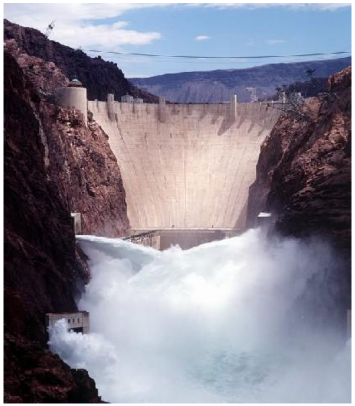
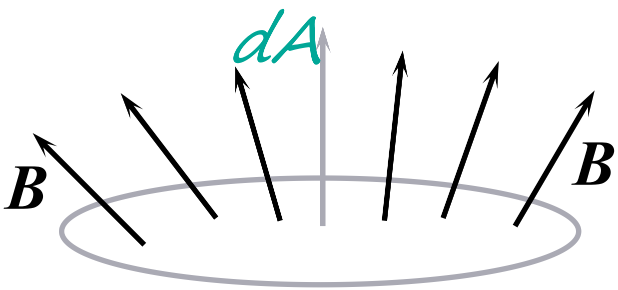
Chapter 34 The Faraday's Law of Induction

(法拉第电磁感应定律)





$$\Phi_B = \iint \vec{B} \circ d\vec{A}$$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

General Review

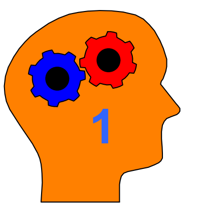
- **Electrostatics (静电学)**
  - » **motion of "q" in external *E*-field**
  - » ***E*-field generated by Σq_i**

- **Magnetostatics (稳恒磁场)**
  - » **motion of "q" and "i" in external *B*-field**
  - » ***B*-field generated by "i"**

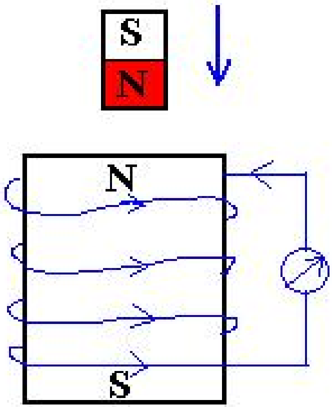
- **Electrodynamics (电动力学)**
  - » **time dependent *B*-field generates *E*-field**
  - » **AC circuits(交流电路), inductors(电感), transformers(变压器), etc.**
  - » **time dependent *E*-field generates *B*-field**
    - **electromagnetic radiation (电磁辐射) - *light!***

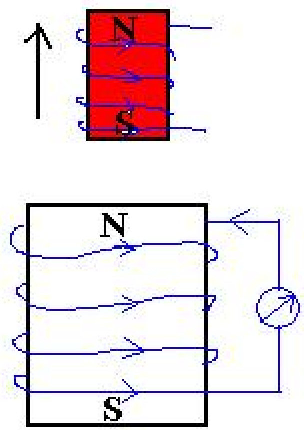
Today...

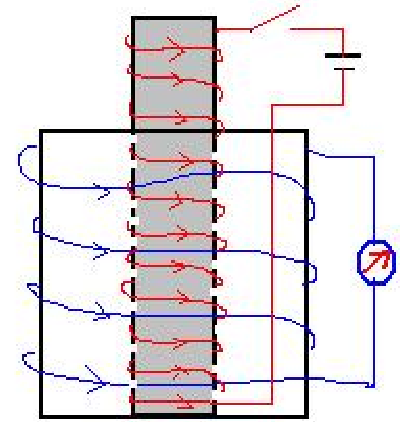
- Induction Effects (电磁感应)
- Faraday's Law (Lenz' Law，楞次定律)
  - Energy Conservation with induced currents?
- Faraday's Law in terms of Electric Fields
- Cool Applications

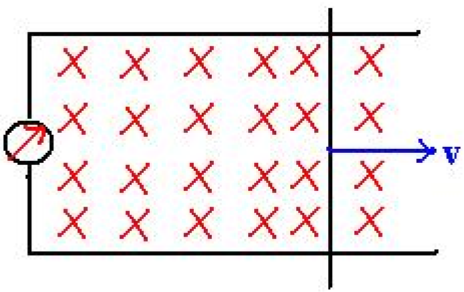


34-1 Basic Phenomena (基本现象)





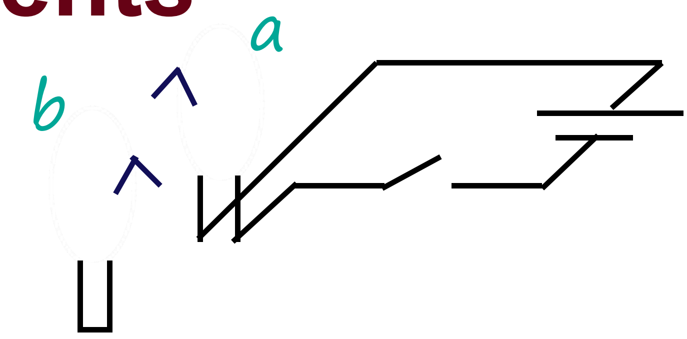




Φ_B change $\rightarrow \varepsilon \rightarrow$ Induced Current

Induced emf (电动势)

Induction Effects from Currents



- **Switch closed (or opened)**

  ⇒ current induced in coil b

- **Steady state current in coil a**

  ⇒ no current induced in coil b

- **Conclusion:**

A current is induced in a loop when:

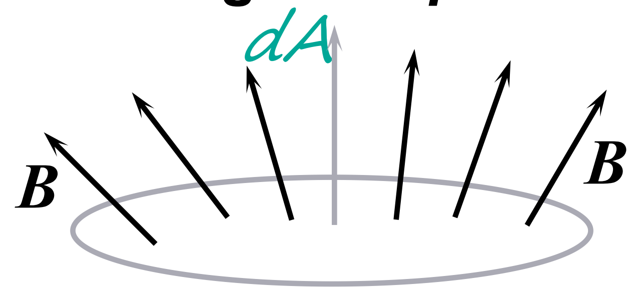
- **there is a change in magnetic field through it**
- **this can happen many different ways**

- **How can we quantify this?**

34-2 Faraday's Law of Induction

- **Define the flux of the magnetic field through an *open* surface as:**

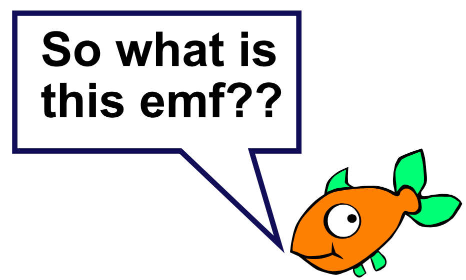
$$\Phi_B = \iint \vec{B} \bullet d\vec{A}$$



- **Faraday's Law:**

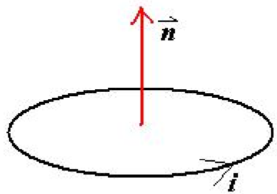
The electromotive force (emf, 电动势) ε induced in a circuit is determined by the time rate of change of the magnetic flux through that circuit.

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$$



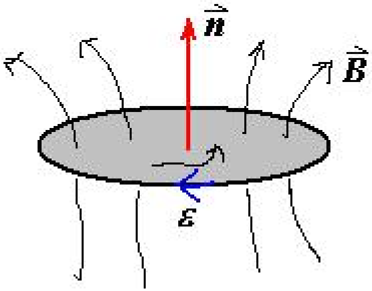
The minus sign indicates direction of induced current (given by Lenz's Law).

Sign of ε

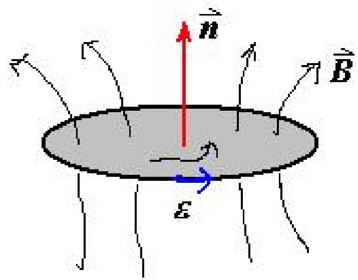


Right Hand Rule

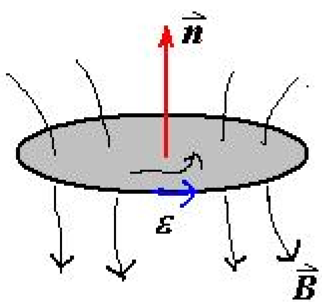
$$\varepsilon = -\frac{d\Phi}{dt}$$



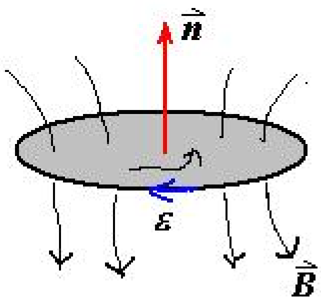
$$\Phi > 0, \Phi \uparrow, \frac{d\Phi}{dt} > 0$$
$$\therefore \varepsilon < 0$$



$$\Phi > 0, \Phi \downarrow, \frac{d\Phi}{dt} < 0$$
$$\therefore \varepsilon > 0$$



$$\Phi < 0, |\Phi| \uparrow, \frac{d\Phi}{dt} < 0$$
$$\therefore \varepsilon > 0$$

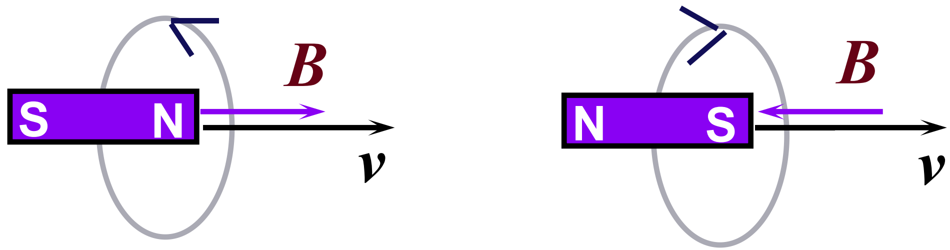


$$\Phi < 0, |\Phi| \downarrow, \frac{d\Phi}{dt} > 0$$
$$\therefore \varepsilon < 0$$

34-3 Lenz's Law (楞次定律)

- **Lenz's Law:**

  The induced current will appear in such a direction that it opposes the change in flux that produced it.

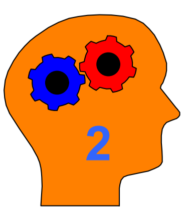


- **Conservation of energy considerations:**

  Claim: Direction of induced current must be so as to oppose the change; otherwise conservation of energy would be violated.

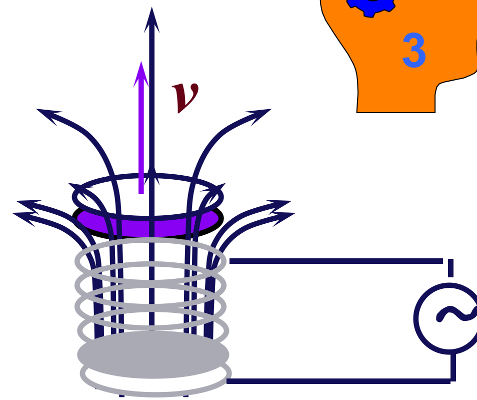
  » **Why???**

  - **If current reinforced the change, then the change would get bigger and that would in turn induce a larger current which would increase the change, etc..**

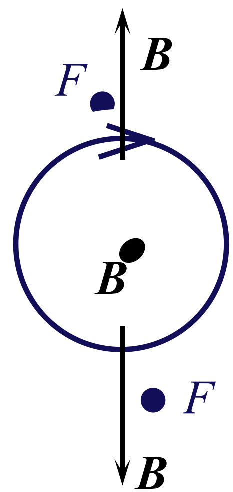


Application of Induction
E-M Cannon(电磁炮)

- Connect solenoid to a source of alternating voltage.
- The flux through the area ⊥ to axis of solenoid therefore changes in time.
- A conducting ring placed on top of the solenoid will have a current induced in it opposing this change.
- There will then be a force on the ring since it contains a current which is circulating in the presence of a magnetic field.



side view



top view

Chapter 34, Act 1

3A

- For this act, we will predict the results of variants of the electromagnetic cannon demo which you just observed.
  – Suppose two aluminum rings are used in the demo; Ring 2 is identical to Ring 1 except that it has a small slit as shown. Let F_1 be the force on Ring 1; F_2 be the force on Ring 2.

(a) $F_2 < F_1$ (b) $F_2 = F_1$ (c) $F_2 > F_1$

3B

– Suppose two identically shaped rings are used in the demo. Ring 1 is made of copper (resistivity = 1.7X10^{-8} Ω-m); Ring 2 is made of aluminum (resistivity = 2.8X10^{-8} Ω-m). Let F_1 be the force on Ring 1; F_2 be the force on Ring 2.

(a) $F_2 < F_1$ (b) $F_2 = F_1$ (c) $F_2 > F_1$

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$\boxed{\textbf{(a) } F_2 < F_1}$ **(b)** $F_2 = F_1$ **(c)** $F_2 > F_1$

- The key here is to realize exactly how the force on the ring is produced.
- A force is exerted on the ring because a current is flowing in the ring and the ring is located in a magnetic field with a component perpendicular to the current.
- An emf is induced in Ring 2 equal to that of Ring 1, but NO CURRENT is induced in Ring 2 because of the slit!
- Therefore, there is NO force on Ring 2!

Chapter 34 ACT 1

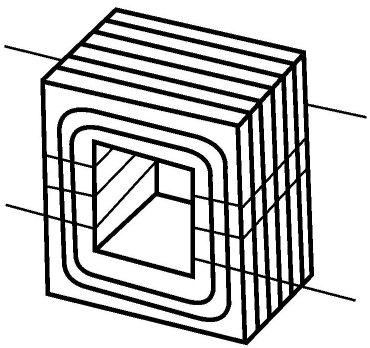
3B

- **For this act, we will predict the results of variants of the electromagnetic cannon demo which you just observed.**
  - **Suppose two identically shaped rings are used in the demo. Ring 1 is made of copper (resistivity = 1.7X10^{-8} Ω-m); Ring 2 is made of aluminum (resistivity = 2.8X10^{-8} Ω-m). Let F_1 be the force on Ring 1; F_2 be the force on Ring 2.**

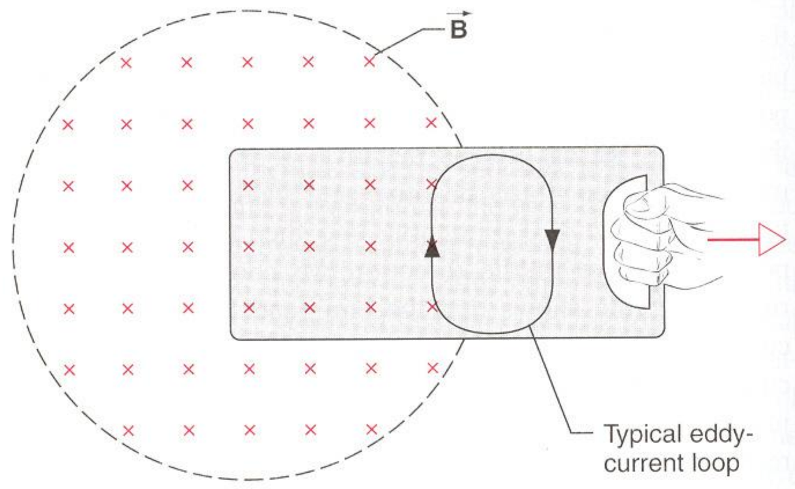
$\boxed{\textbf{(a) } F_2 < F_1}$ **(b)** $F_2 = F_1$ **(c)** $F_2 > F_1$

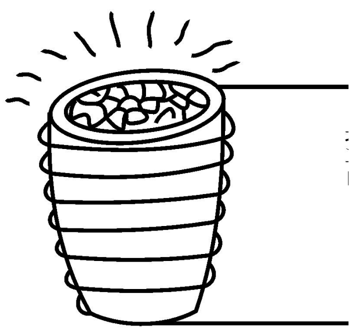
- The emf's induced in each case are equal.
- The currents induced in the ring are NOT equal because of the different resistivities of the materials.
- The copper ring will have a larger current induced (smaller resistance) and therefore will experience a larger force (F proportional to current).

Eddy currents（涡流）

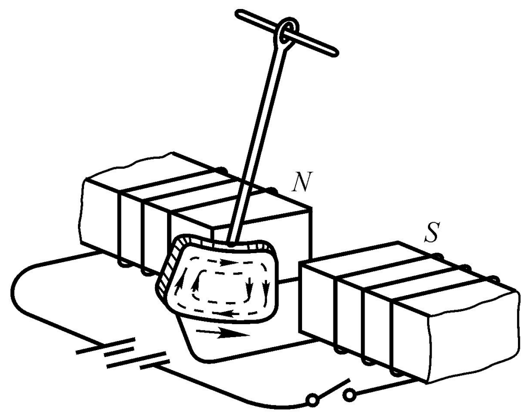


Materials are *laminated*.





Induction furnace



Magnetic breaking

More Applications of Magnetic Induction

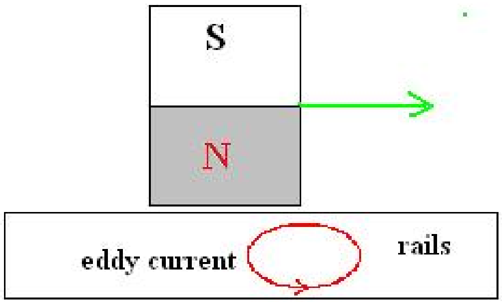
- **Tape / Hard Drive / ZIP Readout**
  - Tiny coil responds to change in flux as the magnetic domains (encoding 0's or 1's) go by.

Question: How can your VCR display an image while paused?

- **Credit Card Reader**
  - Must swipe card
    → generates changing flux
  - Faster swipe → bigger signal

More Applications of Magnetic Induction

- **Magnetic Levitation (Maglev) Trains**
  - Induced surface ("eddy") currents produce field in opposite direction
    - → Repels magnet
    - → Levitates train



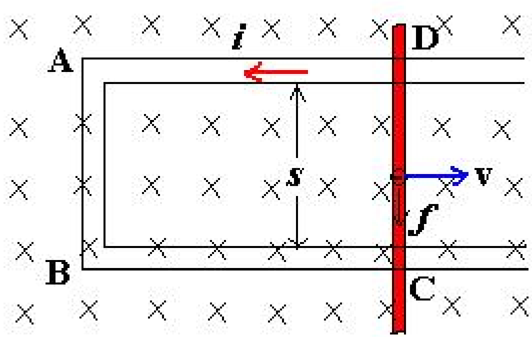
  - Maglev trains today can travel up to *620 km/hr*
    - → Almost Twice the speed of 高铁's fastest conventional train!
  - May eventually use superconducting loops to produce *B*-field
    - → No power dissipation in resistance of wires!

34-4 Motional emf (动生电动势) and Induced emf (感生电动势)

- In a steady magnetic field, moving conductor: motional emf
- Conductor in steady, Changing magnetic field: induced emf

1. Motional emf (动生电动势):

Lorentz force results in a motional emf.



$$\vec{f} = -e(\vec{v} \times \vec{B})$$

Electron moves in the direction of DCBA

Non electrostatic force:
（非静电力）

$$\vec{K} = \frac{\vec{f}}{-e} = \vec{v} \times \vec{B}$$

Motional emf:

$$\varepsilon = \int_{-}^{+} \vec{K} \bullet d\vec{l} = \int_{C}^{D} (\vec{v} \times \vec{B}) \bullet d\vec{l}$$

Notes

- **In high school case:**

$$\vec{v} \perp \vec{B}, \quad v = \text{constant}$$

$$\varepsilon = \int_C^D (\vec{v} \times \vec{B}) \bullet d\vec{l} = Bvl$$

- **Motional emf exists only in the moving conductor**
- **For any cases, (any shape, closed, non-closed)**

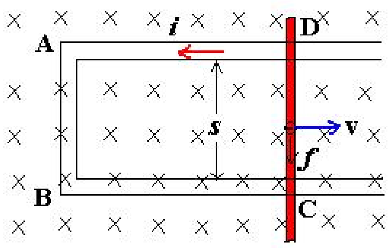
$$\varepsilon = \int_C^D (\vec{v} \times \vec{B}) \bullet d\vec{l}$$

- **Lorentz force can't do work to electron! Why we speak that Lorentz force results in a motional emf? Due to: the velocity of electron** $\vec{u}_d + \vec{v}$

$$\vec{F} = -e(\vec{u}_d + \vec{v}) \times \vec{B}$$

$$\vec{f}_1 = -e\vec{v} \times \vec{B}$$ **Do positive work**

$$\vec{f}_2 = -e\vec{u}_d \times \vec{B}$$ **Do negative work**



Example: Page 781, Sample problem 34-4

Solution 1: $\varepsilon = \int_o^a (\vec{v} \times \vec{B}) \bullet d\vec{l}$

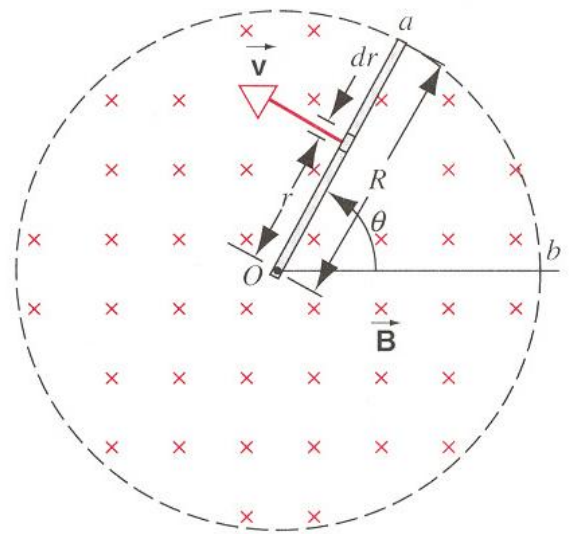
$$d\varepsilon = (\vec{v} \times \vec{B}) \bullet d\vec{r} = -Bvdr,$$

$$\varepsilon = -\int_0^R Bv\mathrm{d}r = -\int_0^R B\omega r dr = -\frac{1}{2}B\omega R^2$$

Solution 2: Suppose <u>*oab*</u> is a loop,

$$\Phi_B = BA = B(\frac{1}{2}R^2\theta)$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{1}{2}BR^2\frac{d\theta}{dt} = -\frac{1}{2}BR^2\omega$$



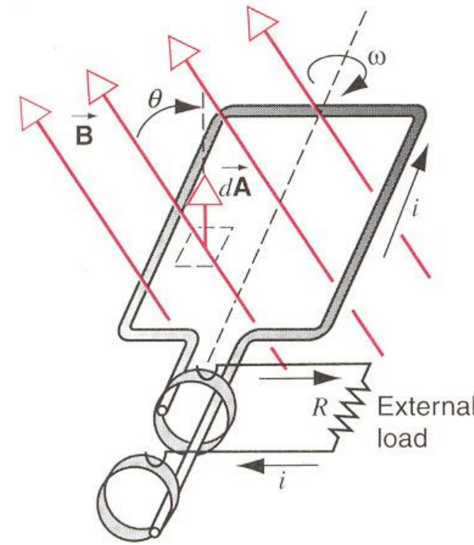
A copper rod of length R, angular frequency ω, in an uniform magnetic field B

What is the direction of induced emf?

+ o, - a

Applications: Generators and Motors (发电机和电动机)

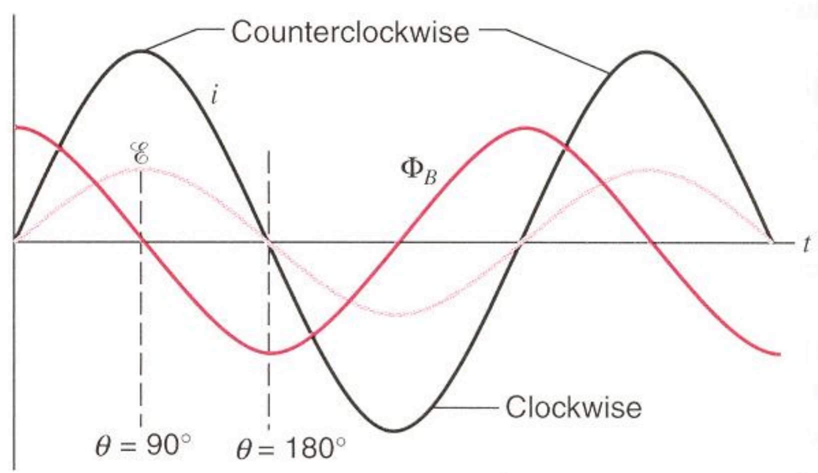
Generator is a device for converting mechanical work (or other) into electrical work in the load.



$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta = BA\cos\omega t$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -BA\frac{d\cos\omega t}{dt} = BA\omega\sin\omega t$$

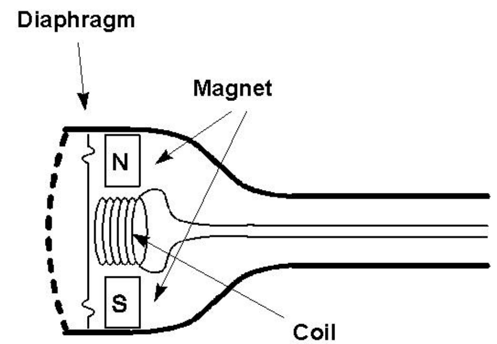
Alternating current (AC)



Applications of Magnetic Induction

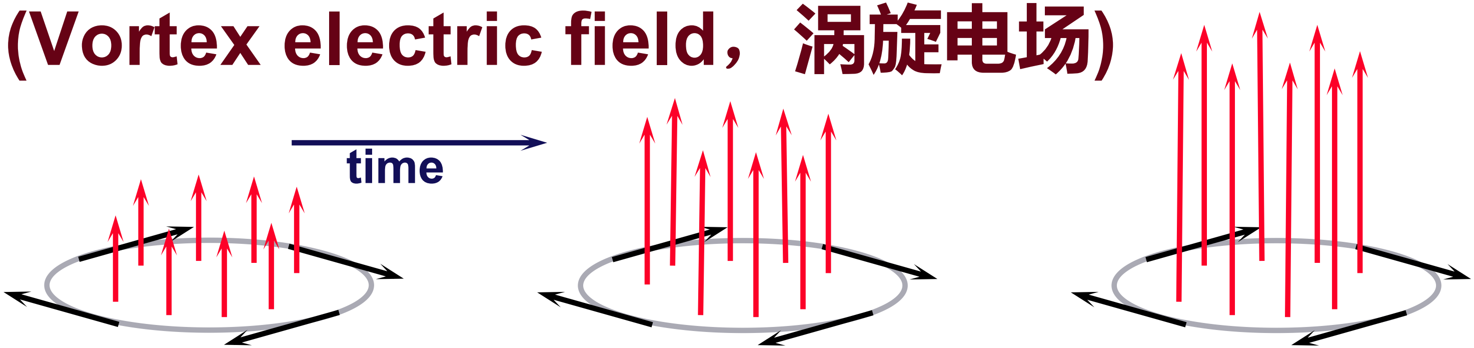
- **AC Generator**
  - Water turns wheel
    → rotates magnet
    → changes flux
    → induces emf
    → drives current

- **"Dynamic" Microphones**
  (E.g., some telephones)
  - Sound
    → oscillating pressure waves
    → oscillating [diaphragm + coil]
    → oscillating magnetic flux
    → oscillating induced emf
    → oscillating current in wire



Question: Do dynamic microphones need a battery?

2. Induced emf 感生电动势 (Vortex electric field，涡旋电场)



A magnetic field, increasing in time, passes through the blue loop

An electric field is generated "ringing" the increasing magnetic field

Circulating *E*-field will drive currents, just like a voltage difference

Loop integral of E-field is the "emf"

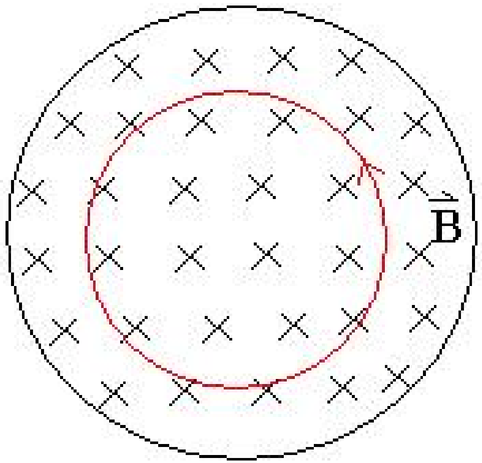
$$\boxed{\varepsilon = \oint \vec{E} \bullet d\vec{l}}$$

Note: The loop does not have to be a wire—**the emf exists even in vacuum**!
When we put a wire there, the electrons respond to the emf → current.

Induced emf (Cont.)

1. Induced electric field *E*

$$\vec{B}\uparrow, \frac{dB}{dt} = cont.$$



If there is a circular loop,

→ induced current *i*,

→ induced emf ε

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt}$$

If there is no circular loop,

→ induced emf ε

→ induced electric field *E*

Notes:

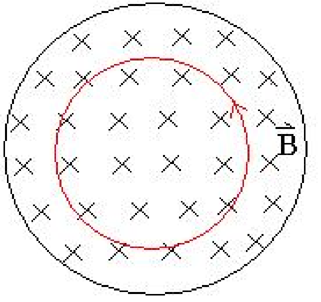
- ***The induced electric field is just real as any that might be set up by static charges, for instance, it exists a force q_0E on a test charge.***
- ***The presence of the induced electric field has nothing to do with the presence of the loop of wire.***

Induced electric field (感应电场)

$$\vec{E}_{induced} = ?$$

- The work *W* done on the charge by the induced electric field *E* in circular is $q_0\varepsilon$

$$\varepsilon q_0 = q_0 E_{induced} \cdot 2\pi r$$
$$\varepsilon = E_{induced} \cdot 2\pi r = \oint \vec{E}_{induced} \cdot d\vec{l}$$



- Faraday's Law:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$
$$\therefore \quad \oint \vec{E}_{induced} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \iint \vec{B} \cdot d\vec{A}$$
$$\oint \vec{E} \cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- For any point in space

$$\vec{E} = \vec{E}_{sta} + \vec{E}_{ind}$$
$$\therefore \quad \oint \vec{E} \cdot d\vec{l} = \oint (\vec{E}_{sta} + \vec{E}_{ind}) \cdot d\vec{l} = 0 + (-\frac{d\Phi_B}{dt}) = -\frac{d\Phi_B}{dt}$$

It is apparent that Faraday's Law implies that a changing magnetic field produces an electric field.

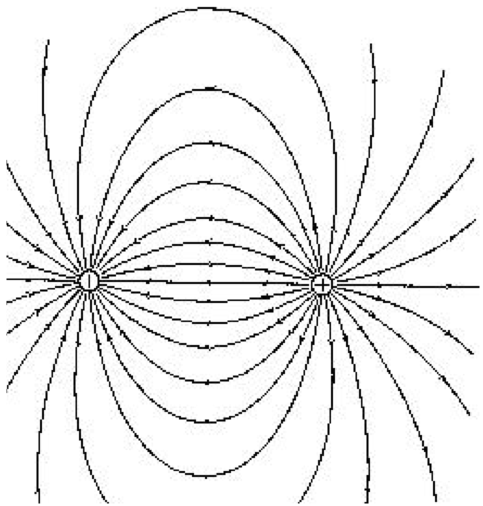
Notes

- $\oint \vec{E}\cdot d\vec{l}$ **can be applied to any shape paths**
- ***E_s* is set up by charges,**
  ***E_{in}* is set up by changing magnetic field.**
  Both kinds of electric field exert on charges.

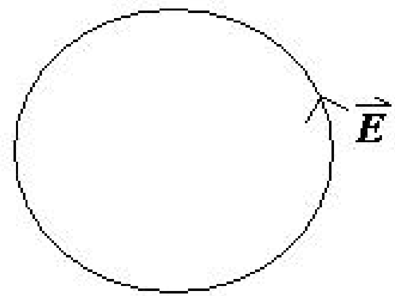
$$\vec{E} = \vec{E}_s + \vec{E}_{in}$$

$$\oint \vec{E}\cdot d\vec{l} = \oint (\vec{E}_s + \vec{E}_{in})\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

  - ***E_s*:**



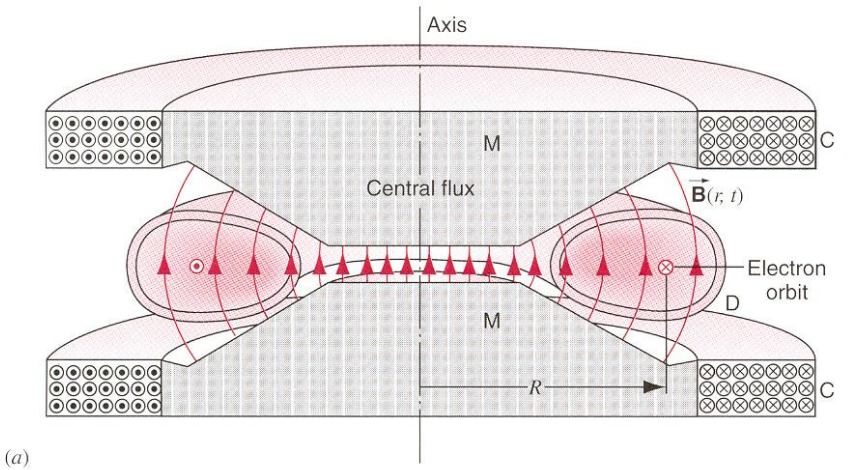
***E_{in}*:**

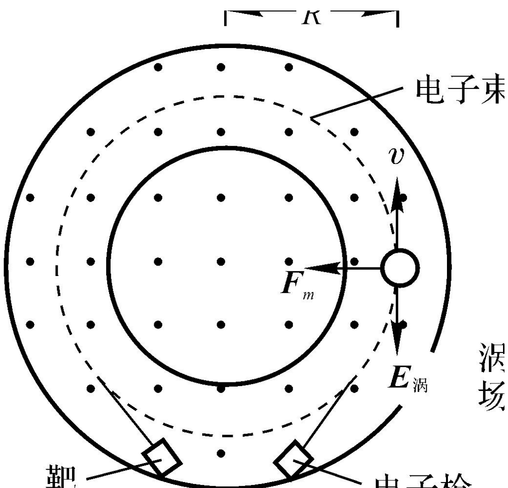


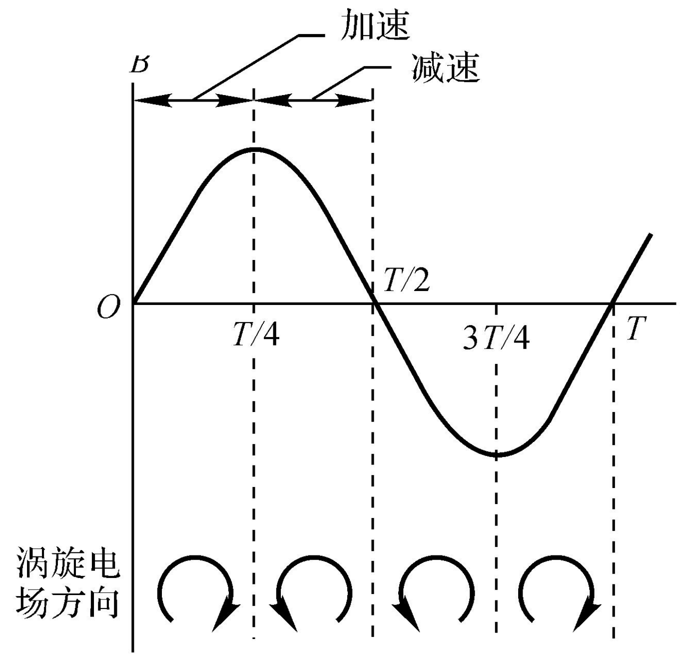
- **Note: In Ch. 28 we claimed $\oint \vec{E}\cdot d\vec{l} = 0$, so we could define a potential independent of path. This holds only for charges at rest (electro*statics*). Forces from changing magnetic fields are *nonconservative*, and no potential can be defined!**

Application: The betatron

（电子感应加速器, 电子回旋加速器）







Betatron produces a pulsed (脉冲) rather than a continuous beam.

34-5. Induction & Relative Motion

- **In the Reference S fixed with B:**

Motional emf:

$$\vec{V} = \vec{v} + \vec{v}_d, \quad \vec{F}_B = \vec{N} + \vec{F}_i$$

$$\mathrm{d}W_N = N(v\mathrm{d}t)$$

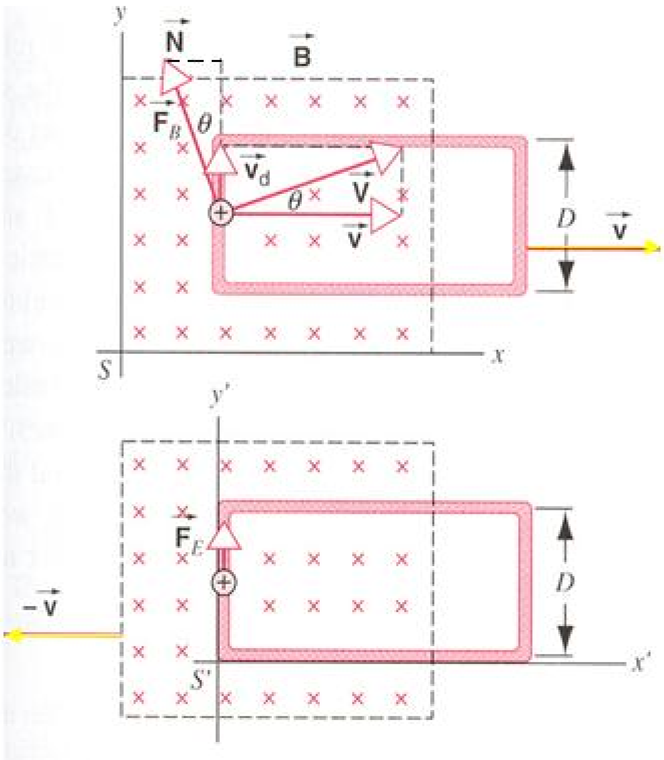
$$= F_B \sin\theta(v\mathrm{d}t)$$

$$= (qVB)(v_d/V)(v\mathrm{d}t)$$

$$= (qBv_d)(v\mathrm{d}t)$$

$$= (qBv)(v_d\mathrm{d}t)$$

$$= qBvdl$$



$$W_N = \int qBvdl = qBvD$$

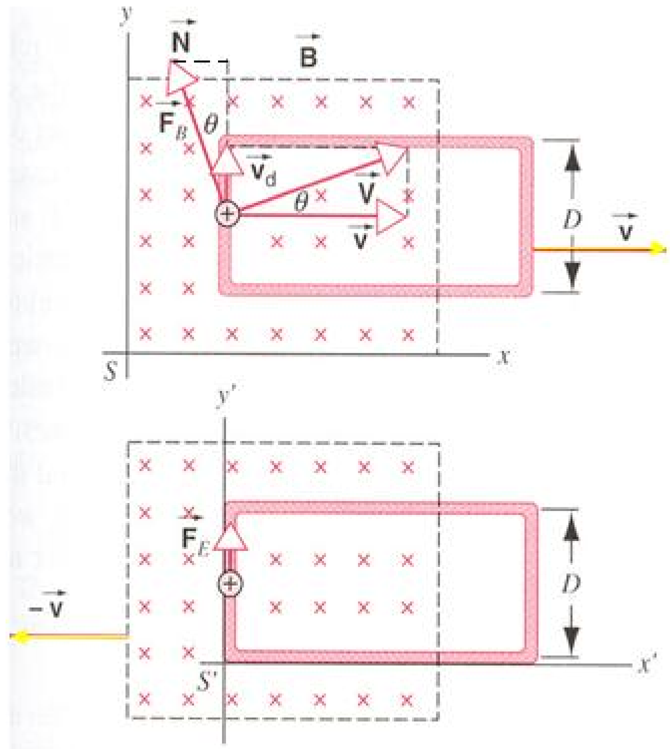
$$\varepsilon = W_N/q = BDv$$

Induction & Relative Motion

$$dW_i = -F_i dl = -F_B \cos\theta dl = -qVB(v/V)dl = -qvBdl$$
$$W_i = -qvBD = -W_N$$

- $W_N + W_i = 0$, the work by force F_L on the charge carrier is zero. It does not apply energy, but play the role of transforming energy.

- Motional *emf* is intimately connected with the sideways deflecting force by a magnetic field.



Induction & Relative Motion

➢ **The Reference *S'* fixed with the loop:**

$$\varepsilon' = \int E' dl = E' D$$

$$\because \varepsilon' = BDv$$

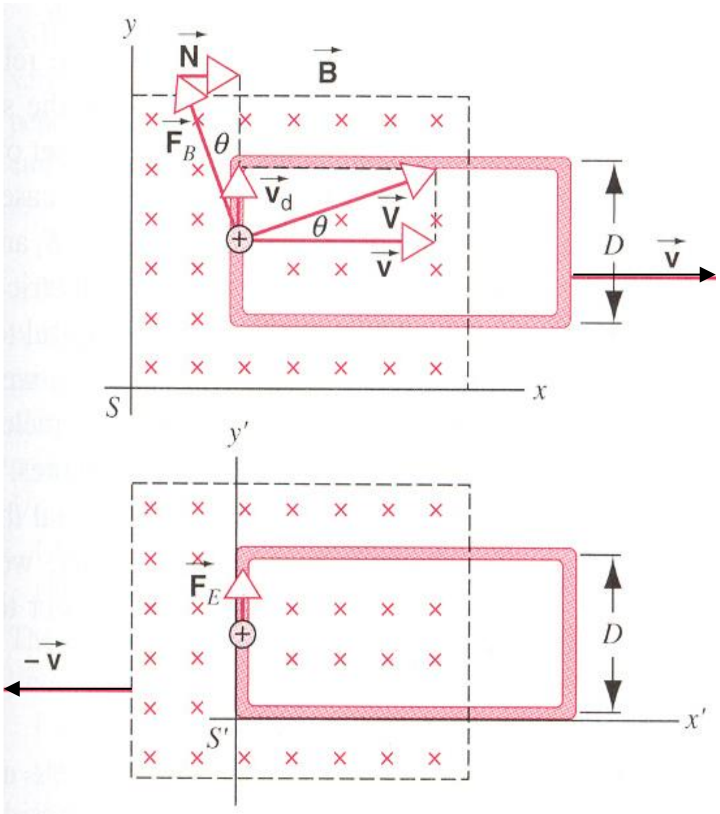
$$\vec{E}' = \vec{v} \times \vec{B}$$

Force is of purely electric origin,

$$\varepsilon = \int \vec{E}' \cdot d\vec{l}$$

In general in S":

$$\varepsilon = \int (\vec{E}' + \vec{v} \times \vec{B}) \cdot d\vec{l}$$



Induction & Relative Motion

- **Electric and magnetic fields are not independent of each other. They are in one unity—electromagnetic field (电磁场).**
- **The basic laws of electromagnetism, including Faraday's law, hold in all inertial reference frames.**
- **Maxwell's equations are invariant with respect to the Lorentz transformation (电动力学).**

$$\vec{B} = \nabla \times \vec{A} \quad \vec{A}\text{磁矢势}$$

$$\vec{E} = \nabla \varphi, \quad \varphi\text{电势}$$

$$A_u = (\vec{A}, \ \frac{i}{c}\varphi) = (A_x, A_y, A_z, \frac{i}{c}\varphi) \quad \text{四维矢量}$$

Summary

- **Faraday's Law (Lenz's Law)**
  - **a changing magnetic flux through a loop induces a current in that loop**

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$$

$$\boxed{\Phi_B \equiv \iint \vec{B} \bullet d\vec{A}}$$

negative sign indicates that the induced EMF opposes the change in flux

- **Faraday's Law in terms of Electric Fields**

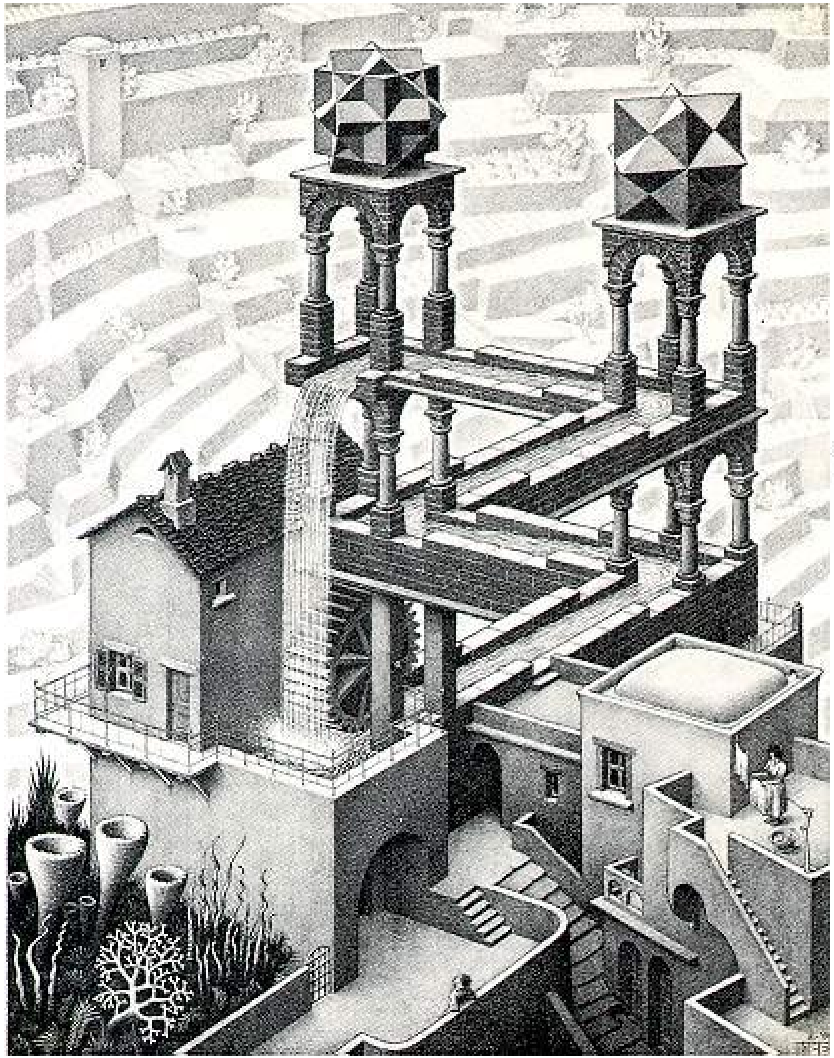
$$\boxed{\oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}}$$

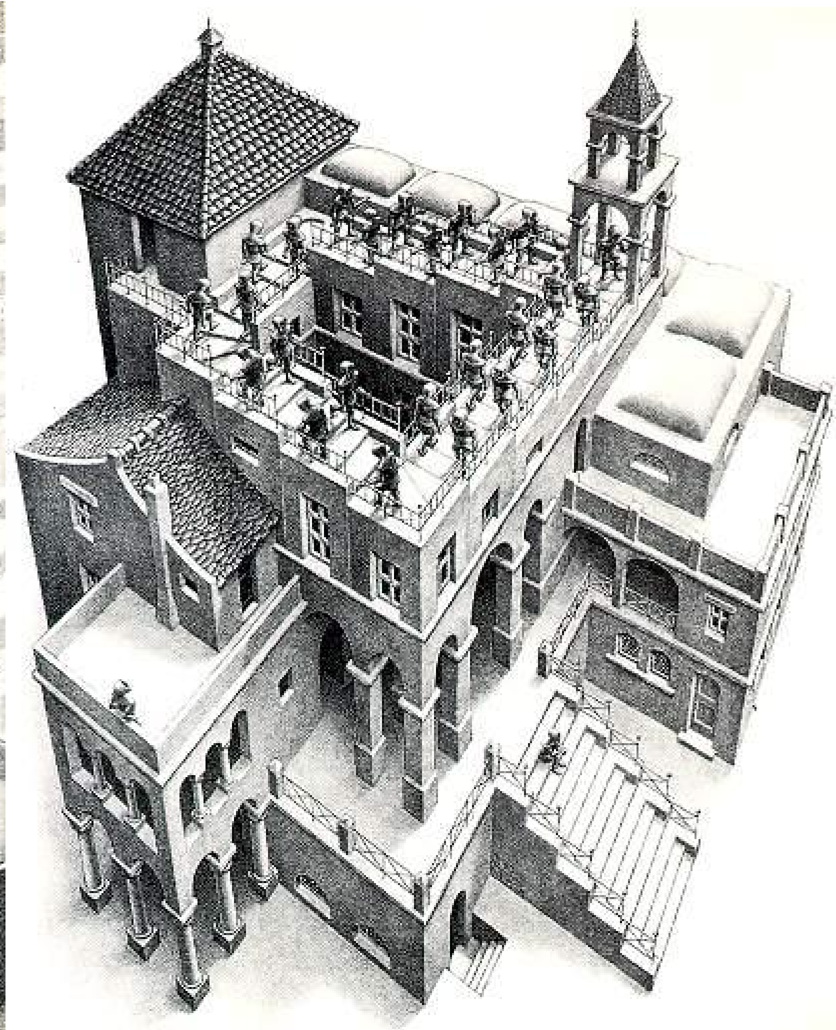
$$\Phi_B = \iint \vec{B} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Escher depiction of nonconservative *emf*





Homework

- Page 793 (questions), 26, 33
- Page 797(Problems), 6, 11,