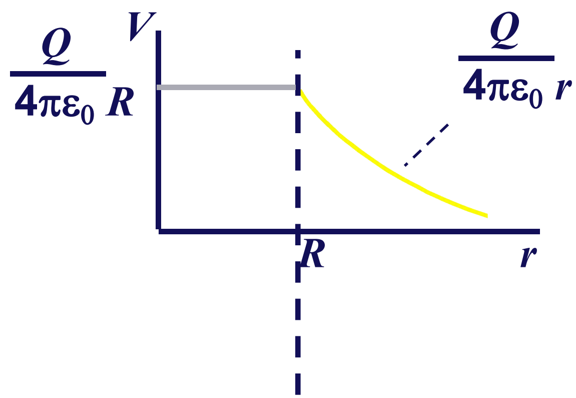
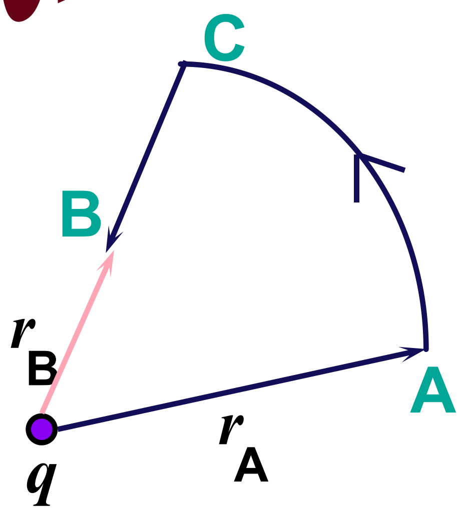
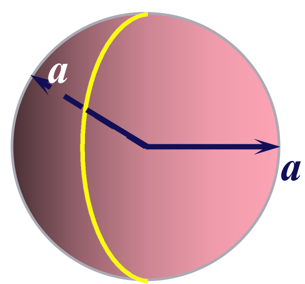
Chapter 28: Electric Potential Energy and Potential(电势能和电势)

Definitions



Examples





path independence

Today…

- Conservative Forces(守恒力) and Energy Conservation
  - Total energy is constant and is sum of kinetic and potential
- Introduce Concept of Electric Potential (电势)
  - A property of the space and sources as is the Electric Field
  - Potential differences drive all biological & chemical reactions, as well as all electric circuits.
- Calculating Electric Potentials *put V(infinity)=0*
  - Charged Spherical Shell
  - N point charges
  - Example: electric potential of a charged sphere
- Electrical Breakdown (击穿)
  - Sparks
  - Lightning!!

28-1 Potential Energy(势能)

1. The advantage of the energy method

A **Although force is a vector, energy is a scalar**

B In problems involving vector forces and field, calculation requiring sums are integrals are often complicated.

C When you introduce the potential energy, the calculations become simplicity, as in mechanics.

2. The similarity between the electrostatic and gravitational forces.

$$\vec{F}_g = -G\frac{m_1 m_2}{r^2}\hat{r}$$ (gravitation)

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$ (electrostatic)

- **Introduce**

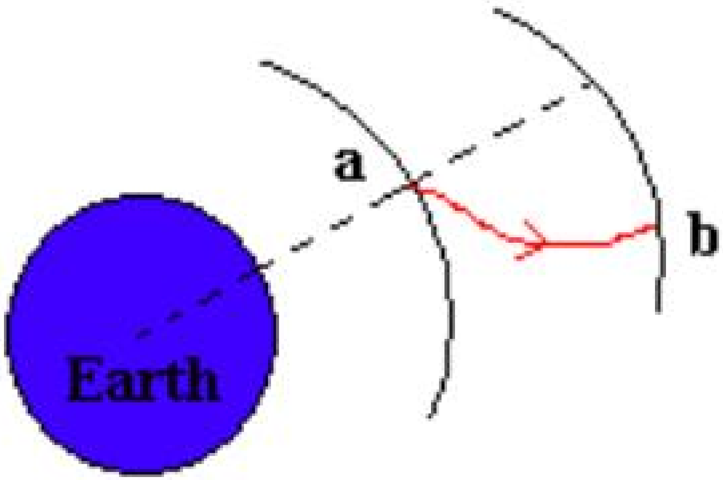
Gravitational field

$$\vec{g} = \frac{\vec{F}_g}{m_0} = -G\frac{M}{r^2}\hat{r}$$

Electric field

$$\vec{E} = \frac{\vec{F}_e}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

In Gravitational Field



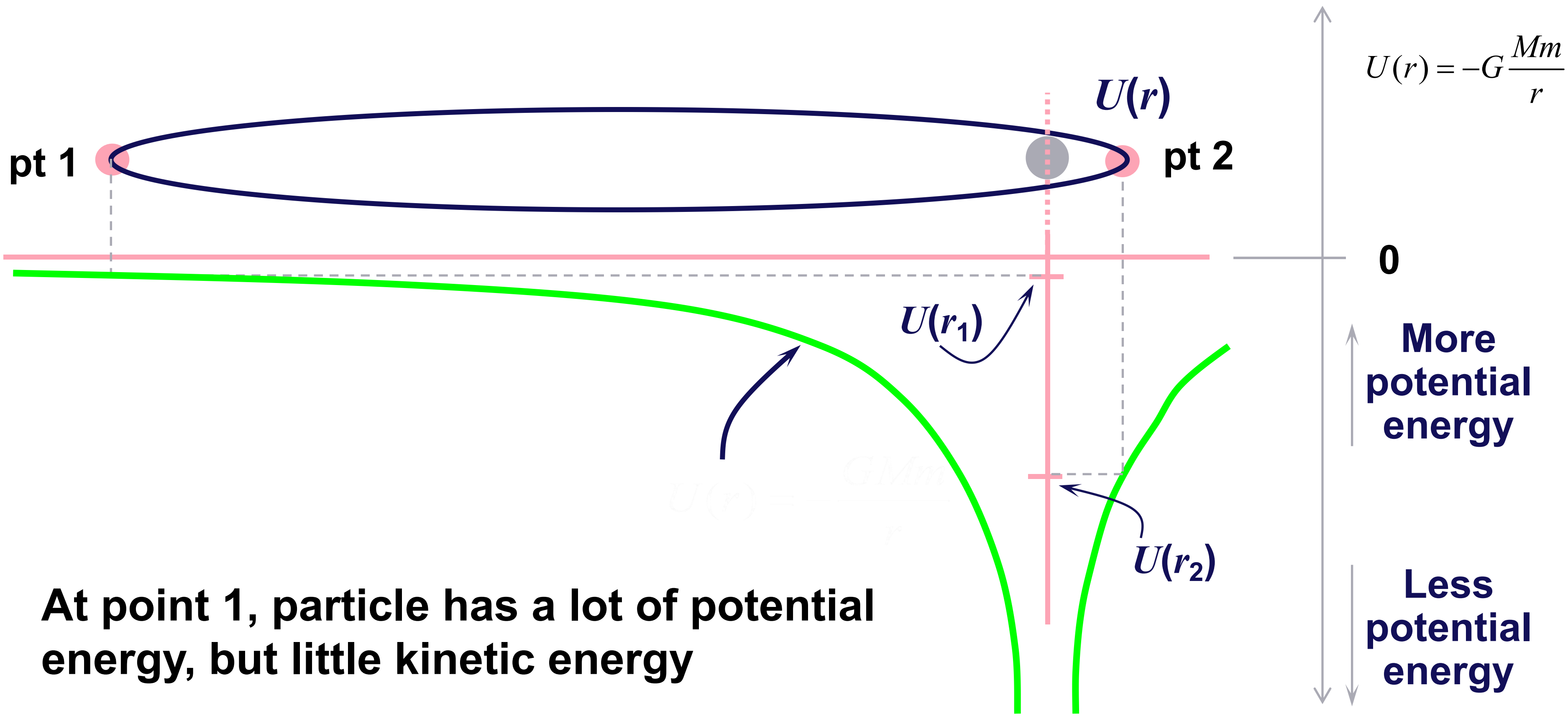
- **A particle moves from *a* to *b***
- $\vec{F}_g$ **does work** $\mathbf{W_{ab}}$
- **The difference in potential energy:** $\Delta U = -W_{ab} = -\int_a^b \vec{F} \bullet d\vec{l}$
- **Only if** $\vec{F}_g$ **is conservative** $\int_a^b \vec{F} \bullet d\vec{l}$ **is independent of path, then define:**

$$r = \infty, \quad U = 0, \quad U(r) = -G\frac{Mm}{r}$$

$$\oint \vec{F} \bullet d\vec{l} = 0$$

Example: Gravitational Force is conservative (and attractive)

- Consider a comet in a highly elliptical orbit



- At point 1, particle has a lot of potential energy, but little kinetic energy
- At point 2, particle has little potential energy, but a lot of kinetic energy

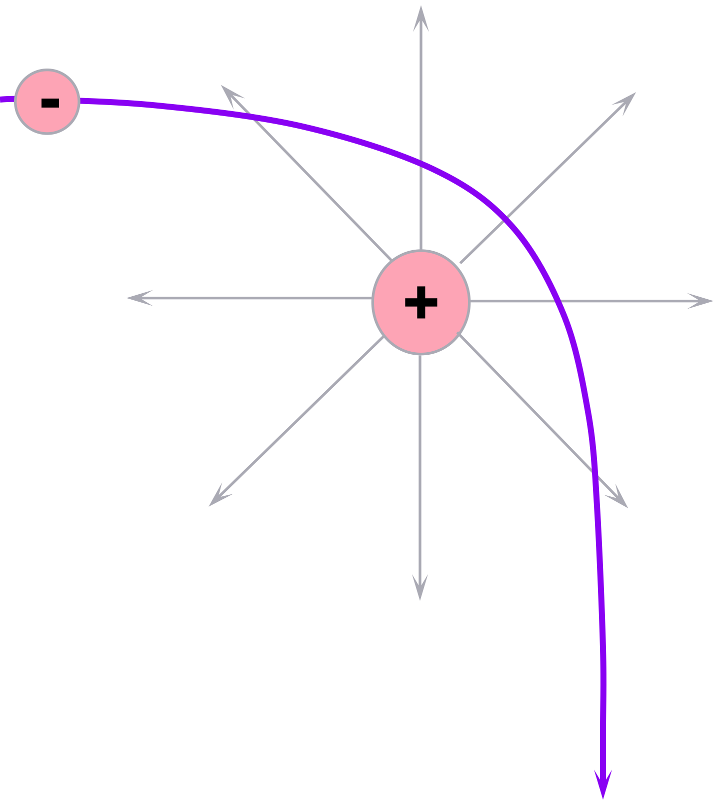
Total energy $= K + U$ is constant!

Conservation of Energy of a particle from phys I

- Kinetic Energy (K) $K = \frac{1}{2}mv^2$
  - non-relativistic
- Potential Energy (U) $U(x, y, z)$
  - determined by force law
- for Conservative Forces: K+U is constant
  - total energy is always constant
- examples of conservative forces
  - gravity; gravitational potential energy
  - springs; coiled spring energy (Hooke's Law): $K = \frac{1}{2}kx^2$
  - electric; electric potential energy (today!)
- examples of non-conservative forces (heat)
  - friction
  - viscous damping (terminal velocity)

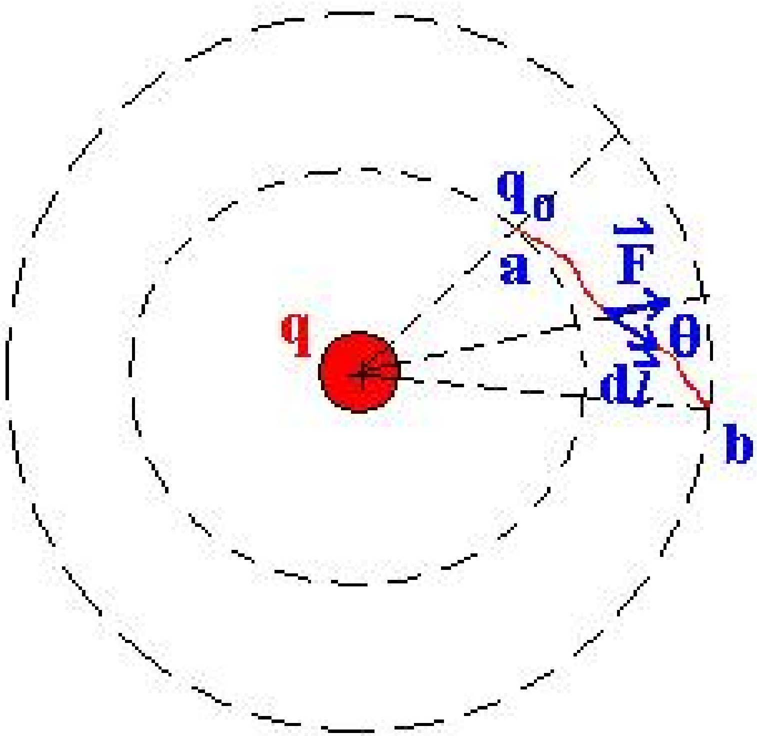
•Electric forces are conservative, too

- **Consider a charged particle traveling through a region of static electric field:**



- A negative charge is attracted to the fixed positive charge
- negative charge has more potential energy and less kinetic energy far from the fixed positive charge, and…
- more kinetic energy and less potential energy near the fixed positive charge.
- But, the total energy is conserved

- **We will now discuss electric potential energy and the electrostatic potential….**



$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}$$

$$W_{ab} = \int_a^b \vec{F}\bullet d\vec{l} = \int_a^b F\cdot dl\cdot\cos\theta$$

$$= \int_{r_a}^{r_b} F\cdot dr = \int_{r_a}^{r_b}\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}dr$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots$$

$$W_{ab} = \int_a^b q_0\vec{E}\bullet d\vec{l}$$

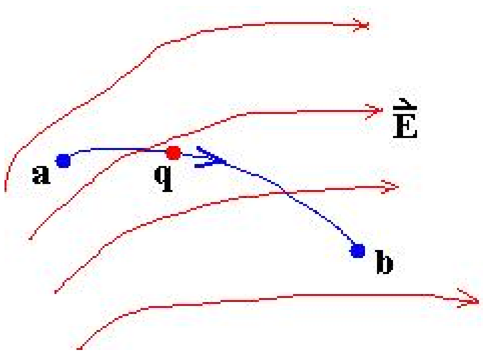
W_{ab} is the function of r_a, r_b, and is independent of the path of q_0 movement.

The electrostatic force (静电力) is a conservative force (守恒力) and it can be represented by a potential energy (势能).

3. Electric Potential Energy(电势能)

- **Because the electrostatic force is conservative,**

$\Longrightarrow$ **a potential energy.**



A charged particle *q* moves from *a* to *b* in an electric field $\vec{E}$

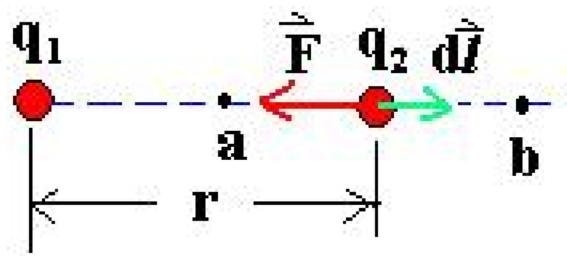
The difference of potential energy:

$$U_b - U_a = -\int_a^b \vec{F} \bullet d\vec{l} = -q\int_a^b \vec{E} \bullet d\vec{l}$$

$\int_a^b \vec{F} \bullet d\vec{l}$ **depends only on the initial and final position *a* and *b*.**

Example: two charges

***q_1, q_2* have opposite signs "attractive force"**



$\vec{F} = q\vec{E}$ **does negative work**

$$\int_a^b \vec{F} \bullet d\vec{l} = q\int_a^b \vec{E} \bullet d\vec{l} < 0$$

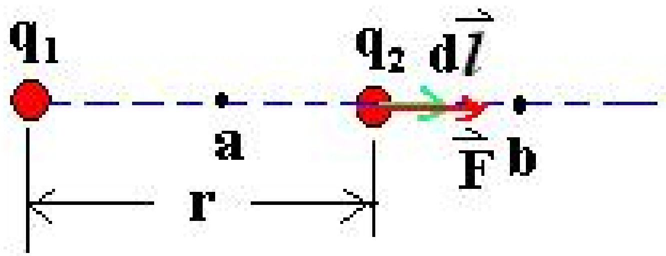
$$U_b - U_a = -\int_a^b \vec{F} \bullet d\vec{l} > 0$$

the potential energy U $\Uparrow$

To release the charge: $\int_b^a \vec{F} \bullet d\vec{l} > 0, \quad U_b - U_a > 0, \;\; U \downarrow \quad E_k \uparrow$

Example: two charges

q_1, q_2 **have the same signs "repulsive force"**



$$\vec{F} = q\vec{E}$$

$$\int_a^b \vec{F} \bullet d\vec{l} = q\int_a^b \vec{E} \bullet d\vec{l} > 0$$

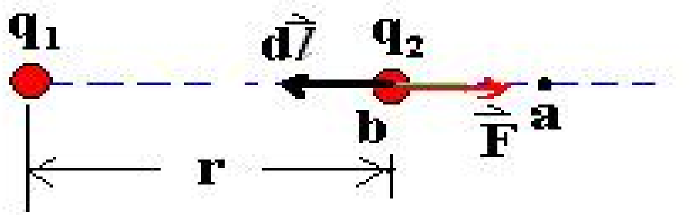
$$U_b - U_a = -W_{ab} = -\int_a^b \vec{F} \bullet d\vec{l} < 0$$

$$\Downarrow$$

The expression for the potential energy of the system of both point charges (结合能).

To assume q_2 moves toward q_1 from a to b along the line connecting the two particles.

The change of electric potential energy



$$U_b - U_a = -\int_a^b \vec{F} \bullet d\vec{l} = -\int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} dr$$

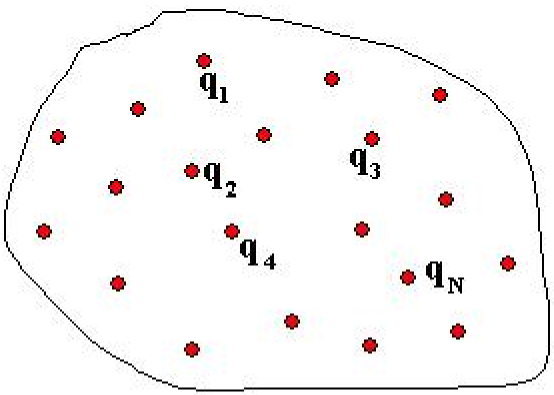
$$= \frac{q_1 q_2}{4\pi\varepsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

If we choose $r_a = \infty, U_\infty = 0$, then $r = r_b$ $\quad U(r) = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r}$

q_1, q_2 have opposite signs, "attractive force" $\quad U(r) = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r} < 0$

q_1, q_2 have the same signs, "repulsive force" $\quad U(r) = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r} > 0$

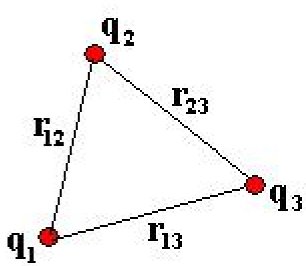
Example 2: Potential Energy of a system of charges



$$U = U_{12} + U_{13} + U_{14} + \ldots + U_{1N}$$
$$+ U_{23} + U_{24} + \ldots + U_{2N}$$
$$+ U_{34} + \ldots + U_{3N}$$
$$+ \ldots\ldots\ldots$$
$$+ U_{N-1N}$$

$$U = \sum_{i \neq j} \frac{1}{2} U_{ij}$$

- For example



$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}$$

Notes

- **The potential energy is a property of the system, not of any individual charge.**
- **You can immediately see the advantage of using an energy method to analyze this system.**
  algebraically Sum of Scalars (标量的代数求和)

  the Sum of a vector

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

- **Another way to interpret the potential energy of this system.**

4.The circuit Law of the electrostatic field (静电场的环路定律)

$$\oint \vec{E} \bullet d\vec{l} = 0$$

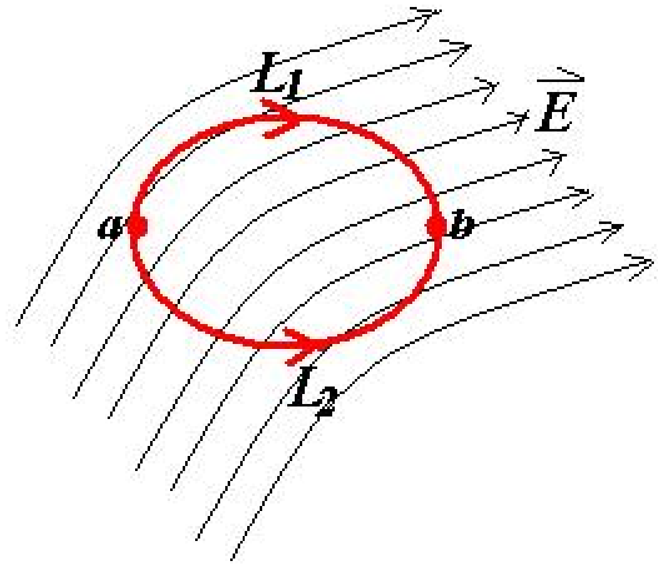
$$\oint \vec{E} \bullet d\vec{l} = \int_a^b \vec{E} \bullet d\vec{l} + \int_b^a \vec{E} \bullet d\vec{l}$$

$$\because q_0 \int_a^b \vec{E} \bullet d\vec{l} = q_0 \int_a^b \vec{E} \bullet d\vec{l}$$

$$\underset{L_1}{\int_a^b} \vec{E} \bullet d\vec{l} = \underset{L_2}{\int_a^b} \vec{E} \bullet d\vec{l}$$

$$\therefore \oint \vec{E} \bullet d\vec{l} = 0$$

$$\nabla \times \vec{E} = 0$$



The Gauss' Law

$$\oiint \vec{E} \bullet d\vec{A} = \frac{q}{\varepsilon_0} = \iiint \frac{\rho}{\varepsilon_0} dv$$

$$\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$$

28-2 Electric Potential (电势)

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{r} \Rightarrow \vec{E} = \frac{\vec{F}}{q_0} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Vector

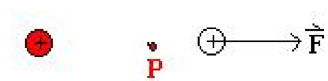
$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r} \Rightarrow V = \frac{U}{q_0} = \frac{q}{4\pi\varepsilon_0 r}$$

Scalar

$$V_p = \frac{U_p}{q_0}$$

The potential energy per unit test charge

$V_p>0,\ U_p>0$



$V_p<0,\ U_p<0$

- **V_p is independent of q_0**
- **q_0 is very small charge**
- **V_p(<0, or >0, or =0)**

$V_p=0,\ U_p=0$

Electric potential and potential energy

- Imagine a positive *test* charge, q_o, in an *external* electric field, $\vec{E}(x,y,z)$, *it's a vector field*
- What is the electric potential energy, $U(x,y,z)$ of the charge in this field?
  - Must define where in space $U(x,y,z)$ is zero, perhaps at infinity *(for charge distributions that are finite)*
  - $U(x,y,z)$ is equal to the work you have to do to take q_o from where U is zero to point (x,y,z)

- Define $V(x,y,z) = U(x,y,z) / q_o$ $\qquad$ $(U = qV)$

U depends on what q_o is, but V is independent of q_o

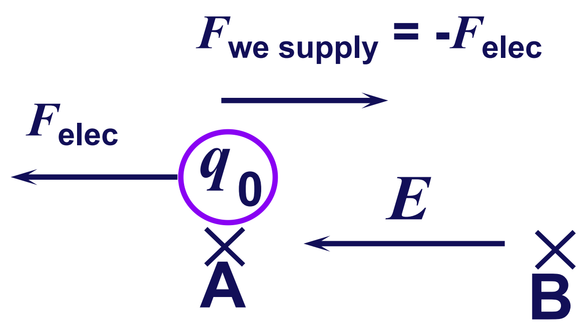
(which can be + or -)

$V(x,y,z)$ is the electric potential in volts associated with

$\vec{E}(x,y,z)$ $(1V = 1\ J/c)$. $\quad$ $V(x,y,z)$ *is a scalar field*

Electric potential difference(电势差)

- Suppose charge q_0 is moved from pt A to pt B through a region of space described by electric field E.

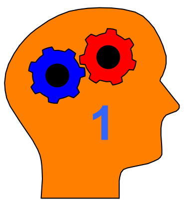


- To move a charge in an E-field, we must supply a force just equal and opposite to that experienced by the charge due to the E-field.

- Since there will be a force on the charge due to E, a certain amount of work $W_{AB} \equiv W_{A\rightarrow B}$ will have to be done to accomplish this task.

Electric potential difference, cont.

- Remember: work is force times distance



$$\therefore \quad W_{AB} = \int_A^B \vec{F}_{we} \bullet d\vec{l} = -\int_A^B \vec{F}_{elec} \bullet d\vec{l} = -\int_A^B q_0 \vec{E} \bullet d\vec{l}$$

$$\Rightarrow \quad \boxed{V_B - V_A \equiv \frac{W_{AB}}{q_0} = -\int_A^B \vec{E} \bullet d\vec{l}}$$

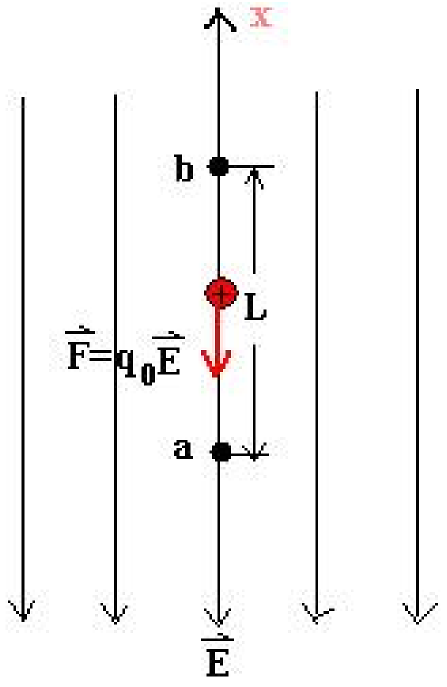
- To get a positive test charge from the lower potential to the higher potential you need to invest energy - you need to do work
- The overall sign of this: A positive charge would “fall” from a higher potential to a lower one
- If a positive charge moves from high to low potential, it can do work on you; you do “negative work” on the charge

Notes

- Electric potential energy (电势能)
- Electric potential **(电势，定义了一个参考点)**
- Electric potential difference **(电势差)**

28-3 Calculating the electric potential from the field

- **Considering an uniform electric field** $\vec{E} \Leftrightarrow V$



To assume a positive test charge q_0 moves from *a* to *b* along the straight line

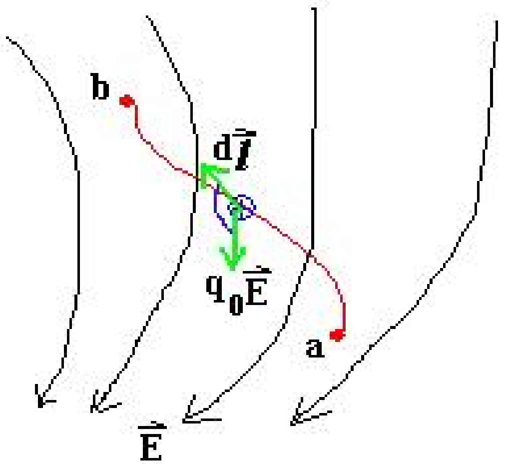
$$F_e = -q_0 E$$

$$W_{ab} = F_e \Delta x = -q_0 EL$$

$$\Delta V = V_b - V_a = \frac{U_b - U_a}{q_0} = \frac{-W_{ab}}{q_0} = EL$$

The potential at *b* point is higher than that at *a* point.

For the more general case, $\vec{E}$ is not uniform.
A test charge q_0(>0) $a \Rightarrow b$



$$W_{ab} = \int_a^b \vec{F} \bullet d\vec{l} = q_0 \int_a^b \vec{E} \bullet d\vec{l}$$

$$\Delta V = V_b - V_a = \frac{U_b - U_a}{q_0} = \frac{-W_{ab}}{q_0} = -\int_a^b \vec{E} \bullet d\vec{l}$$

$$= -\int_a^b \vec{E} \bullet d\underset{\vec{E}}{\vec{l}}$$

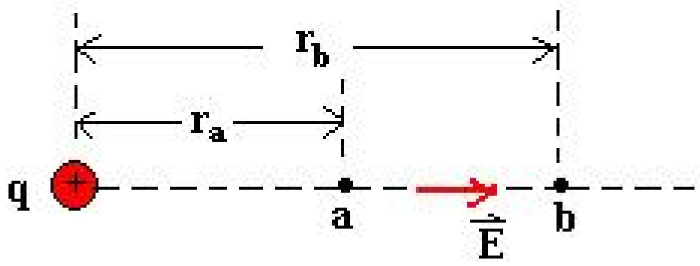
- **If we choose a reference point (for choose *a* point)**

$$r = \infty, V_\infty = 0$$

$$V_p = -\int_\infty^p \vec{E} \bullet d\vec{l} = \int_p^\infty \vec{E} \bullet d\vec{l}$$

The electric potential due to point charge

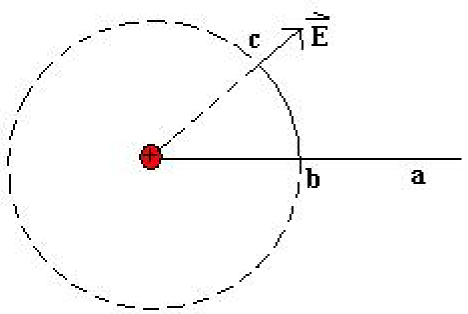
- **For a point charge**



$$V_b - V_a = -\int_a^b \vec{E} \bullet d\vec{l} = -\int_{r_a}^{r_b} \frac{q}{4\pi\varepsilon_0 r^2} dr$$

$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

The potential difference between point *a* and point *b*.

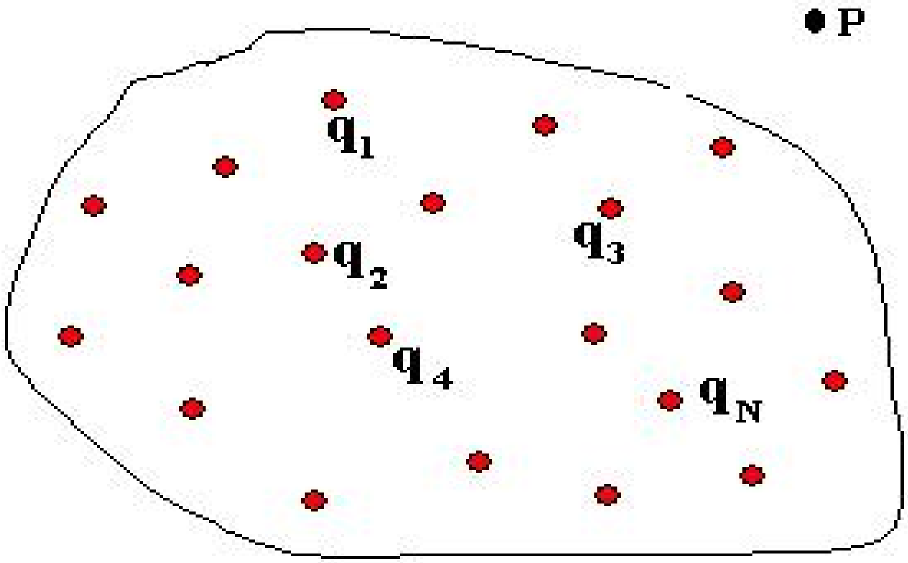


$$V_c - V_a = V_b - V_a = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$\therefore V_c - V_b = 0 \quad (\vec{E} \perp d\vec{l}\,)$$

If we choose $r = \infty, V_a = 0$ At any point, the potential: $V = \dfrac{q}{4\pi\varepsilon_0 r}$

For a collection of point charges

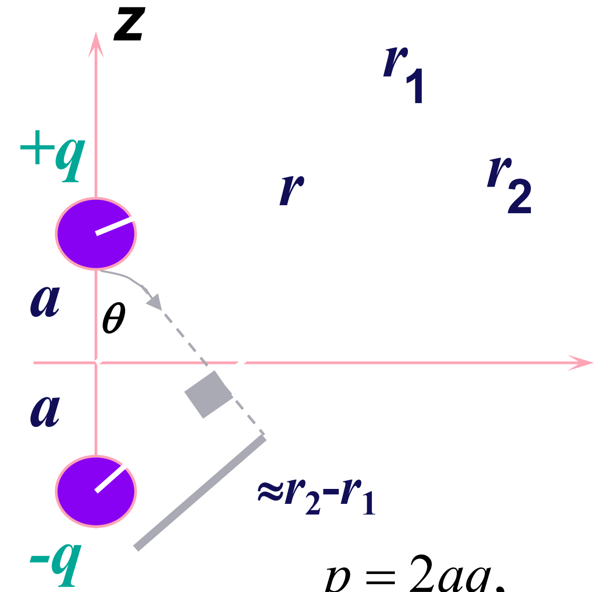


$$V_P = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3} + \ldots.$$

$$= \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_0 r_i}$$

Scalar sum

Example 1: Electric Dipole



$$p = 2aq,$$

$$\vec{p} = q\vec{d}, d = 2a$$

The potential is much easier to calculate than the field since it is an algebraic sum of 2 scalar terms.

- **Rewrite this for special case $r >> a$:**

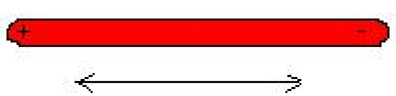
$$\Rightarrow \boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\frac{2aq\cos\theta}{r^2}} = \frac{\vec{p}\bullet\hat{r}}{4\pi\varepsilon_0 r^2}$$

$$\theta = 90^o, V = 0$$

$$\theta = 0, V_{\max} > 0$$

$$\theta = 180^o, V_{\max} < 0$$

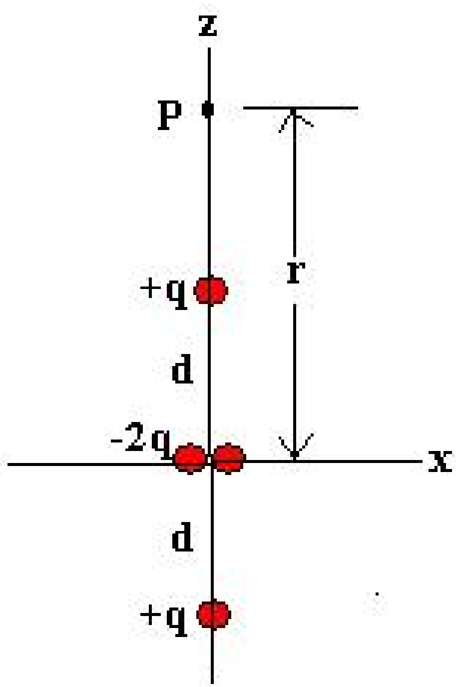
- **Electric dipoles are important in situations other than atomic and molecular ones.**
- **Radio and TV antennas**



$$\vec{p} = \vec{p}_0\cos(\omega t + \phi_0)$$

Example 2: (P_{644}, Problem 28-9) Electric quadrupole （电四偶极矩）

Calculate V(r) for the points on the axis of this quadrupole.



$$V(r)=\sum_i V_i(r_i)$$

$$=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r-d}+\frac{-2q}{r}+\frac{q}{r+d}\right)$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{2qd^2}{r(r^2-d^2)}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{2qd^2}{r^3(1-d^2/r^2)}$$

For $d<<r$, $d^2/r^2<<1$

$$\boxed{V(r)=\frac{2qd^2}{4\pi\varepsilon_0 r^3}=\frac{Q}{4\pi\varepsilon_0 r^3}}$$

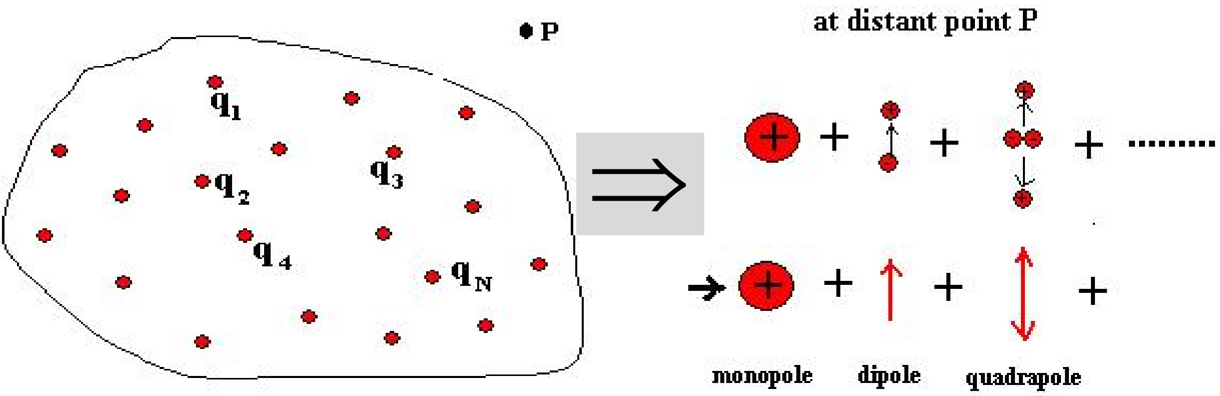
$Q=2qd^2$, **Electric quadrupole moment (电四偶极矩)**

Notes

Point Charge $V(r) \propto \frac{1}{r}$

Dipole $V(r) \propto \frac{1}{r^2}$

Quadrupole $V(r) \propto \frac{1}{r^3}$



$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{A_1}{r} + \frac{A_2}{r^2} + \frac{A_3}{r^3} + ...\right)$$
$$= \frac{1}{4\pi\varepsilon_0}\sum_i \frac{A_i}{r^i}$$

The Electric Potential of continuous charge distribution

- How do we represent the charge “Q” on an extended object?

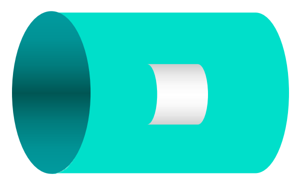
total charge Q

small pieces of charge

- **Line of charge:**
  λ = charge per unit length

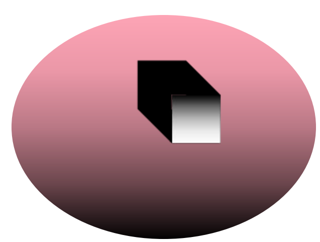
$$dq = \lambda \, dx$$

- **Surface of charge:**
  σ = charge per unit area



$$dq = \sigma \, dA$$

- **Volume of Charge:**
  ρ = charge per unit volume



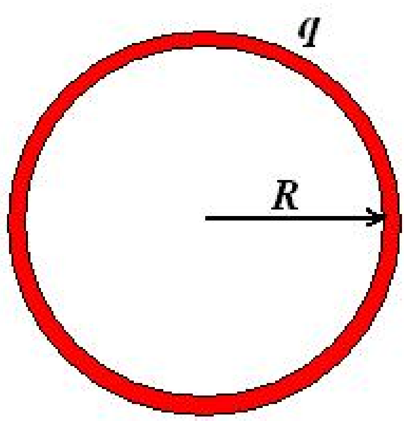
$$dq = \rho \, dV$$

At P point

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

$$V_P = \int \frac{dq}{4\pi\varepsilon_0 r}$$

Example 3 Calculate the electric potential energy and potential of a charged shell.

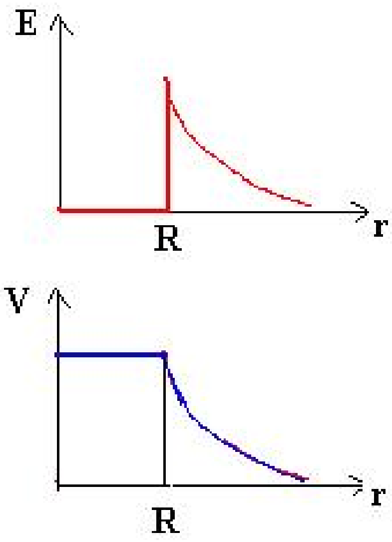


Solution:

From Gauss' Law $E = \dfrac{q}{4\pi\varepsilon_0 r^2}, (r \ge R)$

$E = 0, (r < R)$

The potential



$$r_P > R,\ V(P) = \int_P^{\infty} \vec{E} \bullet d\vec{l} = \frac{q}{4\pi\varepsilon_0 r_P}$$

$$r_P < R,\ V(P) = \int_P^{R} \vec{E} \bullet d\vec{l} + \int_R^{\infty} \vec{E} \bullet d\vec{l}$$

$$= 0 + \frac{q}{4\pi\varepsilon_0 R} = \frac{q}{4\pi\varepsilon_0 R}$$

The electric potential energy

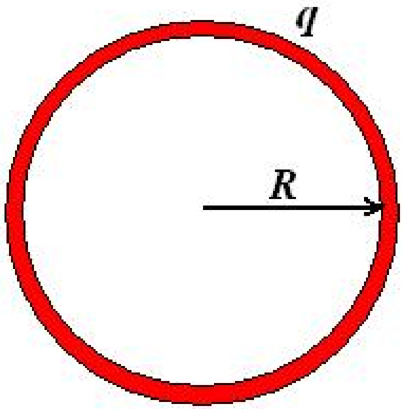
$$U = \sum_{\substack{i,j=1 \\ (j>i)}}^{n} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$

$$= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$

$$= \frac{1}{2}\sum_{i=1}^{n} q_i V_i$$

$$= \frac{1}{2}\int V dq = \frac{1}{2}\cdot\frac{q}{4\pi\varepsilon_0 R}\cdot q$$

$$= \frac{q^2}{8\pi\varepsilon_0 R}$$

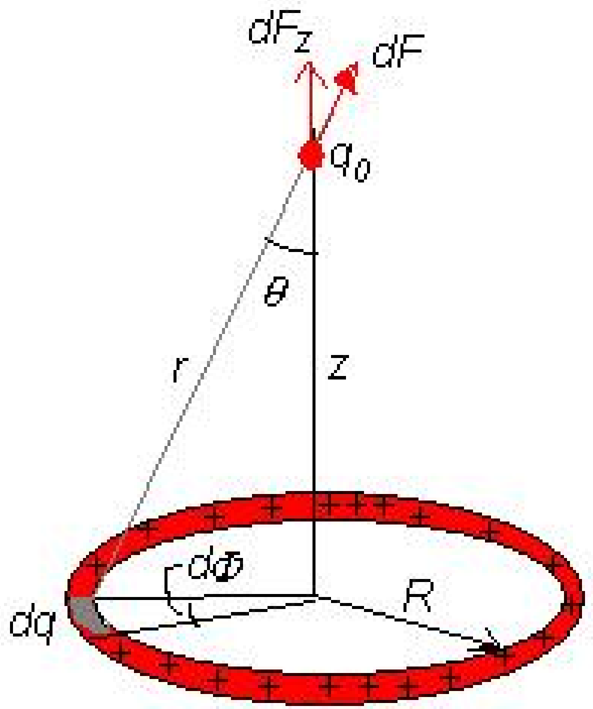


Estimate the radius *R* of an electron

$$W = mc^2 = \frac{e^2}{8\pi\varepsilon_0 R}$$

$$R = \frac{e^2}{8\pi\varepsilon_0 mc^2} \approx 1.4\times 10^{-15}\, m$$

Example 4: an uniform ring of radius R and total charge *q*.

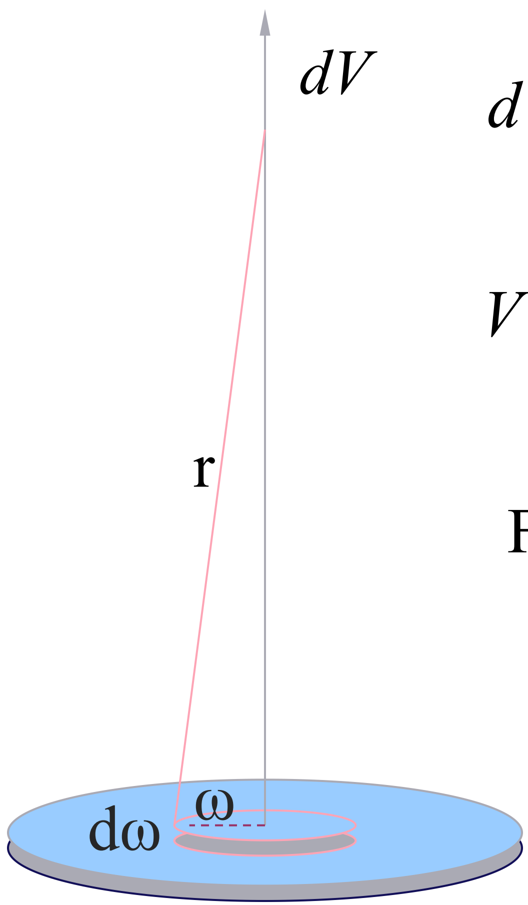


$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda ds}{4\pi\varepsilon_0 r}$$

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\oint\frac{\lambda ds}{r} = \frac{\lambda}{4\pi\varepsilon_0\sqrt{z^2+R^2}}\cdot 2\pi R$$

$$= \frac{q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

Example 5: A circular plastic disk of radius R and the surface charge density σ.



$$dq = 2\pi\omega \cdot d\omega \cdot \sigma$$

$$dV = \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}}$$

$$V = \int_0^R \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}} = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2+R^2}-z)$$

For z>>R

$$\sqrt{z^2+R^2} = z\sqrt{1+(\frac{R}{z})^2} = z(1+\frac{1}{2}\frac{R^2}{z^2}+\ldots)$$

$$V(z) = \frac{\sigma}{2\varepsilon_0}(\sqrt{R^2+z^2}-z)$$

$$\approx \frac{\sigma}{2\varepsilon_0}(z+\frac{R^2}{2z}-z) = \frac{\sigma}{2\varepsilon_0}\cdot\frac{R^2}{2z} = \frac{\sigma\cdot\pi R^2}{4\pi\varepsilon_0 z}$$

$$= \frac{q}{4\pi\varepsilon_0 z}$$

As point charge

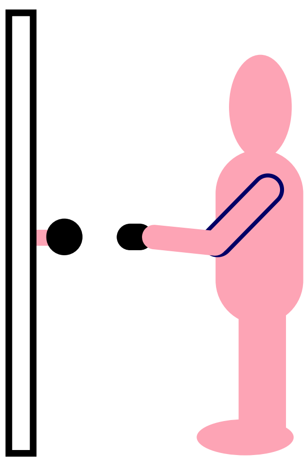
Example in life: Sparks（放电）

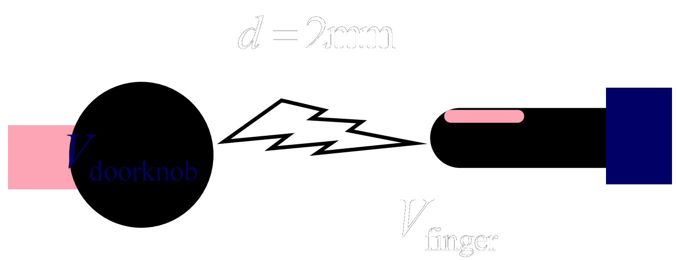
- **High electric fields can ionize nonconducting materials (“dielectrics电介质”）**

$$\text{Insulator} \xrightarrow[\text{Breakdown}]{\text{Dielectric}} \text{Conductor}$$

- **Breakdown can occur when the field is greater than the “*dielectric strength*” of the material.**
  - E.g., in air, $E_{\max} \cong 3\times10^6 \text{ N/C} = 3\times10^6 \text{ V/m} = 30 \text{ kV/cm}$

Ex.





Arc discharge equalizes the potential

What is ΔV?

$$\Delta V = E_{\max} \bullet d$$

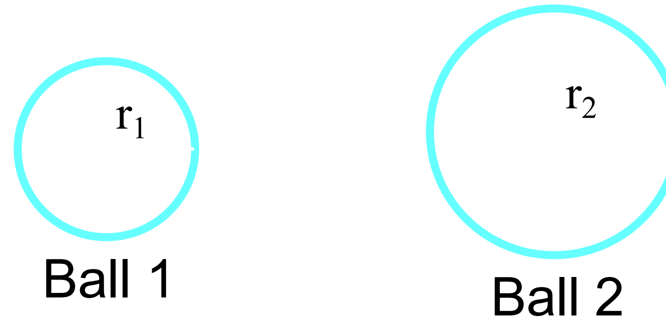
$$= 30 \text{ kV/cm} \times 0.2 \text{ cm}$$

$$= 6 \text{ kV}$$

Note: High humidity can also bleed the charge off → reduce ΔV.

Lecture 5, ACT 4

Two charged balls are each at the same potential V. Ball 2 is twice as large as ball 1.



As V is increased, which ball will induce breakdown first?

$\boxed{\textbf{(a) Ball 1}}$ **(b) Ball 2** **(c) Same Time**

$$E_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 r^2}; \; V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\therefore \quad E_{\text{surface}} = \frac{V}{r}$$

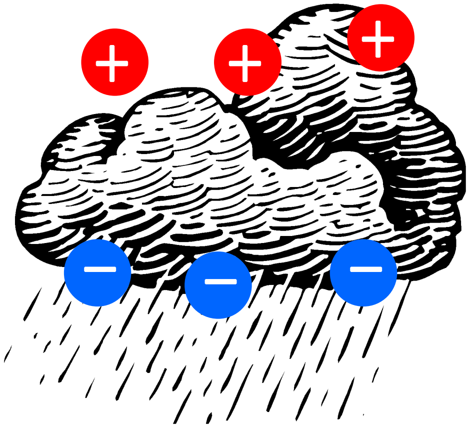
Smaller r → higher E → closer to breakdown

Ex. $V = 100\text{ kV}$

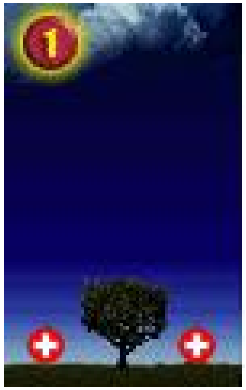
$$r > \frac{100 \bullet 10^3\text{ V}}{3 \bullet 10^6\text{ V/m}} \approx 0.03\text{m} = 3\text{cm}$$

High Voltage Terminals must be big!

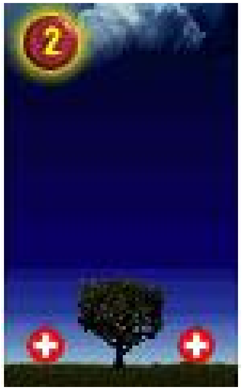
Lightning!（闪电）



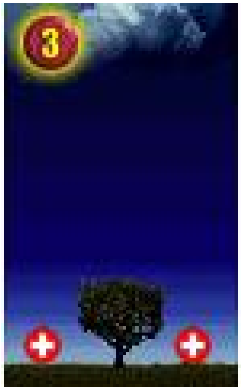
Collisions produce charged particles. The heavier particles (-) sit near the bottom of the cloud; the lighter particles (+) near the top.



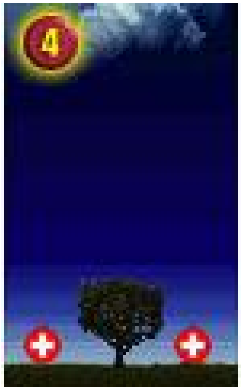
Stepped Leader
Negatively charged electrons begin zigzagging downward.



Attraction
As the stepped leader nears the ground, it draws a streamer of positive charge upward.



Flowing Charge
As the leader and the streamer come together, powerful electric current begins flowing



Contact!
Intense wave of positive charge, a "return stroke," travels upward at 10^8 m/s

Factoids:

$\Delta V \sim 200\ \mathrm{M\ volts}$

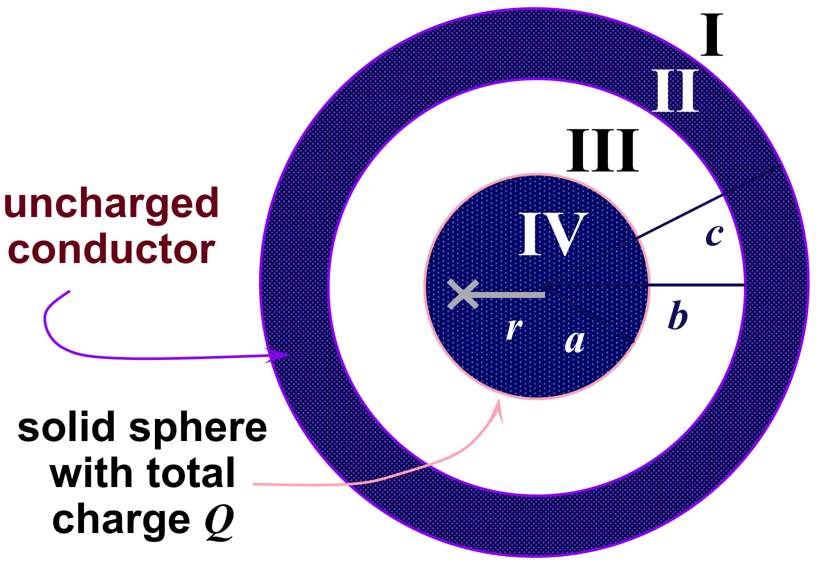
$I \sim 40{,}000\ \mathrm{amp}$

$\Delta t \sim 30\mathrm{ms}$

$P \sim 10^{12}\ \mathrm{W}$

Appendix B

Calculate the potential $V(r)$ at the point shown ($r<a$)

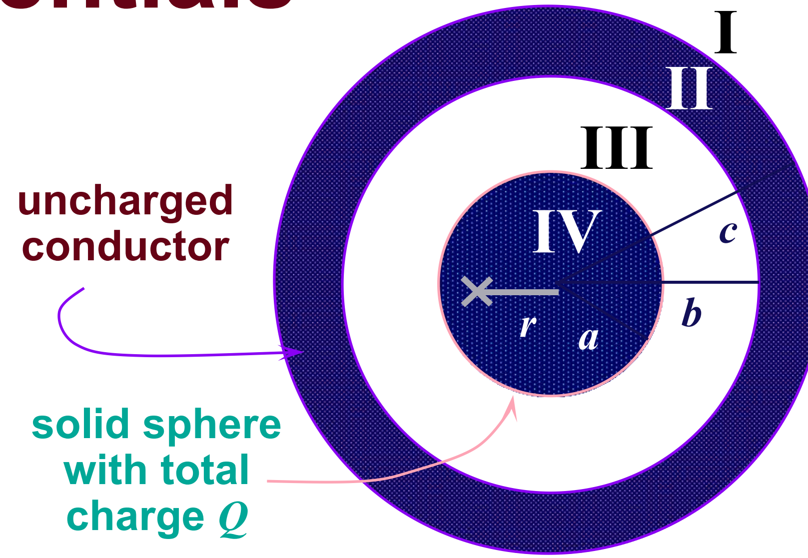


Calculating Electric Potentials

Calculate the potential $V(r)$
at the point shown ($r < a$)

- Where do we know the potential,
  and where do we need to know it?

V=0 at $r = \infty$ ... we need $r < a$ ...



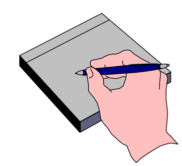
- Determine $E(r)$ for all regions
  in between these two points

$$\vec{E}_I(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \qquad \vec{E}_{II}(r) = 0 \qquad \vec{E}_{III}(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \qquad \vec{E}_{IV}(r) = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{a^3}\hat{r}$$

- Determine ΔV for each region by integration

$$V(r) = V_\infty + \Delta V_{\infty\to c} + \Delta V_{c\to b} + \Delta V_{b\to a} + \Delta V_{a\to r}$$

$$\Delta V_{\infty\to c} = -\int_{r=\infty}^{r=c} \vec{E}_I \bullet d\vec{l} = -\int_\infty^c E_I(dr') = -\int_\infty^c \frac{1}{4\pi\varepsilon_0}\frac{Q}{r'^2}dr'$$



... and so on ...

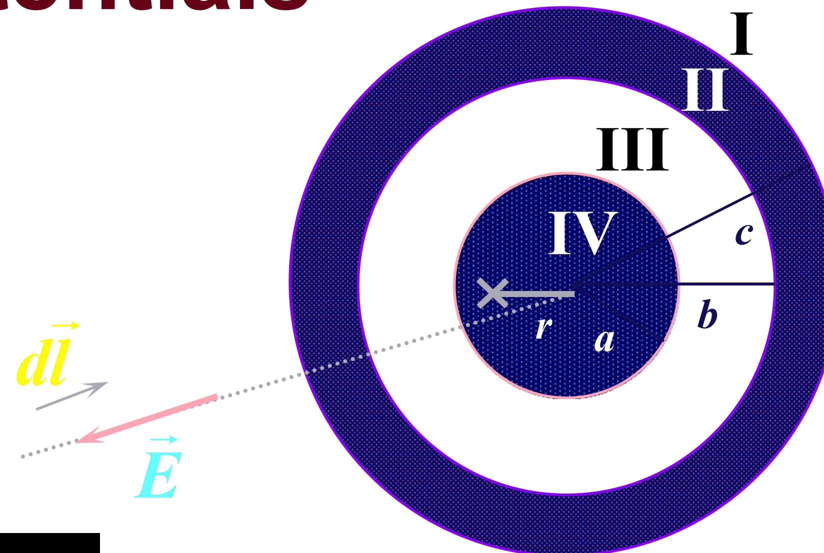
- Check the sign of each potential difference ΔV

$\Delta V > 0$ means we went "uphill"
$\Delta V < 0$ means we went "downhill"

(from the point of view of a positive charge)

Calculating Electric Potentials

Calculate the potential $V(r)$
at the point shown ($r < a$)



- Look at first term:

$$\vec{E}_I(r') = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r'^2}\,\hat{r}$$

- Line integral from infinity to c has to be positive, pushing against a force:

$$\Delta V_{\infty\to c} = -\int_{r'=\infty}^{r'=c} \vec{E}_I \bullet d\vec{l} = -\int_{\infty}^{c} E_I(dr') = -\int_{\infty}^{c} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r'^2}\,dr'$$

Line integral is going "in" which is just the opposite of what usually is done
- controlled by limits

$$= -\frac{1}{4\pi\varepsilon_o}\left(\frac{-Q}{r'}\right]_{\infty}^{c} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{c}$$

- What's left?

$$V(r) = \cancel{V_\infty} + \Delta V_{\infty\to c} + \cancel{\Delta V_{c\to b}} + \Delta V_{b\to a} + \Delta V_{a\to r}$$

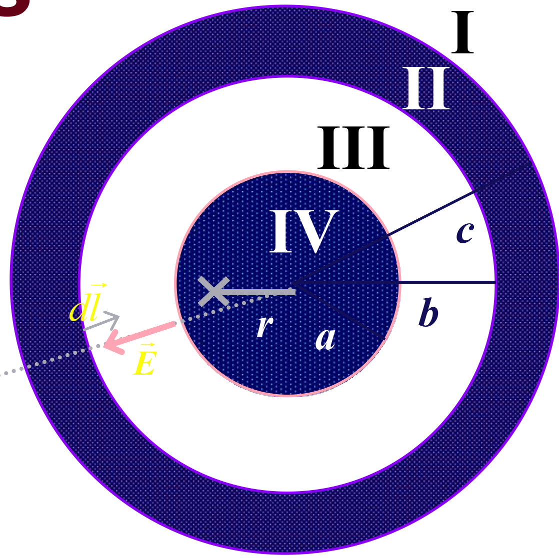
$$\vec{E}_{II}(r') = 0$$

$$\vec{E}_{III}(r') = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r'^2}\,\hat{r}$$

$$\vec{E}_{IV}(r') = \frac{1}{4\pi\varepsilon_0}\frac{Qr'}{a^3}\,\hat{r}$$

Calculating Electric Potentials

Calculate the potential $V(r)$ at the point shown ($r < a$)



- **Look at third term:**

$$\vec{E}_{III}(r') = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r'^2}\hat{r}$$

- **Line integral from b to a, again has to be positive, pushing against a force:**

$$\Delta V_{b\to a} = -\int_{r'=b}^{r'=a} \vec{E}_{III} \bullet d\vec{l} = -\int_b^a E_{III}(dr') = -\int_b^a \frac{1}{4\pi\varepsilon_0}\frac{Q}{r'^2}dr'$$

Line integral is going "in" which is just the opposite of what usually is done
- controlled by limits

$$= -\frac{1}{4\pi\varepsilon_o}\left(\frac{-Q}{r'}\right]_b^a = \frac{1}{4\pi\varepsilon_o}\frac{Q(b-a)}{ab}$$

- **What's left?**

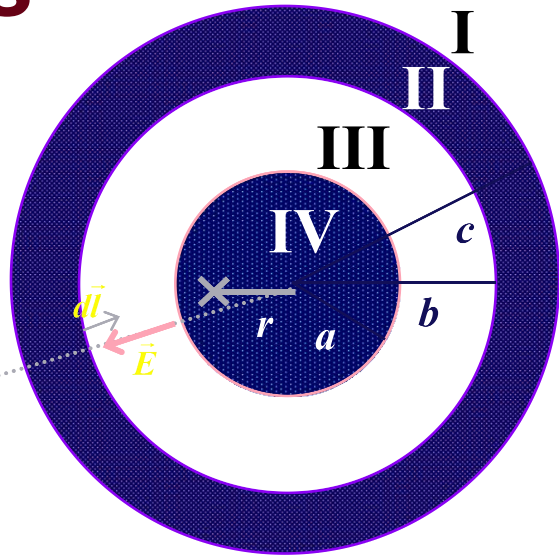
$$V(r) = \cancel{V_\infty} + \Delta V_{\infty\to c} + \cancel{\Delta V_{c\to b}} + \Delta V_{b\to a} + \Delta V_{a\to r}$$

Previous slide
we have calculated this already

$$\vec{E}_{IV}(r') = \frac{1}{4\pi\varepsilon_0}\frac{Qr'}{a^3}\hat{r}$$

Calculating Electric Potentials

Calculate the potential $V(r)$ at the point shown ($r < a$)



- Look at last term: $\vec{E}_{IV}(r') = \frac{1}{4\pi\varepsilon_0}\frac{Qr'}{a^3}\hat{r}$

- Line integral from a to r, again has to be positive, pushing against a force.
  - But this time the force doesn't vary the same way, since "r'" determines the amount of source charge

$$Q\frac{r^3}{a^3} < Q$$

$$\Delta V_{a\to r} = -\int_{r'=a}^{r'=r} \vec{E}_{IV} \bullet d\vec{l} = -\int_a^r E_{IV}(dr') = -\int_a^r \frac{1}{4\pi\varepsilon_0}\frac{Q\frac{r'^3}{a^3}}{r'^2}dr'$$

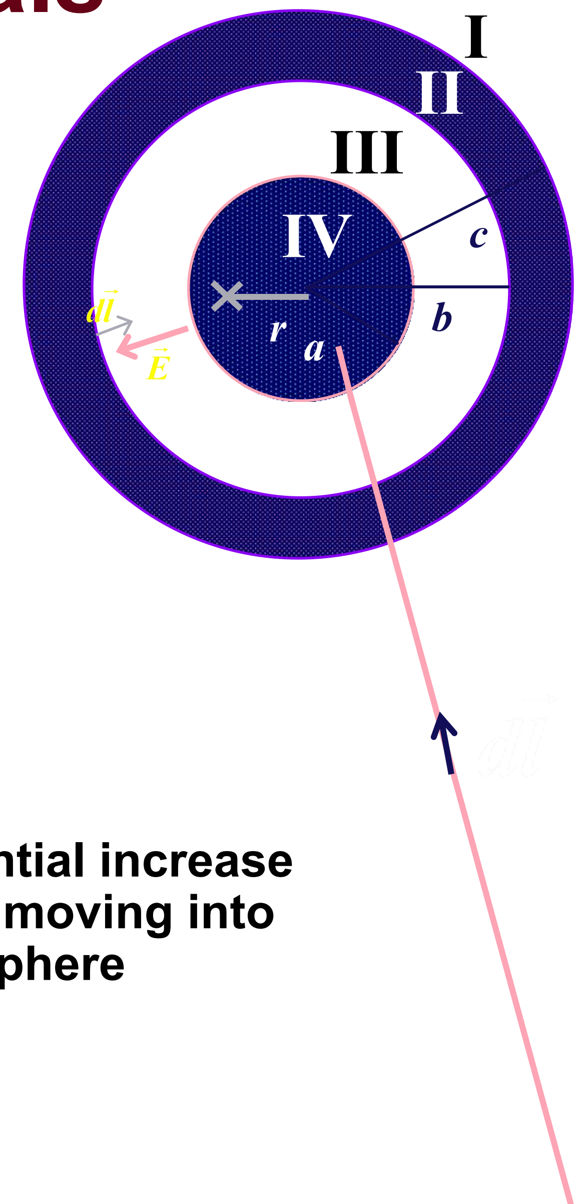
This is the charge that is inside "r" and sources field

- What's left to do?
- ADD THEM ALL UP!
- Sum the potentials

$$= -\frac{1}{4\pi\varepsilon_o}\left(\frac{Q}{a^3}\frac{r'^2}{2}\right]_a^r = \frac{1}{4\pi\varepsilon_o}\frac{Q}{2a}\left(1-\frac{r^2}{a^2}\right)$$

Calculating Electric Potentials

Calculate the potential $V(r)$ at the point shown ($r < a$)



- **Add up the terms from I, III and IV:**

I III IV

$$\Delta V_{\infty\to r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{c} + \frac{1}{4\pi\varepsilon_0}\frac{Q(b-a)}{ab} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2a}\left(1-\frac{r^2}{a^2}\right)$$

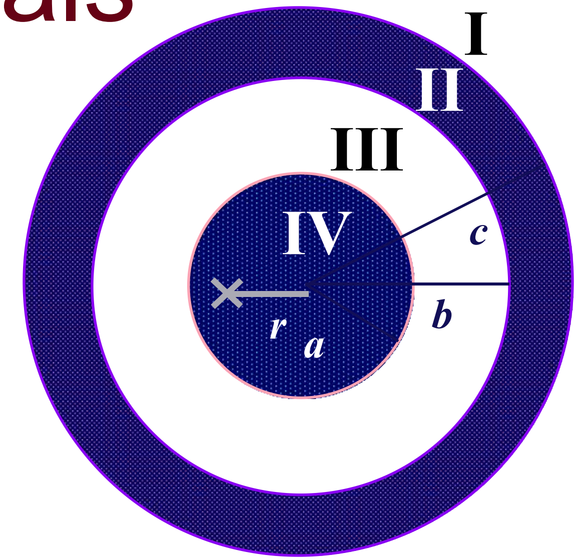
$$\Delta V_{\infty\to r} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \left(\frac{1}{b}-\frac{1}{c}\right) + \frac{1}{2a}\left(1-\frac{r^2}{a^2}\right)\right)$$

The potential difference from infinity to a if the conducting shell weren't there

An adjustment to account for the fact that the conductor is an equipotential, $\Delta V = 0$ from $c \to b$

Potential increase from moving into the sphere

Calculating Electric Potentials Summary



The potential as a function of r for all 4 regions is:

I $\quad r > c$:

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

II $\quad b < r < c$:

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{c}$$

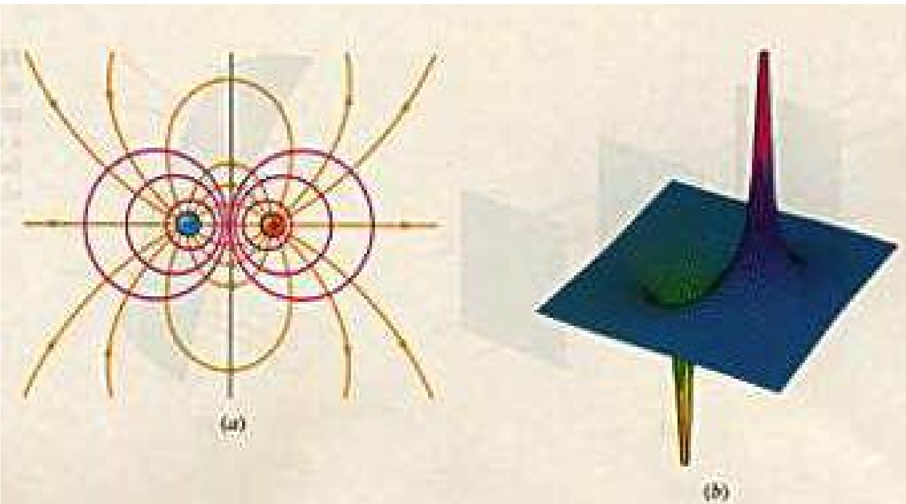
III $\quad a < r < b$:

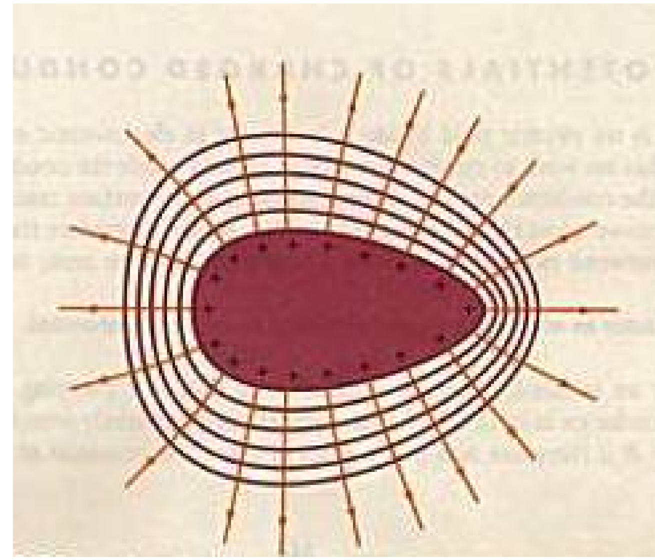
$$V(r) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \left(\frac{1}{b} - \frac{1}{c}\right)\right]$$

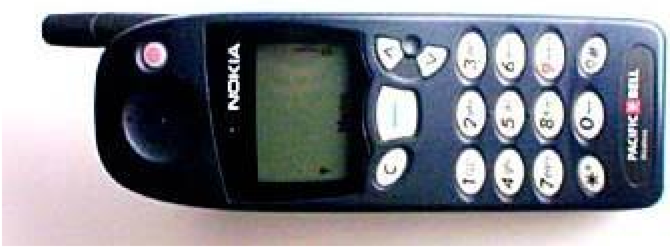
IV $\quad r < a$:

$$V(r) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \left(\frac{1}{b} - \frac{1}{c}\right) + \frac{1}{2a}\left(1 - \frac{r^2}{a^2}\right)\right]$$

28-4 Equipotentials (等势面)



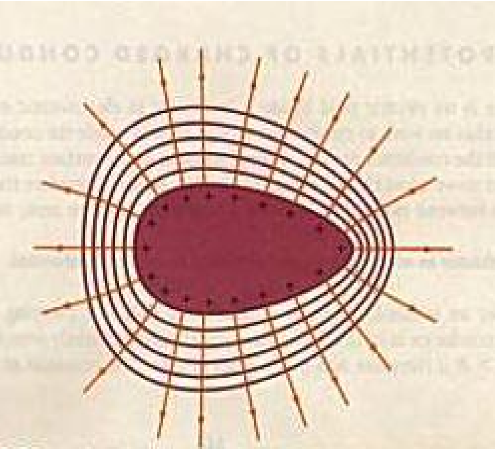




Equipotentials (等势面)

Defined as: The locus of points with the same potential.

- **Example: for a point charge, the equipotentials are spheres centered on the charge.**



The electric field is always perpendicular to an equipotential surface! (电场总是垂直于等势面).

Why??

Along the surface, there is NO change in V (it's an equipotential!)

Therefore, $$-\int_A^B \vec{E} \bullet d\vec{l} = \Delta V = 0$$

We can conclude then, that $\vec{E} \bullet d\vec{l}$ is zero.

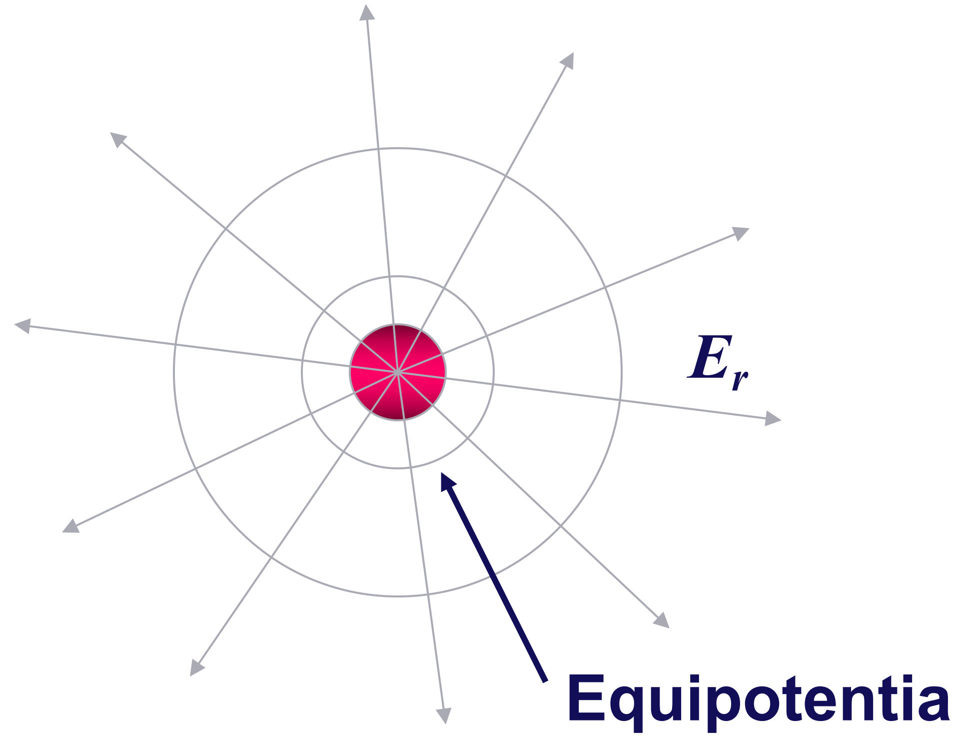
If the dot product of the field vector and the displacement vector is zero, then these two vectors are perpendicular, or the electric field is always perpendicular to the equipotential surface

Potential from a charged sphere

Last time…

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

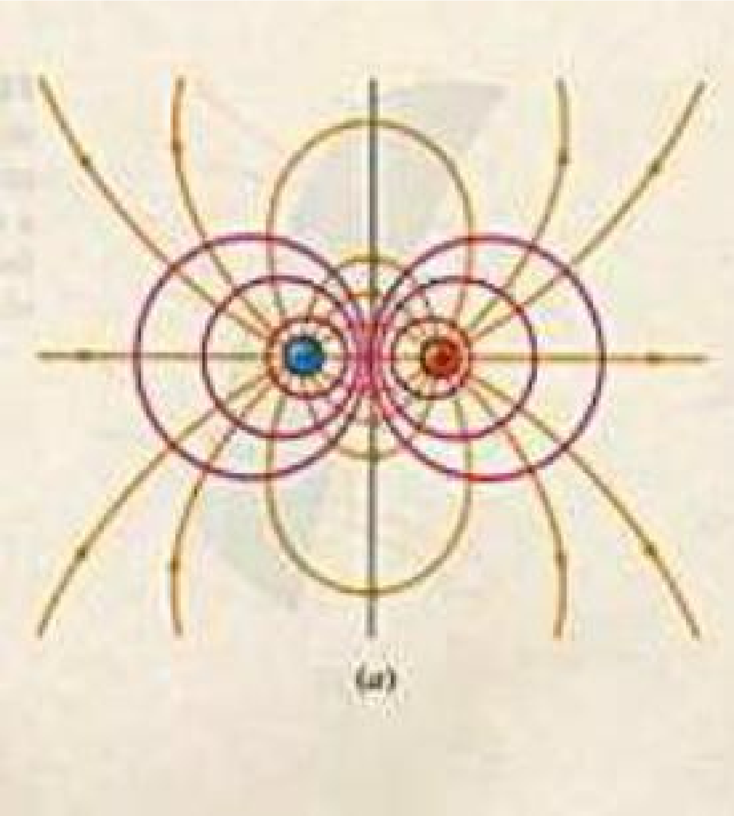
(where $V(\infty) \equiv 0$ **)**

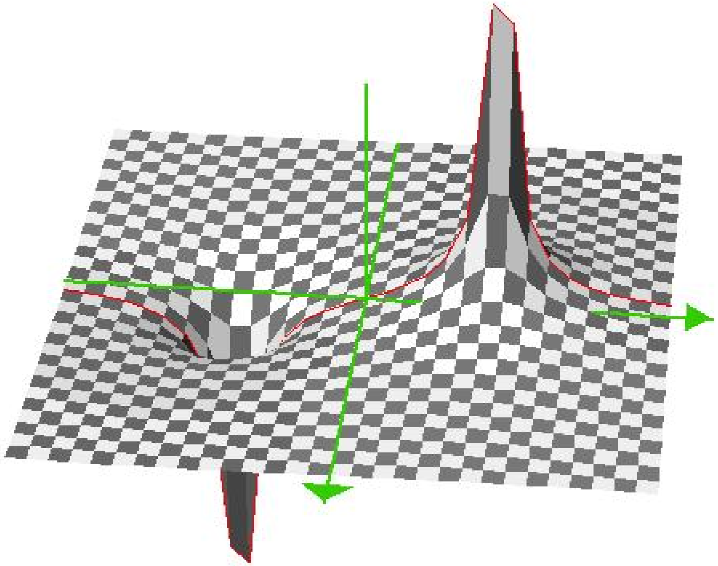


- **The electric field of the charged sphere has spherical symmetry.**
- **The potential depends only on the distance from the center of the sphere, as is expected from spherical symmetry.**
- **Therefore, the potential is constant along a sphere which is concentric with the point charge. These surfaces are called equipotentials.**
- **Notice that the *electric field is perpendicular to the equipotential surface at all points*.**

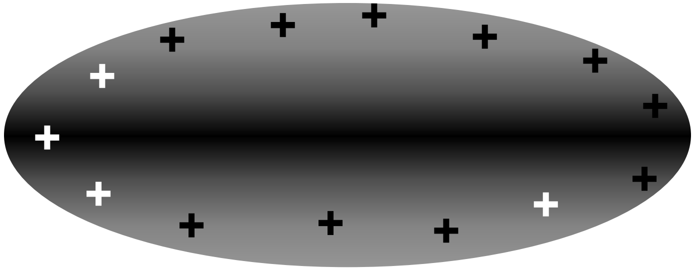
Electric Dipole Equipotentials

- First, let's take a look at the equipotentials:





28-5 The Potential of A charged Conductors



- **Claim**

  The <u>surface</u> of a conductor is *always* an equipotential surface (in fact, the entire conductor is an equipotential).

- **Why??**

  If surface were not equipotential, there would be an electric field component parallel to the surface and the charges would move!!

Preflight 6:

1) The two conductors are now connected by a wire. How do the potentials at the conductor surfaces compare now ?

a) $V_A > V_B$

b) $V_A = V_B$

c) $V_A < V_B$

2) What happens to the charge on conductor A after it is connected to conductor B ?

a) Q_A increases

b) Q_A decreases

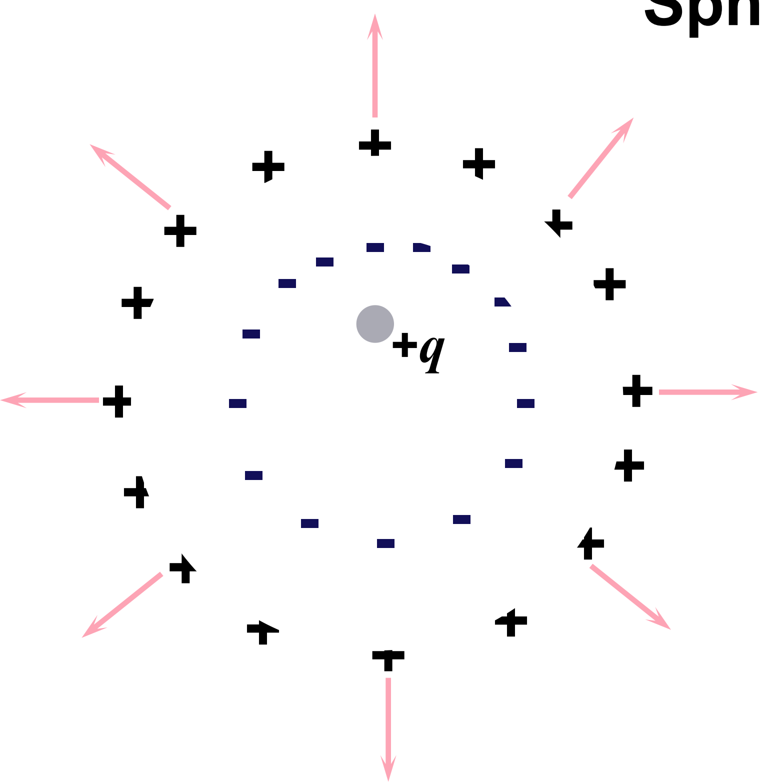
c) Q_A doesn't change

$$\frac{Q_A}{4\pi\varepsilon_0 r_A} = \frac{Q_B}{4\pi\varepsilon_0 r_B}$$

$$\frac{Q_A}{Q_B} = \frac{r_A}{r_B}$$

Charge on Conductors?

- **How is charge distributed on the surface of a conductor?**
  - **KEY: Must produce E=0 inside the conductor and E normal to the surface .**

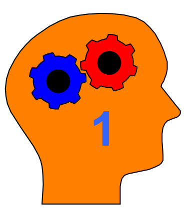


Spherical example **(with little off-center charge):**

E=0 inside conducting shell.

charge density induced on inner surface non-uniform.

charge density induced on outer surface uniform

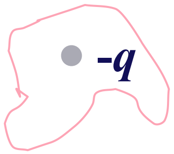


E outside has spherical symmetry centered on spherical conducting shell.

Chapter 28, ACT 1

1A

An uncharged spherical conductor has a weirdly shaped cavity carved out of it. Inside the cavity is a charge -*q*.



How much charge is on the cavity wall?

(a) Less than< *q* (b) Exactly *q* (c) More than *q*

1B

How is the charge distributed on the cavity wall?

(a) Uniformly
(b) More charge closer to –*q*
(c) Less charge closer to -*q*

1C

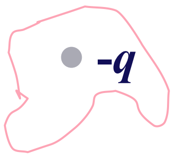
How is the charge distributed on the outside of the sphere?

(a) Uniformly
(b) More charge near the cavity
(c) Less charge near the cavity

Chapter 28, ACT 1

1A

An uncharged spherical conductor has a weirdly shaped cavity carved out of it. Inside the cavity is a charge -*q*.



How much charge is on the cavity wall?

(a) Less than< *q*　　**(b) Exactly *q***　　**(c) More than *q***

By Gauss' Law, since *E*=0 inside the conductor, the total charge on the inner wall must be *q* (and therefore -*q* must be on the outside surface of the conductor, since it has no net charge).

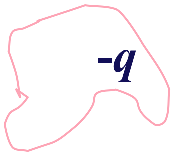
Chapter 28, ACT 1

1B **How is the charge distributed on the cavity wall?**

(a) Uniformly

(b) More charge closer to -*q*

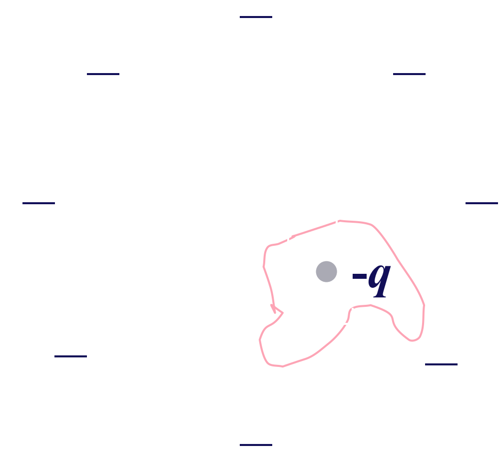
(c) Less charge closer to -*q*



The induced charge will distribute itself nonuniformly to exactly cancel everywhere in the conductor. The surface charge density will be higher near the -*q* charge.

Chapter 28, ACT 1

1C **How is the charge distributed on the outside of the sphere?**



(a) Uniformly

(b) More charge near the cavity

(c) Less charge near the cavity

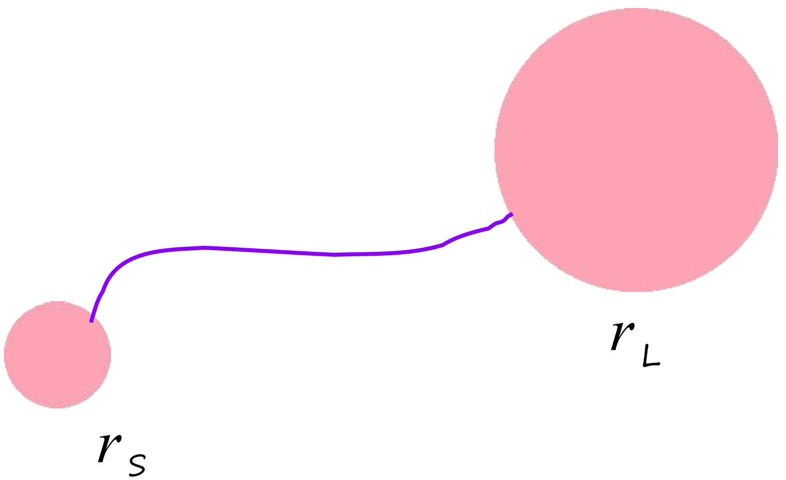
As in the previous example, the charge will be uniformly distributed (because the outer surface is symmetric). Outside the conductor the *E* field always points directly to the center of the sphere, regardless of the cavity or charge.

Note: this is why your radio, cell phone, etc. won’t work inside a metal building!

Corona Discharged (尖端放电)

- How is the charge distributed on a non-spherical conductor?? Claim largest charge density at smallest radius of curvature.
- 2 spheres, connected by a wire, “far” apart
- Both at same potential

$$\frac{Q_S}{4\pi\varepsilon_0 r_S} \approx \frac{Q_L}{4\pi\varepsilon_0 r_L} \Rightarrow \frac{Q_S}{Q_L} \approx \frac{r_S}{r_L}$$



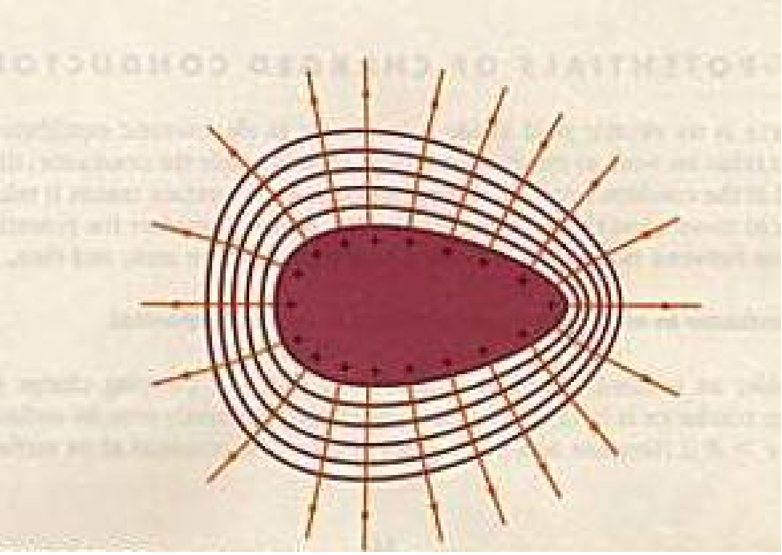
But: $$\frac{\sigma_S}{\sigma_L} \approx \frac{(Q_S / r_S{}^2)}{(Q_L / r_L{}^2)}$$

$$\Rightarrow \boxed{\frac{\sigma_S}{\sigma_L} \approx \frac{r_L}{r_S}}$$

Smaller sphere has the larger surface charge density !

Equipotential Example

- Field lines more closely spaced near end with most curvature – higher E-field
- Field lines ⊥ to surface near the surface (since surface is equipotential).
- Near the surface, equipotentials have similar shape as surface.
- Equipotentials will look more circular (spherical) at large r.

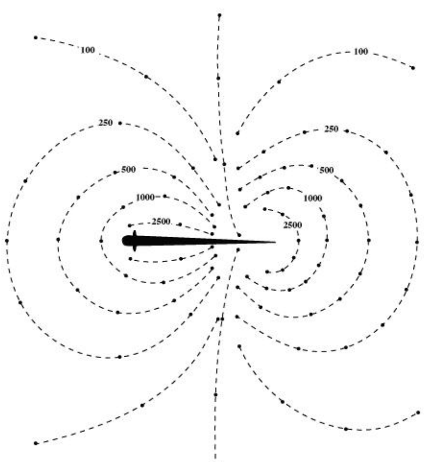


Electric Fish

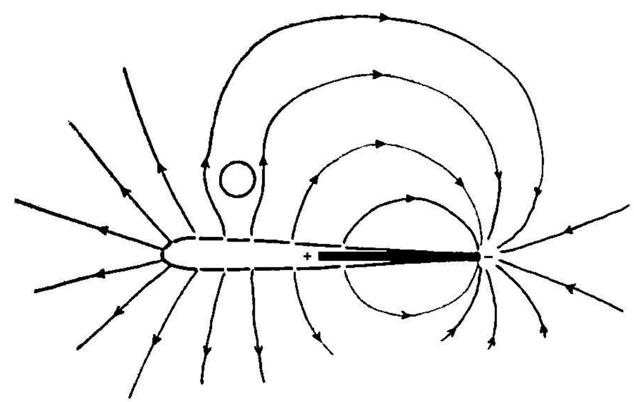
Some fish have the ability to produce & detect electric fields

- **Navigation, object detection, communication with other electric fish**
- **“Strongly electric fish” (eels鰻) can stun their prey (猎物)**

Black ghost knife fish



Dipole-like equipotentials
More info: Prof. Mark Nelson, Beckman Institute, UIUC



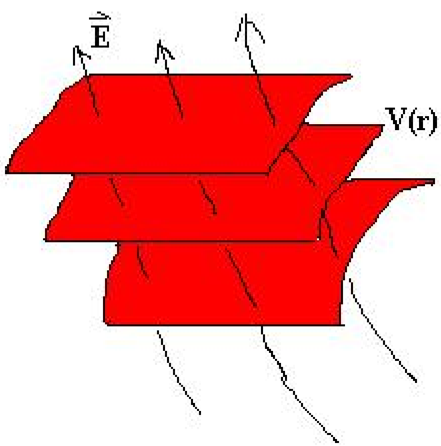
-Electric current flows down the voltage gradient
-An object brought close to the fish alters the pattern of current flow

28-6 Calculating the field from the potential

$$V \Rightarrow \vec{E} \qquad \vec{E} \rightarrow V, \quad V_P = \int_P^\infty \vec{E} \bullet d\vec{l}$$

$$V \rightarrow \vec{E}\,?$$

1. **Graphically (图形法)**

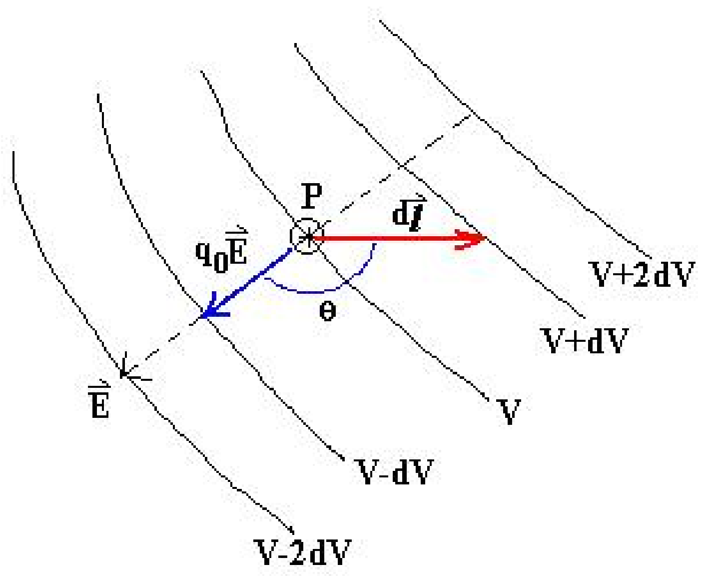


From equipotential surfaces

$\Rightarrow$ draw lines of forces.

Describe the behavior of $\vec{E}$

2. Mathematically



The work done by the electric field force:

$$dW = -q_0 dV$$

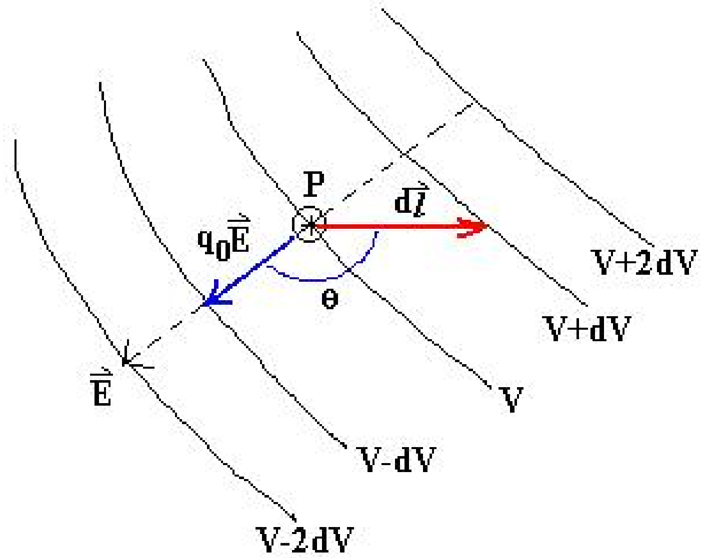
Another

$$dW = \vec{F}\cdot d\vec{l} = q_0\vec{E}\cdot d\vec{l}$$

$$= q_0 E dl \cos\theta$$

$$\therefore -q_0 dV = q_0 E dl \cos\theta$$

$$E\cos\theta = -\frac{dV}{dl}$$



$$E_l = -\frac{dV}{dl}$$

The negative rate of change of the potential with position in any direction is component of $\vec{E}$ in this direction.

$$E = -\left(\frac{dV}{dl}\right)_{\max}$$

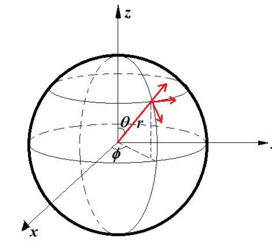
$$\theta = 0$$

The maximum value of $\frac{dV}{dl}$ at a given point is called the potential gradient (**梯度**) at that point.

In the direction $\vec{n}$

Corresponds to the direction of $\vec{E}$

- We can obtain the electric field E from the potential V by inverting our previous relation between E and V:



$$E_l = -\frac{dV}{dl}$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

$$dV = -\vec{E}\cdot\hat{x}dx = -E_x dx$$

- Expressed as a vector, E is the negative gradient of V

$$\vec{\mathrm{E}} = -\vec{\nabla}\,\mathrm{V}$$

- Cartesian coordinates:

$$\vec{\nabla}\,\mathrm{V} = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

- Spherical coordinates:

$$\vec{\nabla}\,\mathrm{V} = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

E from V: an Example

- **Consider the following electric potential:**

$$V(x,y,z) = 3x^2 + 2xy - z^2$$

- **What electric field does this describe?**

$$E_x = -\frac{\partial V}{\partial x} = -6x - 2y \qquad E_y = -\frac{\partial V}{\partial y} = -2x \qquad E_z = -\frac{\partial V}{\partial z} = 2z$$

... expressing this as a vector:

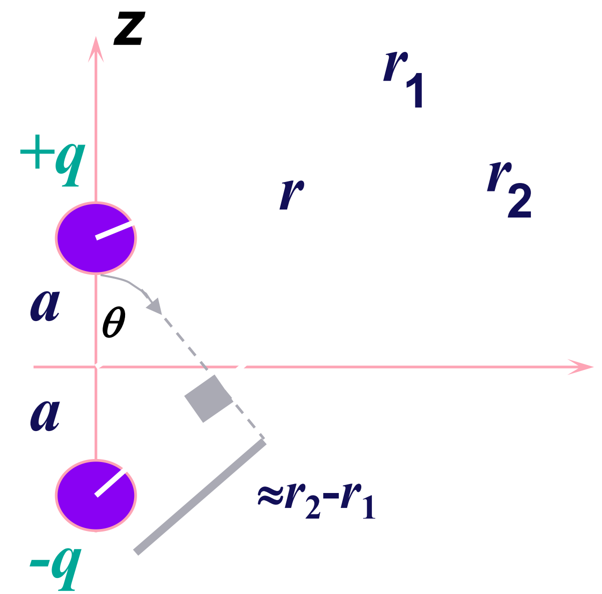
$$\vec{E} = (-6x - 2y)\,\hat{\mathrm{x}} - 2x\hat{y} + 2z\hat{z}$$

- **Something for you to try:**

Can you use the dipole potential to obtain the dipole field? Try it in spherical coordinates ... you should get:

$$\vec{E} = \frac{2aq}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\hat{\theta}\right)$$

Example 1 Electric Dipole



The potential is much easier to calculate than the field since it is an algebraic sum of 2 scalar terms.

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\frac{r_2 - r_1}{r_1 r_2}$$

- **Rewrite this for special case $r >> a$:**

$$r_2 - r_1 \approx 2a\cos\theta$$

$$r_1 r_2 \approx r^2$$

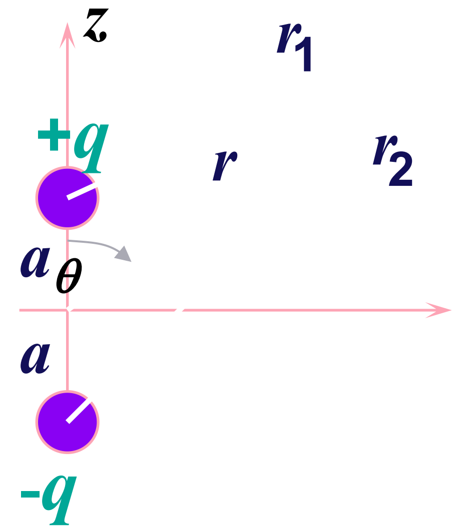
$$\Rightarrow$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{2aq\cos\theta}{r^2}$$

Now we can use this potential to calculate the E field of a dipole (after a picture)

(remember how messy the direct calculation was?)

Electric Dipole



$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{2aq\cos\theta}{r^2}$$

- **Calculate E in spherical coordinates:**

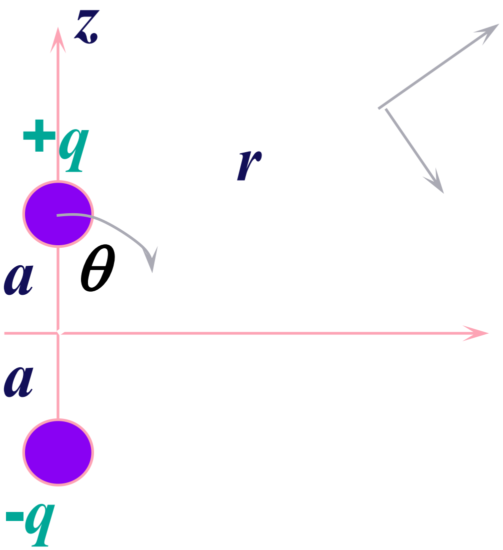
$$E_r = -\frac{\partial V}{\partial r} = -\frac{2aq}{4\pi\varepsilon_0}\left(\frac{-2\cos\theta}{r^3}\right)$$

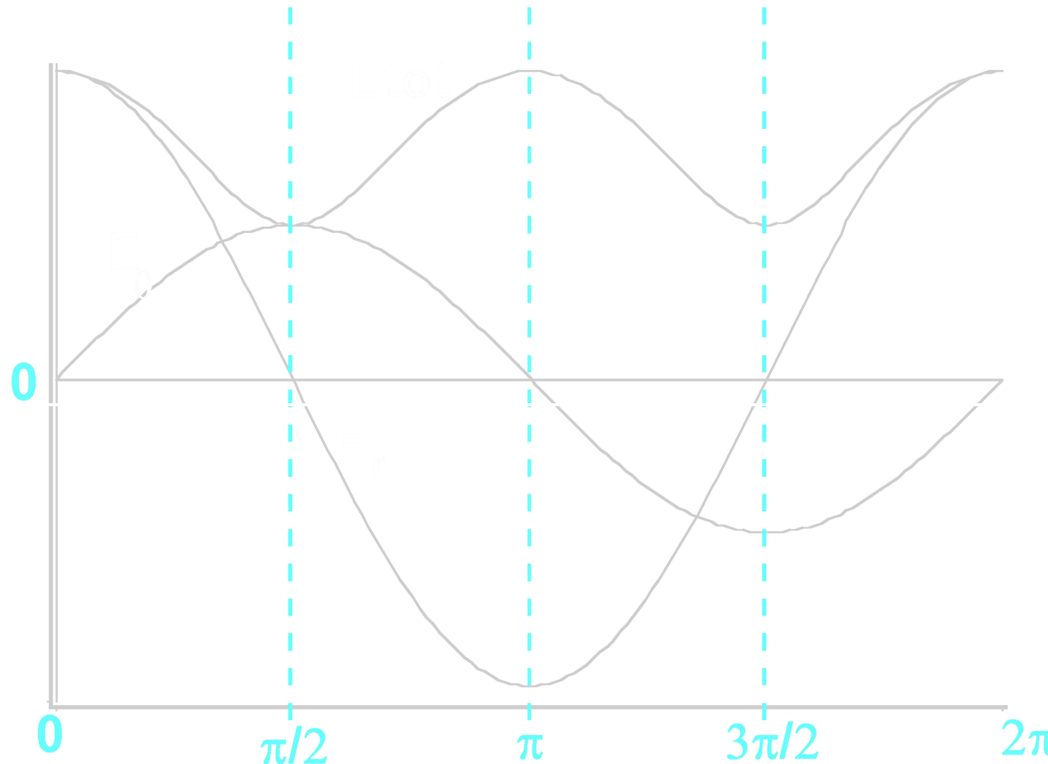
$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} = -\frac{2aq}{4\pi\varepsilon_0}\left(\frac{-\sin\theta}{r^3}\right)$$

the dipole moment

$$\Rightarrow \quad \vec{E} = \frac{2aq}{4\pi\varepsilon_0 r^3}\left((2\cos\theta)\hat{r} + (\sin\theta)\hat{\theta}\right)$$

Dipole Field

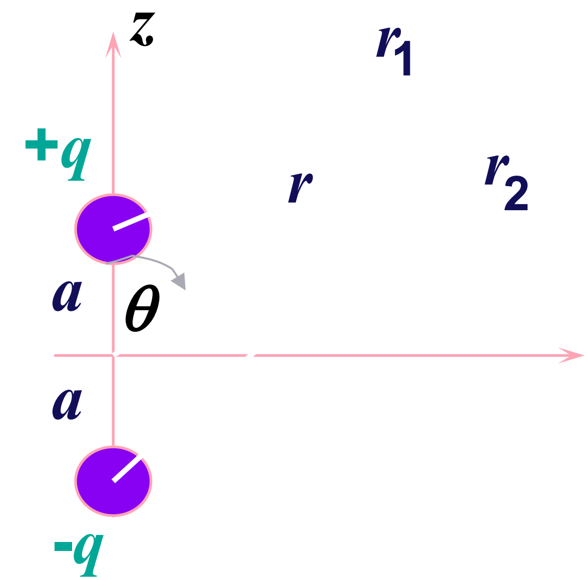




$$\vec{E} = \frac{2\,aq}{4\,\pi\varepsilon_0\,r^3}\Big((2\cos\theta)\hat{r} + (\sin\theta)\hat{\theta}\Big)$$

Sample Problem

- Consider the dipole shown at the right.
  - Fix $r = r_0 >> a$
  - Define θ_{max} such that the polar component E_θ of the electric field has its maximum value (for $r = r_0$).



What is θ_{max}?

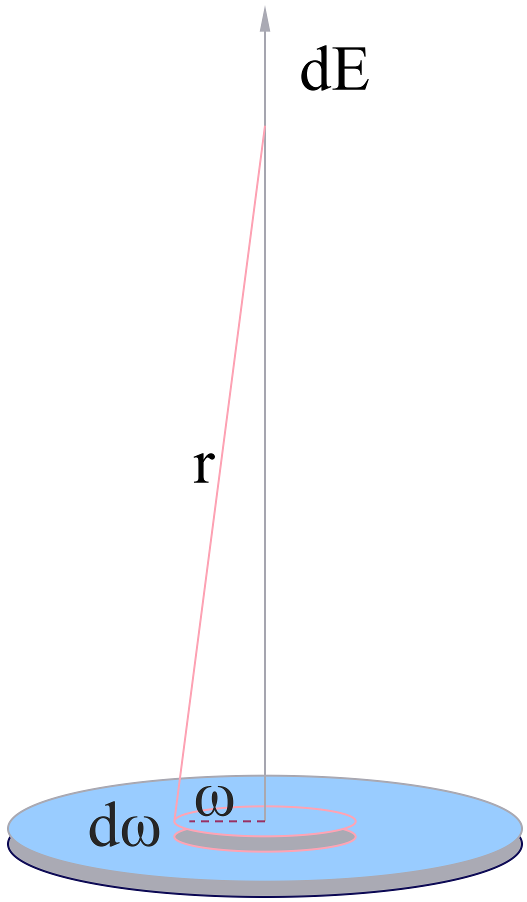
(a) $\theta_{max} = 0$ (b) $\theta_{max} = 45°$ $\boxed{\text{(c) } \theta_{max} = 90°}$

- The expression for the electric field of a dipole ($r >> a$) is:

$$\vec{E} = \frac{2aq}{4\pi\varepsilon_0 r^3}\left((2\cos\theta)\hat{r} + (\sin\theta)\hat{\theta}\right)$$

- The polar component of E is maximum when $\sin\theta$ is maximum.
  - Therefore, E_θ has its maximum value when $\theta = 90°$.

Example 2 A circular plastic disk of radius *R* and the surface charge density σ.



$$dq = 2\pi\omega \cdot d\omega \cdot \sigma$$

$$dV = \frac{dq}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}} = \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}}$$

$$V = \int_0^R \frac{2\pi\omega \cdot d\omega \cdot \sigma}{4\pi\varepsilon_0\sqrt{z^2+\omega^2}} = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2+R^2}-z)$$

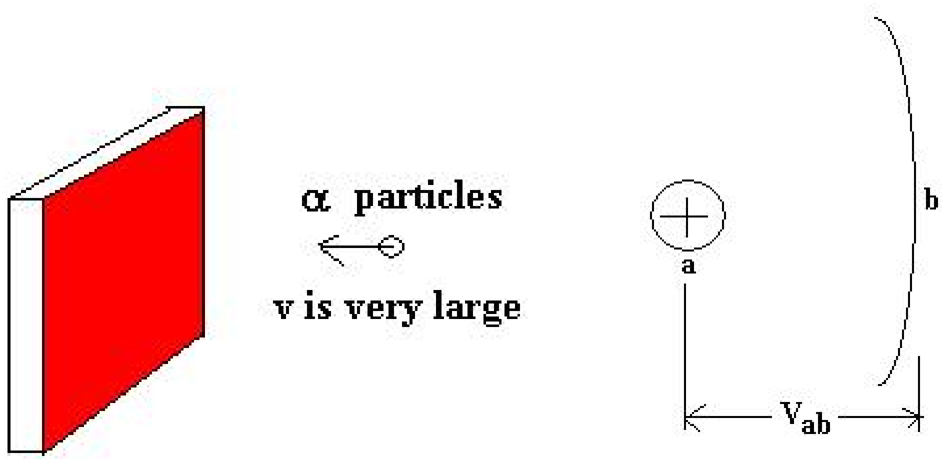
$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\left(\frac{2z}{2\sqrt{R^2+z^2}}-1\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{1+(R/z)^2}}\right)$$

28-7 The Electrostatic Accelerator (P_{651})

Nuclear reactions: How to get large velocity $\vec{v}$

One method is based on an electrostatic technique.



The positive charge q obtain the kinetic energy

$$K = -\Delta U = -q\Delta V > 0$$

$$= q(V_a - V_b)$$

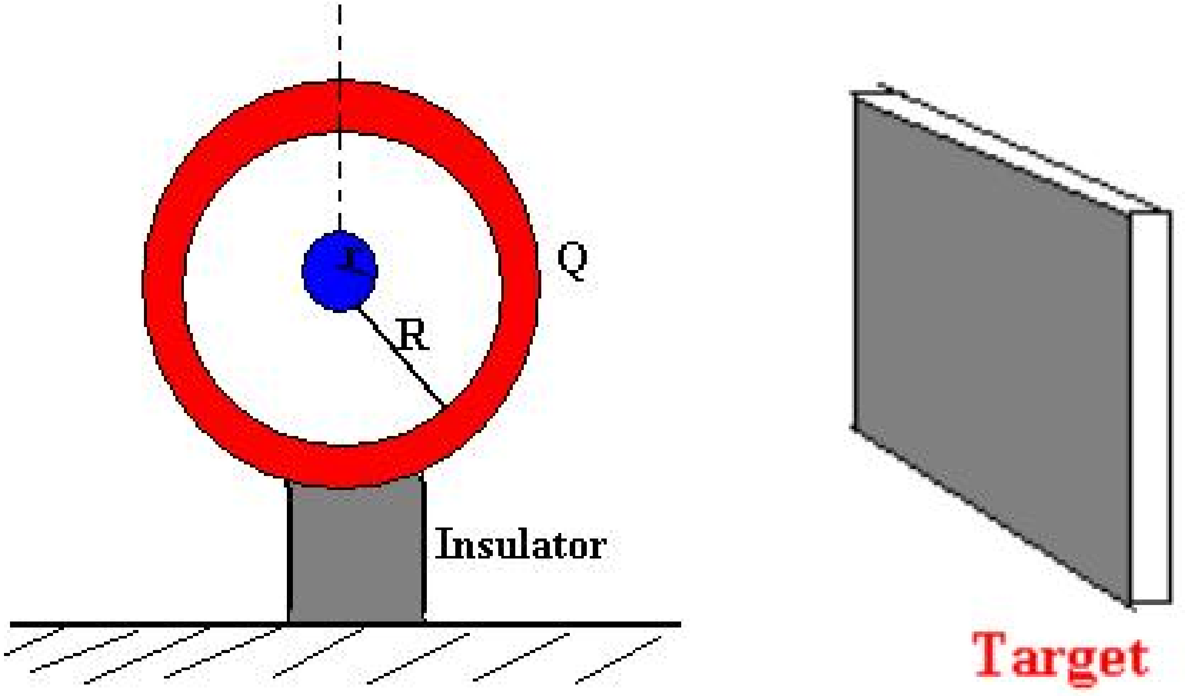
$$= \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2q(V_a - V_b)}{m}}$$

Nuclear: $K \approx MeV(10^6 V)$

No limit, but sparking

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$



The Bottom Line

If we know the electric field E everywhere,

$$V_B - V_A \equiv \frac{W_{AB}}{q_0} \quad \Rightarrow \quad V_B - V_A = -\int_A^B \vec{E} \bullet d\vec{l}$$

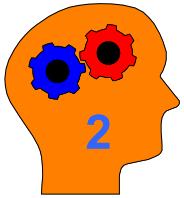
allows us to calculate the potential function V everywhere (keep in mind, we often define $V_A = 0$ at some convenient place)

If we know the potential function V everywhere,

$$\vec{E} = -\vec{\nabla} V$$

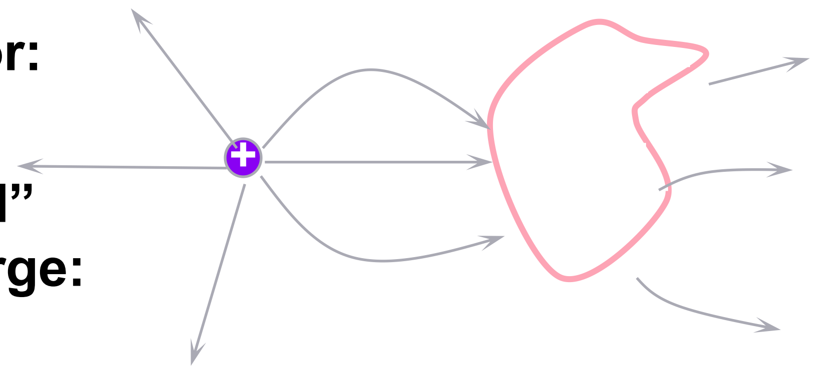
allows us to calculate the electric field E everywhere

- Units for Potential! 1 Joule/Coul = 1 VOLT



Appendix: Induced charge distribution on conductor via "*method of images*(镜像法)"

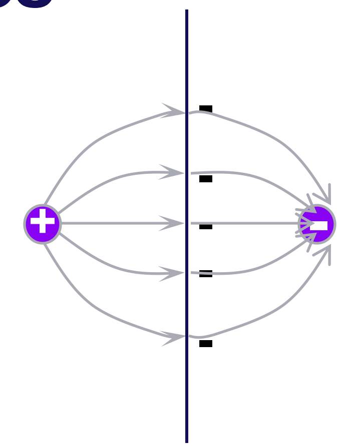
- Consider a source charge brought close to a conductor:



- Charge distribution "induced" on conductor by source charge:
- Induced charge distribution is "real" and sources E-field that is zero inside conductor!
  - resulting E-field is sum of field from source charge and induced charge distribution
  - E-field is locally perpendicular to surface
- With enough symmetry, can solve for σ on conductor
  - how? Gauss' Law

$$E_{normal}(\vec{r}_{surface}) = \left|\vec{E}(\vec{r}_{surface})\right| = \frac{\sigma(\vec{r}_{surface})}{\varepsilon_o}$$

Appendix: Induced charge distribution on conductor via "*method of images*"



- **Consider a source charge brought close to a planar conductor:**
- **Charge distribution "induced" on conductor by source charge**
  - **conductor is equipotential**
  - E**-field is normal to surface**
  - **this is just like a dipole**
- **Method of Images for a charge (distribution) near a flat conducting plane:**
  - **reflect the point charge through the surface and put a charge of opposite sign there**
  - **do this for all source charges**
  - E**-field at plane of symmetry - the conductor surface determines** σ.

Summary

- **If we know the electric field E,**

  allows us to calculate the potential function V everywhere
  (*define V_A = 0 above*)
- **Potential due to *n* charges:**

$$V(r) = \sum_{n=1}^{N} V_n(r) = \frac{1}{4\pi\varepsilon_0} \sum_{n=1}^{N} \frac{q_n}{r_n}$$

- **Equipotential surfaces are surfaces where the potential is constant.**
- **Conductors are equipotentials**
- **Find E from V:**

$$\vec{E} = -\vec{\nabla} V$$

- **Potential Energy**

$$U = qV$$

Homework

P_{657} (Exercises) 34
P_{659}(Problems) 4, 8, 9, 13