# A new proof of claim2

We just give a new proof of claim2 in the appendix of the paper "Pratical decoy state for quantum key distribution,Phys.Rev.A72,012326(2005)".

**Claim2:** The function $F(\nu_2)$ is an increasing function,where $F(\nu_2)$ is:

$$F(\nu_2)=\frac{1}{\mu-\nu_1-\nu_2}[G(\mu)-\frac{\mu}{\mu-\nu_1-\nu_2}[G(\nu_1)-G(\nu_2)]], \tag{1}$$

**Proof.** First,we define two functions and show some useful properties of them.Then,we use these properties to prove claim2 in that paper.
Define functions $H(x)$ and $T(x)$ as

$$H(x)=\begin{cases}\dfrac{G(x)-G(\nu_2)}{x-\nu_2} & x>\nu_2\\ G^{'}(\nu_2) & x=\nu_2\end{cases} \tag{2}$$

$$T(x)=\frac{G(\frac{\mu}{2}+x)-G(\frac{\mu}{2}-x)}{2x} \quad x\geq 0 \tag{3}$$

With Taylor Series,it is easy to show that T(x) is a increasing function,since

$$T(x)=\sum_n\frac{1}{(2n-1)!}G^{(2n-1)}(\frac{\mu}{2})x^{2n-2} \tag{4}$$

So,we have

$$\frac{G(\mu)}{\mu}\geq\frac{G(\mu)-G(0)}{\mu}\geq\frac{G(\mu-\nu_2)-G(\nu_2)}{\mu-\nu_2-\nu_2}=H(\mu-\nu_2) \tag{5}$$

Take derivative of $H(x)$:

$$H^{'}(x)=\frac{G^{'}(x)-H(x)}{x-\nu_2} \tag{6}$$

According to the mean value theorem,$G^{'}(x)\geq H(x)$,so we can have:

$$H^{'}(x)\geq 0 \tag{7}$$

Take the second derivative of $H(x)$:

$$H^{''}(x)=\frac{2}{(x-\nu_2)^2}[H(x)-G^{'}(x)+\frac{1}{2}G^{''}(x)(x-\nu_2)] \tag{8}$$

With Taylor Series,since $\nu_2\leq x$,we can show that:

$$G(\nu_2)\leq G(x)+G^{'}(x)(\nu_2-x)+\frac{1}{2}G^{''}(x)(\nu_2-x)^2 \tag{9}$$

Then,we have

$$H(x)=\frac{G(x)-G(\nu_2)}{x-\nu_2}\geq G^{'}(x)-\frac{1}{2}G^{''}(x)(x-\nu_2) \tag{10}$$

So

$$H^{''}(x)\geq 0 \tag{11}$$

To determine if $F(\nu_2)$ is increasing or decreasing we will need the derivative:

$$F^{'}(\nu_2)=\frac{\mu}{\mu-\nu_1-\nu_2}(\frac{\frac{G(\mu)}{\mu}-H(\nu_1)}{\mu-\nu_1-\nu_2}-\frac{H(\nu_1)-H(\nu_2)}{\nu_1-\nu_2}) \tag{12}$$

$$\geq \frac{\mu}{\mu-\nu_1-\nu_2}(\frac{H(\mu-\nu_2)-H(\nu_1)}{\mu-\nu_1-\nu_2}-\frac{H(\nu_1)-H(\nu_2)}{\nu_1-\nu_2}) \tag{13}$$

$$\geq \frac{\mu}{\mu-\nu_1-\nu_2}(H^{'}(\nu_1)-H^{'}(\nu_1))=0 \tag{14}$$

Here,to prove the first inequality,we have made use of Eq.(5);to prove the second inequality,we have made use of Eq.(11).
In summary,we have proved the claim2.