

实验结果说明

操作系统: Ubuntu 20.04.3 LTS

处理器: Intel(R) Core(TM) i9-10900K CPU @ 3.70GHz

内存: 64GB

SAT 求解器: MiniSAT

算法思想:

数独要求每一行、列都为 1 到 9 的数字序列, 且每个九宫格也需要出现所有 1 到 9 的数字。设 i 和 j 分别表示行数和列数, 且用 n 表示第 i 行、第 j 列的数字。这里使用命题 $p(i, j, n)$ 来表示命题“第 i 行、第 j 列的数字为 n ”, 其中 $i, j, n \in [1, \dots, 9]$ 。则

- 每一行需要出现所有 1 到 9 的数字编码为:

$$\bigwedge_{i=1}^9 \bigwedge_{n=1}^9 \bigvee_{j=1}^9 p(i, j, n)$$

- 每一列需要出现所有 1 到 9 的数字编码为:

$$\bigwedge_{j=1}^9 \bigwedge_{n=1}^9 \bigvee_{i=1}^9 p(i, j, n)$$

- 每一个 3×3 的区域需要出现所有 1 到 9 的数字编码为:

$$\bigwedge_{r=0}^2 \bigwedge_{s=0}^2 \bigwedge_{n=1}^9 \bigvee_{i=1}^3 \bigvee_{j=1}^3 p(3r+i, 3s+j, n)$$

- 每一个单元都不会出现超过一个数字:

$$\bigwedge_{i=1}^9 \bigwedge_{j=1}^9 \bigwedge_{n=1}^9 \bigwedge_{n'=1}^9 (n \neq n') \rightarrow (p(i, j, n) \rightarrow \neg p(i, j, n'))$$

工程运行方式:

进入到代码根目录下, 修改 Global.cpp 文件中的 sudokupath(数独问题文件存放路径), resultpath(最后解得的数独文件存放路径), sudokucnf(MiniSAT 输入文件), resultcnf(MiniSAT 输出文件), 设置数独的大小(row, col)。然后运行 make clean, make 生成 main 可执行文件, ./main 运行可执行文件。

代码解读:

sudoku.cpp 文件中的 ToCnf() 函数读取数独, 然后转化成 MiniSAT 能读取的 CNF 格式。每个 $p(i, j, n)$ 表示第 i 行, 第 j 列, 填入数字 n , $p(i, j, n)$ 是第 $i*(row*row)+j*(row)+n$ 个变量, $\neg p(i, j, n)$ 表示 $\neg p(i, j, n)$ 。

求解结果:

Problem_1.txt

```
0 0 0 1 0 0 3 2 0
5 0 0 0 9 0 4 0 6
2 6 0 0 7 3 0 0 0
0 0 9 0 3 0 0 5 0
0 0 0 0 0 0 0 0 0
0 2 0 0 4 0 9 0 0
0 0 0 3 1 0 0 4 9
8 0 5 0 6 0 0 0 3
0 9 3 0 0 5 0 0 0
```

Result_1.txt

```
9 4 8 1 5 6 3 2 7
5 3 7 8 9 2 4 1 6
2 6 1 4 7 3 8 9 5
7 8 9 2 3 1 6 5 4
1 5 4 6 8 9 7 3 2
3 2 6 5 4 7 9 8 1
6 7 2 3 1 8 5 4 9
8 1 5 9 6 4 2 7 3
4 9 3 7 2 5 1 6 8
```

WARNING: for repeatability, setting FPU to use double precision

===== [Problem Statistics] =====

Number of variables:	729
Number of clauses:	4220
Parse time:	0.00 s
Eliminated clauses:	0.00 Mb
Simplification time:	0.00 s

===== [Search Statistics] =====

Conflicts	Vars	ORIGINAL Clauses	Literals	Limit	LEARNT Clauses	Lit/Cl	Progress
100	373	1692	5723	620	100	11	34.568 %
250	373	1692	5723	682	250	10	34.569 %
475	373	1692	5723	750	475	11	34.568 %
812	371	1685	5698	825	675	11	34.844 %
1318	370	1685	5698	908	622	12	34.980 %
2077	367	1673	5655	999	777	9	35.392 %
3216	365	1642	5585	1099	1259	10	35.667 %

```
restarts          : 17
conflicts         : 3403          (151426 /sec)
decisions        : 5957          (0.00 % random) (265074 /sec)
propagations     : 66365         (2953099 /sec)
conflict literals : 37213         (15.68 % deleted)
Memory used      : 10.63 MB
CPU time         : 0.022473 s
```

SATISFIABLE

Problem_2.txt

```
0 0 0 0 7 0 0 0 0
6 4 0 0 0 3 5 0 0
1 0 0 4 9 0 0 6 0
0 0 6 8 0 0 0 0 0
0 2 0 3 4 1 0 5 0
0 0 0 0 0 9 2 0 0
0 8 0 0 3 4 0 0 6
0 0 2 9 0 0 0 8 3
0 0 0 0 1 0 0 0 0
```

Result_2.txt

```
2 9 8 5 7 6 3 1 4
6 4 7 1 8 3 5 9 2
1 3 5 4 9 2 8 6 7
3 1 6 8 2 5 7 4 9
7 2 9 3 4 1 6 5 8
8 5 4 7 6 9 2 3 1
5 8 1 2 3 4 9 7 6
4 6 2 9 5 7 1 8 3
9 7 3 6 1 8 4 2 5
```

```
WARNING: for repeatability, setting FPU to use double precision
===== [ Problem Statistics ] =====
|
| Number of variables:      729
| Number of clauses:      4280
| Parse time:              0.00 s
| Eliminated clauses:      0.00 Mb
| Simplification time:     0.00 s
|
===== [ Search Statistics ] =====
| Conflicts | ORIGINAL | LEARNT | Progress |
|           | Vars  Clauses Literals | Limit  Clauses Lit/Cl |
|=====|=====|=====|=====|
|    100 |    391   1797   6167 |    658    100    19 | 33.334 % |
|    250 |    391   1797   6167 |    724    250    15 | 33.334 % |
|    475 |    389   1797   6167 |    797    473    13 | 33.609 % |
|    812 |    389   1784   6139 |    876    799    13 | 33.608 % |
|   1318 |    387   1784   6139 |    964    737    12 | 33.882 % |
|   2077 |    386   1765   6084 |   1061    857    12 | 34.019 % |
|   3216 |    385   1757   6058 |   1167   1315    14 | 34.157 % |
|   4924 |    385   1757   6058 |   1284    839    15 | 34.157 % |
|   7486 |    378   1738   5997 |   1412    942    14 | 35.117 % |
|  11330 |    375   1695   5887 |   1553   1135    21 | 35.528 % |
|  17096 |    365   1649   5775 |   1709   1154    13 | 36.900 % |
|  25745 |    330   1464   5241 |   1879   1838    13 | 41.701 % |
|=====|=====|=====|=====|
restarts      : 98
conflicts     : 27722      (162822 /sec)
decisions     : 46874      (0.00 % random) (275308 /sec)
propagations  : 553023     (3248109 /sec)
conflict literals : 378761  (14.90 % deleted)
Memory used   : 10.63 MB
CPU time      : 0.17026 s
```

Problem_3.txt

```
3 0 0 0 6 0 0 2 5
8 0 0 0 0 0 0 0 0
0 0 0 2 0 5 0 0 0
0 0 5 0 1 0 9 0 0
6 0 0 3 9 7 0 0 8
0 0 3 0 5 0 1 0 0
0 0 0 1 0 4 0 0 0
0 0 0 0 0 0 0 0 9
4 1 0 0 7 0 0 0 6
```

Result_3.txt

```
3 9 4 8 6 1 7 2 5
8 5 2 7 4 9 6 3 1
1 7 6 2 3 5 8 9 4
7 8 5 4 1 2 9 6 3
6 2 1 3 9 7 5 4 8
9 4 3 6 5 8 1 7 2
5 6 9 1 2 4 3 8 7
2 3 7 5 8 6 4 1 9
4 1 8 9 7 3 2 5 6
```

```
WARNING: for repeatability, setting FPU to use double precision
===== [ Problem Statistics ] =====
|
|   Number of variables:      729
|   Number of clauses:      4419
|   Parse time:              0.00 s
|   Eliminated clauses:      0.00 Mb
|   Simplification time:     0.00 s
|
===== [ Search Statistics ] =====
| Conflicts | ORIGINAL | LEARNT | Progress | | | |
|   Vars   | Clauses  | Literals | Limit  | Clauses | Lit/Cl |
|=====|=====|=====|=====|=====|=====|
|   100    |   417    |  1945    |   713    |   100    |   13    | 30.864 % |
|   250    |   417    |  1945    |   784    |   250    |   15    | 30.866 % |
|   475    |   417    |  1945    |   862    |   475    |   16    | 30.864 % |
|   812    |   416    |  1937    |   949    |   809    |   16    | 31.003 % |
|  1318    |   415    |  1937    |  1044    |   714    |   18    | 31.139 % |
|  2077    |   414    |  1924    |  1148    |   767    |   15    | 31.276 % |
|  3216    |   412    |  1924    |  1263    |  1183    |   13    | 31.552 % |
|  4924    |   408    |  1884    |  1389    |  1331    |   14    | 32.099 % |
|  7486    |   405    |  1870    |  1528    |  1327    |   14    | 32.511 % |
| 11330    |   405    |  1864    |  1681    |  1561    |   17    | 32.510 % |
| 17096    |   399    |  1822    |  1849    |  1377    |   19    | 33.334 % |
| 25745    |   394    |  1790    |  2034    |  1406    |   14    | 34.021 % |
| 38719    |   378    |  1685    |  2238    |  1410    |   17    | 36.214 % |
|=====|=====|=====|=====|=====|=====|
restarts      : 165
conflicts     : 53950      (146464 /sec)
decisions     : 88366      (0.00 % random) (239897 /sec)
propagations  : 1073653    (2914771 /sec)
conflict literals : 813530  (15.75 % deleted)
Memory used   : 10.63 MB
CPU time      : 0.368349 s
```

Problem_4.txt

```
0 0 1 0 0 0 0 3 14 0 2 0 0 15 7 0
0 10 13 0 1 0 0 4 16 0 0 7 6 12 0 5
12 14 0 11 0 6 0 0 0 0 0 0 13 2 0 3
0 7 0 0 12 11 0 16 15 0 0 13 1 0 4 0
0 6 8 0 5 12 16 15 0 3 11 10 7 0 9 4
3 0 12 0 11 0 0 0 0 2 13 15 0 0 6 0
0 15 7 16 9 0 10 0 6 0 0 5 3 1 0 2
0 0 0 1 6 0 0 2 0 9 4 0 0 0 0 0
5 2 15 0 0 0 0 10 0 0 0 12 0 11 3 1
14 3 0 4 0 2 0 0 11 13 0 6 9 0 0 0
8 13 0 0 0 0 11 0 0 1 5 0 0 4 10 0
1 0 9 0 14 0 0 12 0 16 0 0 5 0 0 8
11 0 0 0 10 0 0 0 0 8 7 1 15 16 0 12
7 12 2 0 0 0 0 8 5 0 0 11 0 0 1 6
16 0 0 0 0 1 0 11 13 0 0 9 4 3 0 0
9 1 0 0 15 0 3 0 0 4 16 0 8 0 0 11
```

Result_4.txt

```
6 16 1 8 13 10 5 3 14 12 2 4 11 15 7 9
15 10 13 9 1 14 2 4 16 11 3 7 6 12 8 5
12 14 4 11 7 6 15 9 10 5 1 8 13 2 16 3
2 7 3 5 12 11 8 16 15 6 9 13 1 10 4 14
13 6 8 2 5 12 16 15 1 3 11 10 7 14 9 4
3 9 12 14 11 4 7 1 8 2 13 15 16 5 6 10
4 15 7 16 9 8 10 13 6 14 12 5 3 1 11 2
10 5 11 1 6 3 14 2 7 9 4 16 12 8 15 13
5 2 15 6 16 13 9 10 4 7 8 12 14 11 3 1
14 3 10 4 8 2 1 5 11 13 15 6 9 7 12 16
8 13 16 12 3 7 11 6 9 1 5 14 2 4 10 15
1 11 9 7 14 15 4 12 2 16 10 3 5 6 13 8
11 4 5 13 10 9 6 14 3 8 7 1 15 16 2 12
7 12 2 3 4 16 13 8 5 15 14 11 10 9 1 6
16 8 14 15 2 1 12 11 13 10 6 9 4 3 5 7
9 1 6 10 15 5 3 7 12 4 16 2 8 13 14 11
```

WARNING: for repeatability, setting FPU to use double precision

===== [Problem Statistics] =====

Number of variables:	4096
Number of clauses:	35963
Parse time:	0.01 s
Eliminated clauses:	0.02 Mb
Simplification time:	0.02 s

===== [Search Statistics] =====

Conflicts	ORIGINAL			LEARNT			Progress
	Vars	Clauses	Literals	Limit	Clauses	Lit/Cl	
100	1475	11270	47226	4132	100	23	48.828 %
250	1475	11270	47226	4545	250	20	48.828 %
475	1475	11270	47226	5000	475	20	48.828 %
812	1475	11270	47226	5500	812	19	48.828 %
1318	1475	11270	47226	6050	1318	19	48.828 %
2077	1475	11270	47226	6655	2077	19	48.828 %
3216	1475	11270	47226	7320	3216	17	48.828 %
4924	1475	11270	47226	8052	4924	17	48.828 %
7486	1474	11258	47167	8858	2654	18	48.853 %
11330	1472	11236	47104	9743	6472	18	48.901 %
17096	1472	11236	47104	10718	6312	24	48.901 %
25745	1471	11222	47076	11790	8455	19	48.926 %
38719	1470	11210	47026	12969	13872	22	48.950 %
58180	1469	11199	46978	14265	9917	22	48.975 %
87372	1465	11151	46786	15692	15042	20	49.072 %
131161	1464	11139	46726	17261	14192	21	49.097 %
196845	1462	11116	46646	18987	11507	23	49.146 %
295371	1439	10899	45786	20886	18013	20	49.707 %
443160	1368	10165	43253	22975	15470	21	51.440 %
664843	1250	9154	39562	25272	23919	23	54.321 %

restarts : 1505
conflicts : 688249 (47428 /sec)
decisions : 931156 (0.00 % random) (64167 /sec)
propagations : 14653236 (1009771 /sec)
conflict literals : 13838756 (18.73 % deleted)
Memory used : 30.80 MB
CPU time : 14.5114 s

Problem_5.txt

```
0 16 1 8 0 0 5 3 14 12 2 4 0 15 7 0
15 0 0 0 1 14 2 4 16 11 3 0 6 12 8 0
12 14 0 0 0 0 0 9 0 0 1 8 0 2 0 0
0 7 3 0 12 0 8 16 0 6 0 0 0 10 0 14
0 0 8 0 0 12 16 15 0 3 0 0 7 14 0 4
3 9 12 14 0 4 7 1 0 2 13 0 0 5 6 10
0 15 7 16 9 8 10 13 6 14 0 0 0 1 11 2
10 5 11 1 6 3 14 2 7 9 4 16 0 8 15 13
0 0 15 0 16 13 0 10 4 0 8 0 14 0 3 1
0 3 0 4 0 0 1 0 0 13 15 0 9 0 12 16
0 0 0 0 3 7 11 6 9 1 0 14 0 4 10 0
1 0 9 0 0 0 0 12 0 0 10 0 0 0 13 8
0 0 0 13 10 9 6 0 3 0 7 1 15 16 0 12
7 0 0 3 4 0 0 8 5 15 14 11 0 9 1 6
0 8 0 15 2 1 0 0 13 10 6 0 4 0 5 7
9 1 6 10 15 0 3 7 12 0 0 0 0 0 14 0
```

Result_5.txt

```
6 16 1 8 13 10 5 3 14 12 2 4 11 15 7 9
15 10 13 9 1 14 2 4 16 11 3 7 6 12 8 5
12 14 4 11 7 6 15 9 10 5 1 8 13 2 16 3
2 7 3 5 12 11 8 16 15 6 9 13 1 10 4 14
13 2 8 6 5 12 16 15 1 3 11 10 7 14 9 4
3 9 12 14 11 4 7 1 8 2 13 15 16 5 6 10
4 15 7 16 9 8 10 13 6 14 12 5 3 1 11 2
10 5 11 1 6 3 14 2 7 9 4 16 12 8 15 13
5 11 15 2 16 13 9 10 4 7 8 12 14 6 3 1
14 3 10 4 8 2 1 5 11 13 15 6 9 7 12 16
8 13 16 12 3 7 11 6 9 1 5 14 2 4 10 15
1 6 9 7 14 15 4 12 2 16 10 3 5 11 13 8
11 4 5 13 10 9 6 14 3 8 7 1 15 16 2 12
7 12 2 3 4 16 13 8 5 15 14 11 10 9 1 6
16 8 14 15 2 1 12 11 13 10 6 9 4 3 5 7
9 1 6 10 15 5 3 7 12 4 16 2 8 13 14 11
```

WARNING: for repeatability, setting FPU to use double precision

===== [Problem Statistics] =====

Number of variables:	4096
Number of clauses:	29936
Parse time:	0.01 s
Eliminated clauses:	0.01 Mb
Simplification time:	0.02 s

===== [Search Statistics] =====

Conflicts		ORIGINAL			LEARNT			Progress
	Vars	Clauses	Literals	Limit	Clauses	Lit/Cl		
100	944	6390	24773	2343	100	14	60.938 %	
250	944	6390	24773	2577	250	14	60.938 %	
475	944	6390	24773	2835	475	14	60.938 %	
812	944	6390	24773	3118	812	15	60.938 %	
1318	944	6390	24773	3430	1318	16	60.938 %	
2077	942	6390	24773	3773	2075	14	60.986 %	
3216	941	6390	24773	4150	3213	14	61.011 %	
4924	940	6354	24650	4565	4853	13	61.035 %	
7486	940	6347	24625	5022	3844	19	61.035 %	
11330	940	6347	24625	5524	3891	24	61.035 %	
17096	939	6337	24581	6077	5436	22	61.060 %	
25745	923	6244	24142	6684	5304	17	61.450 %	
38719	914	6108	23777	7353	7416	20	61.670 %	
58180	887	5877	22720	8088	4642	17	62.329 %	

restarts : 221
conflicts : 72150 (69881 /sec)
decisions : 104557 (0.00 % random) (101269 /sec)
propagations : 1689405 (1636278 /sec)
conflict literals : 1363076 (19.99 % deleted)
Memory used : 12.37 MB
CPU time : 1.03247 s

Problem_6.txt

```
0 16 0 8 0 10 0 0 0 0 2 0 11 0 0 0
15 10 13 0 0 0 0 4 16 11 0 7 0 0 0 5
0 14 0 11 0 6 15 9 0 5 0 0 0 2 16 0
2 7 0 5 12 0 8 16 15 0 9 0 1 0 4 14
0 0 0 2 5 12 0 0 1 0 0 0 0 14 9 0
3 0 0 14 11 4 7 1 0 0 0 15 16 5 6 10
4 0 0 0 9 8 0 0 6 0 0 0 3 1 11 0
10 0 11 1 6 0 0 2 0 0 4 0 12 8 0 13
0 0 0 6 0 0 9 0 4 7 8 0 0 0 0 1
14 3 0 0 8 0 1 5 11 13 0 0 0 0 0 0
0 13 0 12 0 0 0 0 0 1 5 0 2 4 0 15
1 11 0 0 0 0 0 12 0 16 10 3 5 0 13 8
0 4 0 0 0 9 6 14 3 0 7 0 0 16 2 0
0 12 2 0 0 16 0 8 0 15 0 11 0 9 1 0
0 0 14 0 2 1 0 11 0 10 0 9 0 0 0 0
9 1 0 10 0 0 0 0 0 0 0 2 0 13 0 11
```

Result_6.txt

```
6 16 1 8 3 10 5 13 14 12 2 4 11 15 7 9
15 10 13 9 1 14 2 4 16 11 3 7 6 12 8 5
12 14 4 11 7 6 15 9 10 5 1 8 13 2 16 3
2 7 3 5 12 11 8 16 15 6 9 13 1 10 4 14
16 6 8 2 5 12 13 15 1 3 11 10 4 14 9 7
3 9 12 14 11 4 7 1 8 2 13 15 16 5 6 10
4 15 7 13 9 8 16 10 6 14 12 5 3 1 11 2
10 5 11 1 6 3 14 2 7 9 4 16 12 8 15 13
5 2 15 6 16 13 9 3 4 7 8 12 14 11 10 1
14 3 10 4 8 2 1 5 11 13 15 6 9 7 12 16
8 13 16 12 10 7 11 6 9 1 5 14 2 4 3 15
1 11 9 7 14 15 4 12 2 16 10 3 5 6 13 8
11 4 5 15 13 9 6 14 3 8 7 1 10 16 2 12
13 12 2 3 4 16 10 8 5 15 14 11 7 9 1 6
7 8 14 16 2 1 12 11 13 10 6 9 15 3 5 4
9 1 6 10 15 5 3 7 12 4 16 2 8 13 14 11
```

```

WARNING: for repeatability, setting FPU to use double precision
===== [ Problem Statistics ] =====
|
|   Number of variables:      4096
|   Number of clauses:      36599
|   Parse time:              0.01 s
|   Eliminated clauses:      0.02 Mb
|   Simplification time:     0.02 s
|
===== [ Search Statistics ] =====
| Conflicts | ORIGINAL | LEARNT | Progress |
|           | Vars  Clauses Literals | Limit  Clauses Lit/Cl |
=====
|   100   | 1534  11534  47113 | 4229   100    25 | 47.656 % |
|   250   | 1534  11534  47113 | 4652   250    25 | 47.656 % |
|   475   | 1534  11534  47113 | 5117   475    23 | 47.656 % |
|   812   | 1534  11534  47113 | 5628   812    26 | 47.656 % |
|  1318   | 1534  11534  47113 | 6191  1318    27 | 47.656 % |
|  2077   | 1534  11534  47113 | 6811  2077    26 | 47.656 % |
|  3216   | 1534  11534  47113 | 7492  3216    26 | 47.656 % |
|  4924   | 1534  11534  47113 | 8241  4924    26 | 47.656 % |
|  7486   | 1534  11534  47113 | 9065  7486    27 | 47.656 % |
| 11330   | 1534  11534  47113 | 9972  5790    29 | 47.656 % |
| 17096   | 1534  11534  47113 |10969 11556    29 | 47.656 % |
| 25745   | 1534  11534  47113 |12066 7178    20 | 47.656 % |
| 38719   | 1534  11534  47113 |13272 13022    22 | 47.656 % |
| 58180   | 1534  11534  47113 |14600 9575    20 | 47.656 % |
| 87372   | 1533  11521  47062 |16060 12765    24 | 47.681 % |
|131161   | 1532  11509  47034 |17666 11084    19 | 47.705 % |
|196845   | 1526  11440  46784 |19432 11001    24 | 47.852 % |
|295371   | 1516  11326  46401 |21376 11170    21 | 48.096 % |
|443160   | 1512  11284  46248 |23513 16666    31 | 48.193 % |
|664843   | 1484  10996  45260 |25864 12736    20 | 48.877 % |
|997368   | 1430  10473  43531 |28451 26490    34 | 50.195 % |
|1496156  | 1365   9787  41396 |31296 17018    24 | 51.782 % |
|2244338  | 1188   8027  35256 |34426 27721    27 | 56.104 % |
=====
restarts          : 4535
conflicts         : 2621356      (37546 /sec)
decisions         : 3462620      (0.00 % random) (49595 /sec)
propagations      : 58123359     (832504 /sec)
conflict literals : 60441374     (18.35 % deleted)
Memory used       : 58.22 MB
CPU time          : 69.8175 s

```

MiniSAT 求解数独问题的时间随着数独规模的扩大呈现一个递增的态势，数独规模越大，需要的变量越多，9x9 的数独是 729 个变量，16x16 的数独有 4096 个变量，9x9 数独生成的子句一般是 4000 左右的量级，16x16 的数独生成的子句是 30000 左右的量级，两种规模数独的求解时间差了 100 倍左右。