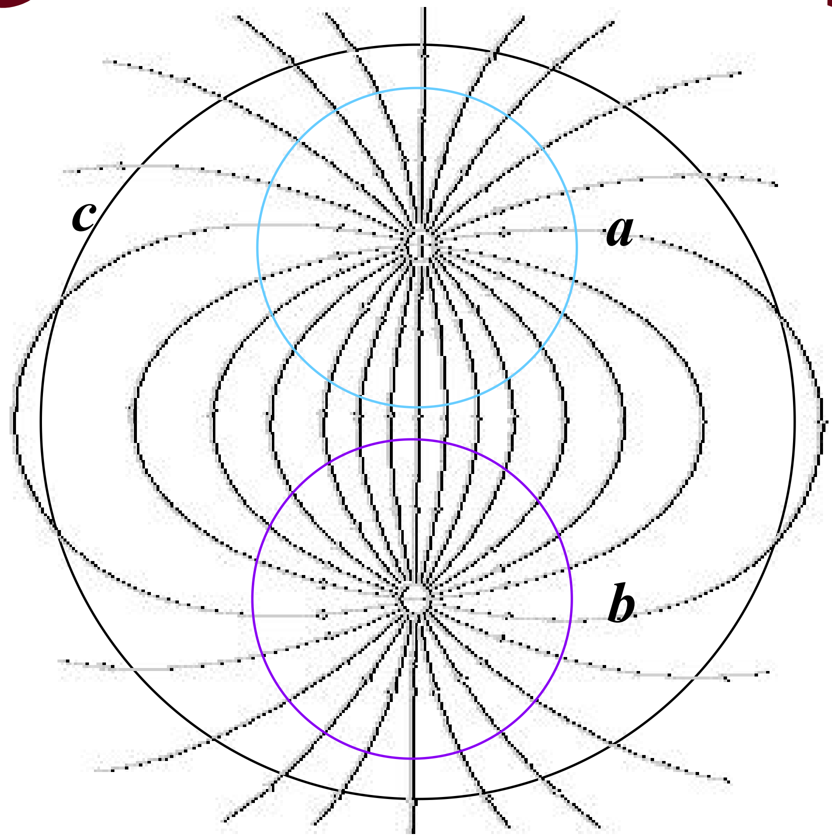
Gauss' Law



$$\varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = q_{enclosed}$$

27-1 Introduction:
Fundamental Law of Electrostatics(静电学)

- Coulomb’s Law: Force between two point charges
- Gauss’ Law: Relationship between Electric Fields
  and charges
- Although Gauss’ law and Coulomb’s law give the identical results.
- Gauss’ Law offers a much simpler way to calculate *E* in situations with a high degree of symmetry (高对称的情况).
- Gauss’ Law as the fundamental law of electrostatics.
- Gauss’ Law is valid in the case of rapidly moving charges.
- Gauss’ Law is more general than Coulomb’s law.

27-2(3) The Flux of a vector field, electric field

1. Flux (Φ) 流量(通量), 流体的流量

- How much of something is passing through some surface

  Ex: How many hairs passing through your scalp.

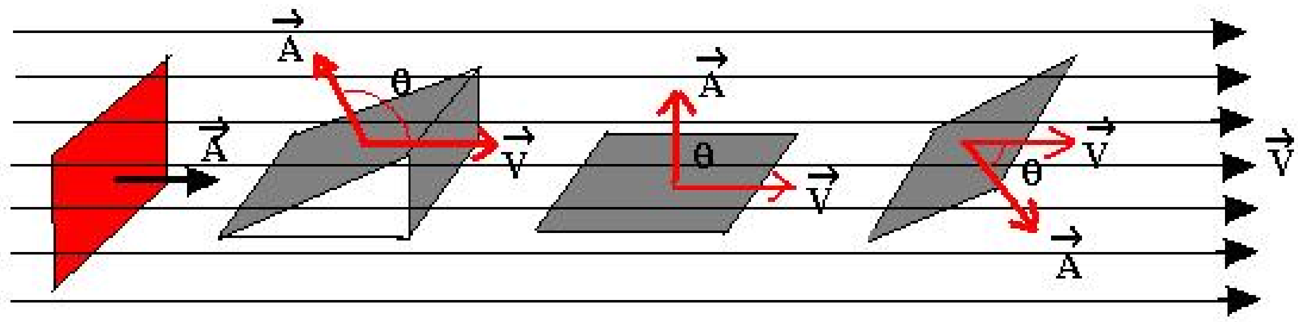
- Two ways to define
  1. Number per unit area (e.g., 10 hairs/mm^2)

     This is *NOT* what we use here.
  2. Number passing through an area of interest

     e.g., 48,788 hairs passing through <u>my</u> scalp.

     This *is* what we are using here.

Example: The velocity field of the fluid flows



incompressible, steady, uniform field

- **A single loop of area A**

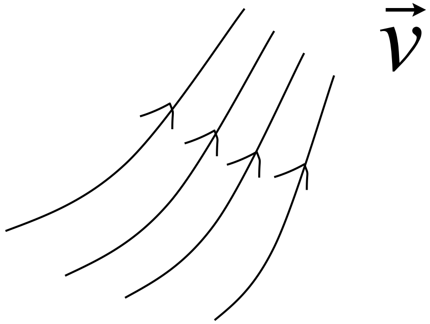
(a) Considering a single loop placed in the stream its plane is perpendicular to the direction of flow, the fluid volume passed through this loop per unit time

(**单位时间流过该回路的体积**).

$$\Phi = \vec{v} \bullet \vec{A} = \frac{L}{t} \bullet A = \frac{LA}{t} = \frac{V}{t}$$

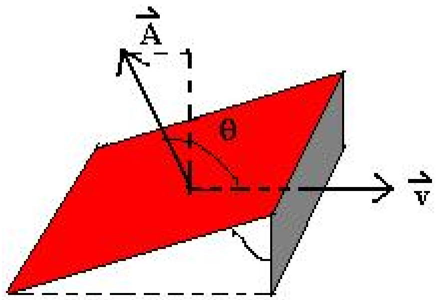
Define Φ is the Flux of the velocity field.

It is convenient to consider it as a measure of the number of field lines passing through the loop.



$$\Phi = \vec{v} \bullet \vec{A} \quad \frac{\Delta\Phi}{\Delta A} \propto \vec{v} \quad E \propto \frac{\Delta N}{\Delta A}$$

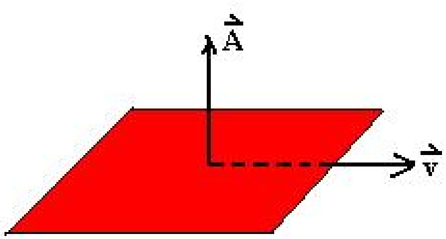
(b)



$$\vec{A} \cap \vec{v} = \theta,$$

$$\Phi = \vec{v} \bullet \vec{A} = vA\cos\theta$$

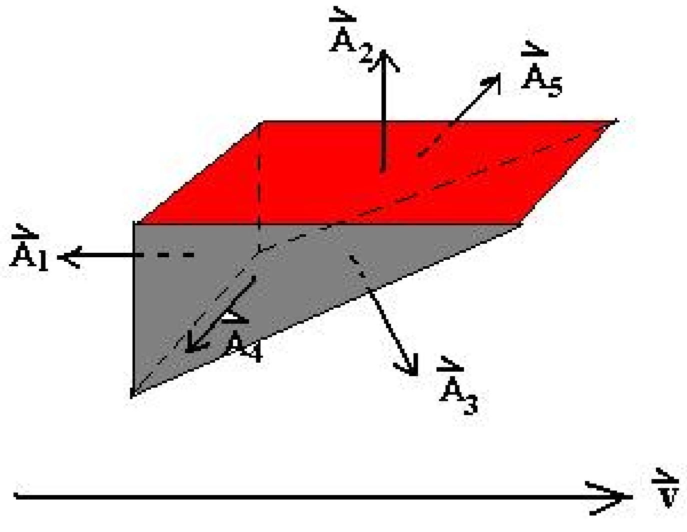
(C)



$$\vec{A} \perp \vec{v}, \theta = 90^o$$

$$\Phi = \vec{v} \bullet \vec{A} = vA\cos\theta = 0$$

• A closed area (闭合曲面)



$$\Phi_1 = \vec{v} \bullet \vec{A}_1 = -vA_1$$

$$\Phi_2 = \vec{v} \bullet \vec{A}_2 = 0$$

$$\Phi_3 = \vec{v} \bullet \vec{A}_3 = vA_3 \cos\theta$$

$$\Phi_4 = \vec{v} \bullet \vec{A}_4 = 0$$

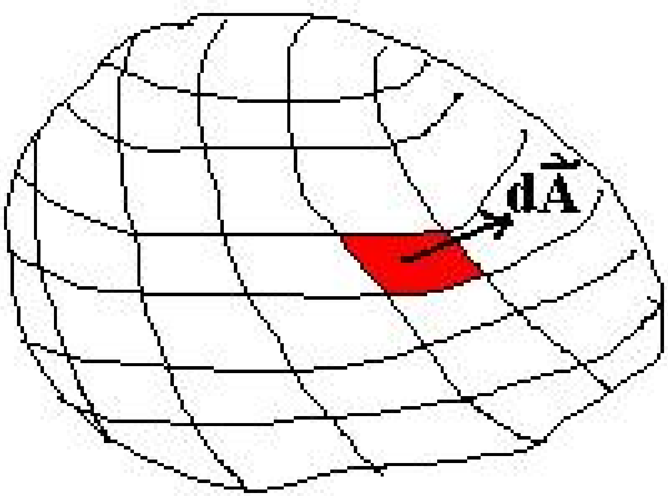
$$\Phi_5 = \vec{v} \bullet \vec{A}_5 = 0$$

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 = -vA_1 + vA_3 \cos\theta = 0$$

The net mount of fluid out the volume:

$$\Phi = \sum \vec{v} \bullet \vec{A}$$

- **For any closed surface**



$$\Phi = \oiint \vec{v} \bullet d\vec{A}$$

$$d\Phi = \vec{v} \bullet d\vec{A}$$

$$d\vec{A} = dydz\vec{i} + dzdx\vec{j} + dxdy\vec{k}$$

$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

$$\vec{v} \bullet d\vec{A} = v_x dydz + v_y dzdx + v_z dxdy$$

If there were within the volume no sources or sink of fluid.

$$\Phi = \oiint \vec{v} \bullet d\vec{A} = 0$$

If there were a source (源) within the volume:

$$\Phi = \oiint \vec{v} \bullet d\vec{A} > 0$$

If there were a sink of fluid:

$$\Phi = \oiint \vec{v} \bullet d\vec{A} < 0$$

2. The flux of the electric Field (电通量)

Even although nothing is flowing in the electrostatic case, we still use the concept of field.

Let's quantify previous discussion about field-line "counting"

Define: electric flux Φ_E through the closed surface *A*

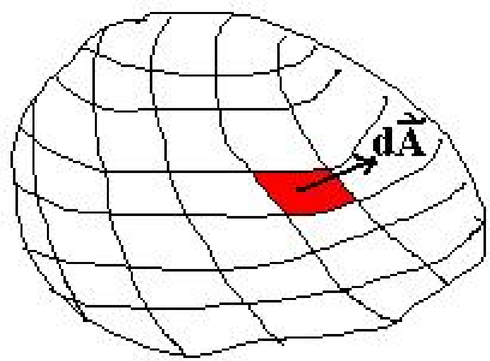
$$\Phi_E = \sum \vec{E} \bullet \vec{A}$$

$$\Phi_E = \oiint \vec{E} \bullet d\vec{A}$$

- The number of lines of electric field (电力线数目) passing through the closed surface A.

Electric Flux (电通量)

$$\Phi_E = \oiint \vec{E} \bullet d\vec{A}$$

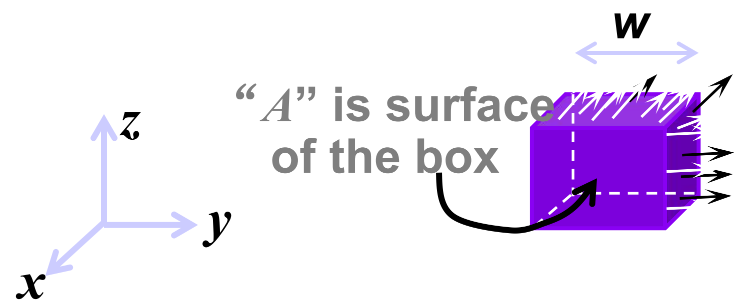


What does this new quantity mean?

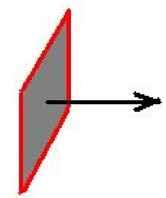
- **The integral is over a CLOSED SURFACE**
- **Since is a SCALAR product, the electric flux is a SCALAR quantity**
- **The integration vector is normal to the surface and points OUT of the surface. is interpreted as the component of E which is NORMAL to the SURFACE**
- **Therefore, the electric flux through a closed surface is the sum of the normal components (垂直分量) of the electric field all over the surface.**
- **The sign matters!!**
  Pay attention to the direction of the normal component as it penetrates the surface… is it "out of" or "into" the surface?
- **"Out of" is "+" "into" is "-"**

How to think about flux

- We will be interested in net flux in or out of a closed surface like this box

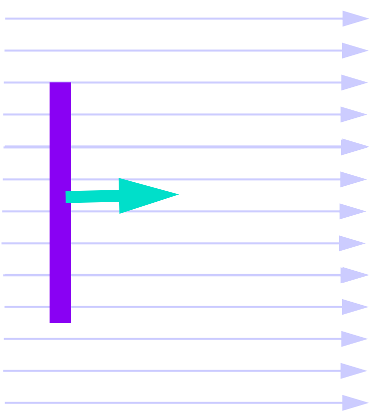


- This is the sum of the flux through each side of the box
  - consider each side separately



$$\vec{A} = Area \bullet \hat{y}$$
$$= w^2 \hat{y}$$

- Let *E*-field point in *y*-direction
  - then $\vec{E}$ and $\vec{A}$ are parallel and $\vec{E} \bullet \vec{A} = Ew^2$

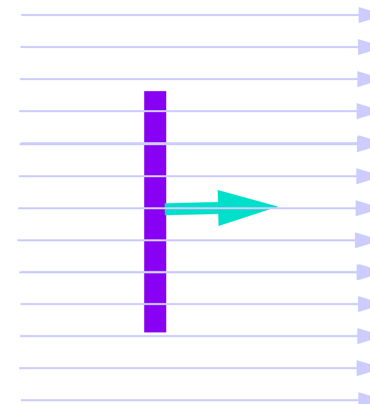


- Look at this from on top
  - down the *z*-axis

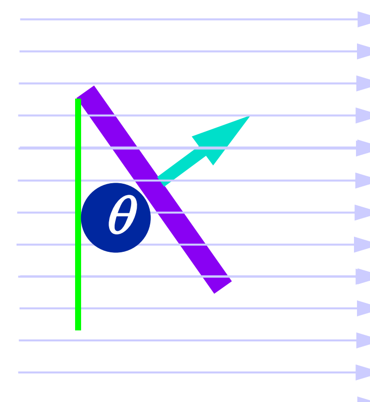
How to think about flux

- **Consider flux through two surfaces that "intercept different numbers of field lines"**
  - **first surface is side of box from previous slide**
  - **Second surface rotated by an angle θ**

case 1



case 2

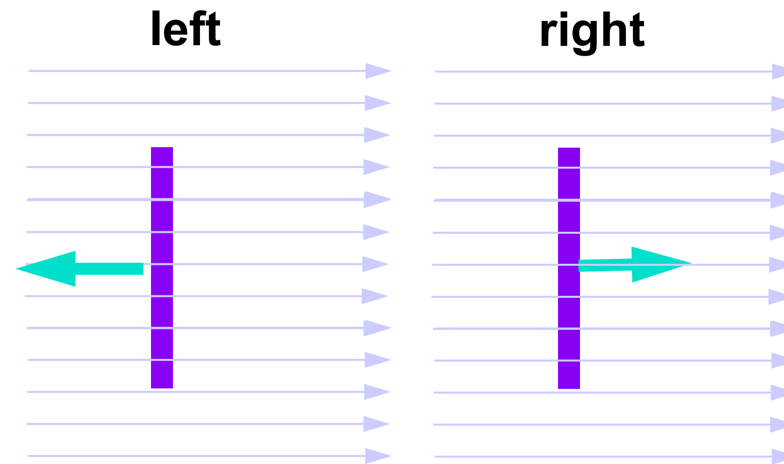


	***E*-field**	**surface area**	**Flux:** $\vec{E} \bullet \vec{A}$
case 1	$\vec{E} = E_o \hat{y}$	w^2	$E_o w^2$
case 2	$\vec{E} = E_o \hat{y}$	w^2	$E_o w^2 \cdot \cos\theta$

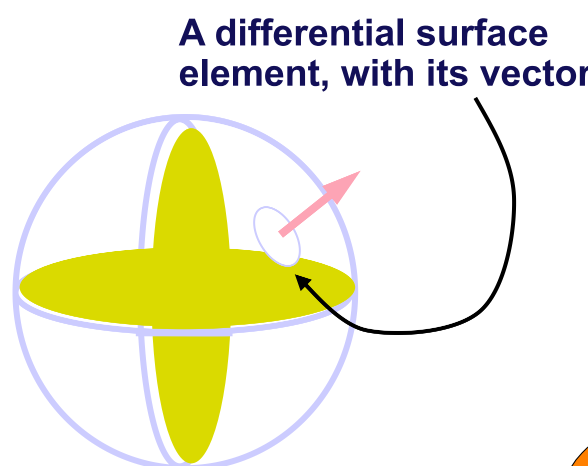
Case 2 is smaller!

The Sign Problem

- For an open surface we can choose the direction of *A*-vector two different ways
  - to the left or to the right
  - what we call flux would be different these two ways
  - different by a minus sign



- For a closed surface we can choose the direction of *A*-vector two different ways
  - pointing "in" or "out"
  - define "out" to be correct
  - Integral of EdA over a closed surface gives net flux "out," but can be + or -



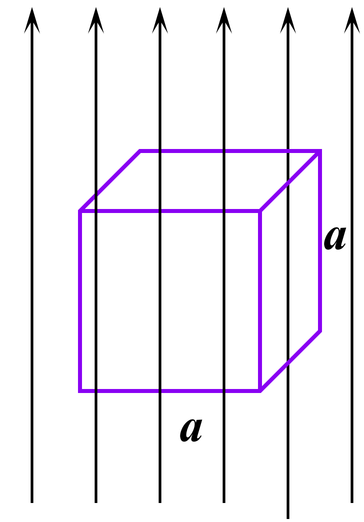
$$\Phi_E = \sum \vec{E} \bullet \vec{A}$$

$$\Phi_E = \oiint \vec{E} \bullet d\vec{A}$$

Chapter 27, ACT 1

3A

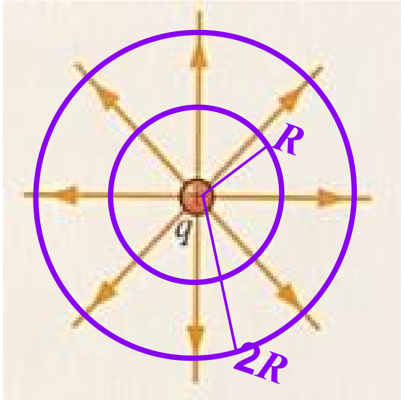
- Imagine a cube of side a positioned in a region of constant electric field as shown
  - Which of the following statements about the net electric flux Φ_E through the surface of this cube is true?



(a) $\Phi_E = 0$ (b) $\Phi_E \propto 2a^2$ (c) $\Phi_E \propto 6a^2$

3B

- Consider 2 spheres (of radius R and $2R$) drawn around a single charge as shown.
  - Which of the following statements about the net electric flux through the 2 surfaces (Φ_{2R} and Φ_R) is true?

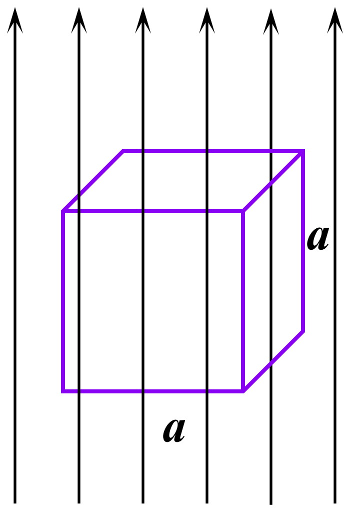


(a) $\Phi_R < \Phi_{2R}$ (b) $\Phi_R = \Phi_{2R}$ (c) $\Phi_R > \Phi_{2R}$

Chapter 27, ACT 1

3A

- Imagine a cube of side a positioned in a region of constant electric field as shown
  - Which of the following statements about the net electric flux Φ_E through the surface of this cube is true?



$\boxed{\text{(a) } \Phi_E = 0}$ (b) $\Phi_E \propto 2a^2$ (c) $\Phi_E \propto 6a^2$

$$\Phi_E = \sum \vec{E} \bullet \vec{A}$$

$$\Phi_E = \oint\!\!\oint \vec{E} \bullet d\vec{A}$$

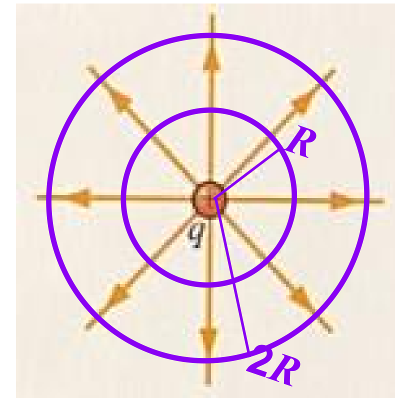
$$\Phi_E = \sum \vec{E} \bullet \vec{A}$$

$$\Phi_E = \oiint \vec{E} \bullet d\vec{A}$$

Chapter 27, ACT 1

3B

- **Consider 2 spheres (of radius R and $2R$) drawn around a single charge as shown.**
  - **Which of the following statements about the net electric flux through the 2 surfaces (Φ_{2R} and Φ_R) is true?**



(a) $\Phi_R < \Phi_{2R}$ **(b) $\boxed{\Phi_R = \Phi_{2R}}$** **(c) $\Phi_R > \Phi_{2R}$**

- **Look at the lines going out through each circle -- each circle has the same number of lines.**
- **The electric field is different at the two surfaces, because E is proportional to $1 / r^2$, but the surface areas are also different. The surface area of a sphere is proportional to r^2.**
- **Since flux = , the r^2 and $1/r^2$ terms will cancel, and the two circles have the same flux!**
- **There is an easier way. Gauss' Law states the net flux is proportional to the NET enclosed charge. The <u>NET charge is the SAME</u> in both cases.**

27-4 Gauss' Law

- **Gauss' Law (a FUNDAMENTAL LAW):**

  The net electric flux through any closed surface is proportional to the charge enclosed by that surface.

$$\oint\!\!\oint \vec{E} \bullet d\vec{A} = \Phi_E = \frac{q_{enclosed}}{\varepsilon_0}$$

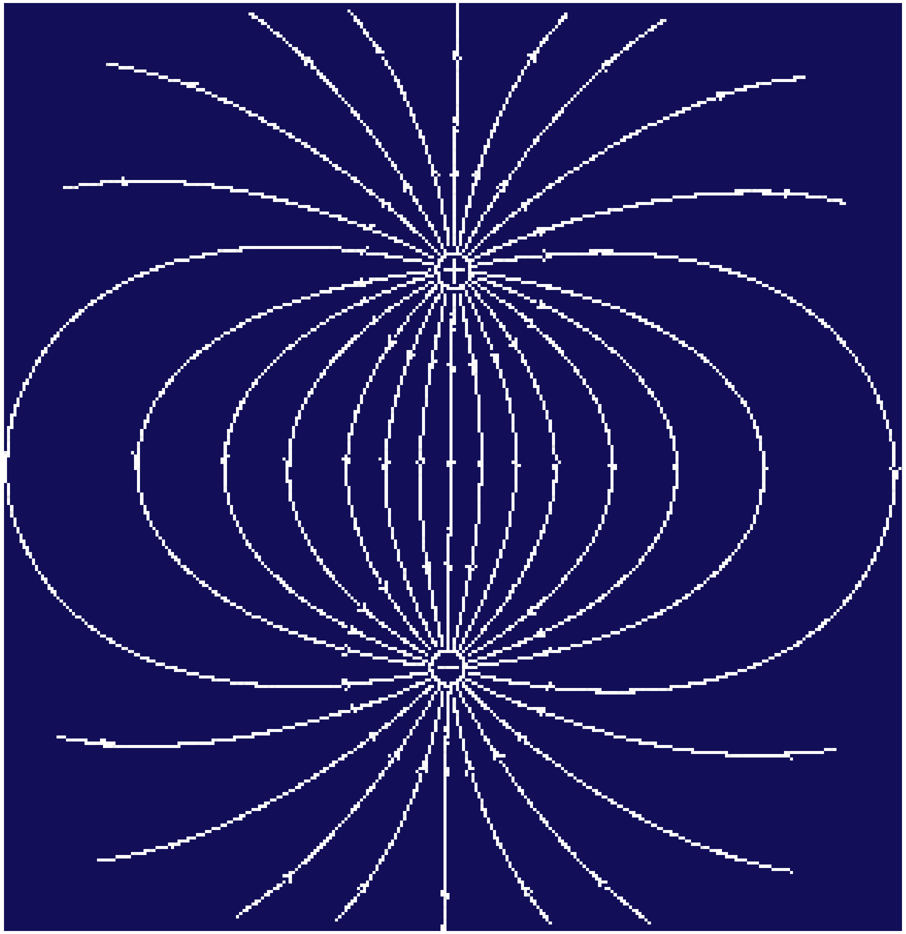
- **How do we use this equation??**
  - **The above equation is *ALWAYS TRUE* but it doesn't look easy to use.**
  - **It is very useful in finding E when the physical situation exhibits massive SYMMETRY.**

Example: a dipole (电偶极矩)

$$a : \varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = q > 0$$

$$b : \varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = -q < 0$$

$$c : \varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = q - q = 0$$



Gauss' Law…made easy

$$\Phi_E = \oiint \vec{E} \bullet d\vec{A} = q_{enclosed} / \varepsilon_0$$

•To solve the above equation for E, you have to be able to CHOOSE A CLOSED SURFACE such that the integral is TRIVIAL.

(1) Direction: surface must be chosen such that E is known to be either parallel or perpendicular to each piece of the surface;

If $\vec{E} \parallel d\vec{A}$ **then** $\vec{E} \bullet d\vec{A} = EdA$

If $\vec{E} \perp d\vec{A}$ **then** $\vec{E} \bullet d\vec{A} = 0$

(2) Magnitude: surface must be chosen such that E has the same value at all points on the surface when E is perpendicular to the surface.

Gauss' Law…made easy

$$\Phi_E = \oiint \vec{E} \bullet d\vec{A} = q_{enclosed} / \varepsilon_0$$

•**With these two conditions we can bring E outside of the integral…and:**

$$\oiint \vec{E} \bullet d\vec{A} = \oiint E dA = E \oiint dA$$

Note that $\oiint dA$ **is just the area of the Gaussian surface over which we are integrating. Gauss' Law now takes the form:**

$$E \oiint dA = q_{enclosed} / \varepsilon_0$$

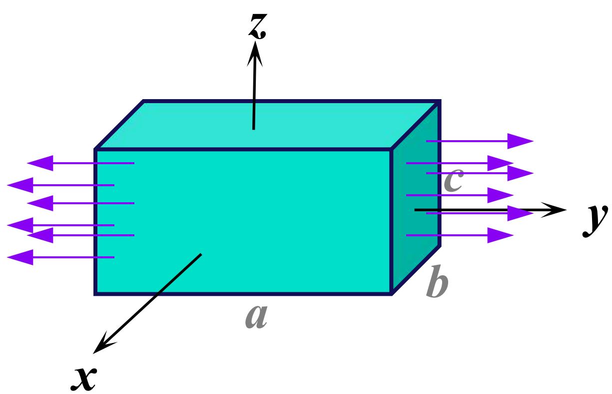
This equation can now be solved for E (at the surface) if we know q_{enclosed} (or for q_{enclosed} if we know E).

Geometry and Surface Integrals

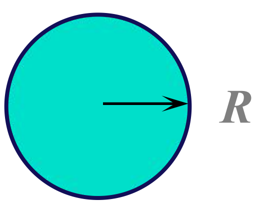
- **If E is constant over a surface, and normal to it everywhere, we can take E outside the integral, leaving only a surface area**

$$\oiint \vec{E} \bullet d\vec{A} = E \oiint dA$$

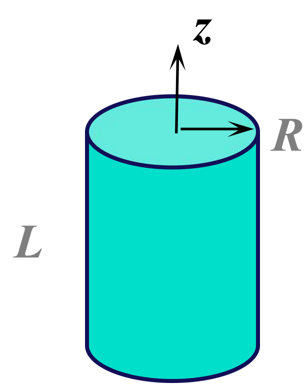
you may use different E'A
for different surfaces
of your "object"



$$\oiint dA = 2ac + 2bc + 2ab$$



$$\oiint dA = 4\pi R^2$$



$$\oiint dA = 2\pi R^2 + 2\pi RL$$

Gauss Law $\Rightarrow$ Coulomb Law

- **We now illustrate this for the field of the point charge and prove that Gauss' Law implies Coulomb's Law.**
- **Symmetry $\Rightarrow$ E-field of point charge is radial and spherically symmetric**
- **Draw a sphere of radius R centered on the charge.**
- **Why?**

  E normal to every point on the surface

  $\Rightarrow \quad \vec{E} \bullet d\vec{A} = EdA$

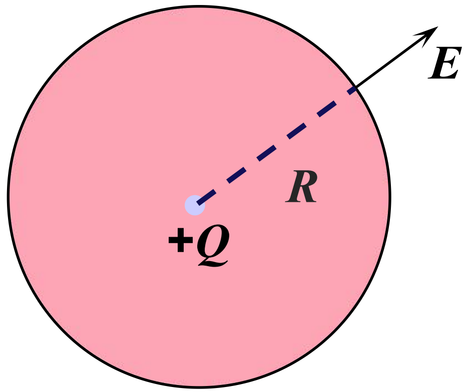
  E has same value at every point on the surface

  $\Rightarrow$ **can take E outside of the integral!**

- **Therefore,** $\oiint \vec{E} \bullet d\vec{A} = \oiint EdA = E \oiint dA = 4\pi R^2 E$ **!**
  - **Gauss' Law** $\quad \varepsilon_0 4\pi R^2 E = Q$

    $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

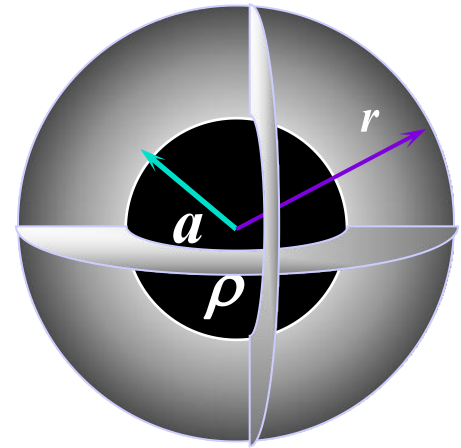
  - **We are free to choose the surface in such problems… we call this a "Gaussian" surface**



27-5, 6 Applications of Gauss' Law

1. Uniform charged sphere

What is the magnitude of the electric field due to a <u>solid</u> sphere of radius a with uniform charge density ρ (C/m³)?



- **Outside sphere: ($r>a$)**
  - **We have spherical symmetry centered on the center of the sphere of charge**
  - **Therefore, choose Gaussian surface = hollow sphere of radius r**

$$\oiint \vec{E} \bullet d\vec{A} = 4\pi r^2 E = \frac{q}{\varepsilon_0}$$

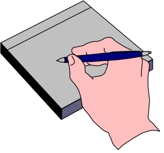
$$q = \frac{4}{3}\pi a^3 \rho$$

$\Rightarrow$ **Gauss' Law**

$$\boxed{E = \frac{\rho a^3}{3\varepsilon_0 r^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

same as point charge!

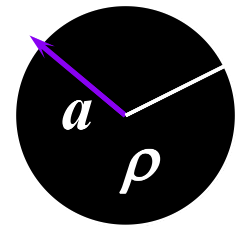


Uniform charged sphere

r

- **Outside sphere: (*r* > *a*)**

$$E = \frac{\rho a^3}{3\varepsilon_0 r^2} = \frac{q}{4\pi\varepsilon_0 r^2}$$



- **Inside sphere: (*r* < *a*)**
  - **We still have spherical symmetry centered on the center of the sphere of charge.**
  - **Therefore, choose Gaussian surface = sphere of radius *r***

Gauss' Law

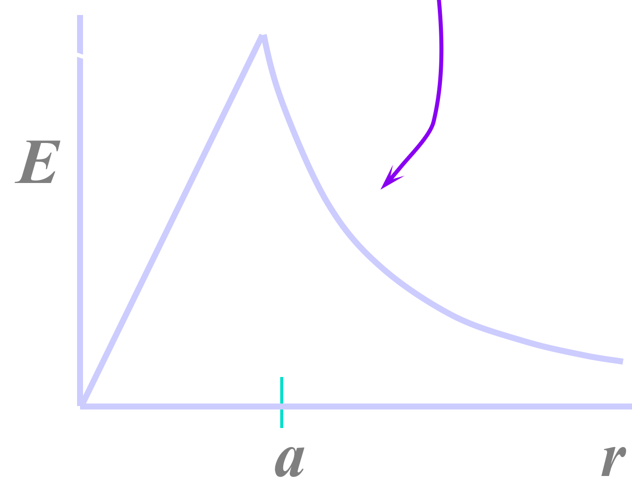
$$\oiint \vec{E} \bullet d\vec{A} = 4\pi r^2 E = \frac{q}{\varepsilon_0}$$

But,

$$q = \frac{4}{3}\pi r^3 \rho$$

Thus:

$$E = \frac{\rho}{3\varepsilon_0} r$$

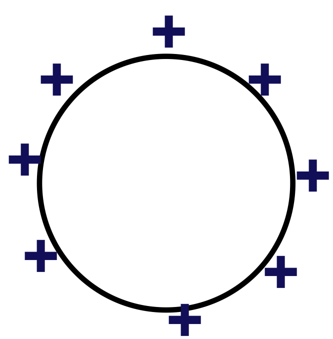


2. Gauss' Law and Conductors

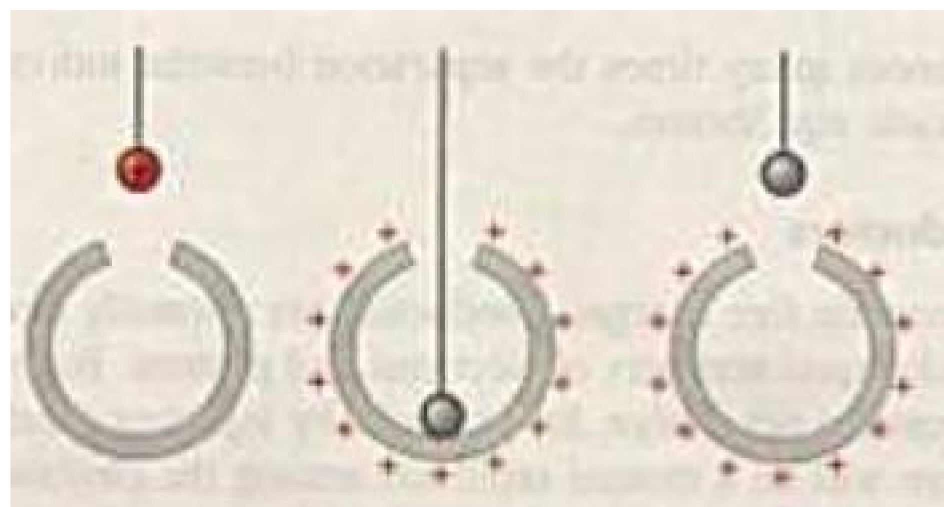
- **We know that *E*=0 inside a conductor (otherwise the charges would move).**

- **But since** $\oiint \vec{E} \bullet d\vec{A} = 0 \rightarrow \boxed{Q_{\text{inside}} = 0}$ **.**

Charges on a conductor only reside on the surface(s)!

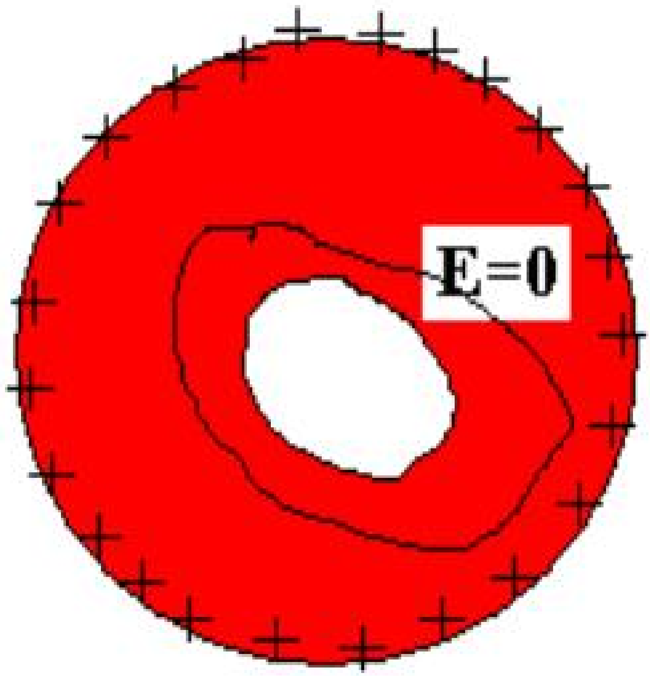


Conducting sphere

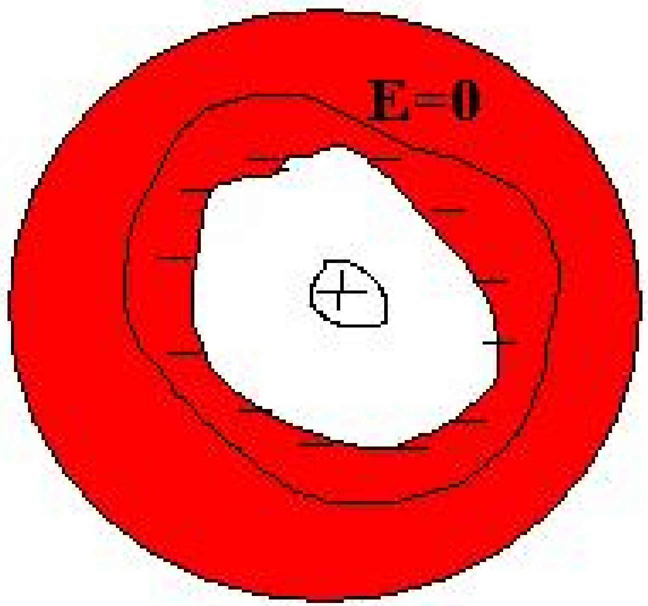


A charged isolated conductor (带电的孤立导体)

- An isolated conductor with a cavity.

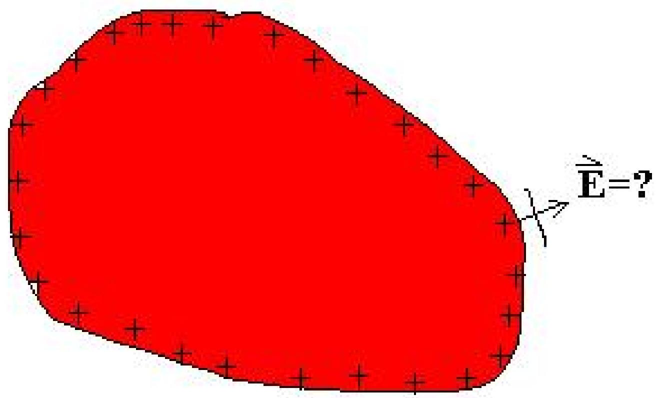


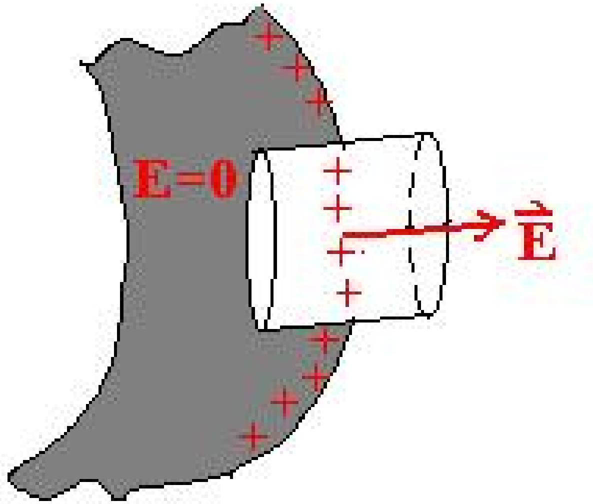
- Charge in cavity



- The distribution of excess charge is not changed

The outer electric field for a charged conductor



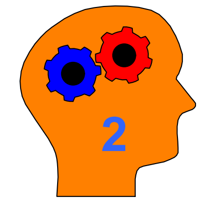


$$\varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = \sigma \cdot \Delta A$$

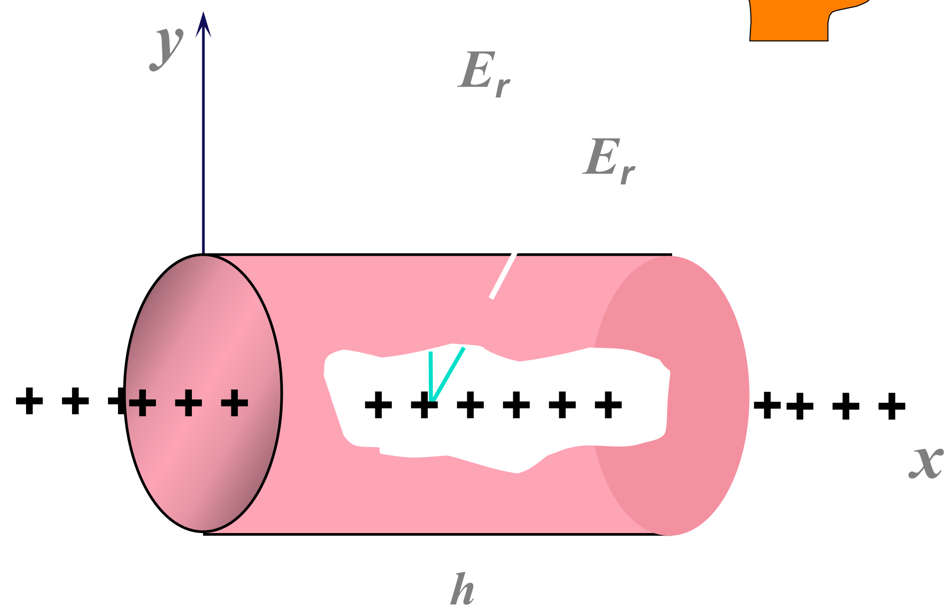
$$\varepsilon_0 E \Delta A + 0 + 0 = \sigma \cdot \Delta A$$

$$E = \frac{\sigma}{\varepsilon_0}$$

3. Infinite Line of Charge



- Symmetry $\Rightarrow$ E-field must be $\perp$ to line and can only depend on distance from line
- Therefore, CHOOSE Gaussian surface to be a cylinder of radius r and length h aligned with the x-axis.



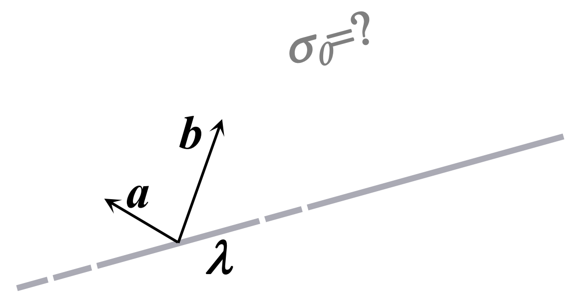
•Apply Gauss' Law:

- On the ends, $\vec{E} \bullet d\vec{A} = 0$
- On the barrel, $\oiint \vec{E} \bullet d\vec{A} = 2\pi rhE$ AND $q = \lambda h$ $\Rightarrow$ $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

NOTE: we have obtained here the same result as we did last lecture using Coulomb's Law. The symmetry makes today's derivation easier.

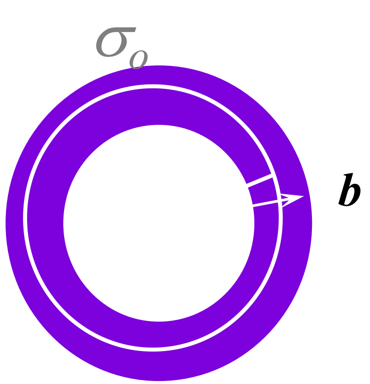
Chapter 27, ACT 2

- **A line charge λ (C/m) is placed along the axis of an uncharged conducting cylinder of inner radius $r_i = a$, and outer radius $r_o = b$ as shown.**
  - **What is the value of the charge density σ_o (C/m²) on the outer surface of the cylinder?**



(a) $\sigma_o = -\dfrac{\lambda}{2\pi b}$ **(b)** $\sigma_o = 0$ **(c)** $\boxed{\sigma_o = +\dfrac{\lambda}{2\pi b}}$

View end on:
Draw Gaussian tube which surrounds only the outer edge

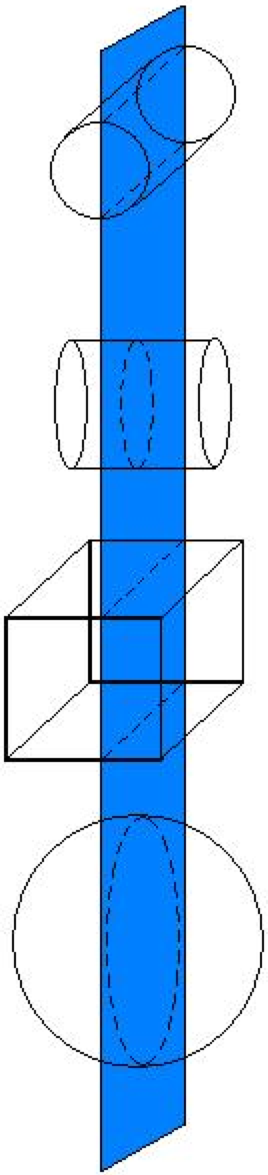


$$E_{outside} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\oint\!\!\oint \vec{E} \bullet d\vec{A} = (2\pi rL)E_{conductor} + (2\pi rL)E_{outside} = \frac{q}{\varepsilon_0} = \frac{\sigma_o 2\pi bL}{\varepsilon_0}$$

$$E_{outside} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma_o b}{\varepsilon_0 r} \Rightarrow \sigma_o = \frac{\lambda}{2\pi b}$$

Preflight 4:



5) Given an infinite sheet of charge as shown in the figure. You need to use Gauss' Law to calculate the electric field near the sheet of charge. Which of the following Gaussian surfaces are best suited for this purpose?

Note: you may choose more than one answer

a) a cylinder with its axis along the plane

b) a cylinder with its axis perpendicular to the plane

c) a cube

d) a sphere

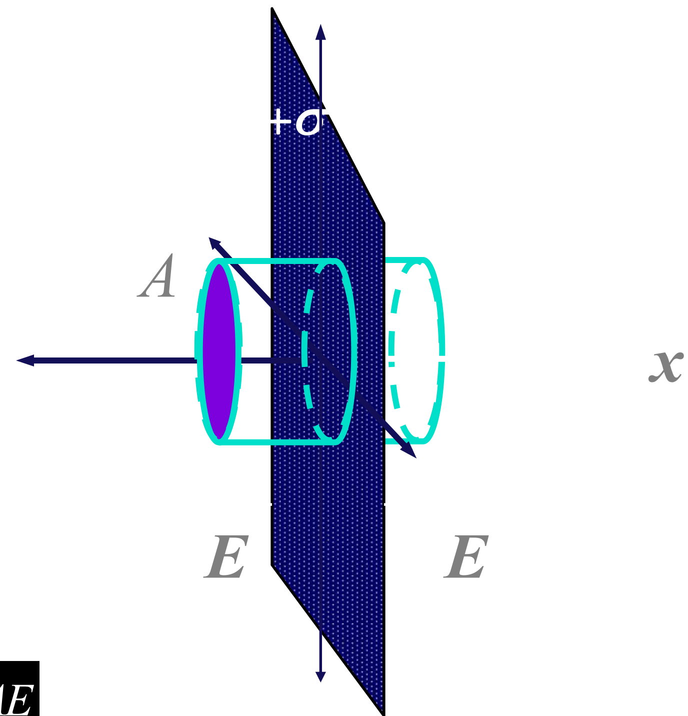
$$\varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = q_{enclosed}$$

4. Infinite sheet of charge

- **Symmetry:**

  direction of E = x-axis

- **Therefore, CHOOSE Gaussian surface to be a cylinder whose axis is aligned with the x-axis.**



- **Apply Gauss' Law:**
  - **On the barrel,** $\vec{E} \bullet d\vec{A} = 0$
  - **On the ends,** $\oiint \vec{E} \bullet d\vec{A} = 2AE$
  - **The charge enclosed =** σA

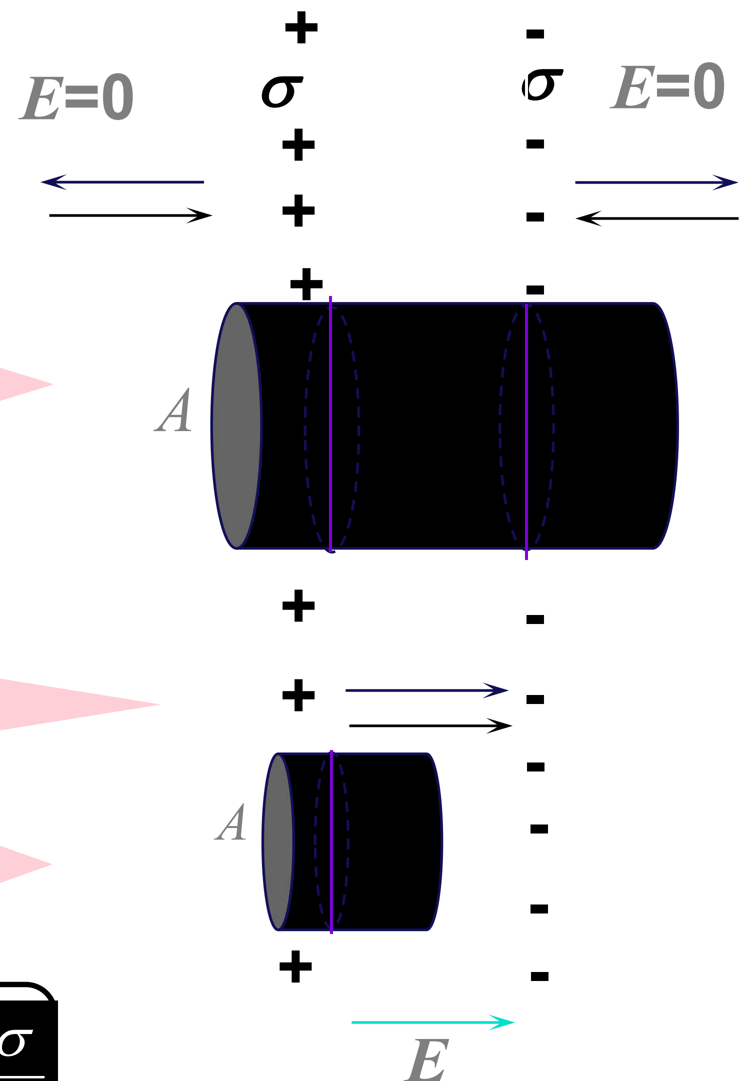
Therefore, Gauss' Law $\Rightarrow \varepsilon_0 (2EA) = \sigma A$ $\Rightarrow$ $\boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

Conclusion: An infinite plane sheet of charge creates a CONSTANT electric field .

Two Infinite Sheets

(into the screen)

- Field outside must be zero. Two ways to see:
  - Superposition
  - Gaussian surface encloses zero charge
- Field inside is NOT zero:
  - Superposition
  - Gaussian surface encloses non-zero charge

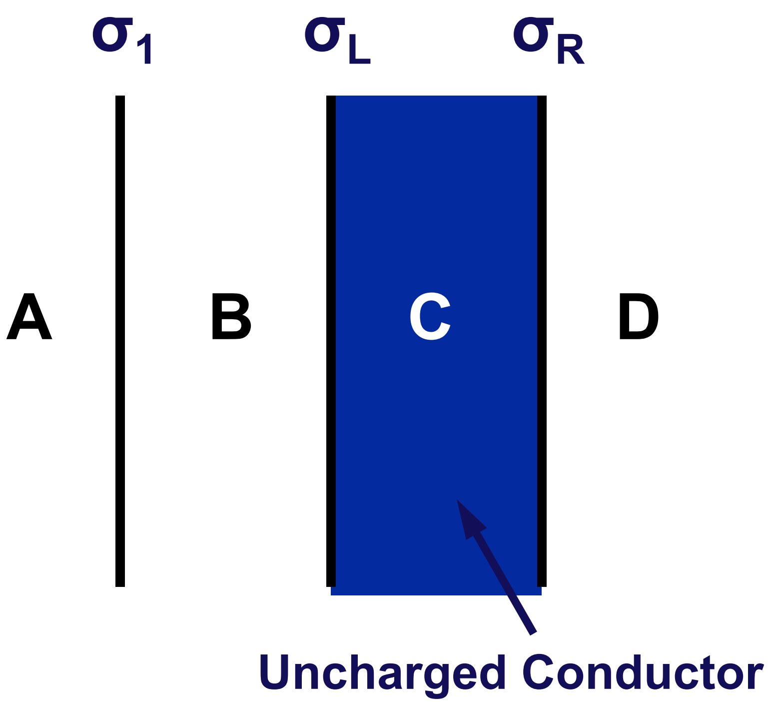


$$Q = \sigma A$$

$$\oiint \vec{E} \bullet d\vec{A} = \cancelto{0}{AE_{outside}} + AE_{inside}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Sheets of Charge



$$\vec{E}_A = \frac{-\sigma_1}{2\varepsilon_0}\hat{x},$$

$$\vec{E}_B = \frac{+\sigma_1}{2\varepsilon_0}\hat{x},$$

$$E_C = 0,$$

$$\vec{E}_D = \frac{+\sigma_1}{2\varepsilon_0}\hat{x}$$

Hints:

1. Assume σ is positive. If it's negative, the answer will still work.
2. Assume $\hat{x}$ to the right.
3. Use superposition, but keep signs straight
4. Think about which way a test charge would move.

Gauss' Law: Help for the Problems

- **How to do practically all of the homework problems**
  - **Gauss' Law is ALWAYS VALID!**

$$\varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = q_{enclosed}$$

  - **What Can You Do With This?**

    If you have (a) spherical, (b) cylindrical, or (c) planar symmetry AND:
    - **If you know the charge, you can calculate the electric field**
    - **If you know the field (usually because E=0 inside conductor), you can calculate the charge.**

- **Spherical Symmetry: Gaussian surface = Sphere of radius r**

$$\varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = 4\pi\varepsilon_0 r^2 E = q$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

- **Cylindrical symmetry: Gaussian surface = cylinder of radius r**

$$\varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = \varepsilon_0 2\pi r L E = q$$

$$E = \frac{q}{2\pi\varepsilon_0 r}$$

- **Planar Symmetry: Gaussian surface = Cylinder of area A**

$$\varepsilon_0 \oiint \vec{E} \bullet d\vec{A} = \varepsilon_0 2 A E = q$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Summary

- Gauss' Law: Electric field flux through a closed surface is proportional to the net charge enclosed

$$\varepsilon_0 \oint\!\!\oint \vec{E} \bullet d\vec{A} = \varepsilon_0 \Phi = q_{enclosed}$$

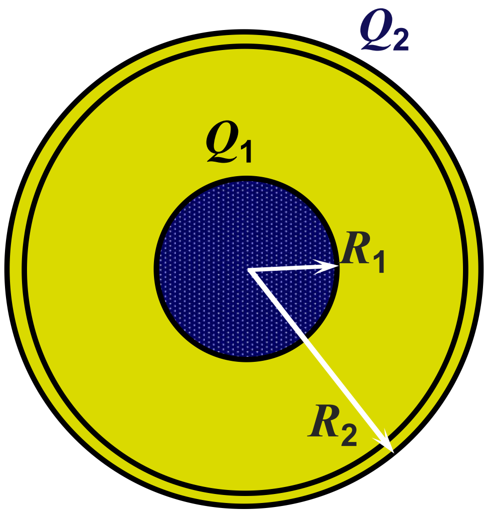
  Gauss' Law is exact and *always* true….

- Gauss' Law makes solving for E-field easy when the symmetry is sufficient
  - spherical, cylindrical, planar

- Gauss' Law proves that electric fields vanish in conductor
  - extra charges reside on surface

Example 1: spheres

- A solid *conducting* sphere is concentric with a thin *conducting* shell, as shown
- The inner sphere carries a charge Q_1, and the spherical shell carries a charge Q_2, such that $Q_2 = -3Q_1$.



A

- How is the charge distributed on the sphere?

B

- How is the charge distributed on the spherical shell?

C

- What is the electric field at $r < R_1$? Between R_1 and R_2? At $r > R_2$?

D

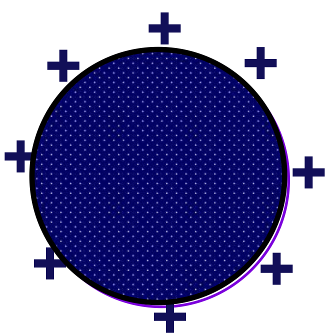
- What happens when you connect the two spheres with a wire? (What are the charges?)

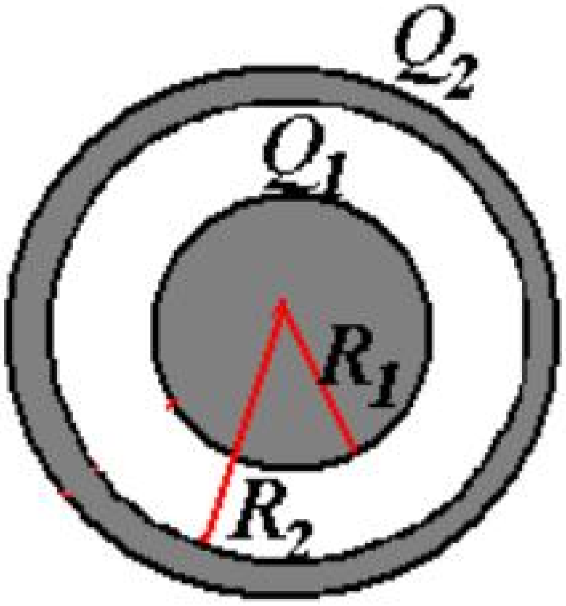
A

- **How is the charge distributed on the sphere?**

* The electric field <u>inside</u> a conductor is zero.

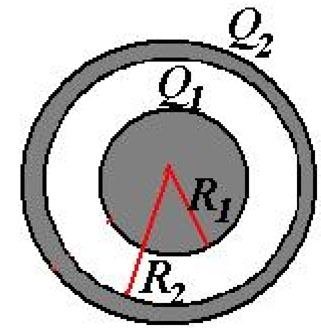
(A) By Gauss's Law, there can be no net charge inside the conductor, and the charge must reside on the outside surface of the sphere





B

- **How is the charge distributed on the spherical shell?**



* **The electric field <u>inside</u> the conducting shell is zero.**

(B) **There can be no net charge inside the conductor, therefore the inner surface of the shell must carry a net charge of -Q_1, and the outer surface must carry the charge +Q_1 + Q_2, so that the net charge on the shell equals Q_2.**

The charges are distributed uniformly over the inner and outer surfaces of the shell, hence

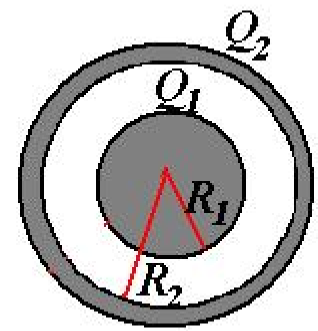
$$\boxed{\sigma_{inner} = -\frac{Q_1}{4\pi R_2{}^2}}$$

and

$$\boxed{\sigma_{outer} = \frac{Q_2 + Q_1}{4\pi R_2{}^2} = \frac{-2Q_1}{4\pi R_2{}^2}}$$

C

- What is the Electric Field at $r < R_1$? Between R_1 and R_2? At $r > R_2$?



* The electric field <u>inside</u> a conductor is zero.

(C) $r < R_1$:
Inside the conducting sphere

$$\vec{E} = 0.$$

(C) Between R_1 and R_2 : $R_1 < r < R_2$
Charge enclosed = Q_1

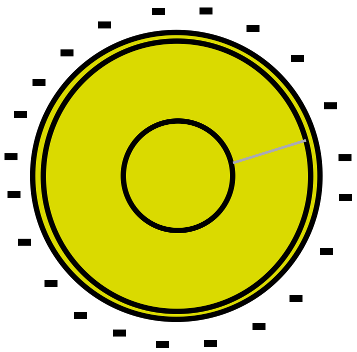
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}\hat{r}$$

(C) $r > R_2$
Charge enclosed = $Q_1 + Q_2$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 + Q_2}{r^2}\hat{r} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q_1}{r^2}\hat{r}$$

D

- What happens when you connect the two spheres with a wire? (What are the charges?)



After electrostatic equilibrium is reached, there is no charge on the inner sphere, and none on the inner surface of the shell. The charge $Q_1 + Q_2$ on the outer surface remains.

Also, for $r < R_2$

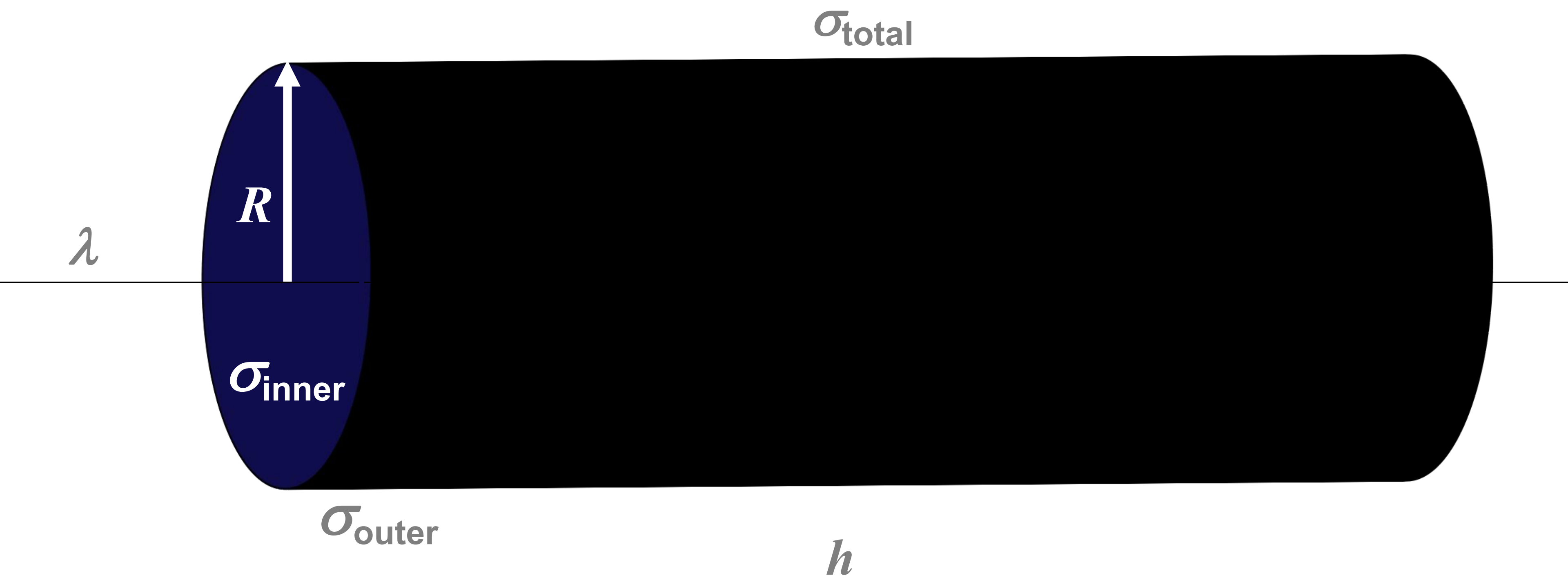
$$\vec{E} = 0.$$

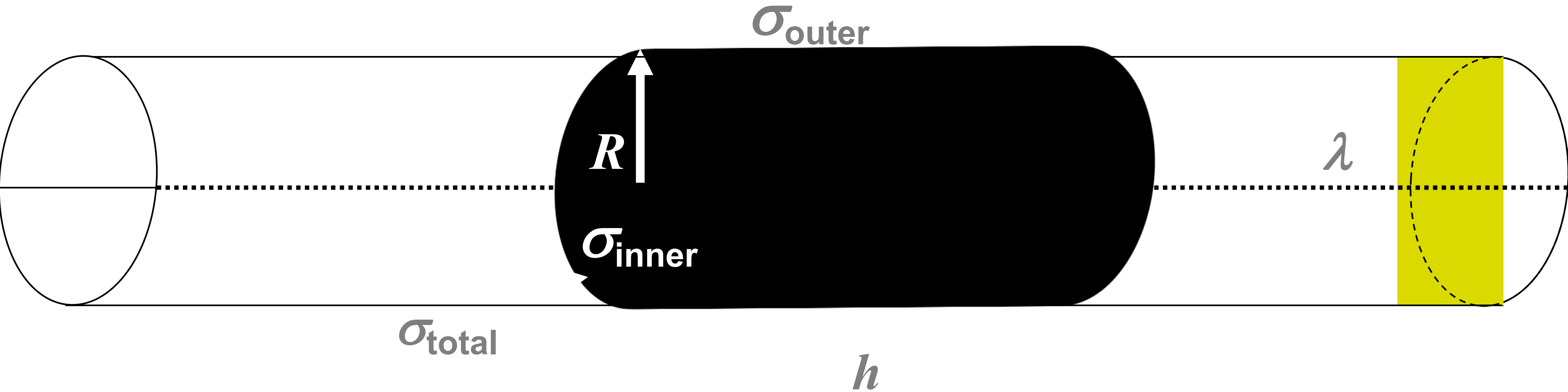
and for $r > R_2$

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q_1}{r^2}\hat{r}$$

Example 2: Cylinders

An infinite line of charge passes directly through the middle of a hollow, charged, CONDUCTING infinite cylindrical shell of radius R. We will focus on a segment of the cylindrical shell of length h. The line charge has a linear charge density λ, and the cylindrical shell has a net surface charge density of σ_{total}.





A

- **How is the charge distributed on the cylindrical shell?**
  - **What is σ_{inner}?**
  - **What is σ_{outer}?**

B

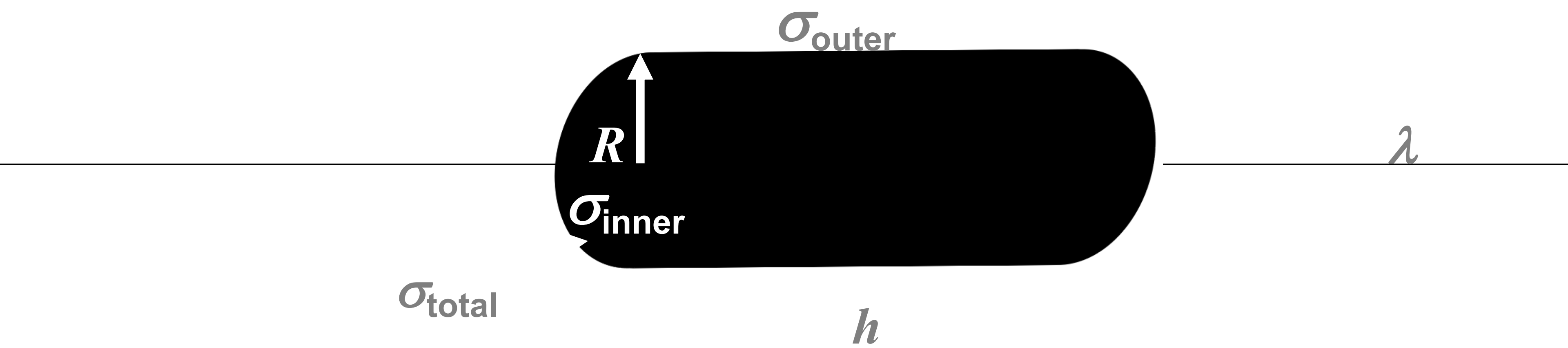
- **What is the electric field at $r<R$?**

C

- **What is the electric field for $r>R$?**

A1

What is σ_{inner}?



The electric field inside the cylindrical shell is zero. Hence, if we choose as our Gaussian surface a cylinder, which lies inside the cylindrical shell, we know that the net charge enclosed is zero. Therefore, there will be a surface charge density on the inside wall of the cylinder to balance out the charge along the line.

•The total charge on the enclosed portion (of length h) of the line charge is:

Total line charge enclosed = λh

•Therefore, the charge on the inner surface of the conducting cylindrical shell is

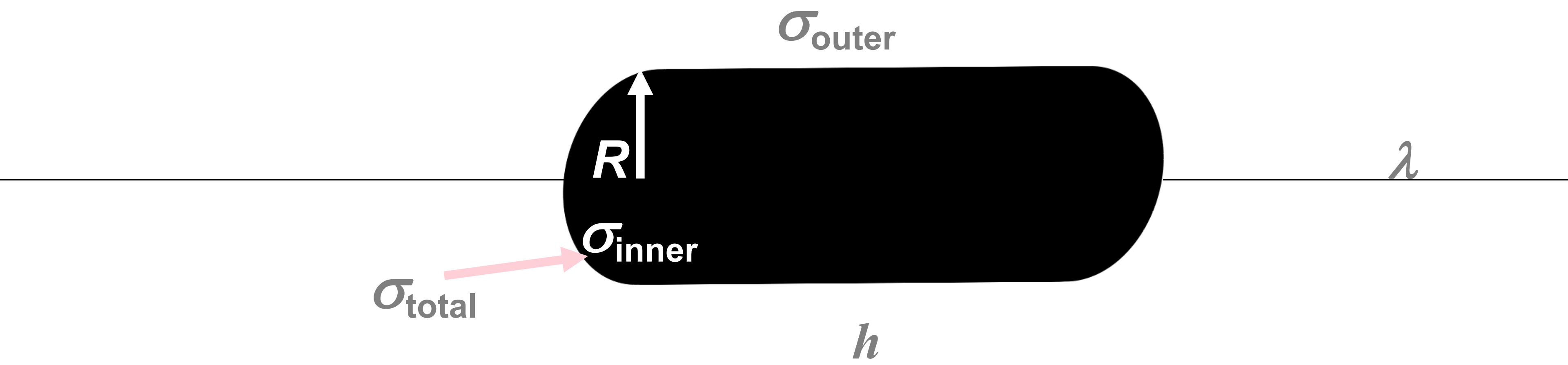
$$Q_{inner} = -\lambda h$$

The total charge is evenly distributed along the inside surface of the cylinder.

Therefore, the inner surface charge density σ_{inner} **is just** Q_{inner} **divided by total area of the cylinder:** $\sigma_{inner} = -\lambda h / 2\pi Rh = -\lambda / 2\pi R$

•Notice that the result is independent of h**.**

A2 What is σ_{outer}?

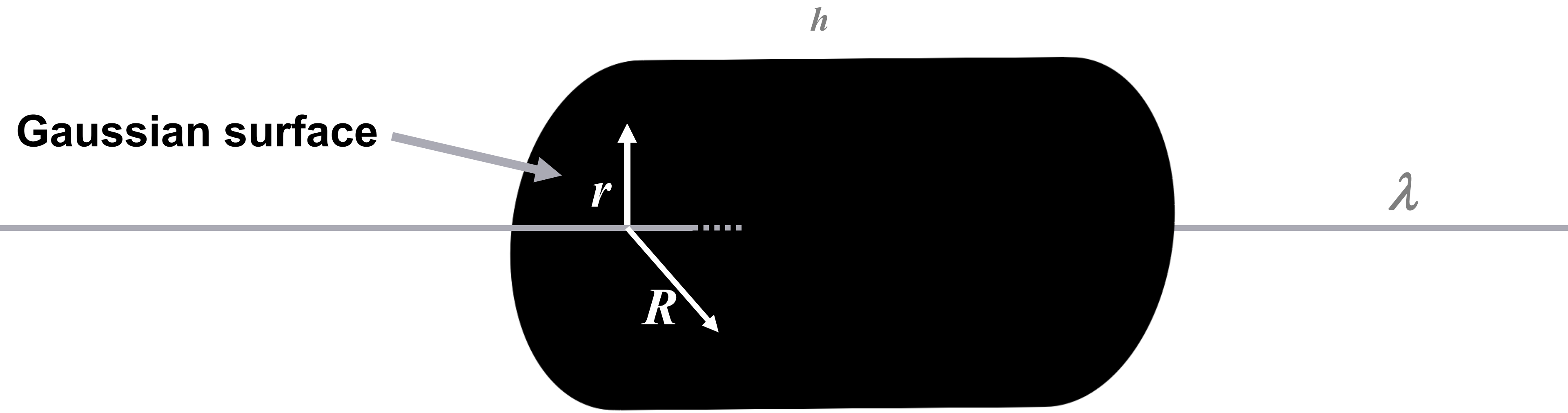


•We know that the net charge density on the cylinder is σ_{total}. The charge densities on the inner and outer surfaces of the cylindrical shell have to add up to σ_{total}. Therefore,

$$\sigma_{outer} = \sigma_{total} - \sigma_{inner} = \sigma_{total} + \lambda/(2\pi R).$$

B

What is the Electric Field at $r<R$?



•Whenever we are dealing with electric fields created by symmetric charged surfaces, we must always first chose an appropriate Gaussian surface. In this case, for $r < R$, the surface surrounding the line charge is actually a cylinder of radius r.

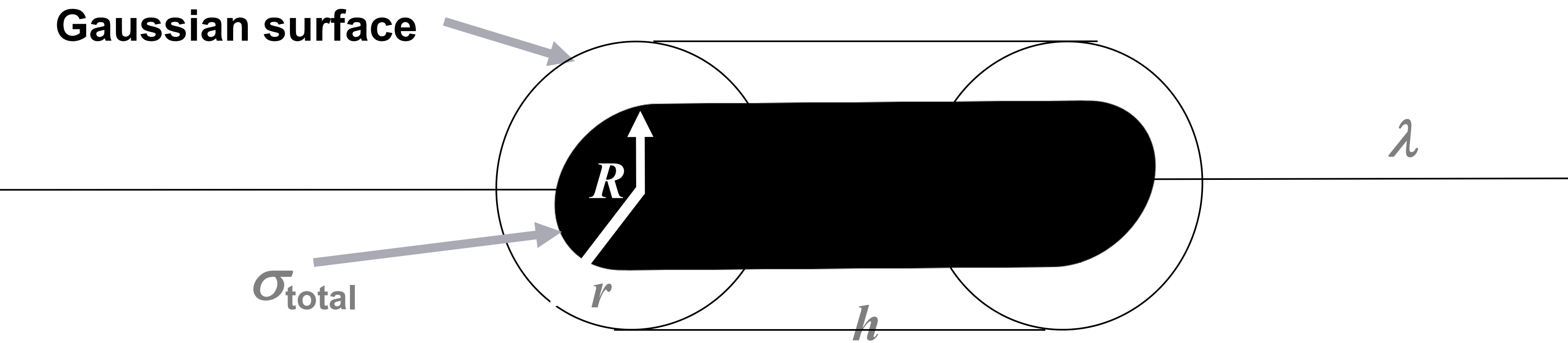
•Using Gauss' Law, the following equation determines the E-field:

$$2\pi r h E_r = q_{\text{enclosed}} / \varepsilon_o$$

q_{enclosed} is the charge on the enclosed line charge, which is λh, and $(2\pi r h)$ is the area of the barrel of the Gaussian surface.

The result is: $E_r = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

C What is the Electric field for $r>R$?



- As usual, we must first chose a Gaussian surface as indicated above. We also need to know the net charge enclosed in our Gaussian surface. The net charge is a sum of the following:
  - Net charge enclosed on the line: λh
  - Net charge enclosed within Gaussian surface, residing on the cylindrical shell: $Q = 2\pi R h\, \sigma_{total}$
- Therefore, net charge enclosed is $Q + \lambda h$
- The surface area of the barrel of the Gaussian surface is $2\pi r h$
- Now we can use Gauss' Law: $2\pi r h\, E = (Q + \lambda h) / \varepsilon_o$
  - You have all you need to find the Electric field now.

Solve for E_r to find

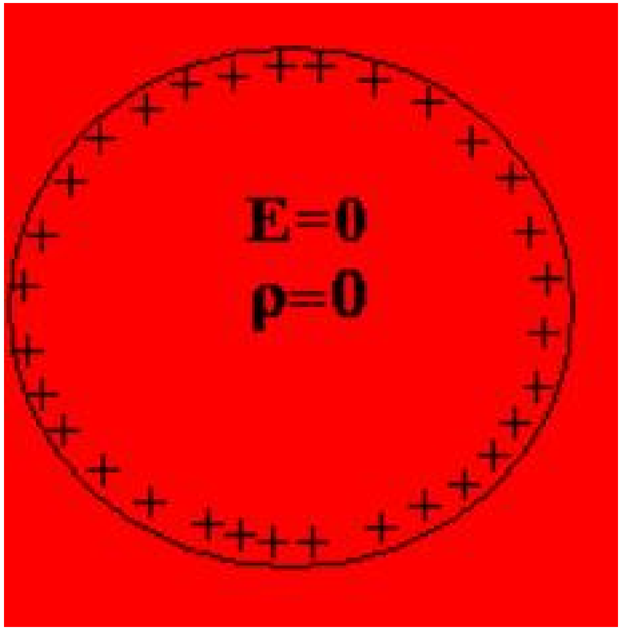
$$E_r = \frac{\sigma}{\varepsilon_0}\frac{R}{r} + \frac{\lambda}{2\pi\varepsilon_0 r}$$

27-7 Experimental Tests of Gauss'Law and Coulomb's Law

- Gauss' Law
- Coulomb's Law

$$E = \frac{q}{4\pi\varepsilon_0 r^{2+\delta}}$$

$$\delta = 0$$



See Page 625, Table 27-1

Homework

- **P_{629} (Exercises): 7, 27,**
- **P_{631}(Problems): 3, 6, 14**