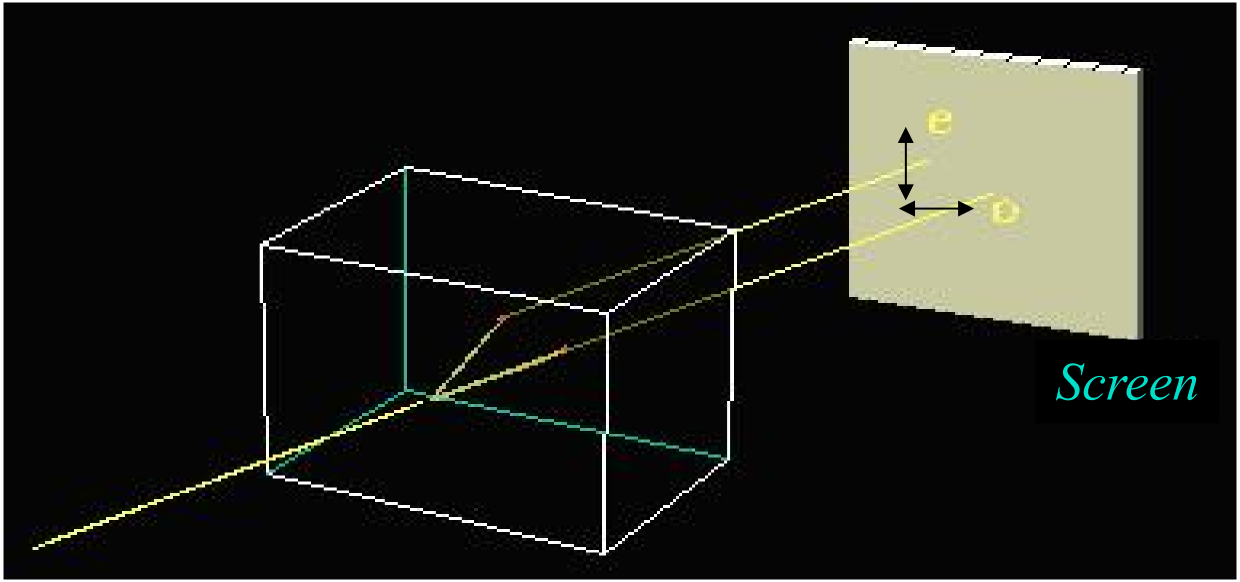
$$\begin{cases} \vec{E} \perp \vec{H} \\ \vec{k} \parallel \vec{E}\times\vec{H} \\ \sqrt{\kappa_e\varepsilon_0}E = \sqrt{\kappa_m\mu_0}H \\ k = \dfrac{2\pi}{\lambda} = \dfrac{n}{c}\omega \\ n = \sqrt{\kappa_e\kappa_m} \approx \sqrt{\kappa_e} \end{cases}$$

$$\begin{cases} D_{2n} = D_{1n} & \kappa_{e2}\varepsilon_0 E_{2n} = \kappa_{e1}\varepsilon_0 E_{1n} \\ E_{2t} = E_{1t} & \\ B_{2n} = B_{1n} & \kappa_{\mathrm{m2}}\mu_0 H_{2n} = \kappa_{\mathrm{m1}}\mu_0 H_{1n} \\ H_{2t} = H_{1t} & \end{cases}$$

44-4 Birefringence (双折射)

- Optical anisotropy of crystalline solids (光学晶体)

Calcite ($CaCO_3$)



Ordinary light (寻常光)—single index of refraction

Extraordinary light (非寻常光)—the index of refraction varies with direction of light.

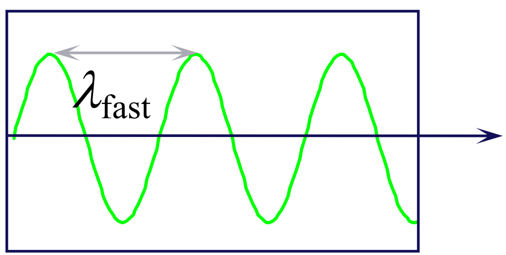
Explanation

-----Birefringence (双折射)

- Birefringent materials (crystals or stressed plastics) have the property that the speed of light is different in the two transverse dimensions.
- Since the frequency of the wave must remain constant as the wave passes through the birefringent material, the wavelength must be different in the two dimensions, which allows for a phase change.

Fast Axis:

$$v_{fast} = f \cdot \lambda_{fast}$$

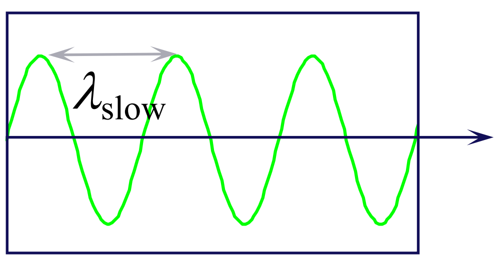


$\lambda_{slow} < \lambda_{fast}$

The relative phase change (fast vs slow) is determined by the thickness

Slow Axis:

$$v_{slow} = f \cdot \lambda_{slow}$$



Explanation (Con.)

- **The o-ray travels in the crystal with the same speed v_0 *in all directions.***

  --------- a single index of refraction n_0.

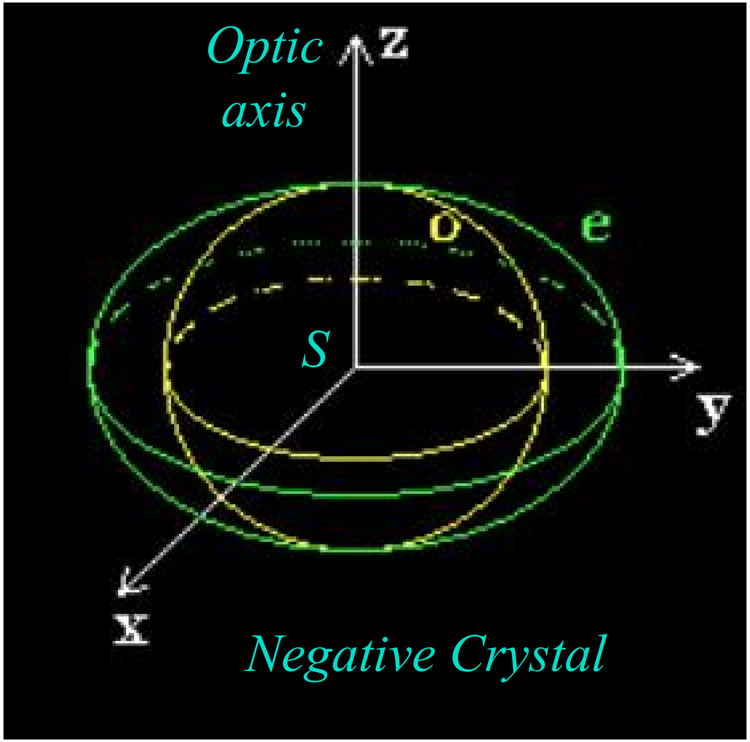
- ***The e-ray travels in the crystal with a speed that varies with the direction from v_0 to v_e.***

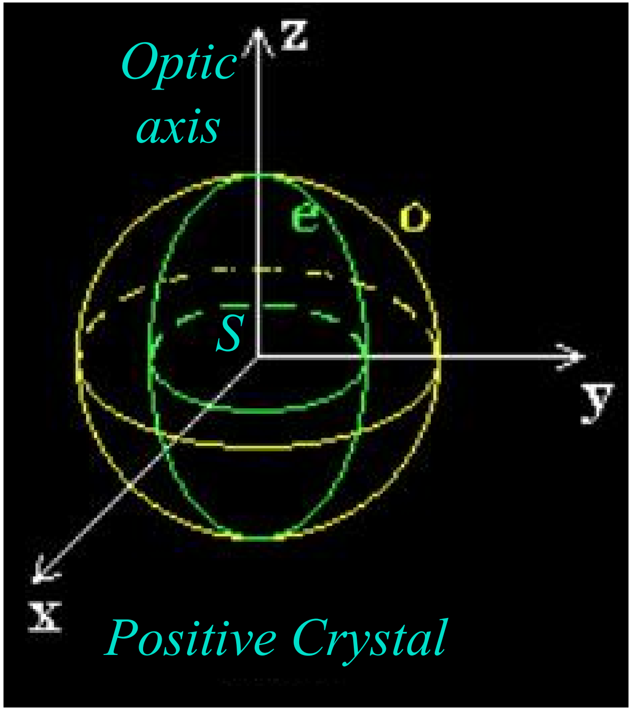
$$\Rightarrow n = \frac{c}{v} \quad \text{varies with direction from } n_0 \text{ to } n_e.$$

n_o, n_e: **The principle indices of refraction.**

(主折射率)

Explanation (Con.，光轴)





$CaCO_3$ (方解石)

$v_e > v_o$，$n_e < n_o$

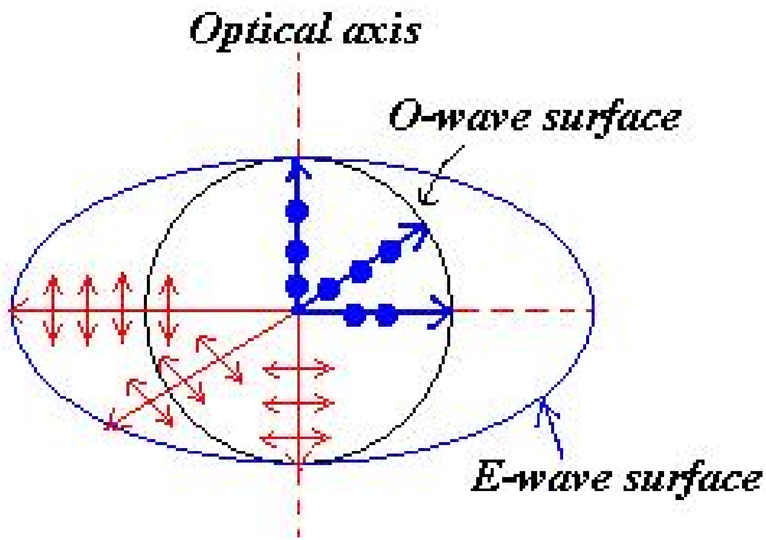
负晶体

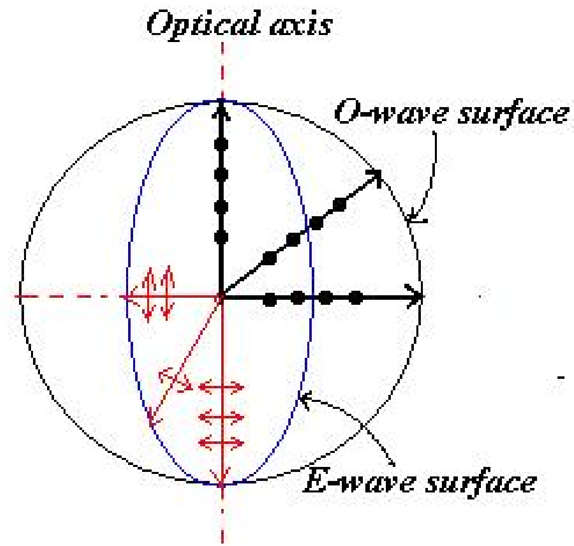
SiO_2 (石英，水晶)

$v_e < v_o$，$n_e > n_o$

正晶体

Explanation (Con. 光轴)





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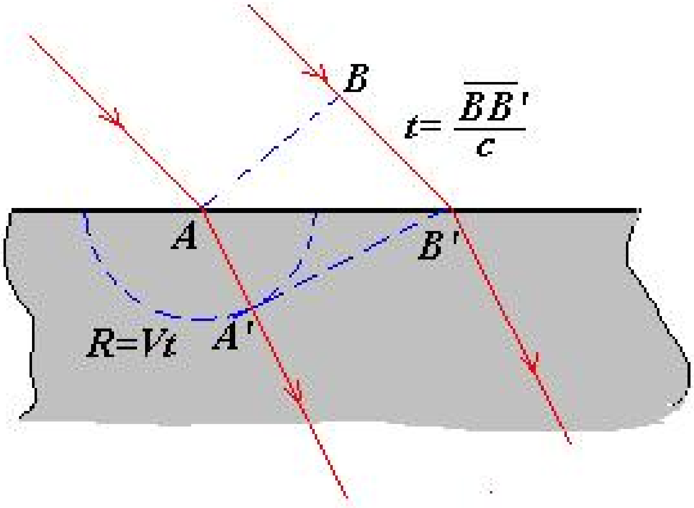
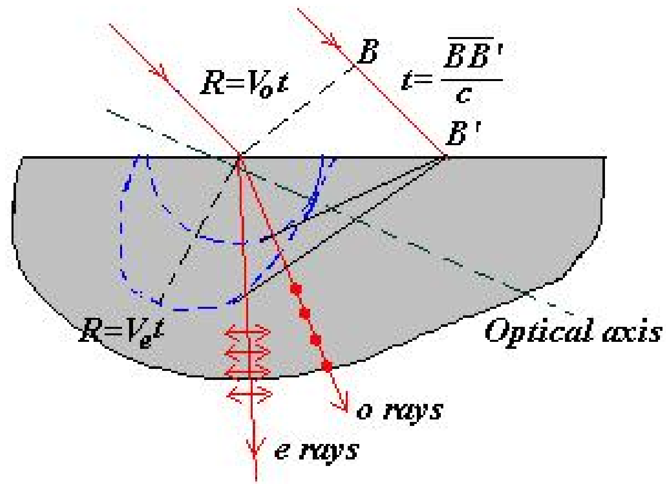
Birefringence (双折射)

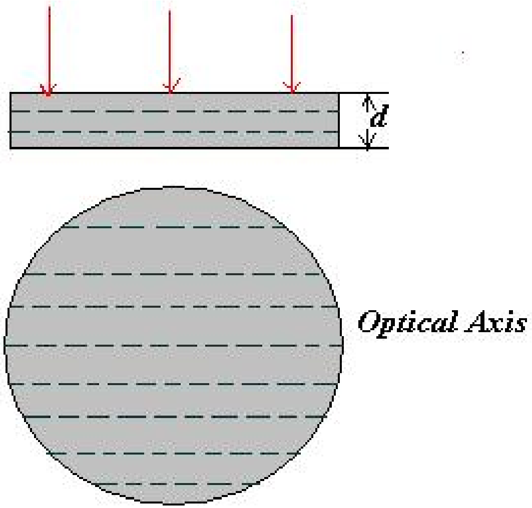


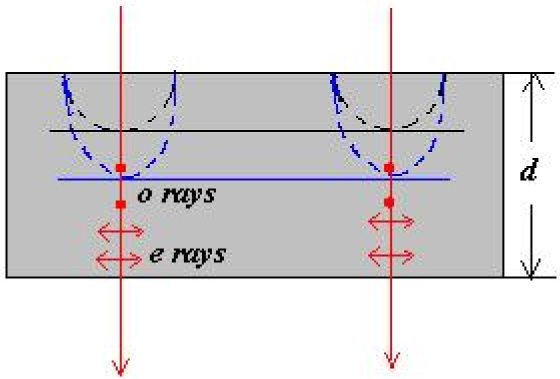


Isotropy (各向同性) Crystal

Anisotropy (各向异性) Crystal

Wave Plates (波片)





O-rays optical path: $L_o = n_o d$

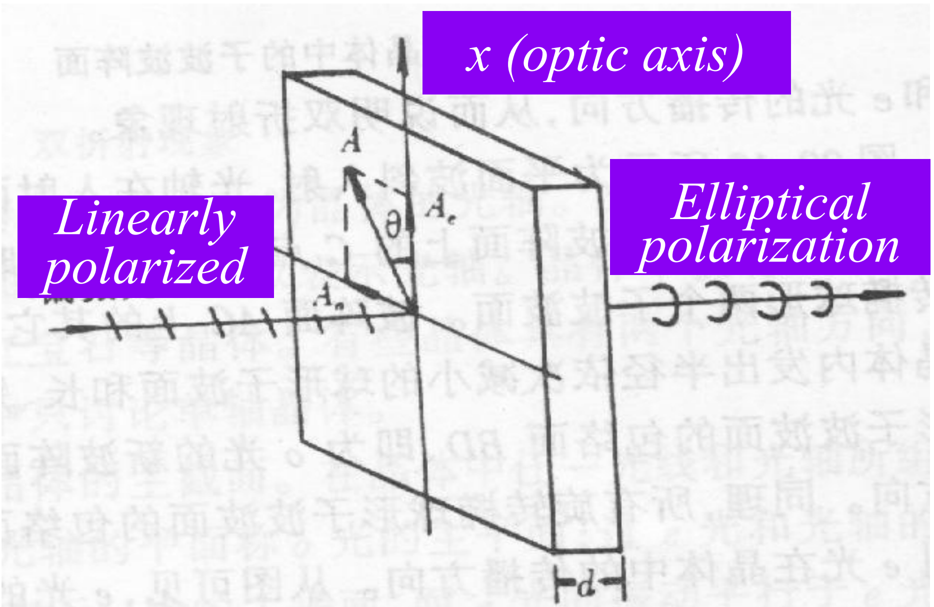
E-rays optical path: $L_e = n_e d$

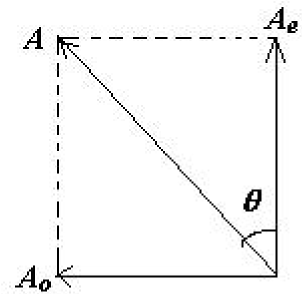
The difference of phase: $$\Delta\phi = \frac{2\pi}{\lambda}(n_o - n_e)d$$

$$\text{If } (n_o - n_e)d = \pm\frac{1}{4}\lambda, \quad \Delta\phi = \pm\frac{\pi}{2} \quad \text{Quarter - wave Plate (QWP, } \frac{1}{4}\lambda\text{片)}$$

$$\text{If } (n_o - n_e)d = \pm\frac{1}{2}\lambda, \quad \Delta\phi = \pm\pi \text{ or } 2\pi \text{ Full - wave Plate (FWP, } \frac{1}{2}\lambda\text{片)}$$

How to distinguish between circularly polarized and unpolarized?





$$A_e = A\cos\theta$$

$$A_o = A\sin\theta$$

a quarter wave plate $\dfrac{\lambda}{4}$: $\Delta\varphi = \dfrac{\pi}{2}$

If $\theta = 45^{\circ}$, Linearly polarized light $\Rightarrow$ Circularly polarized light

Example: Wave Plates (波片)

- **Birefringent crystals with precise thicknesses**

  Ex.: Crystal which produces a phase change of $\pi/2$ → "quarter wave plate" (a "full wave plate" produces a relative shift of $4\times\frac{\pi}{2}=2\pi$ → no effect).

Light polarized along the fast or slow axis merely travels through at the appropriate speed → polarization is unchanged.

Light linearly polarized at 45° to the fast or flow axis will acquire a relative phase shift between these two components → alter the state of polarization.

The phase of the component along the fast axis advances $\pi/2$ less than the phase of the component along the slow axis. E.g.,

Before QWP

$$E_x = E_0 \sin(kz-\omega t)$$
$$E_y = E_0 \sin(kz-\omega t)$$

After QWP

$$E_x = E_0 \sin(kz-\omega t)$$
$$E_y = E_0 \sin(kz-\omega t-\frac{\pi}{2})$$

Passage through QWP:

$\alpha \equiv kz - \omega t$

$\alpha_{\text{slow}} \rightarrow \alpha_{\text{slow}}$

$\alpha_{\text{fast}} \rightarrow \alpha_{\text{fast}} - \pi/2$

RCP

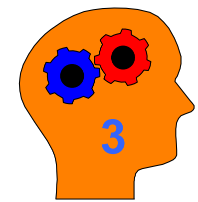
Quarter Wave Plate → circular polarization

Quarter Wave Plates

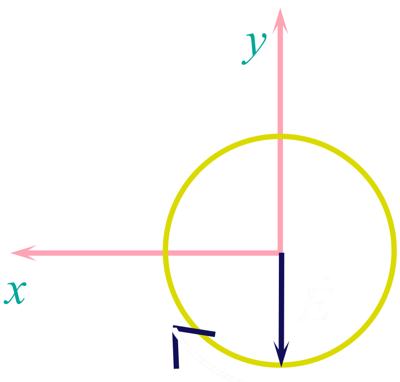
- **Light linearly polarized at 45° incident on a quarter wave plate produces the following wave after the quarter wave plate:**

Fast axis: $E_y = -E_0 \cos(kz - \omega t)$

Slow axis: $E_x = E_0 \sin(kz - \omega t)$

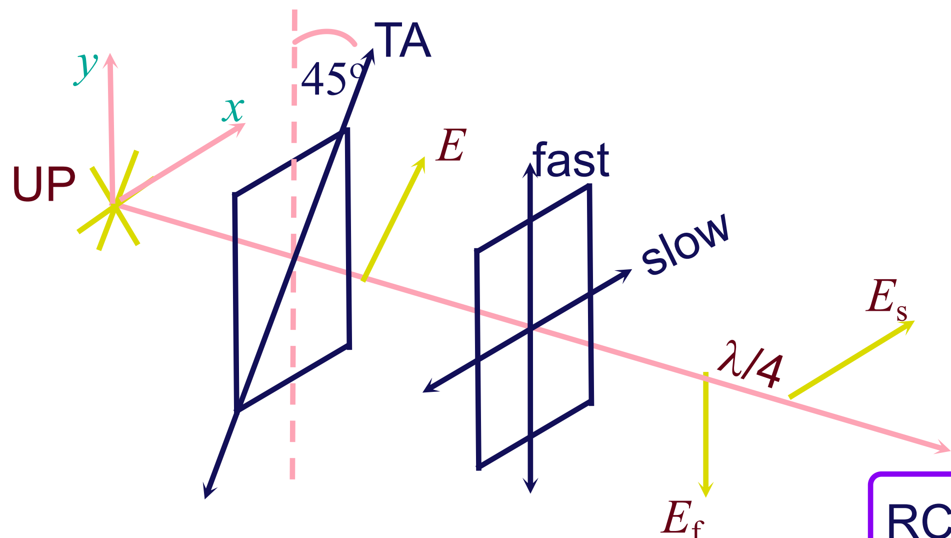


Rotation at $t=0$:



RCP = CW

Max vectors at $t=0$:

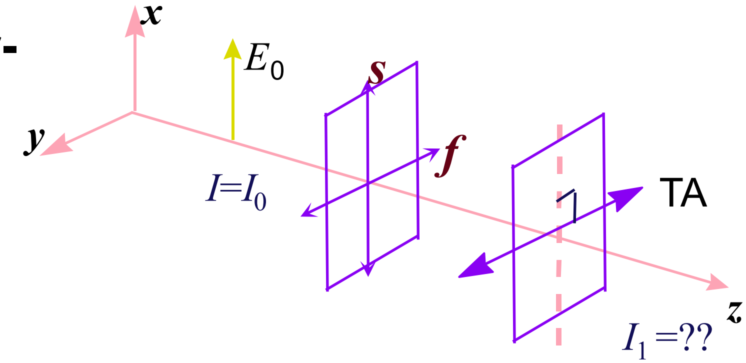


RCP

QWP: slow (no change), fast (ahead $\lambda/4$)

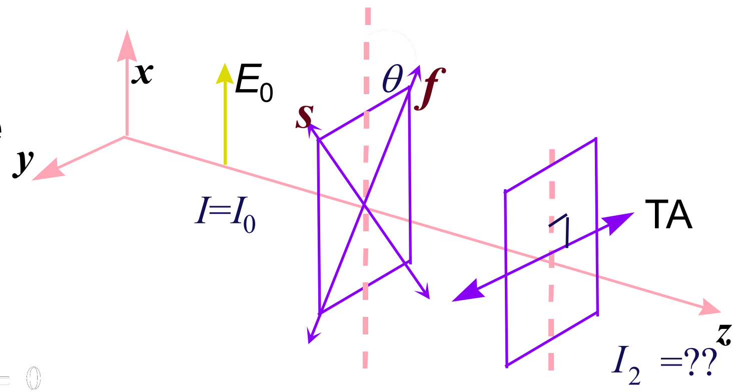
Chapter 44, ACT 2

- **Light of intensity I_0, polarized along the x direction is incident on a quarter-wave plate (fast axis= y-axis) and a linear polarizer as shown.**



3A – **What is the relation between I_1, the intensity at the exit of the system, in terms of I_0?**

(a) $I_1 = \frac{1}{2} I_0$ **(b)** $I_1 = \frac{1}{4} I_0$ **(c)** $I_1 = 0$

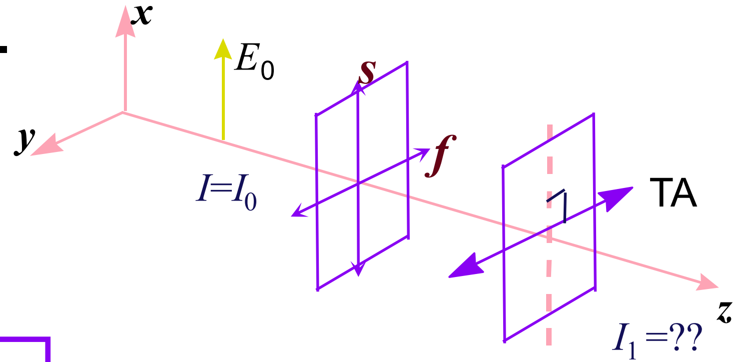


3B • **What is the relation between I_2, the final intensity when the fast axis makes an angle $\theta = 45^\circ$ with the x-axis, in terms of I_0?**

(a) $I_2 = \frac{1}{2} I_0$ **(b)** $I_2 = \frac{1}{4} I_0$ **(c)** $I_2 = 0$

Chapter 44, ACT 2

- Light of intensity I_0, polarized along the x direction is incident on a quarter-wave plate (fast axis= y-axis) and a linear polarizer as shown.



3A – What is the relation between I_1, the intensity at the exit of the system, in terms of I_0?

(a) $I_1 = \frac{1}{2} I_0$

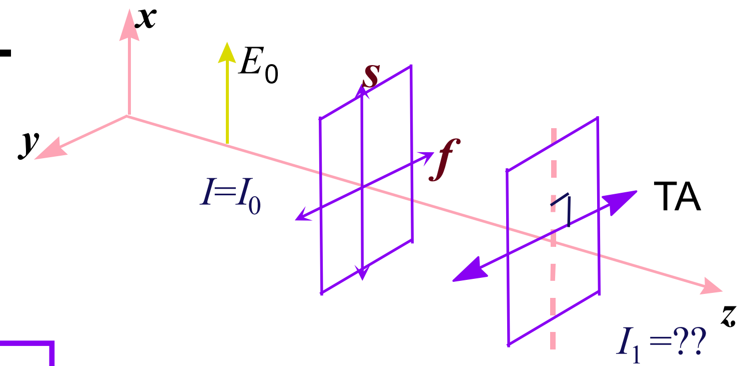
(b) $I_1 = \frac{1}{4} I_0$

(c) $\boxed{I_1 = 0}$

- The quarter wave plate aligned with the initial axis of polarization will not change the polarization of the wave at all!!!
- The next polarizer, being at right angles, will eliminate the wave completely!

Chapter 44, ACT 2

- Light of intensity I_0, polarized along the x direction is incident on a quarter-wave plate (fast axis= y-axis) and a linear polarizer as shown.



3A – What is the relation between I_1, the intensity at the exit of the system, in terms of I_0?

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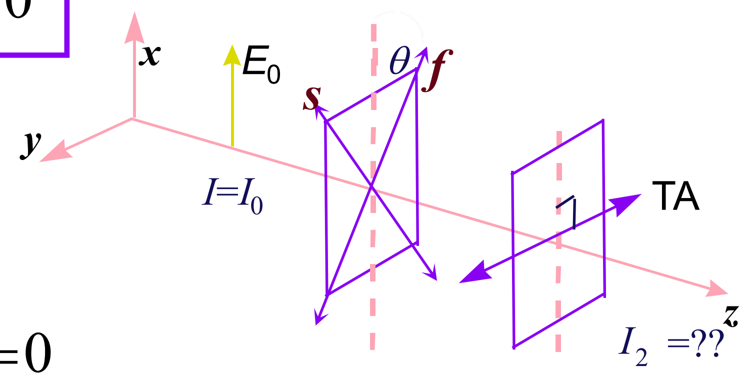


3B • What is I_2, the final intensity when the fast axis makes an angle $\theta = 45^\circ$ with the x-axis, in terms of I_0?

$\boxed{\text{(a) } I_2 = \frac{1}{2} I_0}$ (b) $I_2 = \frac{1}{4} I_0$ (c) $I_2 = 0$

- The quarter wave plate produces circularly polarized light. $E_f = E_s = \frac{E_0}{\sqrt{2}}$
- The fast and slow components are attenuated independently since they are out of phase!

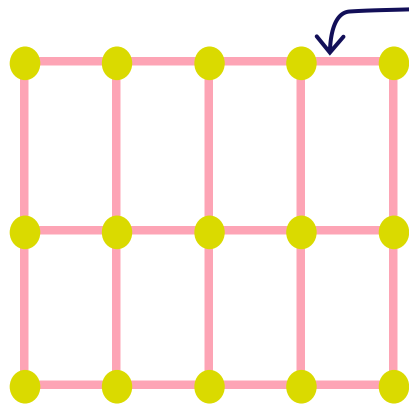
fast slow

$$\boxed{I_2 = \left(\left(\frac{E_0}{2} \right)^2 + \left(\frac{E_0}{2} \right)^2 \right) = \frac{1}{2} I_0}$$

What Causes Birefringence?

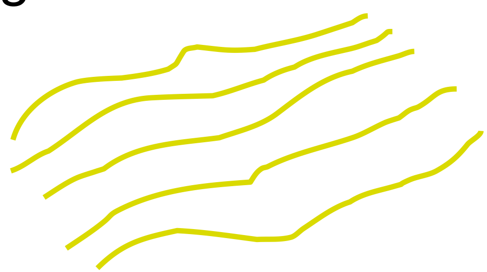
Birefringence can occur in any material that possesses some asymmetry (**不对称**) in its structure, so that the material is more “springy” in one direction than another.

<u>Examples</u>: **Crystals (晶体): quartz, calcite**



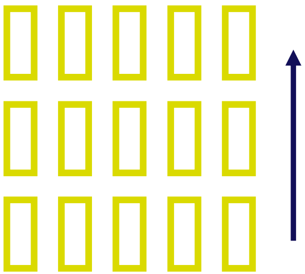
Different atom spacings in $\hat{x}$ and $\hat{y}$.

Long stretched molecular chains: saran wrap (**保鲜膜**), cellophane tape (**玻璃纸**)



Birefringence, cont.

Oblong molecules (长方形分子): “liquid crystals (液晶)”



Dipoles of the molecules orient along an externally applied electric field. Change the field → change the birefringence → change the polarization of transmitted light → pass through polarization analyzer to change the intensity

→ Digital displays (数值显示)
→ LCD monitors, etc.

Stress-induced birefringence:

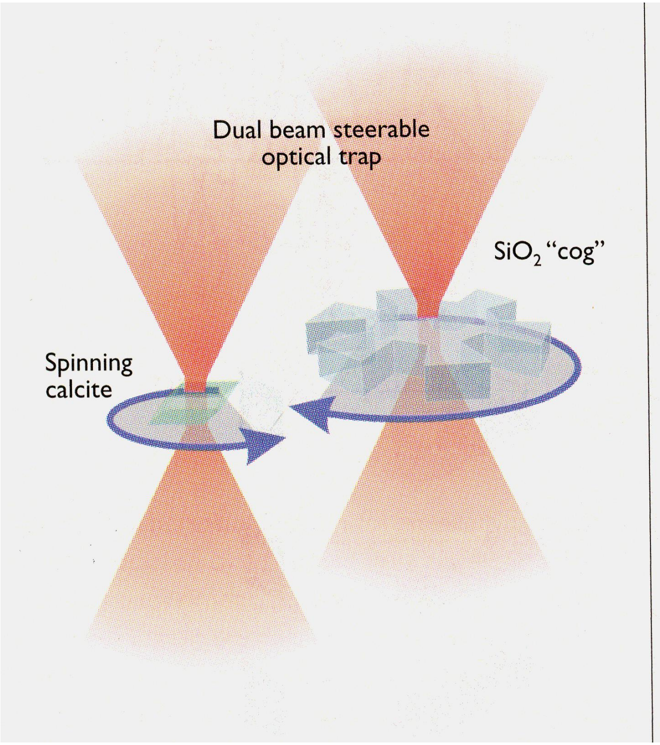
Applying a mechanical stress to a material will often produce an asymmetry → birefringence.
This is commonly used to measure stress.

A Really Cool Application: Light-Driven Micro Machines

(April 2002)

Fact 1: Small particles are attracted to regions of high *E* field gradient (induced dipole force) → laser "tweezers" (激光镊子)

Fact 2: Because birefringent crystals convert linear ↔ circular polarization, they acquire angular momentum → rotation



Parts are only 10μm across!

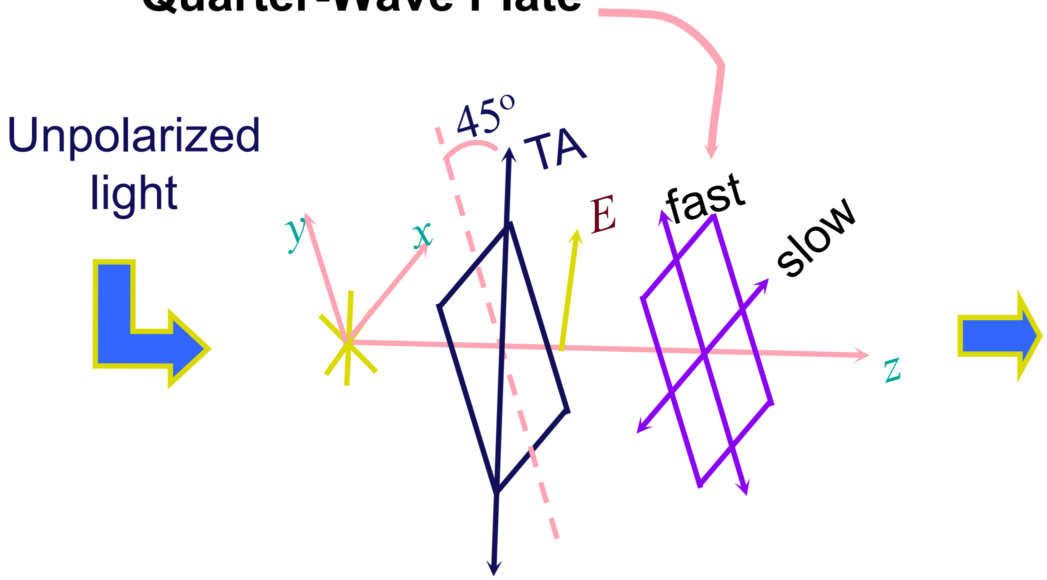
Uses: Biophysics (生物物理学): Manipulating DNA, proteins, etc. Microscopic fluid pumps (微液泵)

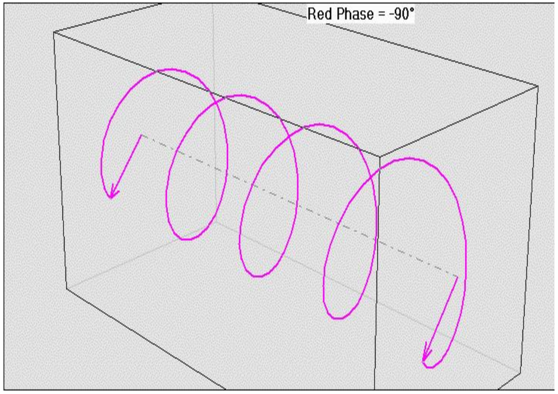
Summary

- **Linear Polarizers**
  - **Law of Malus**

$$\boxed{I_2 = I_1 \cos^2\theta}$$

- **Circular Polarization**
  - **Quarter-Wave Plate**





Unpolarized Light (无偏振光、 自然光)

- **We have primarily been considering light that has a definite polarization (e.g., linear or circular). Most sources – a candle, the sun, any light bulb – produce light that is unpolarized :**

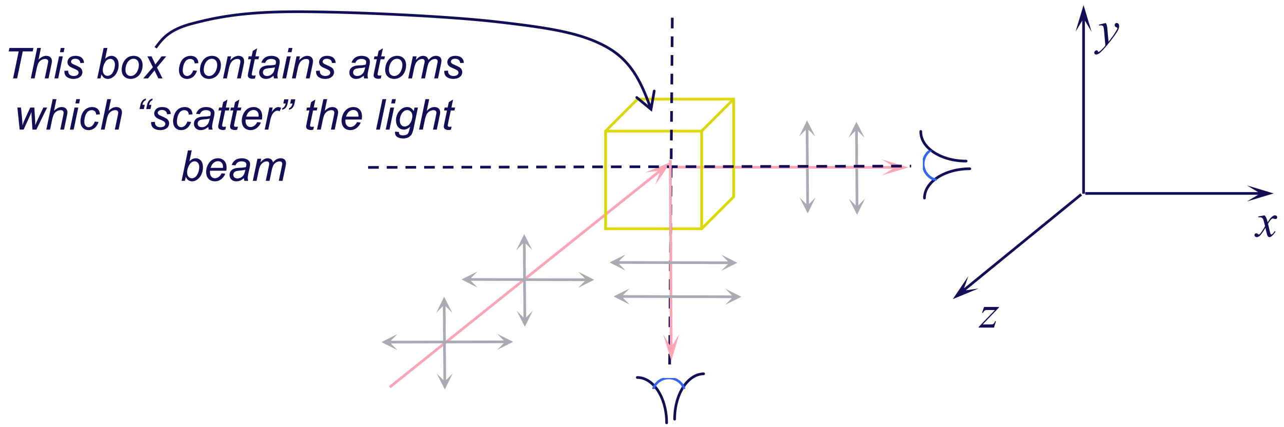
  - **it does not have a definite direction of the electric field**
  - **there is no definite phase between orthogonal components**
  - **the atomic or molecular dipoles that emit the light are randomly oriented in the source**
  - **the intensity of light transmitted through a polarizer is always half the intensity of the unpolarized input, regardless of the orientation of the polarizer**

    (though of course the output *is* polarized!)

These are all equivalent ways of describing the same thing.

44-5 Polarization by Scattering (散射产生的偏振)

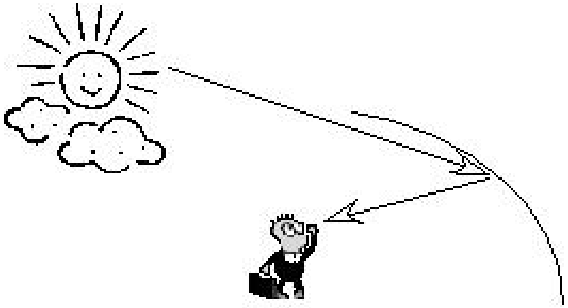
- Suppose unpolarized light encounters an atom and scatters (energy absorbed & reradiated).
  - What happens to the polarization of the scattered light?
  - The scattered light is preferentially polarized perpendicular to the plane of the scattering.
    - » For example, assume the incident unpolarized light is moving in the z-direction.
    - » Scattered light observed along the x-direction (scattering plane = x-z) will be polarized along the y-direction.
    - » Scattered light observed along the y-direction (scattering plane = y-z) will be polarized along the x-direction.



Applications

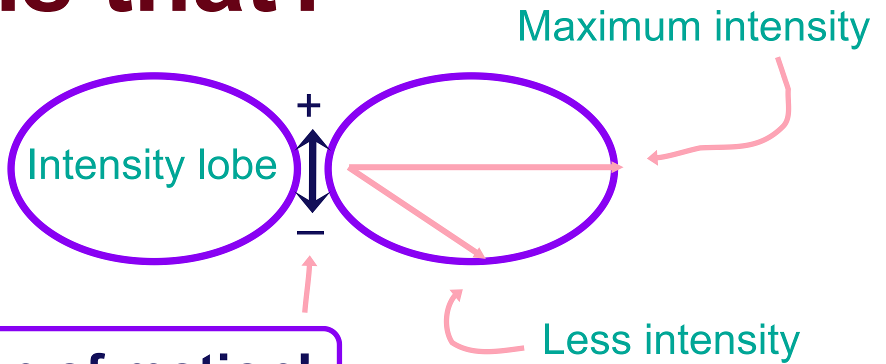
- **Sunglasses**
  - The reflection off a horizontal surface (e.g., water, the hood of a car, etc.) is strongly polarized. Which way?
  - A perpendicular polarizer can preferentially reduce this glare.

- **Polarized sky**
  - The same argument applies to light scattered off the sky:



Why is that?

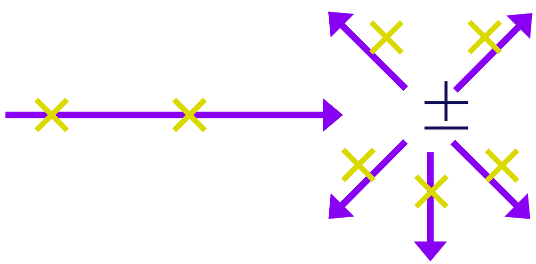
In many cases light is radiated/scattered by oscillating electric dipoles.

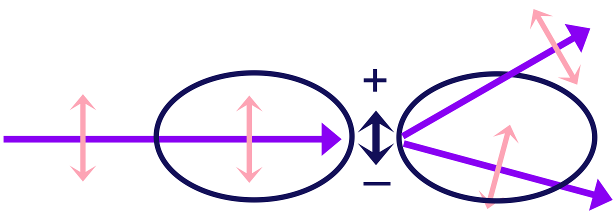


***No* radiation along direction of motion!**

Start with sunlight with all polarizations & randomly oriented dipoles.

2 cases:





Dipole oscillates *into* the paper.

Horizontal dipoles reradiate H-polarized light downward.

(Do not respond to incident V -light.)

Dipole oscillates vertically.

Vertical dipoles reradiate V-polarized light to the sides (*not* downward).

(Do not respond to incident H -light.)

Homework

- Page 1012   Exercises   5, 8, 14, 15
- Page 1013   Problems, 4, 6