```
1
     > set.seed(1)
 2
     > #install.packages("stats")
 3
     > #install.packages("DAAG")
 4
 5
     > library(stats)
     > library(DAAG)
 7
     > input = data.frame(read.table("uscrime.txt", header = TRUE)) #read in data
 8
9
     > mydata = input[c(16, 1:15)] #reorder so that crime is the first column (for formula)
10
11
    > # Plot predictors vs. response
12
     > predictors = mydata[-1]
13
     > headers = list(
14
         "M",
15
     +
         "So",
16
         "Ed",
17
         "Po1",
         "Po2",
18
         "LF",
19
20
         "M.F",
21
     +
         "Pop",
22
         "NW",
     +
23
         "U1",
         "U2",
24
25
         "Wealth",
     +
         "Ineq",
26
     +
         "Prob",
27
     +
         "Time"
28
    +
29
     + )
30
     > par(mfrow = c(4, 4))
31
     > for (i in 1:15) {
32
         plot(predictors[, i], mydata$Crime, xlab = headers[i])
     +
33
    + }
34
     >
35
36
     > point = data.frame(
        M = 14.0,
37
38
         So = 0,
39
     +
         Ed = 10.0,
        Po1 = 12.0,
40
     +
41
        Po2 = 15.5,
42
        LF = 0.640,
     +
        M.F = 94.0,
43
     +
44
        Pop = 150,
     +
45
        NW = 1.1,
     +
46
     +
        U1 = 0.120,
47
     +
         U2 = 3.6,
48
         Wealth = 3200,
49
         Ineq = 20.1,
     +
50
         Prob = 0.04,
     +
51
     +
         Time = 39.0
52
     + )
53
     >
54
55
     > f1 = formula(mydata)
56
     > model1 = lm(f1, mydata)
57
     > summary(model1)
58
59
     Call:
60
     lm(formula = f1, data = mydata)
61
    Residuals:
62
63
                1Q Median
        Min
                               3Q
                                     Max
64
    -395.7 -98.1 -6.7 113.0 512.7
65
66
   Coefficients:
```

```
67
                  Estimate Std. Error t value Pr(>|t|)
                                       -3.68 0.00089 ***
 68
      (Intercept) -5.98e+03 1.63e+03
                            4.17e+01
 69
                  8.78e+01
                                         2.11 0.04344 *
                                       -0.03 0.97977
 70
     So
                 -3.80e+00 1.49e+02
 71
     Ed
                  1.88e+02 6.21e+01
                                        3.03 0.00486 **
 72
     Po1
                  1.93e+02 1.06e+02
                                        1.82 0.07889 .
 73
                                       -0.93 0.35883
                  -1.09e+02
                             1.17e+02
     Po2
 74
                                        -0.45
     _{
m LF}
                  -6.64e+02
                              1.47e+03
                                               0.65465
 75
                  1.74e+01
                                        0.86
     M.F
                             2.04e+01
                                               0.39900
 76
     Pop
                  -7.33e-01 1.29e+00 -0.57 0.57385
 77
     NW
                  4.20e+00 6.48e+00
                                        0.65 0.52128
 78
     U1
                 -5.83e+03 4.21e+03
                                       -1.38 0.17624
 79
                  1.68e+02 8.23e+01
                                         2.04 0.05016 .
     112
 80
     Wealth
                   9.62e-02
                             1.04e-01
                                        0.93 0.36075
 81
                  7.07e+01
                              2.27e+01
                                         3.11 0.00398 **
     Ineq
 82
                  -4.86e+03
                             2.27e+03
                                        -2.14 0.04063 *
     Prob
 83
     Time
                  -3.48e+00
                            7.17e+00
                                       -0.49 0.63071
 84
      Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
 85
 86
 87
     Residual standard error: 209 on 31 degrees of freedom
 88
      Multiple R-squared: 0.803, Adjusted R-squared: 0.708
 89
     F-statistic: 8.43 on 15 and 31 DF, p-value: 3.54e-07
 90
 91
      > coef1 = model1$coefficients
 92
 93
     > par(mfrow = c(2, 2))
 94
     > plot(model1)
 95
 96
      > crime_prediction = predict.lm(model1, point)
 97
      > crime_prediction
 98
 99
      155
      > #that answer of 155 is really low, we are probably overfit, so let's look at the
100
     p-values of each point.
101
102
103
     > Pvalues = summary(model1)$coefficients[, 4]
104
    > coef = model1$coefficients
105
106
107
      > # Eliminate those predictors with a p-value > 0.08. I know 0.05 is usually the rule,
108
      > # but the U2 factor (unemployment rate of urban males 35-39) and Pol
      > # should be left in as it was considered important by the summary() function.
109
110
      > mydata_fit = mydata[1]
111
      > n = 2
     > for (i in 2:16) {
112
113
          if (Pvalues[i] < 0.08) {
114
           mydata_fit[n] = mydata[i]
115
      +
           n = n + 1
116
          }
      +
     + }
117
118
119
      > f2 = formula(mydata_fit)
120
      > model2 = lm(f2, mydata_fit)
      > summary(model2)
121
122
123
124
      lm(formula = f2, data = mydata_fit)
125
126
     Residuals:
127
         Min
                10 Median
                              30
                                    Max
128
      -470.7 \quad -78.4 \quad -19.7 \quad 133.1 \quad 556.2
129
130
      Coefficients:
131
                  Estimate Std. Error t value Pr(>|t|)
```

```
132
     (Intercept) -5040.5
                             899.8
                                     -5.60 1.7e-06 ***
                                            0.0031 **
133
                   105.0
                              33.3
                                       3.15
134
     Ed
                   196.5
                               44.8
                                       4.39 8.1e-05 ***
                                       8.36 2.6e-10 ***
135
     Po1
                   115.0
                               13.8
136
                    89.4
                               40.9
     U2
                                    2.18
                                            0.0348 *
137
                     67.7
                               13.9
                                       4.85 1.9e-05 ***
     Ineq
138
     Prob
                 -3801.8
                             1528.1 -2.49
                                            0.0171 *
139
140
     Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
141
142
     Residual standard error: 201 on 40 degrees of freedom
     Multiple R-squared: 0.766, Adjusted R-squared: 0.731
143
144
     F-statistic: 21.8 on 6 and 40 DF, p-value: 3.42e-11
145
146
     > plot(model2)
147
     >
148
149
     > crime_prediction_adj = predict.lm(model2, point)
150
151
     > crime_prediction_adj
152
        1
153
     1304
154
155
     > #Try cross-validation as well:
156
     > par(mfrow = c(1, 1))
157
     > c1 = cv.lm(mydata, model1, m = 5)
158
     Analysis of Variance Table
159
160
    Response: Crime
161
               Df Sum Sq Mean Sq F value Pr(>F)
162
                   55084 55084
                                   1.26 0.2702
     Μ
                1
163
                1
                   15370
                           15370
                                   0.35 0.5575
     So
164
     Ed
                1
                  905668 905668
                                   20.72 7.7e-05 ***
165
               1 3076033 3076033 70.38 1.8e-09 ***
     Po1
166
     Po2
               1 153024 153024
                                   3.50 0.0708 .
167
                   61134
                                   1.40 0.2459
     _{
m LF}
               1
                          61134
               1 111000 111000
168
                                   2.54 0.1212
     M.F
169
     Pop
                1
                   42649
                           42649
                                   0.98 0.3309
170 NW
               1 14197 14197
                                    0.32 0.5728
171
               1
                   7065
                           7065 0.16 0.6904
     U1
172
               1 269663 269663
                                  6.17 0.0186 *
    U2
173
     Wealth
               1
                   34748
                          34748
                                  0.79 0.3795
174
                1 547423 547423
                                   12.52 0.0013 **
     Ineq
175
     Prob
                1 222620
                                   5.09
                          222620
                                         0.0312 *
176
     Time
                1
                                    0.24 0.6307
                  10304 10304
177
     Residuals 31 1354946 43708
178
179
     Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
180
181
182
183
     fold 1
184
     Observations in test set: 9
185
                      4
                              9 18
                                           20
                                               23
                          8
186
                 755 1791 1362 689 844 1227.84 958 807.8 992
     Predicted
187
                 658 1690 1300 617 792 1220.22 814 804.9 1077
     cvpred
                 791 1969 1555 856 929 1225.00 1216 754.0 849
188
     Crime
189
     CV residual 133 279 255 239 137
                                        4.78 402 -50.9 -228
190
191
     Sum of squares = 453204
                              Mean square = 50356
                                                    n = 9
192
193
     fold 2
194
     Observations in test set: 10
195
                      13
                           15 17
                                     25
                                           34
                                                39
                                                       40
                                                             42
                           903 393 606 971.5 839.3 1131.5 326.3 827
196
     Predicted
                 1167 733
197
     cvpred
                 1132 926 977 152 740 902.7 918.1 1248.5 62.3 1004
```

```
1234 511 798 539 523 923.0 826.0 1151.0 542.0 508
198
199
     CV residual 102 -415 -179 387 -217 20.3 -92.1 -97.5 479.7 -496
200
201
     Sum of squares = 906384 Mean square = 90638
                                                      n = 10
202
203
     fold 3
204
     Observations in test set: 10
205
                              11
                                             22
                                                        31
                      2
                          3
                                   14
                                       16
                                                  28
206
                 1473.7 322 1161
                                  780 1006 657 1258 388.0
                                                            841 562.693
     Predicted
207
                 1566.9 313 953 782 1129 876 1368 321.7 700 566.231
     cvpred
208
     Crime
                 1635.0 578 1674 664 946 439 1216 373.0 1072 566.000
209
     CV residual
                   68.1 265 721 -118 -183 -437 -152 51.3 372 -0.231
210
211
     Sum of squares = 997216 Mean square = 99722
                                                      n = 10
212
213
     fold 4
214
     Observations in test set: 9
215
                                   27
                   19
                         21
                              26
                                        29
                                              30
                                                     36
                 1146 774.9 1977 279 1287 702.7 1137.6 1121
216
                                                             617
     Predicted
                 1529 802.3 1673 467 1673 629.6 1191.9 1298
217
     cvpred
                  750 742.0 1993 342 1043 696.0 1272.0 1030 455
218
     Crime
219
     CV residual -779 -60.3 320 -125 -630 66.4 80.1 -268 -247
220
221
     Sum of squares = 1269688
                                 Mean square = 141076
222
223
     fold 5
224
     Observations in test set: 9
225
                    6
                          7
                               10
                                     12 24
                                               35
                                                    37 41
                  793 934.2 736.5 722.0 869 737.8 971 824 1134
226
     Predicted
                  819 950.9 758.1 772.5 802 690.5 1227 891 1267
227
     cvpred
                  682 963.0 705.0 849.0 968 653.0 831 880 823
228
     Crime
229
     CV residual -137 12.1 -53.1 76.5 166 -37.5 -396 -11 -444
230
231
     Sum of squares = 410109
                              Mean square = 45568
232
233
     Overall (Sum over all 9 folds)
234
        ms
235
     85885
236
     Warning message:
237
     In cv.lm(mydata, model1, m = 5):
238
239
      As there is >1 explanatory variable, cross-validation
240
      predicted values for a fold are not a linear function
241
      of corresponding overall predicted values. Lines that
242
      are shown for the different folds are approximate
243
244
     > c2 = cv.lm(mydata, model2, m = 5)
245
     Analysis of Variance Table
246
247
     Response: Crime
248
               Df Sum Sq Mean Sq F value Pr(>F)
249
                   55084
                           55084
                1
                                    1.37 0.24914
250
                   725967 725967 18.02 0.00013 ***
     Ed
                1
251
                1 3173852 3173852
                                    78.80 5.3e-11 ***
     Po1
252
                           217386
     U2
                1
                   217386
                                    5.40 0.02534 *
253
                1
                   848273
                           848273
                                    21.06 4.3e-05 ***
     Ineq
254
     Prob
                1
                  249308 249308
                                    6.19 0.01711 *
255
     Residuals 40 1611057
                           40276
256
257
     Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
258
259
260
261
     fold 1
262
     Observations in test set: 9
263
                     1
                          4
                               8
                                   9 18
                                             20
                                                  23
                                                        32
                                                             47
```

```
Predicted 810.8 1897 1354 719 800 1203.0 938 773.7
264
                 762.1 1858 1282 657 672 1210.8 871 777.6 998
265
     cvpred
266
     Crime
                 791.0 1969 1555 856 929 1225.0 1216 754.0
267
     CV residual 28.9 111 273 199 257 14.2 345 -23.6 -149
268
269
     Sum of squares = 335463 Mean square = 37274
270
271
     fold 2
272
     Observations in test set: 10
273
                               15
                                          25
                    5
                       13
                                     17
                                              34
                                                    39
                                                         40 42
274
     Predicted
                 1270 739 828.34 527.4 579 998 786.7 1141 369
                                                                 748
275
     cvpred
                 1337 842 804.73 469.3 671 1032 810.3 1187 302 839
276
     Crime
                 1234 511 798.00 539.0 523 923 826.0 1151 542 508
277
     CV residual -103 -331 -6.73 69.7 -148 -109 15.7 -36 240 -331
278
279
     Sum of squares = 327423 Mean square = 32742 n = 10
280
281
     fold 3
282
     Observations in test set: 10
283
                    2 3 11
                                  14
                                      16
                                             22
                                                     28
                                                          31
                 1388 386 1118 713.6 1004.4 728 1259.0 440.4 874 544.4
284
     Predicted
                 1368 390 1019 711.8 985.8 767 1252.6 423.8 850 511.2
285
     cvpred
286
     Crime
                 1635 578 1674 664.0 946.0 439 1216.0 373.0 1072 566.0
287
     CV residual 267 188 655 -47.8 -39.8 -328 -36.6 -50.8
                                                             222 54.8
288
289
     Sum of squares = 702726
                               Mean square = 70273
290
291
     fold 4
292
     Observations in test set: 9
293
                   19
                         21
                                26
                                       27
                                            29
                                                  30
                                                      36
                 1221 783.3 1789.1 312.20 1495 668.0 1102 1178 622
294
     Predicted
                 1316 836.4 1895.7 334.15 1693 631.2 1163 1191
295
     cvpred
296
     Crime
                  750 742.0 1993.0 342.00 1043 696.0 1272 1030 455
297
     CV residual -566 -94.4 97.3 7.85 -650 64.8 109 -161 -157
298
299
     Sum of squares = 827924 Mean square = 91992
                                                     n = 9
300
301
     fold 5
     Observations in test set: 9
302
303
                           10 12
                                      24
                                         35
                                                37
                                                     41
                                                          43
                   6
                     7
304
     Predicted
                 730 733 787.3 673 919.4 808 992 796.4 1017
305
     cvpred
                 707 694 776.8 660 879.7 777 1115 812.6 1091
306
                 682 963 705.0 849 968.0 653 831 880.0 823
     Crime
307
     CV residual -25 269 -71.8 189 88.3 -124 -284 67.4 -268
308
309
     Sum of squares = 294201 Mean square = 32689 n = 9
310
311
     Overall (Sum over all 9 folds)
312
        ms
313
     52931
314
     Warning message:
315
     In cv.lm(mydata, model2, m = 5) :
316
317
      As there is >1 explanatory variable, cross-validation
318
      predicted values for a fold are not a linear function
319
      of corresponding overall predicted values. Lines that
320
      are shown for the different folds are approximate
321
322
323
     > # Now compare the models using the R^2 values. From the summaries printed earlier
324
     > # we know Model 1's R2 was 0.803 and Model 2's R2 was .766.
325
326
     > SStot = sum((mydata$Crime - mean(mydata$Crime)) ^ 2)
327
     > SSc1 = attr(c1, "ms") * nrow(mydata)
328
     > SSc2 = attr(c2, "ms") * nrow(mydata)
329
```

```
330
331
     > R2_cvm1 = 1 - SSc1 / SStot
    > R2_{cvm2} = 1 - SSc2 / SStot
332
333
    > R2_cvm1
334
    [1] 0.413
335
     > R2_cvm2
336
     [1] 0.638
337
     > # So we see that the first model was overfit to the data. While the R2 of
338
    > # model 1 was initially higher than model 2 using all of the data, by using
339
     > # 5 fold cross validation we see that model 2 has a better fit, though it is
340
     > # still probably over-fit as we only have a small set of data. As expected, the R2
341
> # of model 2 using cross validation is lower than that of the whole data set.
```