Definition 1 For any ordinal number α, the Hardy hierarchy $H_\alpha(n)$ is defined as the following function:

1. For any natural number n, $H_0(n) = n$.
2. For any ordinal number α and natural number n, $H_{\alpha+1}(n) = H_\alpha(n+1)$.
3. For any limit ordinal α and natural number n, $H_\alpha(n) = H_{\alpha[n]}(n)$, where $\alpha[n]$ is its fundamental sequence.

Definition 2 For any ordinal number α, the medium growth hierarchy $m_\alpha(n)$ is defined as the following function:

1. For any natural number n, $m_0(n) = n + 1$.
2. For any ordinal number α and natural number n, $m_{\alpha+1}(n) = m_\alpha(m_\alpha(n))$.
3. For any limit ordinal α and natural number n, $m_\alpha(n) = m_{\alpha[n]}(n)$, where $\alpha[n]$ is its fundamental sequence.

Definition 3 For any ordinal number α, the slow growth hierarchy $g_\alpha(n)$ is defined as the following function:

1. For any natural number n, $g_0(n) = 0$.
2. For any ordinal number α and natural number n, $g_{\alpha+1}(n) = g_\alpha(n) + 1$.
3. For any limit ordinal α and natural number n, $g_\alpha(n) = g_{\alpha[n]}(n)$, where $\alpha[n]$ is its fundamental sequence.