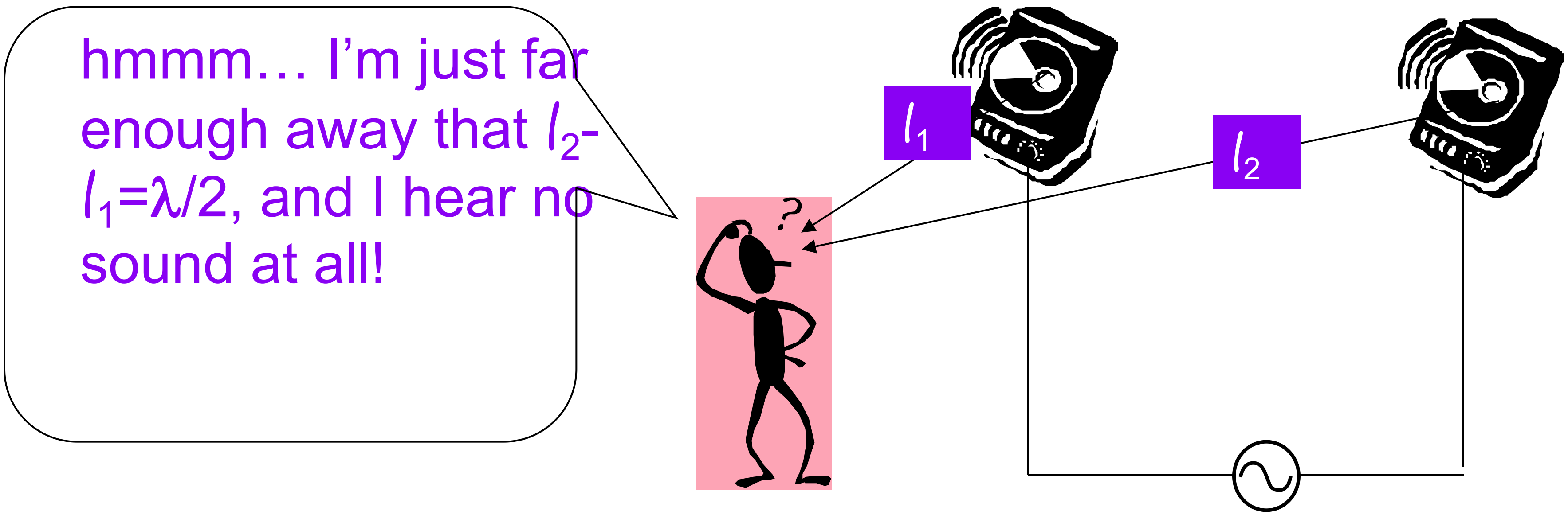
Chapter 41 Wave Optics (1)
Interference(干涉)

41-1 Introduction

- **Geometrical Optics: $\lambda \ll d$**
- **Wave Optics (波动光学): $\lambda \approx d$**
- **Interference, Diffraction: all kinds of wave (Sound wave, water wave, light wave, matter wave……)**
- **The same mathematics (相同的数学方法).**

Interference for Sound …

For example, a pair of speakers, driven in phase, producing a tone of a single f and λ:



But this won't work for light--can't get coherent sources

41-2 Steady Light Wave (定态光波)

1. Wave: Space-Time periodicity ($\vec{r}$, t 周期性)

Scalar Wave: $\rho(\vec{r},t),\ T(\vec{r},t)$

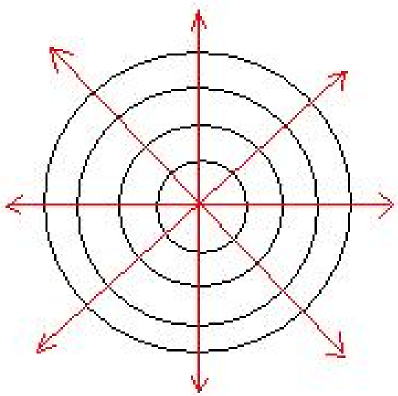
Vector Wave: EM Wave $\vec{E}(\vec{r},t),\ \vec{H}(\vec{r},t)$

Wave Plane (波面)

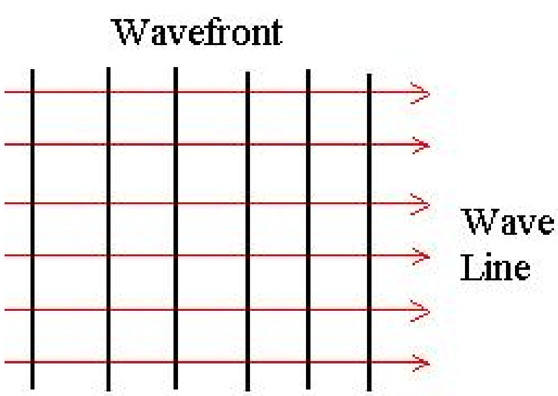
Wave ray (波线)

Spherical plane waves (球面波)

Plane Waves (平面波)



Spherical Wave



Plane Wave

2. Steady Wave (定态波)

- **Steady Scalar Wave**

$$U(P,t) = A(P)\cos[\omega t - \varphi(P)]$$

$A(P)$ and $\varphi(P)$ are only the function of space, independent of t.

- **For example:**

Steady Plane Wave:

(定态平面波)

$$\begin{cases} A(P) = \text{constant, It is independent of (x, y, z)} \\ \varphi(P) = \vec{k} \bullet \vec{r} + \varphi_0 = k_x x + k_y y + k_z z + \varphi_0 \end{cases}$$

$$k = \frac{2\pi}{\lambda}, \quad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

Steady Spherical Wave:

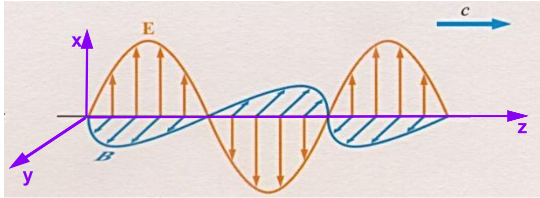
(定态球面波)

$$\begin{cases} A(P) = \dfrac{a}{r} \\ \varphi(P) = kr + \varphi_0 \end{cases}$$

- **Light Wave (EM Wave)**

$$\vec{E}(P,t) = \vec{E}_0(P)\cos[\omega t - \varphi(P)]$$

$$\vec{H}(P,t) = \vec{H}_0(P)\cos[\omega t - \varphi(P)]$$



3. Complex Number Description (复数描述)

$$U(P,t)=A(P)\cos[\omega t-\varphi(P)]$$

To Choose "-"

$$\Leftrightarrow \tilde{U}(P,t)=A(P)e^{\pm i[\omega t-\varphi(P)]}$$

$$\tilde{U}(P,t)=A(P)e^{i[\varphi(P)-\omega t]}=A(P)e^{i\varphi(P)}e^{-i\omega t}$$

$$\tilde{U}(P)=A(P)e^{i\varphi(P)}$$ 复振幅

Plane Wave:

$$\tilde{U}(P)=Ae^{i\varphi(P)}=Ae^{i(\vec{k}\bullet\vec{r}+\varphi_0)}=Ae^{i(k_x x+k_y y+k_z z+\varphi_0)}$$

Spherical Wave:

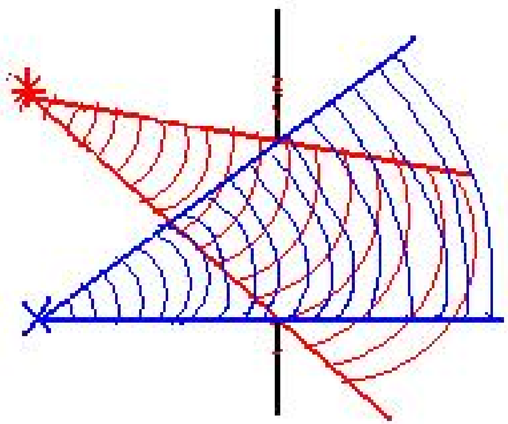
$$\tilde{U}(P)=\frac{a}{r}e^{i(kr+\varphi_0)}$$

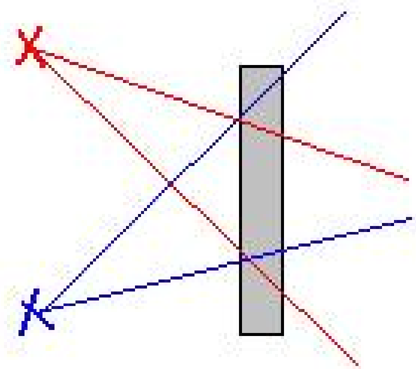
Wave Intensity:

$$I(P)=[A(P)]^2=\tilde{U}^*(P)\cdot\tilde{U}(P)$$

41-3 Wave Superposition and Interference (波的叠加和干涉)

1. Wave superposition principle (波的叠加原理)





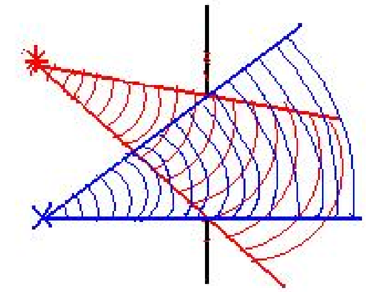
线性叠加

Scalar wave: $U(P,t) = U_1(P,t) + U_2(P,t) + U_3(P,t) + \ldots$

Vector wave: $\vec{U}(P,t) = \vec{U}_1(P,t) + \vec{U}_2(P,t) + \vec{U}_3(P,t) + \ldots$

2. Wave interference and the condition of interference (波的干涉和相干条件)

$$\tilde{U}_1(P,t) = A_1 e^{i\varphi_1(P)} e^{-i\omega t}$$
$$\tilde{U}_2(P,t) = A_2 e^{i\varphi_2(P)} e^{-i\omega t}$$
$$\tilde{U}(P,t) = \tilde{U}_1(P,t) + \tilde{U}_2(P,t) = [A_1 e^{i\varphi_1(P)} + A_1 e^{i\varphi_1(P)}] e^{-i\omega t}$$



Intensity:

$$\begin{aligned} I(P) &= \tilde{U}^*(P)\cdot\tilde{U}(P) \\ &= [A_1 e^{-i\phi_1(P)} + A_2 e^{-i\phi_2(P)}][A_1 e^{i\phi_1(P)} + A_2 e^{i\phi_2(P)}] \\ &= A_1^2 + A_2^2 + A_1A_2[e^{i(\phi_1-\phi_2)} + e^{-i(\phi_1-\phi_2)}] \\ &= A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1-\phi_2) \end{aligned}$$

If:

$$I_1(P) = A_1^2(P)$$
$$I_2(P) = A_2^2(P)$$

$$I(P) = I_1(P) + I_2(P) + 2\sqrt{I_1(P)I_2(P)}\cos(\varphi_1-\varphi_2)$$

In general:

$$I(P) \neq I_1(P) + I_2(P)$$

$$\cos(\varphi_1-\varphi_2) > 0, \quad I(P) > I_1(P) + I_2(P)$$
$$\cos(\varphi_1-\varphi_2) < 0, \quad I(P) < I_1(P) + I_2(P)$$

The wave superposition results in the re-distribution of intensity in space. Wave Interference.

Discussion

$$I(P)=I_1(P)+I_2(P)+2\sqrt{I_1(P)I_2(P)}\cos(\varphi_1-\varphi_2)$$

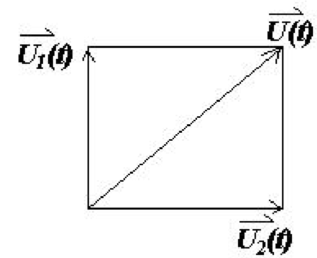
- If $\delta(P)=\varphi_1-\varphi_2$ is not steady, $\overline{\cos\ \delta(P)}=\overline{cos(\varphi_1-\varphi_2\ )}=0$

  No Interference

- For Vector Wave,

  No interference

$$\text{For example: light wave, } \vec{E}(P,t)$$
$$\vec{U}_1(P,t)\perp\vec{U}_2(P,t)$$
$$\vec{U}(P,t)=\vec{U}_1(P,t)+\vec{U}_2(P,t)$$
$$U^2(P,t)=U_1^2(P,t)+U_1^2(P,t)$$



- If $\omega_1\neq\omega_2$

$$\tilde{U}(P,t)=A_1e^{i\varphi_1(P)}e^{-i\omega_1t}+A_2e^{i\varphi_2(P)}e^{-i\omega_2t}$$
$$\begin{aligned}I(P,t)&=\tilde{U}^*(P,t)\cdot\tilde{U}(P,t)\\&=[A_1e^{-i\varphi_1(P)}e^{i\omega_1t}+A_2e^{-i\varphi_2(P)}e^{i\omega_2t}]\cdot[A_1e^{i\varphi_1(P)}e^{-i\omega_1t}+A_2e^{i\varphi_2(P)}e^{-i\omega_2t}]\\&=A_1^2+A_2^2+2A_1A_2\cos[(\varphi_1-\varphi_2)-(\omega_1-\omega_2)t]\end{aligned}$$

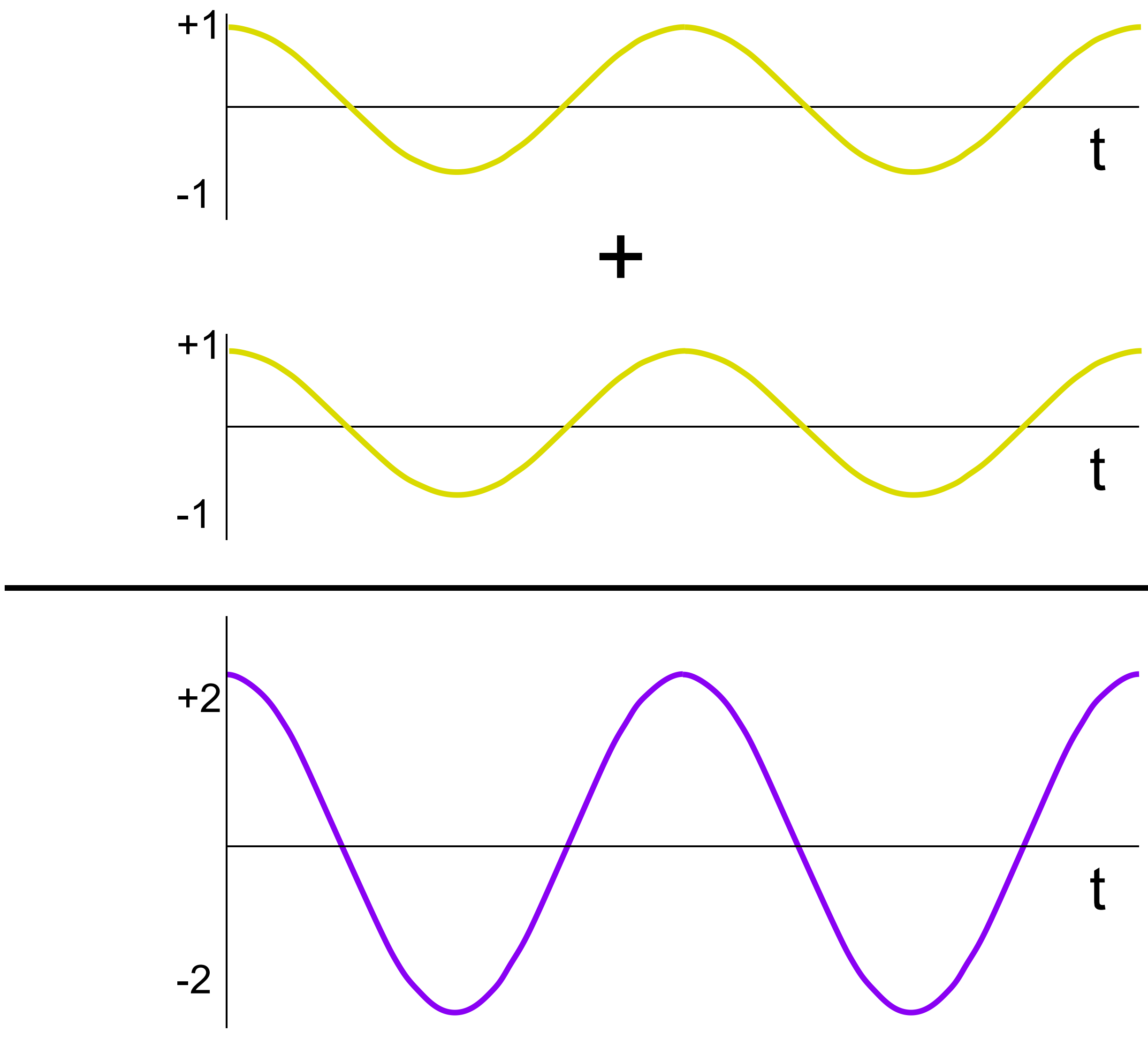
$\omega_1-\omega_2\neq0$ $\quad\overline{cos[(\varphi_1-\varphi_2\ )-(\omega_1-\omega_2)t]}=0$

$I(P)=I_1(P)+I_2(P)$ **No interference.**

Interference Requirements	$\omega_1=\omega_2=\omega$	$\vec{U}_1\parallel\vec{U}_2$	$\varphi_1(P)-\varphi_2(P)$ ***steady***

Superposition

Constructive Interference

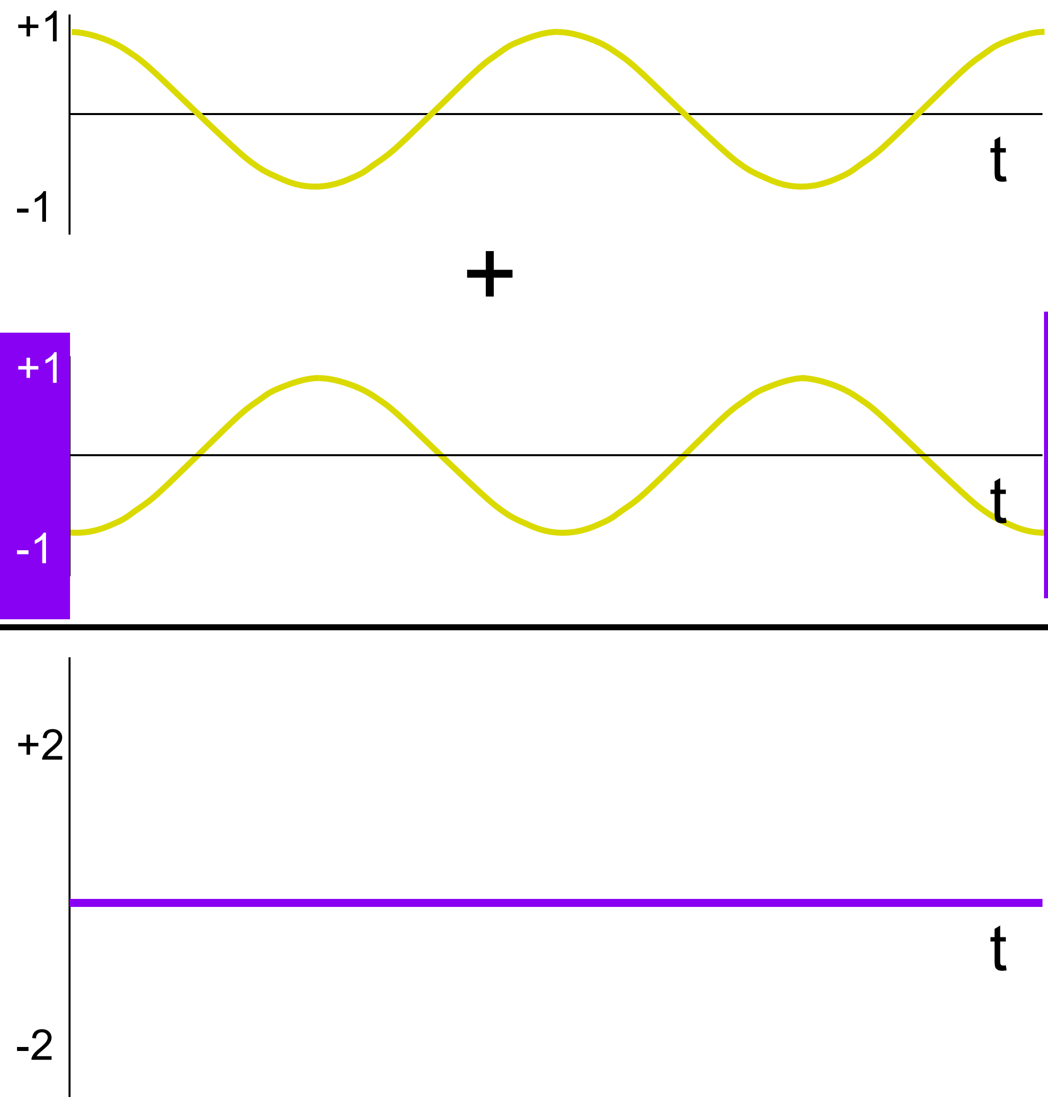


In Phase

$$
\begin{aligned}
I(P) &= \widetilde{U}^*(P)\cdot\widetilde{U}(P) \\
&= A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_1-\varphi_2) \\
&= (A_1 + A_2)^2
\end{aligned}
$$

Superposition

Destructive Interference

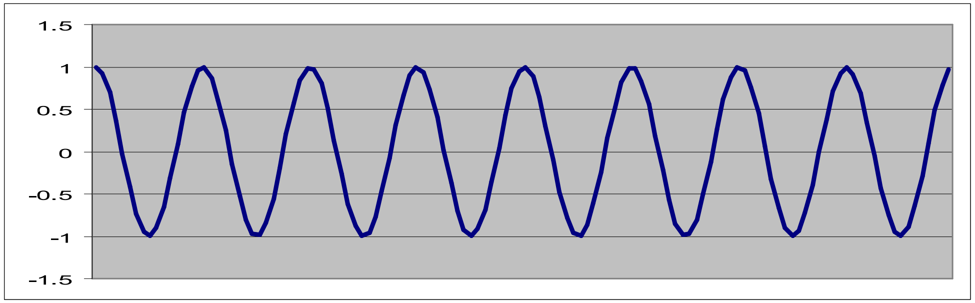


Out of Phase

180 degrees

$$
\begin{aligned}
I(P) &= \widetilde{U}^*(P)\cdot\widetilde{U}(P) \\
&= A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_1-\varphi_2) \\
&= (A_1 - A_2)^2
\end{aligned}
$$

Superposition ACT



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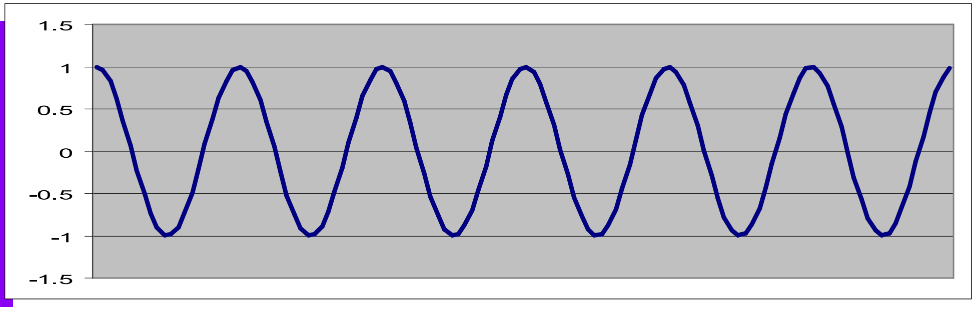
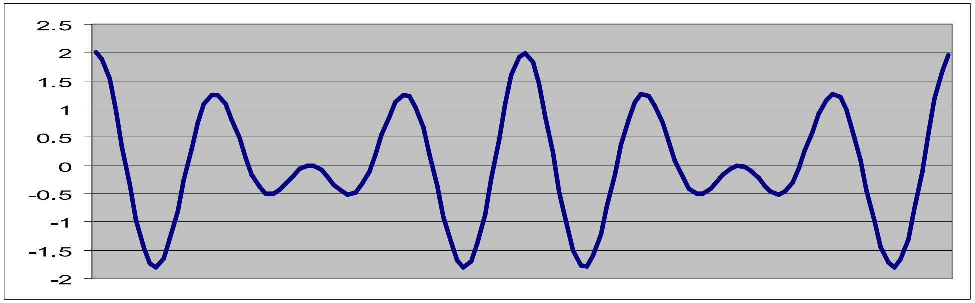


Different ω

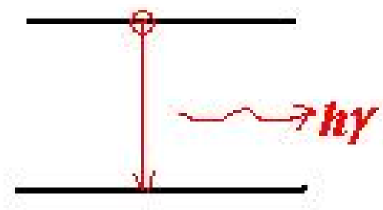


拍频

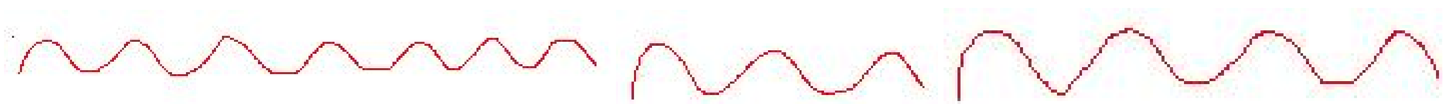
1) Constructive   2) Destructive   3) Neither

3. Coherence（相关性, 相干性）

The phase difference at points in space must not change with time.



Wave train (波列)



$$I(P)=I_1(P)+I_2(P)+2\sqrt{I_1(P)I_2(P)}\cos(\varphi_1-\varphi_2)$$

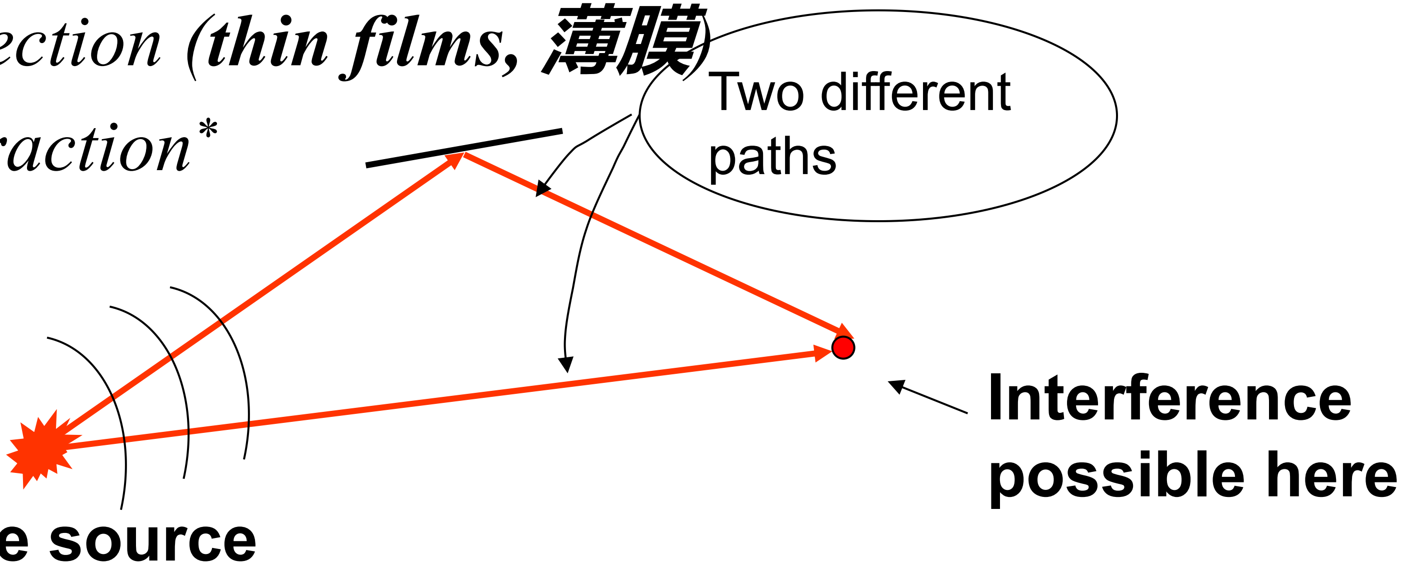
$$\delta=\varphi_1-\varphi_2 \quad \text{change with time,} \quad \cos\delta(P)=\cos(\varphi_1-\varphi_2)=0$$

$$I(P)=I_1(P)+I_2(P), \quad \text{Incoherent(不相干)}$$

Laser(激光)!

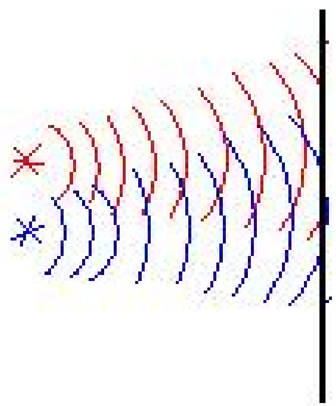
Interference for Light …

- **Can't produce coherent light from separate sources. ($f \approx 10^{14}$ Hz)**
- **Need two waves from <u>single source</u> taking <u>two different paths</u>**
  - *Two slits (**双缝**)*
  - *Reflection (**thin films, 薄膜**)*
  - *Diffraction**



41-4 The interference of two waves

1. The interference of two spherical plane waves:



$$I(P)=I_1(P)+I_2(P)+2\sqrt{I_1(P)I_2(P)}\cos\delta(P)$$

$$I_1(P)=[A_1(P)]^2,\quad A_1(P)\propto\frac{1}{r_1}$$

$$I_2(P)=[A_2(P)]^2,\quad A_2(P)\propto\frac{1}{r_2}$$

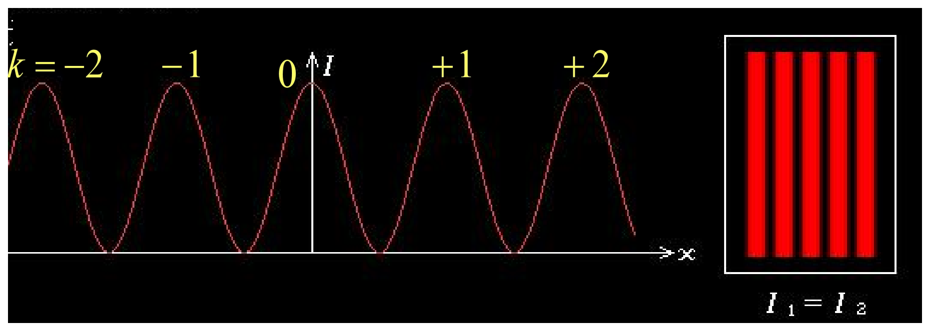
$$\delta(P)=\varphi_1(P)-\varphi_2(P)$$

If $r_1 >> d, r_2 >> d \Rightarrow A_1(P)\approx A_2(P)=A$

Intensity: $I(P)=A^2+A^2+2A\cos\delta(P)$

$$=2A^2[1+\cos\delta(P)]$$

$$=4A^2\cos^2\frac{\delta(P)}{2}$$



Phase: $\delta(P)=\varphi_1(P)-\varphi_2(P)$

$$\varphi_1(P)=\varphi_{10}+kr_1=\varphi_{10}+\frac{2\pi}{\lambda}r_1$$

$$\varphi_2(P)=\varphi_{20}+kr_2=\varphi_{10}+\frac{2\pi}{\lambda}r_2$$

$$\therefore\delta(P)=\varphi_{10}-\varphi_{20}+\frac{2\pi}{\lambda}(r_1-r_2)$$

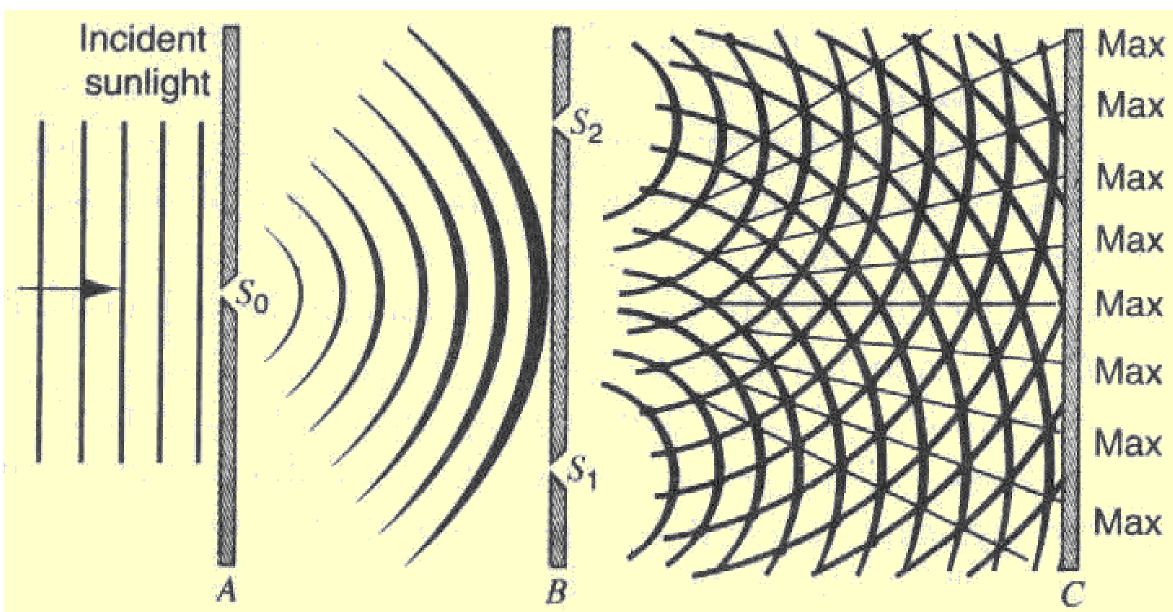
if $\varphi_{10}-\varphi_{20}=0,\quad \delta(P)=\frac{2\pi}{\lambda}(r_1-r_2)$

$$\begin{cases}\Delta L=r_1-r_2=m\lambda, & \text{I(P) maximum}\\ \Delta L=r_1-r_2=(m+\frac{1}{2})\lambda, & \text{I(P) minimum}\end{cases}$$

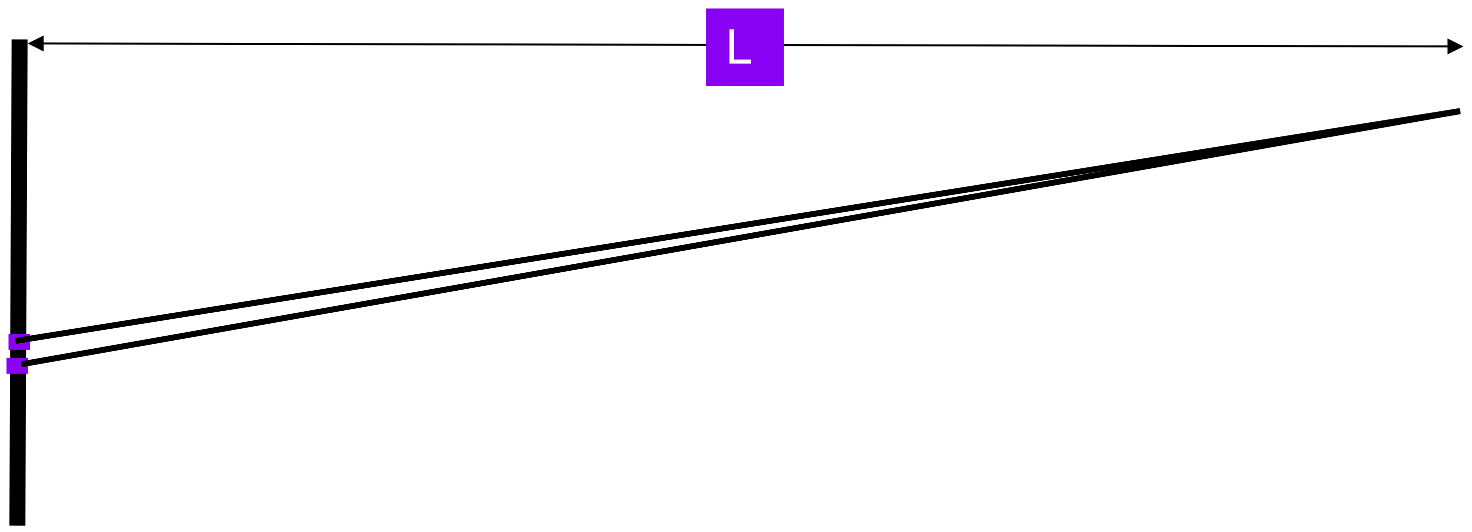
$$m=0,\pm1,\pm2\ldots\ldots\ldots$$

2. Young's Double Slit Interference

（杨氏双缝干涉）



Young's Double Slit Key Idea



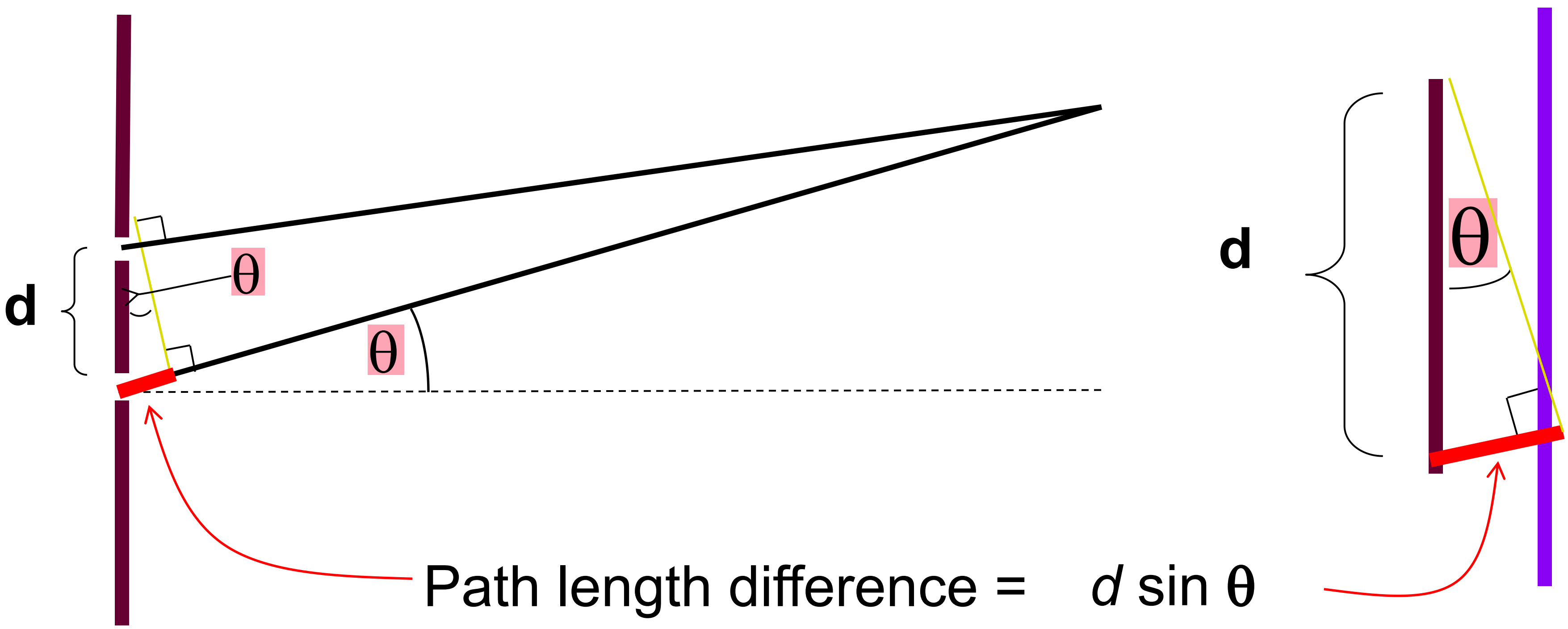
Two rays travel almost exactly the same distance.

(screen must be *very* far away: L >> a)

Bottom ray travels a little further.

Key for interference is this small extra distance.

Young's Double Slit Quantitative



Path length difference = $d \sin \theta$

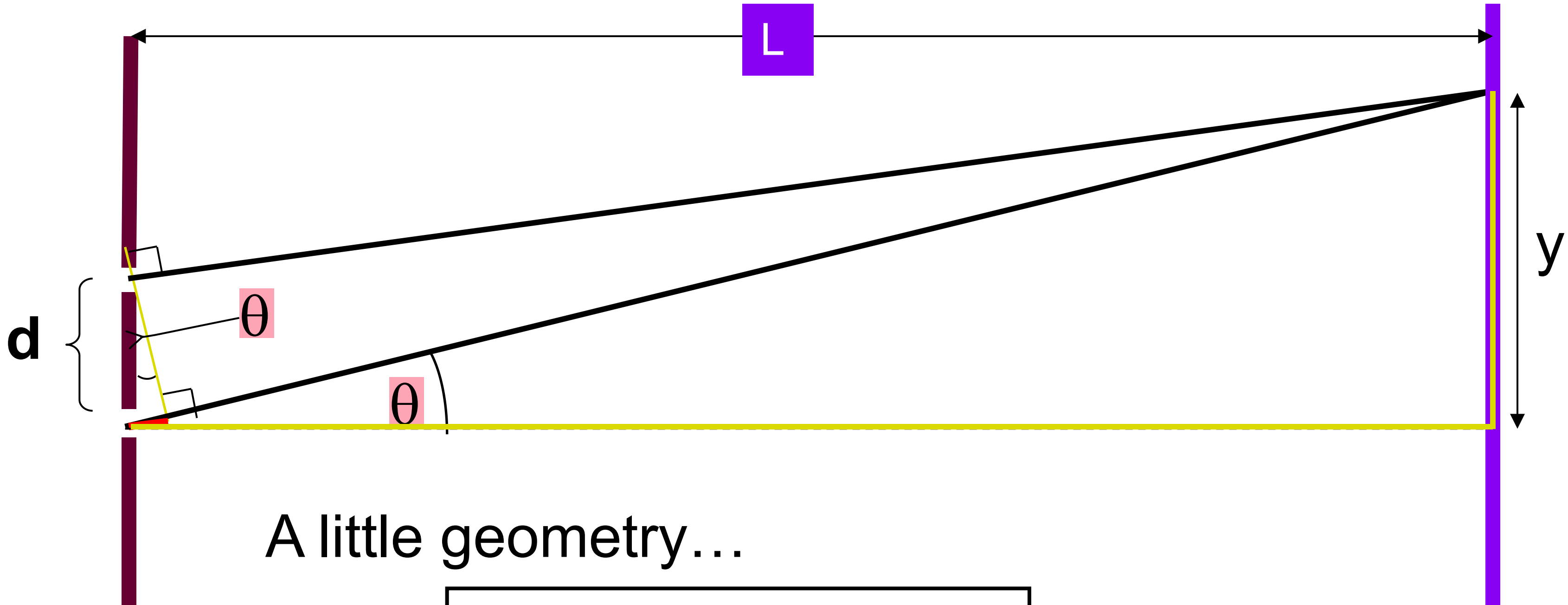
Constructive interference $d \sin \theta = m\lambda$

Destructive interference $d \sin \theta = (m + \frac{1}{2})\lambda$

where m = 0, or 1, or 2, ...

Need λ < d

Young's Double Slit Quantitative



A little geometry…

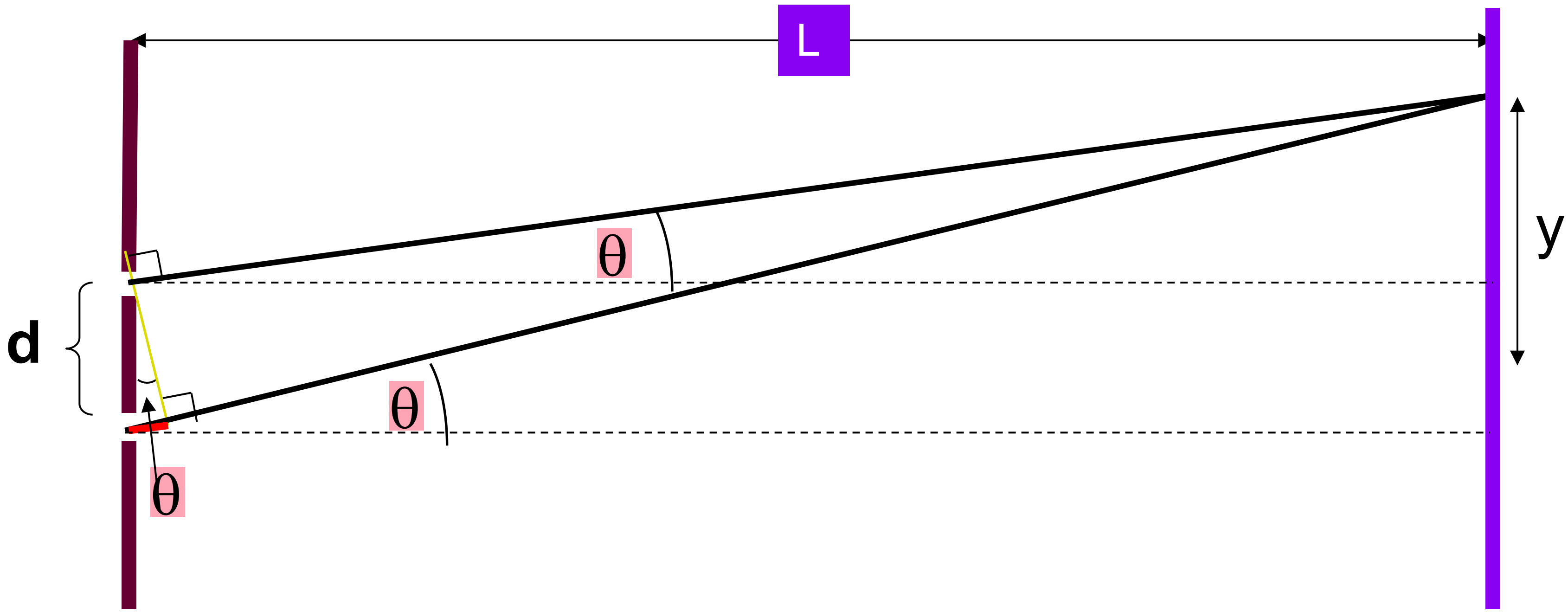
$$\boxed{\sin(\theta) \approx \tan(\theta) = y/L}$$

Constructive interference $\quad d\sin\theta = m\lambda \quad y = \dfrac{m\lambda L}{d}$

Destructive interference $\quad d\sin\theta = (m + \frac{1}{2})\lambda \quad y = \dfrac{\left(m + \frac{1}{2}\right)\lambda L}{d}$

where m = 0, or 1, or 2, ...

Preflight



When this Young's double slit experiment is placed under water. The separation y between minima and maxima

1) increases 27%
2) same 21%
3) decreases 52%

Under water λ decreases so y decreases

Note

In the Young double slit experiment, is it possible to see interference maxima when the distance between slits is smaller than the wavelength of light?

1) Yes 2) No

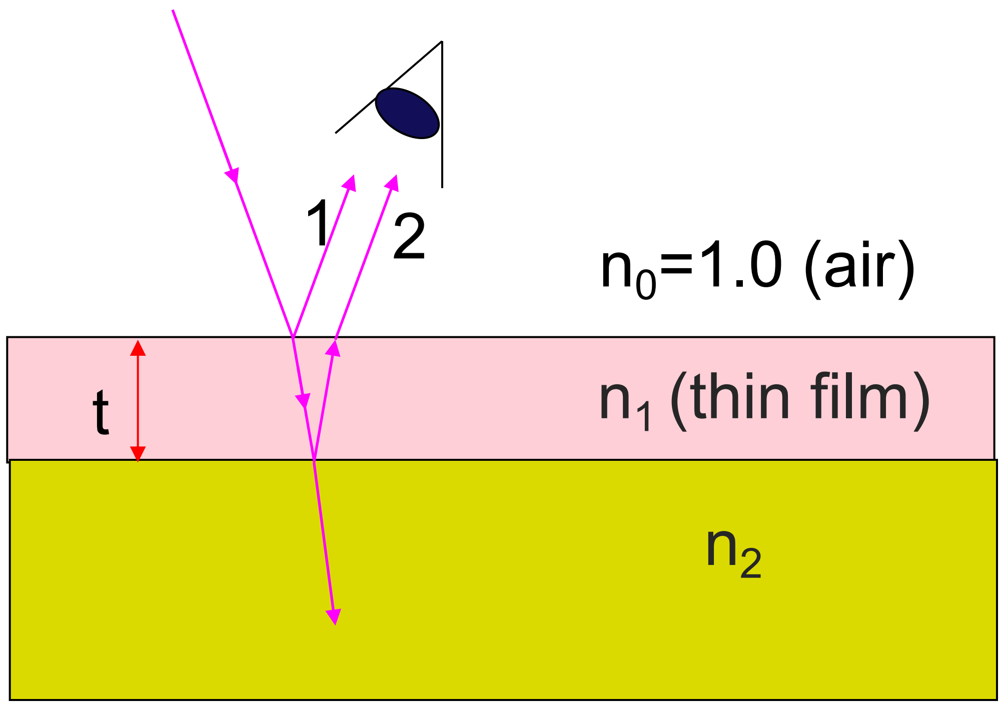
Need: $d \sin\theta = m\lambda$ => $\sin\theta = m\lambda / d$

If $\lambda > d$ then $\lambda / d > 1$

so $\sin\theta > 1$

Not possible!

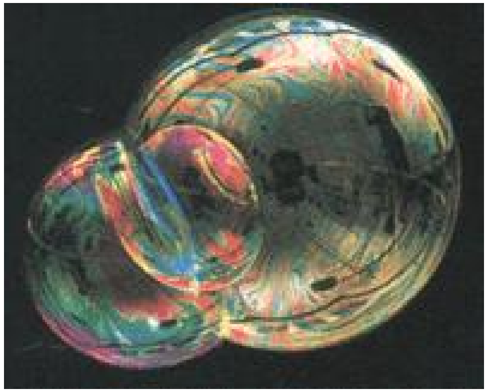
41-5 Thin Film Interference(薄膜干涉)

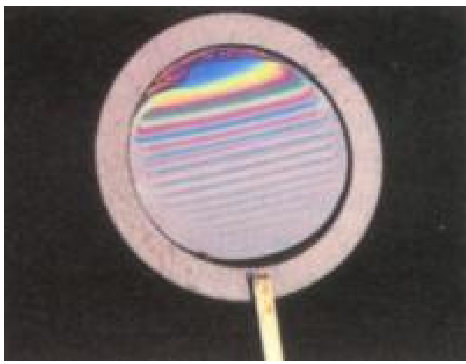


Get two waves by reflection off of two different interfaces.

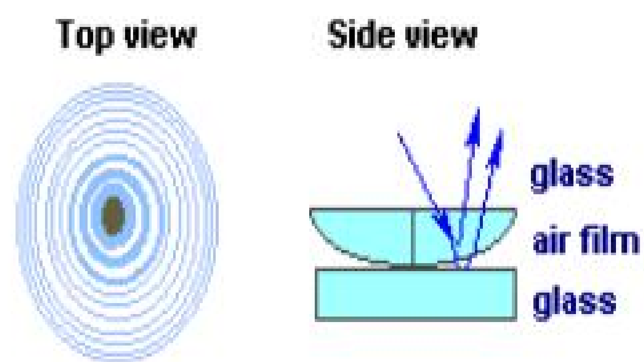
Ray 2 travels approximately *2t* further than ray 1.

Thin Film Interference



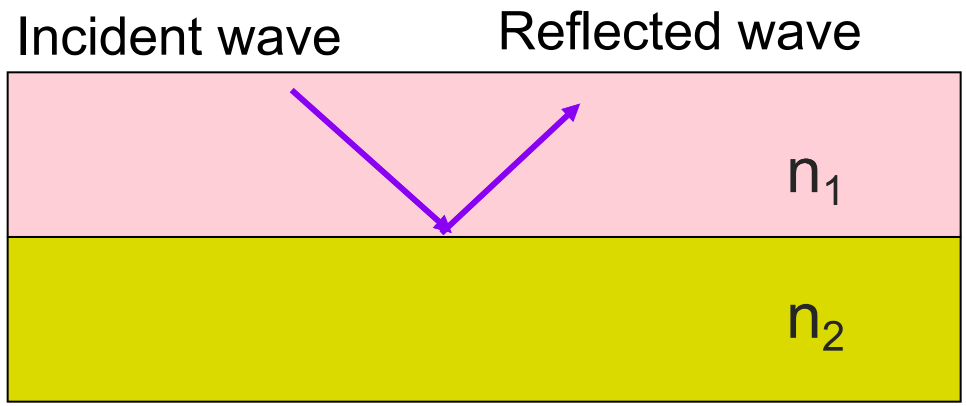


The color bands in a CD



Newton's rings

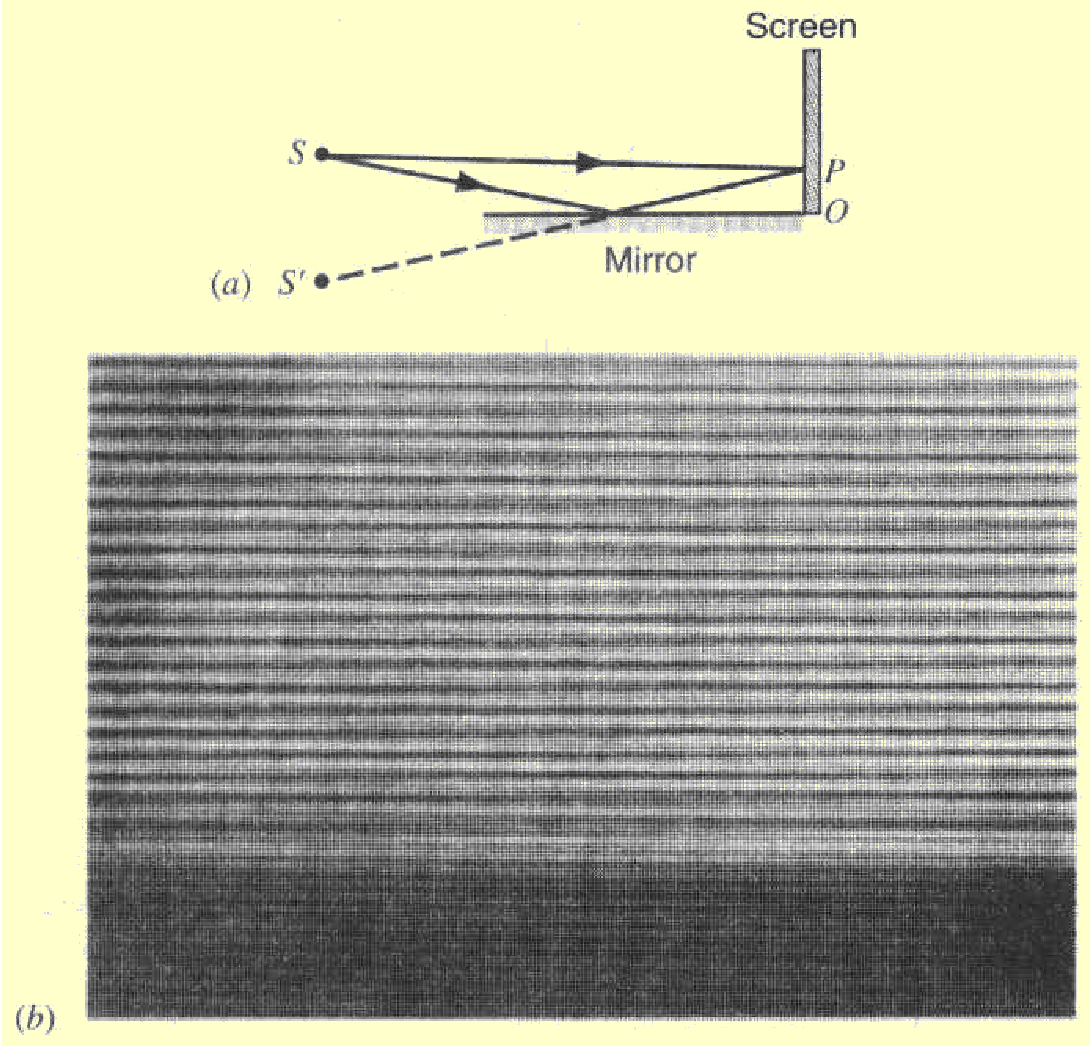
Reflection + Phase Shifts



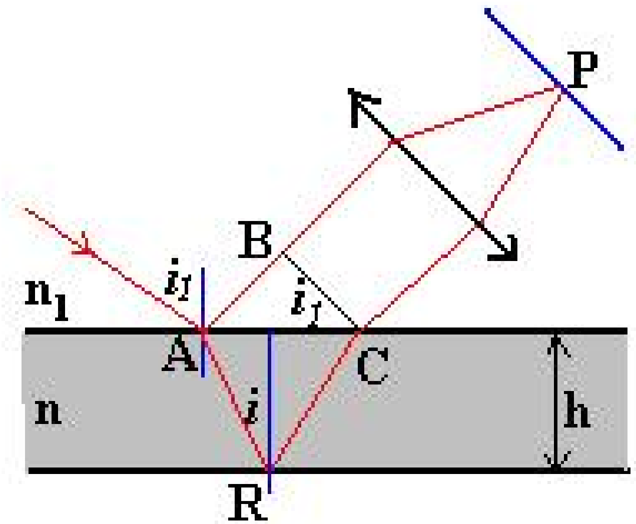
Upon reflection from a boundary between two transparent materials, the phase of the reflected light *may* change.

- If $n_1 > n_2$ - no phase change upon reflection.
- If $n_1 < n_2$ - phase change of 180º upon reflection.
  (equivalent to the wave shifting by $\lambda/2$.) **半波损失**

Optical reversibility and phase changes on reflection



1. The thickness is the same at every point（等倾干涉条纹）



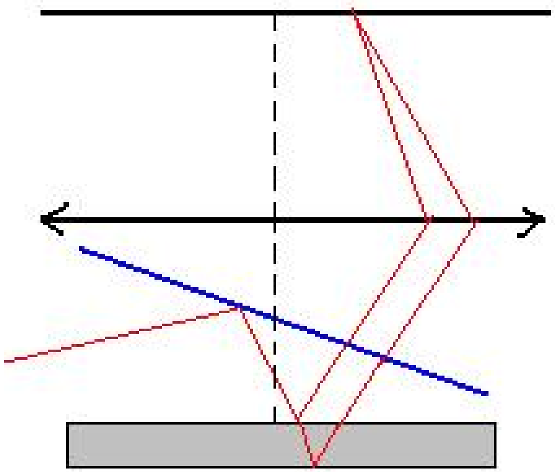
$$
\begin{aligned}
\Delta L &= (ARC) - (AB) \\
&= n\left(\frac{2h}{\cos i}\right) - n_1 \overline{A}\,\overline{C} \cdot \sin i_1 \\
&= n\frac{2h}{\cos i} - n_1 2h \cdot tgi \cdot \sin i_1 \\
&= 2h\left(\frac{n}{\cos i} - \frac{n_1 \sin i_1 \cdot \sin i}{\cos i}\right) \qquad n_1 \sin i_1 = n \sin i \\
&= 2nh\left(\frac{1}{\cos i} - \frac{\sin i \cdot \sin i}{\cos i}\right) \\
&= 2nh\cos i
\end{aligned}
$$

$$\Delta L = 2nh\cos i = m\lambda, \qquad \text{maxmum}$$

$$\Delta L = 2nh\cos i = \left(m + \frac{1}{2}\right)\lambda, \qquad \text{minimum}$$

$i\uparrow$, $\cos i\downarrow$, $\therefore$ at the center, m is the biggest

Discussion



$$\Delta L_m = 2nh\cos i_m = m\lambda, \qquad \cos i_m = \frac{m\lambda}{2nh}$$

$$\Delta L_{m+1} = 2nh\cos i_{m+1} = (m+1)\lambda, \qquad \cos i_{m+1} = \frac{(m+1)\lambda}{2nh}$$

$$\cos i_{m+1} - \cos i_m = \frac{\lambda}{2nh}$$

$$\cos i_{m+1} - \cos i_m = \left(\frac{\partial \cos i}{\partial i}\right)_{i=i_m} (i_{m+1} - i_m) = -\sin i_m (i_{m+1} - i_m) = \frac{\lambda}{2nh}$$

$$\Delta r_m = r_{m+1} - r_m \propto i_{m+1} - i_m = -\frac{\lambda}{2nh\sin i_m}$$

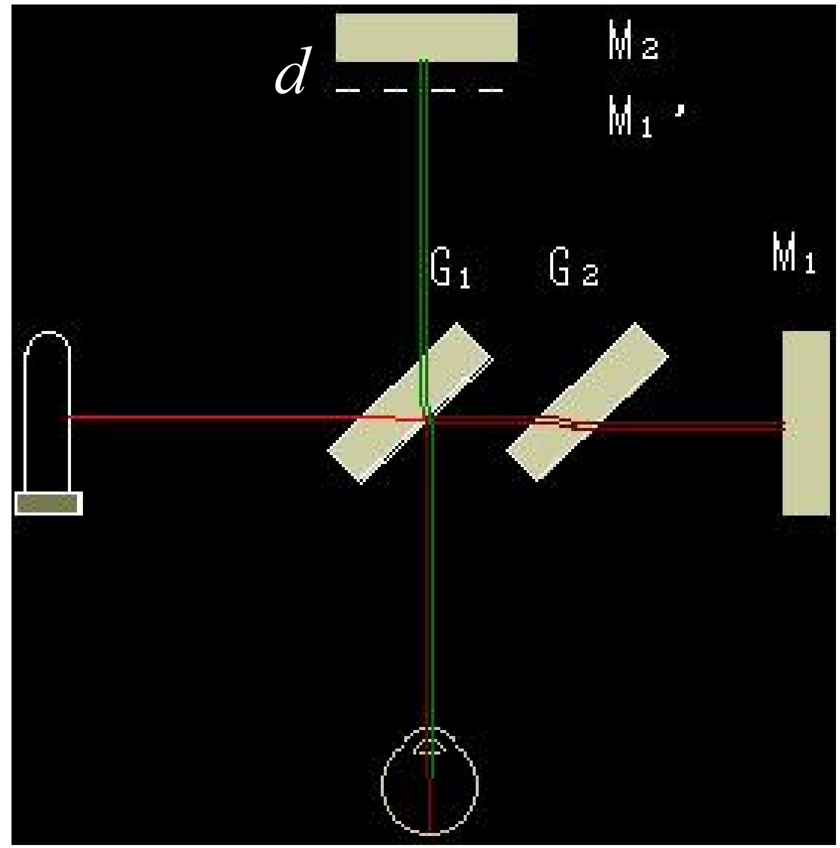
- "-" , $r_{m+1} < r_m$
- *n is bigger,* $\Delta r = r_{m+1} - r_m$ *is smaller*
- *h is bigger,* $\Delta r = r_{m+1} - r_m$ *is smaller*

Michelson's interferometer (迈克尔逊干涉仪)

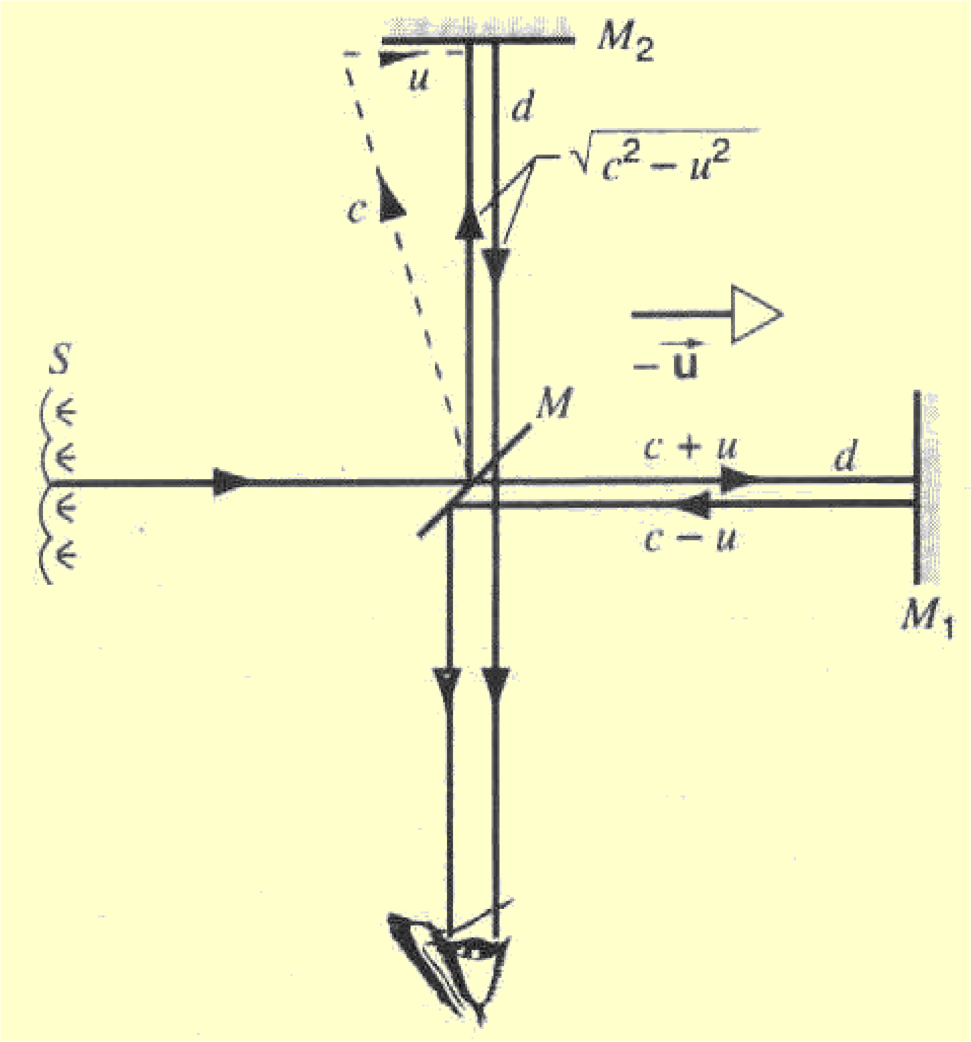
➢ **Constant angle interference:**

$\Delta L_m = 2d = m\lambda,$ maxima

➢ **Originally built in 1881 for proving if the ether, the medium the light propagates with respect to，exists.**



Michelson's interferometer and light propagation



Demonstration!

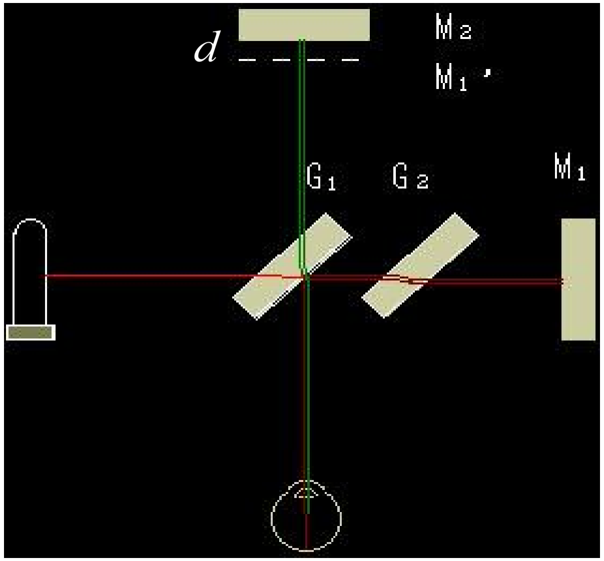
Example:Yellow light (wave length = 589 nm) illuminates a Mechelson's interferometer. How many bright fringes will be counted as the mirror is moved through 1.0 cm?

$$L_m = 2d = m\lambda, \quad \text{maxima}$$

$$\Delta L_m = 2\Delta d = \Delta m\lambda$$

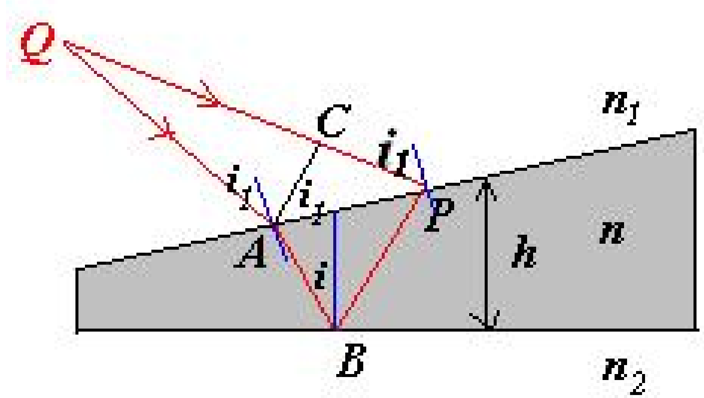
$$\Delta m = \frac{2\Delta d}{\lambda} = \frac{2\times 0.01}{589\times 10^{-9}}$$

$$= 33956 \text{ fringes}$$



2. The thickness is not the same at different point

（等厚干涉）



Note, A and P is almost the same point.

The difference of Optical Path:

$$\Delta L = (QABP) - (QP)$$
$$= (QA) - (QP) + (ABP)$$
$$(QA) - (QP) \approx -(CP) = -n_1 \overline{AP} \sin i_1 \qquad n_1 \sin i_1 = n \sin i$$
$$= -n\overline{AP} \sin i$$
$$= -n(2h \cdot tgi) \sin i$$
$$= -2nh \frac{\sin^2 i}{\cos i}$$

$$(ABP) \approx 2(AB) \approx \frac{2nh}{\cos i}$$

$$\therefore \Delta L = 2nh\left(\frac{1}{\cos i} - \frac{\sin^2 i}{\cos i}\right) = 2nh \cos i$$

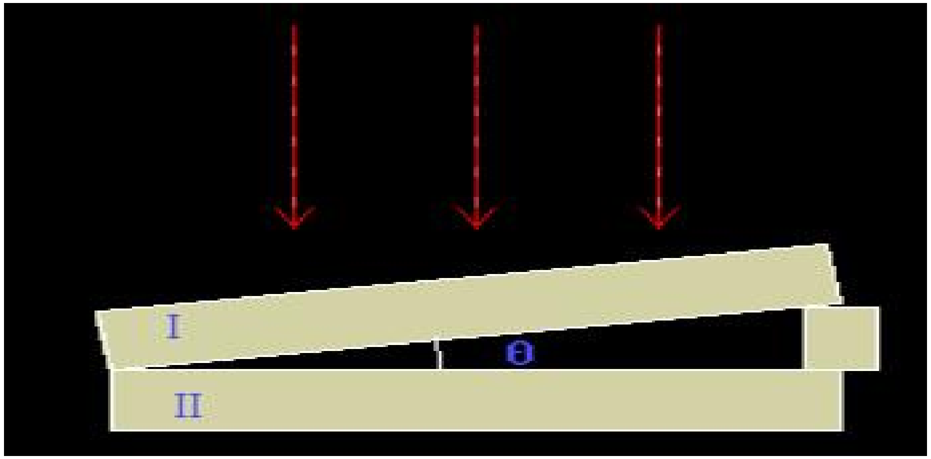
$$\begin{cases} \Delta L = 2nh\cos i = m\lambda & h = \dfrac{m\lambda}{2n\cos i} \quad \text{Maximum} \\ \Delta L = 2nh\cos i = (m + \dfrac{1}{2})\lambda & h = \dfrac{(m+\frac{1}{2})\lambda}{2n\cos i} \quad \text{Minimum} \end{cases}$$

Discussion

If $i = 0, \quad \mathit{\Delta L} = 2nh = m\lambda$

where h is the same,

the m value is the same.

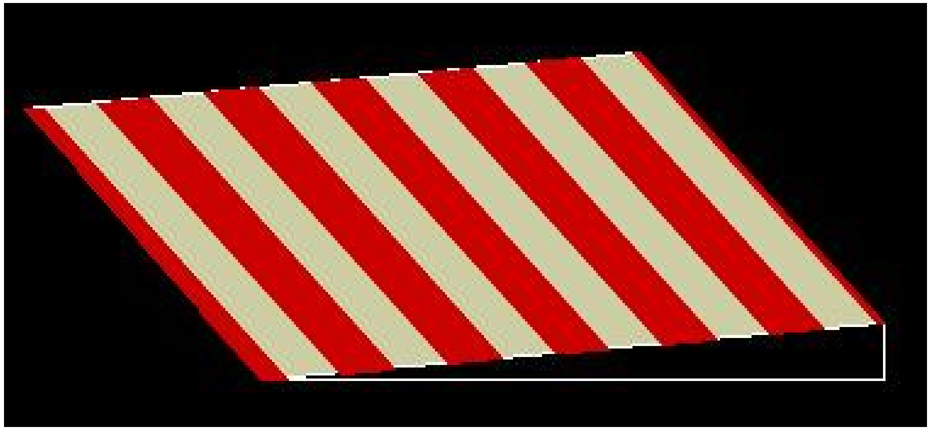


$$2nh = m\lambda$$

$$2n(h + \Delta h) = (m+1)\lambda$$

$$2n\Delta h = \lambda$$

$$\Delta h = \frac{\lambda}{2n}$$



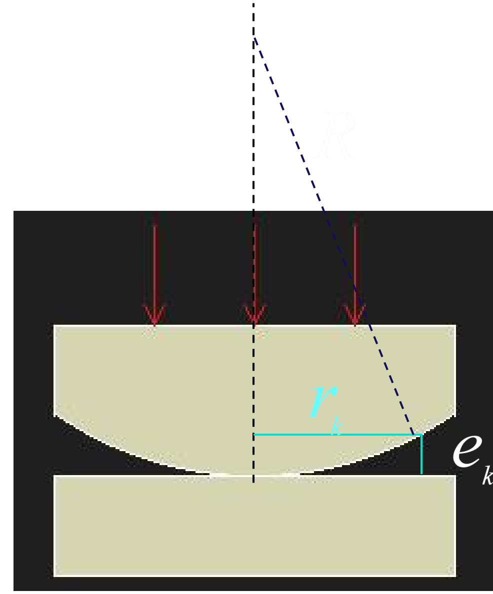
For the air film, the gap between fringe: $\mathit{\Delta x} = \dfrac{\lambda}{2\theta} \Rightarrow \theta = \dfrac{\lambda}{2\Delta x}$

➢ The Newton's ring (牛顿环):

$$\Delta L = 2h + \frac{\lambda}{2}$$

$$= \begin{cases} m\lambda & \text{maxima} \\ (2m+1)\dfrac{\lambda}{2} & \text{minima} \end{cases}$$

$m = 0,\ 1,\ 2,\ 3,\ \cdots\cdots$

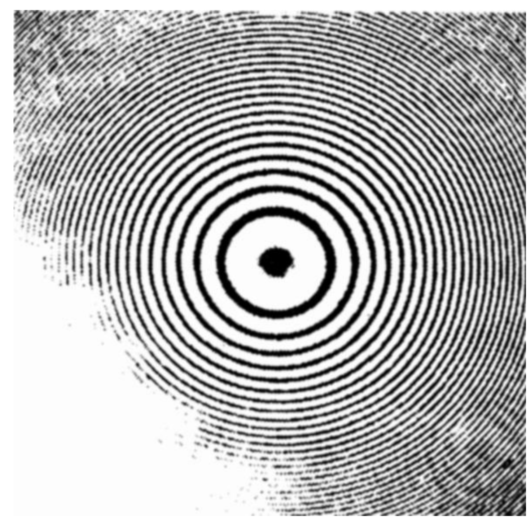


For the minimum fringes:

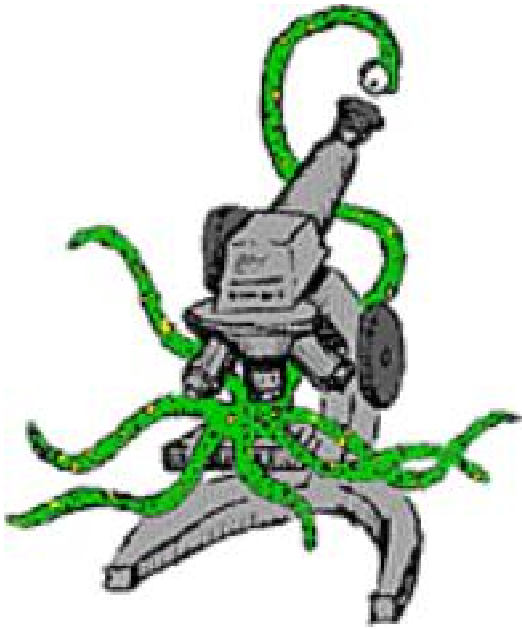
$$2h_m = m\lambda, \quad m = 0,\ 1,\ 2, \cdots$$

$$h_m = R - \sqrt{R^2 - r_m^2} \approx \frac{1}{2}\frac{r_m{}^2}{R} = \frac{1}{2}m\lambda$$

$$r_m = \sqrt{m\lambda R}$$



Did you know?



You should never get immersion oil on microscope lenses that are not designed for it!

The oil is a thin film that can create light and dark spots all over your image!

(and it’s really hard to clean off, too!)

Homework:

Exercises

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Problems

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