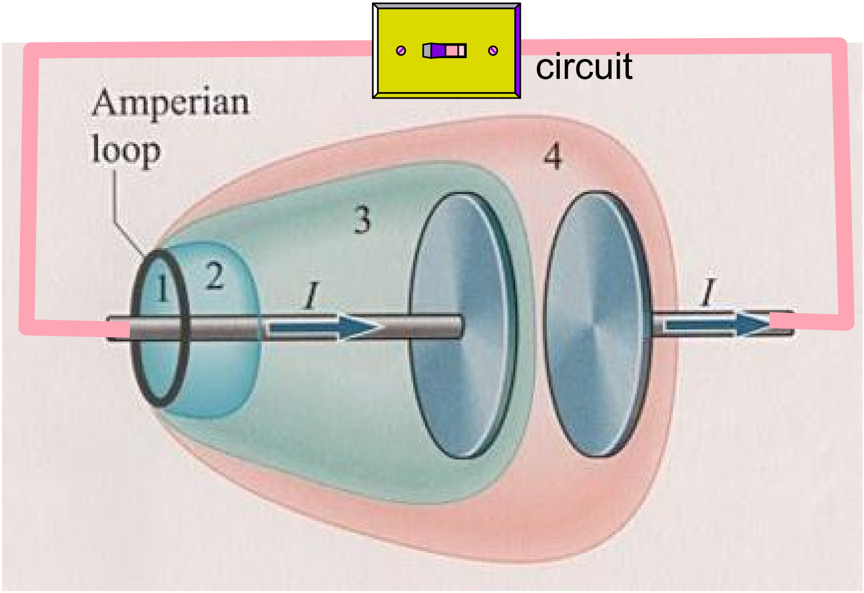
Chapter 38 Maxwell's Equations



38-1 The basic equations of electromagnetisms

1. Introduction:

- Classical Mechanics: the Newton's Laws
- Thermodynamics: the three thermodynamics Law
- Electromagnetism, Optics: Maxwell's Equations (麦克斯韦方程组)

the prediction of electromagnetic waves that travel with the speed of light.

2. The tentative basic equations of electromagnetism

- **In vacuum:**

$$\text{The Gauss' Law of Electricity}: \oiint \vec{E} \bullet d\vec{A} = \frac{q_0}{\varepsilon_0}$$

$$\text{The Gauss' Law of Magnetism}: \oiint \vec{B} \bullet d\vec{A} = 0$$

$$\text{Faraday's Law of Induction}: \oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt} = -\iint \frac{\partial \vec{B}}{\partial t} \bullet d\vec{A}$$

$$\text{Ampere's Loop Law}: \oint \vec{B} \bullet d\vec{l} = \mu_0 i$$

- **In the dielectric and magnetic materials**

$$\begin{cases} \oiint \vec{D} \bullet d\vec{A} = q_0 \\ \oiint \vec{B} \bullet d\vec{A} = 0 \\ \oint \vec{E} \bullet d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \bullet d\vec{A} \\ \oint \vec{H} \bullet d\vec{l} = i_0 = \iint \vec{j} \bullet d\vec{A} \end{cases}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \kappa_e \varepsilon_0 \vec{E}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \kappa_m \mu_0 \vec{H}$$

$$\vec{j}_0 = \sigma \vec{E}$$

$$\begin{cases} \nabla \bullet \vec{D} = \rho_{e0} \\ \nabla \bullet \vec{B} = 0 \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} = \vec{j}_0 \end{cases}$$

The principle of symmetry (对称性原则)

How it permeates physics (渗透了整个物理学)!

How it has often lead to new insight or discovery!

positive electrons, relativity, Maxwell's Equations

Notes: (a) magnetic monopoles: q_m

$$\begin{cases} \oiint \vec{E}\bullet d\vec{A} = \dfrac{q_0}{\varepsilon_0} \\ \oiint \vec{B}\bullet d\vec{A} = 0 \\ \oint \vec{E}\bullet d\vec{l} = -\iint \dfrac{\partial \vec{B}}{\partial t}\bullet d\vec{A} \\ \oint \vec{B}\bullet d\vec{l} = \mu_0 i \end{cases}$$

$$\begin{cases} \oiint \vec{B}\bullet d\vec{A} = 0 \\ \oint \vec{E}\bullet d\vec{l} = -\iint \dfrac{\partial \vec{B}}{\partial t}\bullet d\vec{A} \end{cases} \longrightarrow \begin{cases} \oiint \vec{B}\bullet d\vec{A} = q_m \\ \oint \vec{E}\bullet d\vec{l} = \dfrac{dq_m}{dt} - \iint \dfrac{\partial \vec{B}}{\partial t}\bullet d\vec{A} \end{cases}$$

(b)

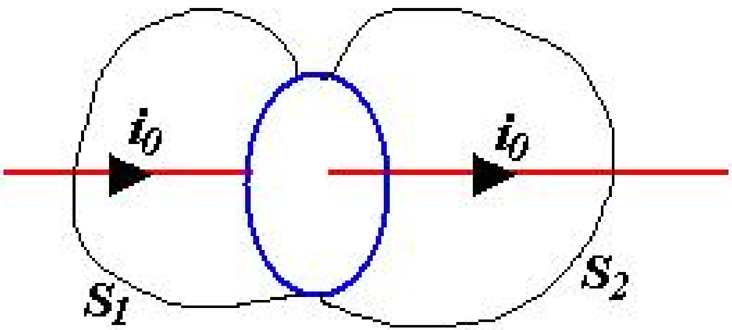
$$\begin{cases} \oint \vec{E}\bullet d\vec{l} = -\iint \dfrac{\partial \vec{B}}{\partial t}\bullet d\vec{A} \\ \oint \vec{H}\bullet d\vec{l} = i_0 = \dfrac{dq}{dt} = \iint \vec{j}\bullet d\vec{A} \end{cases}$$

Changing *B* $\dfrac{d\Phi_B}{dt}$, produce *E*

Changing E $\dfrac{d\Phi_E}{dt}$, *Produce B* ?

Displacement Current (位移电流）

Ampere's loop law



$$\oint \vec{H} \bullet d\vec{l} = i_0 = \iint_{S_2} \vec{j}_0 \bullet d\vec{A}$$

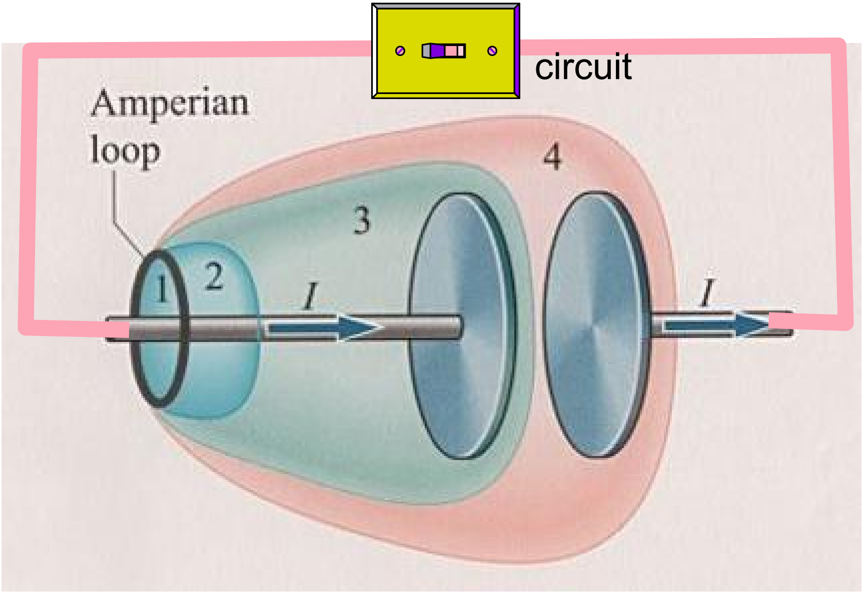
$$-\iint_{S_1} \vec{j}_0 \bullet d\vec{A} = \iint_{S_2} \vec{j}_0 \bullet d\vec{A} = i_0$$

$$\oiint_{S} \vec{j}_0 \bullet d\vec{A} = \iint_{S_1} \vec{j}_0 \bullet d\vec{A} + \iint_{S_2} \vec{j}_0 \bullet d\vec{A} = 0$$

- **At steady condition, current is continuous.**
- **Not at steady condition *$i_0(t)$***

Maxwell's Displacement Current

- Consider applying Ampere's Law to the current shown in the diagram.
  - If the surface is chosen as 1, 2 or 4, the enclosed current = I
  - If the surface is chosen as 3, the enclosed current = 0! (i.e., there is no current between the plates of the capacitor)



Big Idea: In order to have

$\oint \vec{H} \bullet d\vec{l}$ for surface 1 = $\oint \vec{H} \bullet d\vec{l}$ for surface 3

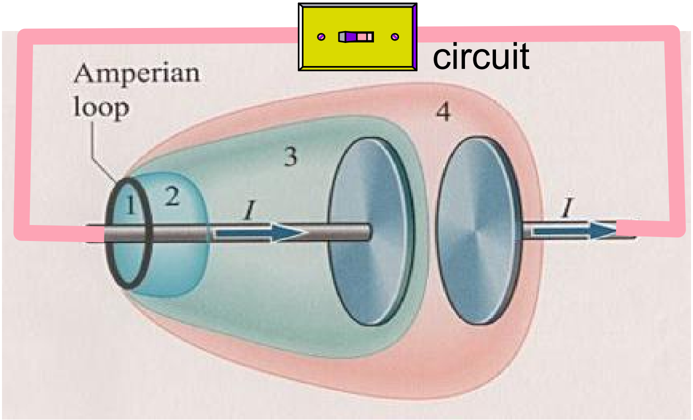
Maxwell proposed there was an extra "displacement current" in the region between the plates, equal to the current in the wire→

Modified Ampere's law: $$\oint \vec{H} \bullet d\vec{l} = i_0 + i_D$$

38-2 Induced Magnetic Field (感应磁场) and the Displacement current (位移电流)

Not at steady condition

$$\oint\!\!\oint_S \vec{j}_0 \bullet d\vec{A} = -\frac{dq_0}{dt}$$



From Gauss' Law:

$$\oint\!\!\oint_S \vec{D} \bullet d\vec{A} = q_0$$

$$\frac{dq_0}{dt} = \frac{d}{dt}\oint\!\!\oint_S \vec{D} \bullet d\vec{A} = \oint\!\!\oint_S \frac{\partial \vec{D}}{\partial t} \bullet d\vec{A}$$

$$\therefore \oint\!\!\oint_S \vec{j}_0 \bullet d\vec{A} = -\oint\!\!\oint_S \frac{\partial \vec{D}}{\partial t} \bullet d\vec{A}$$

$$\oint\!\!\oint_S (\vec{j}_0 + \frac{\partial \vec{D}}{\partial t}) \bullet d\vec{A} = 0$$

$$-\iint_{S_1} (\vec{j}_0 + \frac{\partial \vec{D}}{\partial t}) \bullet d\vec{A} = \iint_{S_2} (\vec{j}_0 + \frac{\partial \vec{D}}{\partial t}) \bullet d\vec{A}$$

$$\boxed{\vec{j}_0 + \frac{\partial \vec{D}}{\partial t}}$$

Is continuous.

Maxwell's Displacement Current (位移电流)

$$\begin{cases} \Phi_D = \iint \vec{D} \bullet d\vec{A} & \text{electric displacement flux} \\ i_D = \dfrac{d\Phi_D}{dt} = \iint \dfrac{\partial \vec{D}}{\partial t} \bullet d\vec{A} & \text{displacement current} \\ \vec{j}_D = \dfrac{\partial \vec{D}}{\partial t} & \text{displacement current density} \end{cases}$$

电位移通量

位移电流

位移电流密度

New Ampere's Loop Law:

$$\oint \vec{H} \bullet d\vec{l} = i_0 + i_D = \iint (\vec{j}_0 + \frac{\partial \vec{D}}{\partial t}) \bullet d\vec{A}$$

Notes

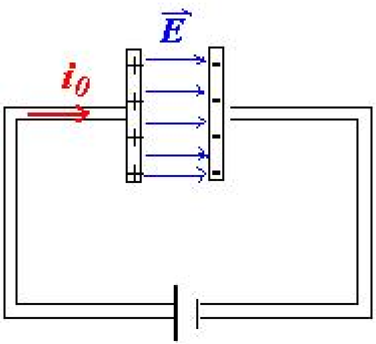
$$\oint \vec{H} \bullet d\vec{l} = i_0 + i_D = \iint (\vec{j}_0 + \frac{\partial \vec{D}}{\partial t}) \bullet d\vec{A}$$

- **Both ways of producing a magnetic field:**
  by a conducting current (*q* motion)
  by a changing electric displacement flux Φ_D

- **The displacement current in the gap equals to the conduction current in the wires, which shows the current is continuous.**

Consider in vacuum: **In the wires, there is only conduction current i_0.**

In the gap, there is only displacement current i_D.



$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}, \quad \therefore q = \varepsilon_0 AE = \varepsilon_0 \Phi_E = AD$$

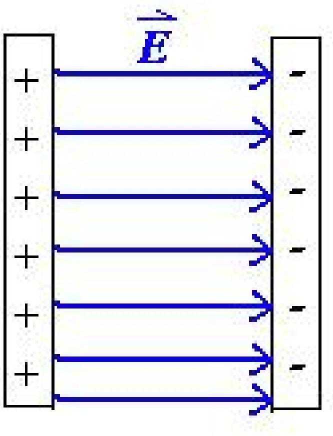
$$\therefore i_0 = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{d\Phi_D}{dt} = i_D, \quad \vec{D} = \varepsilon_0 \vec{E}$$

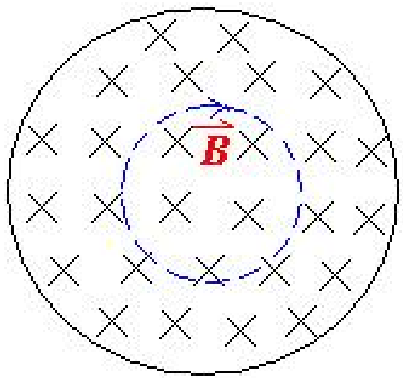
$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$
$$\vec{P} = 0$$

When the capacitor is fully charged, then $i_0=0$, $i_D=0$

Notes (con.)

- **The induced magnetic field *B* is produced by the changing electric filed *E* inside the capacitor.**





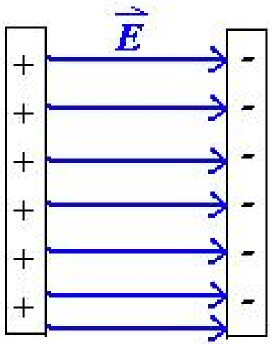
$$\oint \vec{H} \bullet d\vec{l} = \iint \frac{\partial \vec{D}}{\partial t} \bullet d\vec{A}$$

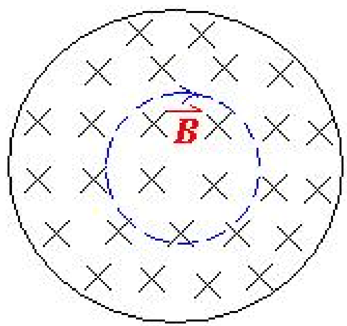
$$\oint \frac{\vec{B}}{\mu_0} \bullet d\vec{l} = \iint \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \bullet d\vec{A}$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \varepsilon_0 \iint \frac{\partial \vec{E}}{\partial t} \bullet d\vec{A}$$

$\dfrac{\partial \vec{E}}{\partial t} > 0$, $\vec{\mathrm{B}}$ is clockwise, eddy magnetic field (涡旋磁场)

Example Page863, problem 38-1





A Parallel-plate capacitor

(a) Derive an expression for the induced magnetic field at radius r in the region between the plates. Consider both $r \le R$ and $r \ge R$.

(b) Find B at $r = R$ for $dE/dt = 10^{12}$V/m· s and $R = 5.0$ cm.

Solution:

$$(a).\ \oint \vec{H} \bullet d\vec{l} = \iint (\vec{j}_0 + \frac{\partial \vec{D}}{\partial t}) \bullet d\vec{A}$$

$$\text{in vacuum}: \ \vec{j}_0 = 0$$

$$\oint \frac{\vec{B}}{\mu_0} \bullet d\vec{l} = \iint \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \bullet d\vec{A}$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \varepsilon_0 \iint \frac{\partial \vec{E}}{\partial t} \bullet d\vec{A}$$

$$r \le R, \quad B \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{dE}{dt} \cdot \pi r^2, \ B = \frac{1}{2} \varepsilon_0 \mu_0 r \frac{dE}{dt}$$

$$r \ge R, \quad B \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{dE}{dt} \cdot \pi R^2, \ B = \frac{1}{2} \varepsilon_0 \mu_0 \frac{R^2}{r} \frac{dE}{dt}$$

$$(b) \quad r = R, \ \frac{dE}{dt} = 10^{12} V/m \cdot s$$

$$B = \frac{1}{2} \varepsilon_0 \mu_0 R \frac{dE}{dt}$$

$$= \frac{1}{2}(8.9 \times 10^{-12} C^2/Nm^2) \times (4\pi \times 10^{-7} T.m/A)$$

$$\times (5.0 \times 10^{-2})(10^{12})$$

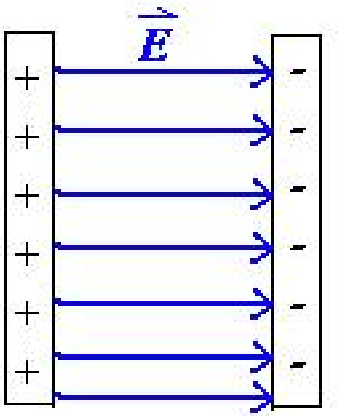
$$= 2.8 \times 10^{-7} T$$

$$= 280 nT$$

They can scarcely be measured with simple apparatus.

Example Page864, problem 38-2

- **What is the displacement current for the situation of Sample Problem 38-1?**



Solution:

$$i_D = \frac{d\Phi_D}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \pi R^2 \cdot \frac{dE}{dt}$$
$$= 8.9\times10^{-12} \times \pi \times (5\times10^{-2})^2 \times 10^{12}$$
$$= 0.07\mathrm{A} = 70\mathrm{mA}$$

i_D **is a reasonably large current, but** B=280 nT, **Why?**

Under the same conditions, both kinds of current are equally effective in generating magnetic field.

38-3 Maxwell's Equations

- **In vacuum:**

The Gauss' Law of Electricity: $\oiint \vec{E} \bullet d\vec{A} = \dfrac{q_0}{\varepsilon_0}$

The Gauss' Law of Magnetism: $\oiint \vec{B} \bullet d\vec{A} = 0$

Faraday's Law of Induction: $\oint \vec{E} \bullet d\vec{l} = -\dfrac{d\Phi_B}{dt} = -\iint \dfrac{\partial \vec{B}}{\partial t} \bullet d\vec{A}$

Ampere's Loop Law: $\oint \vec{B} \bullet d\vec{l} = \mu_0 i + \mu_0 \varepsilon_0 \iint \dfrac{\partial \vec{E}}{\partial t} \bullet d\vec{A}$

(As extended by Maxwell)

- **In the dielectric and magnetic materials**

$$\begin{cases} \oiint \vec{D} \bullet d\vec{A} = q_0 \\ \oiint \vec{B} \bullet d\vec{A} = 0 \\ \oint \vec{E} \bullet d\vec{l} = -\iint \dfrac{\partial \vec{B}}{\partial t} \bullet d\vec{A} \\ \oint \vec{H} \bullet d\vec{l} = i_0 + \iint \dfrac{\partial \vec{D}}{\partial t} \bullet d\vec{A} \end{cases}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \kappa_e \varepsilon_0 \vec{E}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \kappa_m \mu_0 \vec{H}$$

$$\vec{j}_0 = \sigma \vec{E}$$

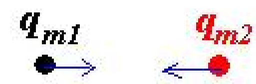
$$\begin{cases} \nabla \bullet \vec{D} = \rho_{e0} \\ \nabla \bullet \vec{B} = 0 \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} = \vec{j}_0 + \dfrac{\partial \vec{D}}{\partial t} \end{cases}$$

Notes for Maxwell's Equations

- **Symmetry: There are still not completely symmetric.**

  If the existence of individual magnetic charges (monopoles) were confirmed, a completely symmetric set would result.

$$\begin{cases} \oiint \vec{B} \bullet d\vec{A} = 0 \\ \oint \vec{E} \bullet d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \bullet d\vec{A} \end{cases} \Rightarrow \begin{cases} \oiint \vec{B} \bullet d\vec{A} = q_m \\ \oint \vec{E} \bullet d\vec{l} = \frac{dq_m}{dt} - \iint \frac{\partial \vec{B}}{\partial t} \bullet d\vec{A} \end{cases}$$



$$\vec{F}_{12} = k' \frac{q_{m1} \bullet q_{m2}}{r^2}$$

- **Electromagnetic waves.**

$$\text{Changing } \vec{E} \Rightarrow \vec{B}$$

$$\text{Changing } \vec{B} \Rightarrow \vec{E}$$

Before Maxwell's time (1864), no new predictions arise from from 4 equations. 1888, EW was discovered by Hertz.

Electromagnetic waves.

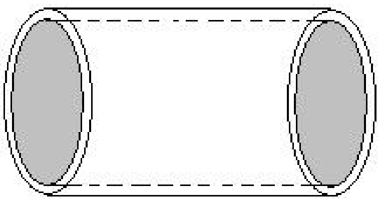
- **Electromagnetism and relativity:**

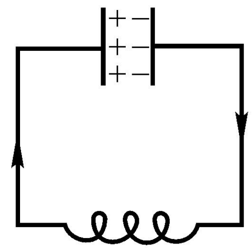
Newton's Law 17th c

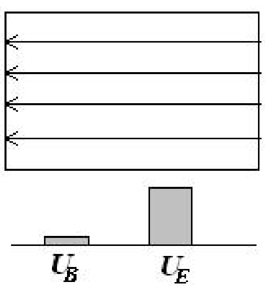
Maxwell Eqs. 19th

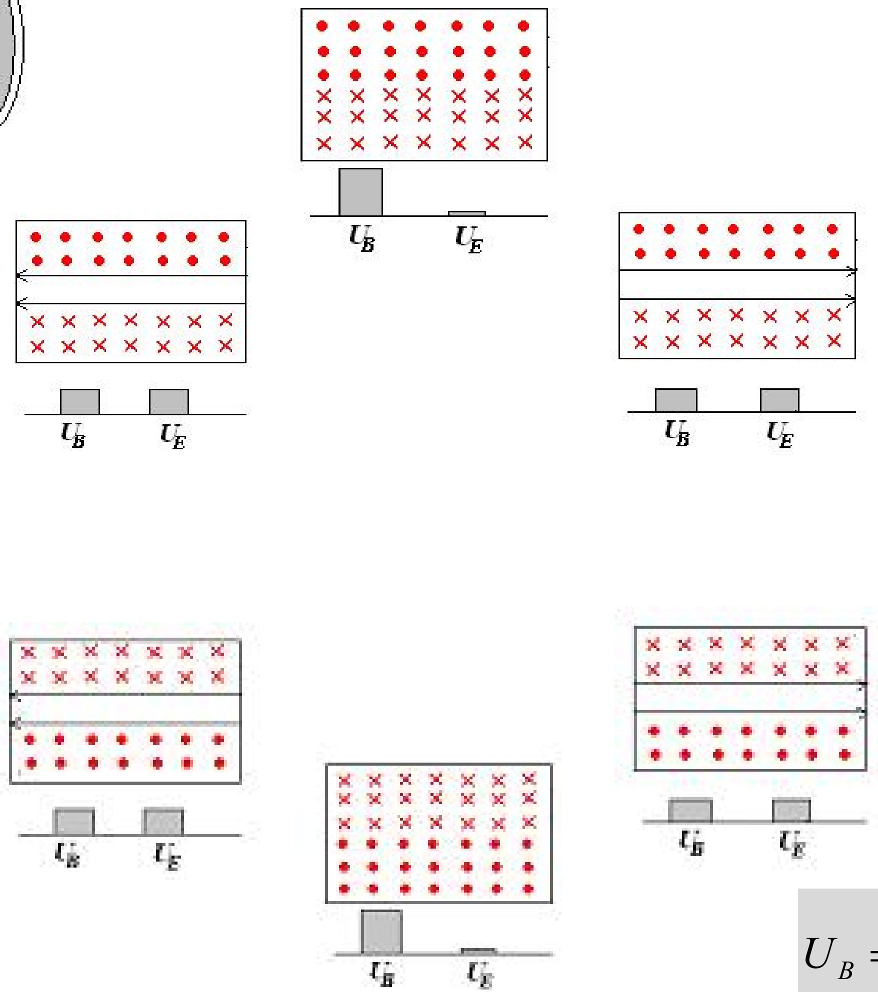
Relativity, 1905 Einstein

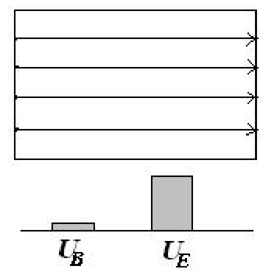
Maxwell's Equations and Cavity Oscillations (谐振腔)







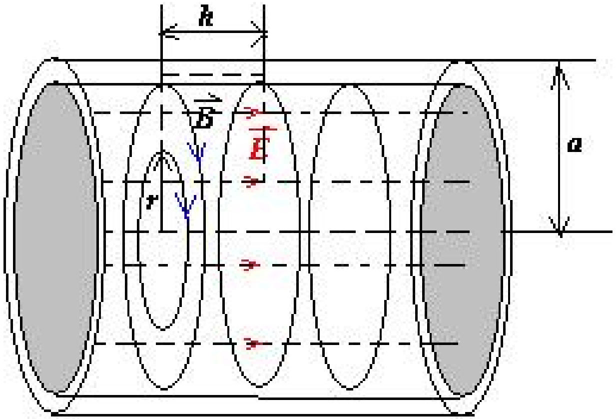




Oscillation

$$U_B = \frac{B^2}{2\mu_0} \Leftrightarrow U_E = \frac{1}{2}\varepsilon_0 E^2$$

Cavity Oscillations (Con.)



A more detailed representation of a cylindrical electromagnetic resonant cavity

- **From Faraday's Law**

$$\oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$B \text{ increasing}, E \cdot h = -\frac{d\Phi_B}{dt}$$

$$E = -\frac{1}{h}\frac{d\Phi_B}{dt}$$

- **From Ampere's Loop Law**

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \varepsilon_0 \iint \frac{\partial \vec{E}}{\partial t} \bullet d\vec{A}$$

$$B \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$B = \frac{\mu_0 \varepsilon_0}{2\pi r}\frac{d\Phi_E}{dt}$$