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**xtreg** — Fixed-, between-, and random-effects and population-averaged linear models

Quick start

Description Syntax Options for FE model Remarks and examples Acknowledgments

Options for RE model Options for MLE model Stored results References Menu Options for BE model Options for PA model Methods and formulas Also see

# **Description**

xtreg fits regression models to panel data. In particular, xtreg with the be option fits random-effects models by using the between regression estimator; with the fe option, it fits fixed-effects models (by using the within regression estimator); and with the re option, it fits random-effects models by using the GLS estimator (producing a matrix-weighted average of the between and within results). See [XT] xtdata for a faster way to fit fixed- and random-effects models.

## **Quick start**

Random-effects linear regression by GLS of y on x1 and xt2 using xtset data xtreg y x1 x2

As above, but estimate by maximum likelihood

xtreg y x1 x2, mle

Fixed-effects model with cluster-robust standard errors for panels nested within cvar xtreg y x1 x2, fe vce(cluster cvar)

Population-averaged model with an exchangeable within-panel correlation structure xtreg y x1 x2, pa

As above, but specify an autoregressive correlation structure of order 1 xtreg y x1 x2, pa corr(ar 1)

Between-effects model

xtreg y x1 x2, be

#### Menu

Statistics > Longitudinal/panel data > Linear models > Linear regression (FE, RE, PA, BE)

# **Syntax**

```
GLS random-effects (RE) model
    xtreg depvar [indepvars] [if] [in] [, re RE_options]
 Between-effects (BE) model
    xtreg depvar [indepvars] [if] [in], be [BE_options]
 Fixed-effects (FE) model
    xtreg depvar [indepvars] [if] [in] [weight], fe [FE_options]
 ML random-effects (MLE) model
    xtreg depvar [indepvars] [if] [in] [weight], mle [MLE_options]
 Population-averaged (PA) model
    xtreg depvar [indepvars] [if] [in] [weight], pa [PA_options]
 RE_options
                     Description
Model
                     use random-effects estimator: the default
 re
                     use Swamy-Arora estimator of the variance components
 sa
SE/Robust
 vce(vcetype)
                     vcetype may be conventional, robust, cluster clustvar, bootstrap, or
                        jackknife
Reporting
 level(#)
                     set confidence level; default is level (95)
 theta
                     report \theta
 display_options
                     control columns and column formats, row spacing, line width,
                        display of omitted variables and base and empty cells, and
                       factor-variable labeling
 coeflegend
                     display legend instead of statistics
```

BE_options	Description
Model	
be	use between-effects estimator
$\underline{\mathtt{w}}\mathtt{ls}$	use weighted least squares
SE	
vce(vcetype)	vcetype may be conventional, bootstrap, or jackknife
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<u>coefl</u> egend	display legend instead of statistics
FE_options	Description
Model	
fe	use fixed-effects estimator
SE/Robust	
vce( <i>vcetype</i> )	vcetype may be conventional, robust, cluster clustvar, bootstrap, or jackknife
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<u>coefl</u> egend	display legend instead of statistics
MLE_options	Description
Model	
$\underline{\mathtt{nocons}}\mathtt{tant}$	suppress constant term
mle	use ML random-effects estimator
SE/Robust	
vce( <i>vcetype</i> )	vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
maximize_options	control the maximization process; seldom used
<u>coefl</u> egend	display legend instead of statistics

#### 4 xtreg — Fixed-, between-, and random-effects and population-averaged linear models

PA_options	Description
Model	
<u>nocons</u> tant	suppress constant term
pa	use population-averaged estimator
offset(varname)	include varname in model with coefficient constrained to 1
Correlation	
$\underline{c}$ orr( $correlation$ )	within-panel correlation structure
force	estimate even if observations unequally spaced in time
SE/Robust	
vce( <i>vcetype</i> )	vcetype may be conventional, <u>r</u> obust, <u>boot</u> strap, or jackknife
nmp	use divisor $N-P$ instead of the default $N$
rgf	multiply the robust variance estimate by $(N-1)/(N-P)$
<u>s</u> cale(parm)	overrides the default scale parameter; parm may be x2, dev, phi, or #
Reporting	
$\underline{1}$ evel(#)	set confidence level; default is level(95)
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
optimize_options	control the optimization process; seldom used
<u>coefl</u> egend	display legend instead of statistics
correlation	Description
<u>exc</u> hangeable	exchangeable
$\underline{\mathtt{ind}}\mathtt{ependent}$	independent
$\underline{\mathtt{uns}}\mathtt{tructured}$	unstructured
<u>fix</u> ed matname	user-specified
ar#	autoregressive of order #
$\underline{\mathtt{sta}}\mathtt{tionary}~\#$	stationary of order #
<pre>nonstationary #</pre>	nonstationary of order #

A panel variable must be specified. For xtreg, pa, correlation structures other than exchangeable and independent require that a time variable also be specified. Use xtset; see [XT] xtset.

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayes, by, collect, mi estimate, and statsby are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: xtreg. fp is allowed for the between-effects, fixed-effects, and maximum-likelihood random-effects models.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

aweights, fweights, and pweights are allowed for the fixed-effects model. iweights, fweights, and pweights are allowed for the population-averaged model. iweights are allowed for the maximum-likelihood random-effects (MLE) model. See [U] 11.1.6 weight. Weights must be constant within panel.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

# **Options for RE model**

Model

re, the default, requests the GLS random-effects estimator.

sa specifies that the small-sample Swamy-Arora estimator individual-level variance component be used instead of the default consistent estimator. See xtreg, re in Methods and formulas for details.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Specifying vce(robust) is equivalent to specifying vce(cluster panelvar); see xtreg, re in Methods and formulas.

Reporting

level(#); see [R] Estimation options.

theta specifies that the output include the estimated value of  $\theta$  used in combining the between and fixed estimators. For balanced data, this is a constant, and for unbalanced data, a summary of the values is presented in the header of the output.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
 allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt),
 sformat(%fmt), and nolstretch; see [R] Estimation options.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

# Options for BE model

Model

be requests the between regression estimator.

wls specifies that, for unbalanced data, weighted least squares be used rather than the default OLS. Both methods produce consistent estimates. The true variance of the between-effects residual is  $\sigma_{\nu}^2 + T_i \sigma_{\epsilon}^2$  (see xtreg, be in Methods and formulas below). WLS produces a "stabilized" variance of  $\sigma_{\nu}^2/T_i + \sigma_{\epsilon}^2$ , which is also not constant. Thus the choice between OLS and WLS amounts to which is more stable.

Comment: xtreg, be is rarely used anyway, but between estimates are an ingredient in the random-effects estimate. Our implementation of xtreg, re uses the OLS estimates for this ingredient, based on our judgment that  $\sigma^2_{\nu}$  is large relative to  $\sigma^2_{\epsilon}$  in most models. Formally, only a consistent estimate of the between estimates is required.

SE

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

\_\_\_\_\_ Reporting

level(#); see [R] Estimation options.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

# Options for FE model

\_\_ Model

fe requests the fixed-effects (within) regression estimator.

SE/Robust L

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from
asymptotic theory (conventional), that are robust to some kinds of misspecification (robust),
that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods
(bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Specifying vce(robust) is equivalent to specifying vce(cluster panelvar); see xtreg, fe in Methods and formulas.

Reporting

level(#); see [R] Estimation options.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
 allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt),
 sformat(%fmt), and nolstretch; see [R] Estimation options.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

# Options for MLE model

Model noconstant; see [R] Estimation options.

mle requests the maximum-likelihood random-effects estimator.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

Reporting

level(#); see [R] Estimation options.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize\_options: iterate(#), [no]log, trace, tolerance(#), ltolerance(#), and from(init\_specs); see [R] Maximize. These options are seldom used.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

# Options for PA model

Model

noconstant; see [R] Estimation options.

pa requests the population-averaged estimator. For linear regression, this is the same as a random-effects estimator (both interpretations hold).

xtreg, pa is equivalent to xtgee, family (gaussian) link (id) corr(exchangeable), which are the defaults for the xtgee command. xtreg, pa allows all the relevant xtgee options such as vce(robust). Whether you use xtreg, pa or xtgee makes no difference. See [XT] xtgee.

offset(varname); see [R] Estimation options.

Correlation

corr(correlation) specifies the within-panel correlation structure; the default corresponds to the equal-correlation model, corr(exchangeable).

When you specify a correlation structure that requires a lag, you indicate the lag after the structure's name with or without a blank; for example, corr(ar 1) or corr(ar1).

If you specify the fixed correlation structure, you specify the name of the matrix containing the assumed correlations following the word fixed, for example, corr(fixed myr).

force specifies that estimation be forced even though the time variable is not equally spaced. This is relevant only for correlation structures that require knowledge of the time variable. These correlation structures require that observations be equally spaced so that calculations based on lags correspond to a constant time change. If you specify a time variable indicating that observations are not equally spaced, the (time dependent) model will not be fit. If you also specify force, the model will be fit, and it will be assumed that the lags based on the data ordered by the time variable are appropriate.

```
SE/Robust
```

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

nmp; see [XT] vce\_options.

rgf specifies that the robust variance estimate is multiplied by (N-1)/(N-P), where N is the total number of observations and P is the number of coefficients estimated. This option can be used with family(gaussian) only when vce(robust) is either specified or implied by the use of pweights. Using this option implies that the robust variance estimate is not invariant to the scale of any weights used.

scale(x2|dev|phi|#); see [XT] vce\_options.

```
Reporting
```

level(#); see [R] Estimation options.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
 allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt),
 sformat(%fmt), and nolstretch; see [R] Estimation options.

```
Optimization
```

optimize\_options control the iterative optimization process. These options are seldom used.

<u>iter</u>ate(#) specifies the maximum number of iterations. When the number of iterations equals #, the optimization stops and presents the current results, even if convergence has not been reached. The default is iterate(100).

<u>tolerance(#)</u> specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to #, the optimization process is stopped. tolerance(1e-6) is the default.

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

 $\underline{\mathtt{tr}}$  ace specifies that the current estimates be printed at each iteration.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

# Remarks and examples

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If you have not read [XT] xt, please do so.

See Baltagi (2013, chap. 2) and Wooldridge (2020, chap. 14) for good overviews of fixed-effects and random-effects models. Allison (2009) provides perspective on the use of fixed-versus random-effects estimators and provides many examples using Stata.

Consider fitting models of the form

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it} \tag{1}$$

In this model,  $\nu_i + \epsilon_{it}$  is the error term that we have little interest in; we want estimates of  $\beta$ .  $\nu_i$  is the unit-specific error term; it differs between units, but for any particular unit, its value is constant. In the pulmonary data of [XT] xt, a person who exercises less would presumably have a lower forced expiratory volume (FEV) year after year and so would have a negative  $\nu_i$ .

 $\epsilon_{it}$  is the "usual" error term with the usual properties (mean 0, uncorrelated with itself, uncorrelated with x, uncorrelated with  $\nu$ , and homoskedastic), although in a more thorough development, we could decompose  $\epsilon_{it} = \upsilon_t + \omega_{it}$ , assume that  $\omega_{it}$  is a conventional error term, and better describe  $\upsilon_t$ .

Before making the assumptions necessary for estimation, let's perform some useful algebra on (1). Whatever the properties of  $\nu_i$  and  $\epsilon_{it}$ , if (1) is true, it must also be true that

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta} + \nu_i + \overline{\epsilon}_i \tag{2}$$

where  $\overline{y}_i = \sum_t y_{it}/T_i$ ,  $\overline{\mathbf{x}}_i = \sum_t \mathbf{x}_{it}/T_i$ , and  $\overline{\epsilon}_i = \sum_t \epsilon_{it}/T_i$ . Subtracting (2) from (1), it must be equally true that

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\boldsymbol{\beta} + (\epsilon_{it} - \overline{\epsilon}_i)$$
(3)

These three equations provide the basis for estimating  $\beta$ . In particular, xtreg, fe provides what is known as the fixed-effects estimator—also known as the within estimator—and amounts to using OLS to perform the estimation of (3). xtreg, be provides what is known as the between estimator and amounts to using OLS to perform the estimation of (2). xtreg, re provides the random-effects estimator and is a (matrix) weighted average of the estimates produced by the between and within estimators. In particular, the random-effects estimator turns out to be equivalent to estimation of

$$(y_{it} - \theta \overline{y}_i) = (1 - \theta)\alpha + (\mathbf{x}_{it} - \theta \overline{\mathbf{x}}_i)\beta + \{(1 - \theta)\nu_i + (\epsilon_{it} - \theta \overline{\epsilon}_i)\}$$
(4)

where  $\theta$  is a function of  $\sigma_{\nu}^2$  and  $\sigma_{\epsilon}^2$ . If  $\sigma_{\nu}^2=0$ , meaning that  $\nu_i$  is always 0,  $\theta=0$  and (1) can be estimated by OLS directly. Alternatively, if  $\sigma_{\epsilon}^2=0$ , meaning that  $\epsilon_{it}$  is  $0,\,\theta=1$  and the within estimator returns all the information available (which will, in fact, be a regression with an  $R^2$  of 1).

For more reasonable cases, few assumptions are required to justify the fixed-effects estimator of (3). The estimates are, however, conditional on the sample in that the  $\nu_i$  are not assumed to have a distribution but are instead treated as fixed and estimable. This statistical fine point can lead to difficulty when making out-of-sample predictions, but that aside, the fixed-effects estimator has much to recommend it.

More is required to justify the between estimator of (2), but the conditioning on the sample is not assumed because  $\nu_i + \bar{\epsilon}_i$  is treated as an error term. Newly required is that we assume that  $\nu_i$  and  $\bar{\mathbf{x}}_i$ are uncorrelated. This follows from the assumptions of the OLS estimator but is also transparent: were  $\nu_i$  and  $\overline{\mathbf{x}}_i$  correlated, the estimator could not determine how much of the change in  $\overline{y}_i$ , associated with an increase in  $\bar{\mathbf{x}}_i$ , to assign to  $\boldsymbol{\beta}$  versus how much to attribute to the unknown correlation. (This, of course, suggests the use of an instrumental-variable estimator,  $\overline{z}_i$ , which is correlated with  $\overline{x}_i$  but uncorrelated with  $\nu_i$ , though that approach is not implemented here.)

The random-effects estimator of (4) requires the same no-correlation assumption. In comparison with the between estimator, the random-effects estimator produces more efficient results, albeit ones with unknown small-sample properties. The between estimator is less efficient because it discards the over-time information in the data in favor of simple means; the random-effects estimator uses both the within and the between information.

All of this would seem to leave the between estimator of (2) with no role (except for a minor, technical part it plays in helping to estimate  $\sigma_{\nu}^2$  and  $\sigma_{\epsilon}^2$ , which are used in the calculation of  $\theta$ , on which the random-effects estimates depend). Let's, however, consider a variation on (1):

$$y_{it} = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta}_1 + (\mathbf{x}_{it} - \overline{\mathbf{x}}_i) \boldsymbol{\beta}_2 + \nu_i + \epsilon_{it}$$
 (1')

In this model, we postulate that changes in the average value of  $\mathbf{x}$  for an individual have a different effect from temporary departures from the average. In an economic situation, y might be purchases of some item and  $\mathbf{x}$  income; a change in average income should have more effect than a transitory change. In a clinical situation, y might be a physical response and  $\mathbf{x}$  the level of a chemical in the brain; the model allows a different response to permanent rather than transitory changes.

The variations of (2) and (3) corresponding to (1') are

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta}_1 + \nu_i + \overline{\epsilon}_i \tag{2'}$$

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\boldsymbol{\beta}_2 + (\epsilon_{it} - \overline{\epsilon}_i) \tag{3'}$$

That is, the between estimator estimates  $\beta_1$  and the within  $\beta_2$ , and neither estimates the other. Thus even when estimating equations like (1), it is worth comparing the within and between estimators. Differences in results can suggest models like (1'), or at the least some other specification error.

Finally, it is worth understanding the role of the between and within estimators with regressors that are constant over time or constant over units. Consider the model

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta}_1 + \mathbf{s}_i\boldsymbol{\beta}_2 + \mathbf{z}_t\boldsymbol{\beta}_3 + \nu_i + \epsilon_{it}$$
 (1")

This model is the same as (1), except that we explicitly identify the variables that vary over both time and i ( $\mathbf{x}_{it}$ , such as output or FEV); variables that are constant over time ( $\mathbf{s}_i$ , such as race or sex); and variables that vary solely over time ( $\mathbf{z}_t$ , such as the consumer price index or age in a cohort study). The corresponding between and within equations are

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta}_1 + \mathbf{s}_i \boldsymbol{\beta}_2 + \overline{\mathbf{z}} \boldsymbol{\beta}_3 + \nu_i + \overline{\epsilon}_i \tag{2''}$$

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\boldsymbol{\beta}_1 + (\mathbf{z}_t - \overline{\mathbf{z}})\boldsymbol{\beta}_3 + (\epsilon_{it} - \overline{\epsilon}_i)$$

$$(3'')$$

In the between estimator of (2''), no estimate of  $\beta_3$  is possible because  $\overline{\mathbf{z}}$  is a constant across the i observations; the regression-estimated intercept will be an estimate of  $\alpha + \overline{\mathbf{z}}\beta_3$ . On the other hand, it can provide estimates of  $\beta_1$  and  $\beta_2$ . It can estimate effects of factors that are constant over time, such as race and sex, but to do so it must assume that  $\nu_i$  is uncorrelated with those factors.

The within estimator of (3''), like the between estimator, provides an estimate of  $\beta_1$  but provides no estimate of  $\beta_2$  for time-invariant factors. Instead, it provides an estimate of  $\beta_3$ , the effects of the time-varying factors. The within estimator can also provide estimates  $u_i$  for  $\nu_i$ . More correctly, the estimator  $u_i$  is an estimator of  $\nu_i + \mathbf{s}_i \boldsymbol{\beta}_2$ . Thus  $u_i$  is an estimator of  $\nu_i$  only if there are no time-invariant variables in the model. If there are time-invariant variables,  $u_i$  is an estimate of  $\nu_i$ plus the effects of the time-invariant variables.

Remarks are presented under the following headings:

Assessing goodness of fit xtreg and associated commands

## Assessing goodness of fit

 $R^2$  is a popular measure of goodness of fit in ordinary regression. In our case, given  $\widehat{\alpha}$  and  $\widehat{\beta}$ estimates of  $\alpha$  and  $\beta$ , we can assess the goodness of fit with respect to (1), (2), or (3). The prediction equations are, respectively,

$$\widehat{y}_{it} = \widehat{\alpha} + \mathbf{x}_{it}\widehat{\boldsymbol{\beta}} \tag{1}^{""}$$

$$\widehat{\overline{y}}_i = \widehat{\alpha} + \overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}} \tag{2'''}$$

$$\widehat{\widetilde{y}}_{it} = (\widehat{y}_{it} - \widehat{\overline{y}}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}$$

$$(3''')$$

xtreg reports "R-squares" corresponding to these three equations. R-squares is in quotes because the R-squares reported do not have all the properties of the OLS  $\mathbb{R}^2$ .

The ordinary properties of  $R^2$  include being equal to the squared correlation between  $\hat{y}$  and y and being equal to the fraction of the variation in y explained by  $\hat{y}$ —formally defined as  $Var(\hat{y})/Var(y)$ . The identity of the definitions is from a special property of the OLS estimates; in general, given a prediction  $\hat{y}$  for y, the squared correlation is not equal to the ratio of the variances, and the ratio of the variances is not required to be less than 1.

xtreg reports  $R^2$  values calculated as correlations squared, calling them  $R^2$  overall, corresponding to (1"");  $R^2$  between, corresponding to (2""); and  $R^2$  within, corresponding to (3""). In fact, you can think of each of these three numbers as having all the properties of ordinary  $R^2$ 's, if you bear in mind that the prediction being judged is not  $\widehat{y}_{it}$ ,  $\widehat{\overline{y}}_{i}$ , and  $\widehat{\widetilde{y}}_{it}$ , but  $\gamma_1 \widehat{y}_{it}$  from the regression  $y_{it} = \gamma_1 \widehat{y}_{it}$ ;  $\gamma_2 \widehat{\overline{y}}_i$  from the regression  $\overline{y}_i = \gamma_2 \widehat{\overline{y}}_i$ ; and  $\gamma_3 \widehat{\widetilde{y}}_{it}$  from  $\widetilde{y}_{it} = \gamma_3 \widehat{\widetilde{y}}_{it}$ .

In particular, xtreg, be obtains its estimates by performing OLS on (2), and therefore its reported  $R^2$  between is an ordinary  $R^2$ . The other two reported  $R^2$ 's are merely correlations squared, or, if you prefer,  $R^2$ 's from the second-round regressions  $y_{it} = \gamma_{11} \hat{y}_{it}$  and  $\tilde{y}_{it} = \gamma_{13} \hat{\tilde{y}}_{it}$ .

xtreg, fe obtains its estimates by performing OLS on (3), so its reported  $\mathbb{R}^2$  within is an ordinary  $R^2$ . As with be, the other  $R^2$ 's are correlations squared, or, if you prefer,  $R^2$ 's from the second-round regressions  $\overline{y_i} = \gamma_{22} \widehat{\overline{y}}_i$  and, as with be,  $\widetilde{y}_{it} = \gamma_{23} \widehat{\widetilde{y}}_{it}$ .

xtreg, re obtains its estimates by performing OLS on (4); none of the  $R^2$ 's corresponding to (1'''), (2'''), or (3''') correspond directly to this estimator (the "relevant"  $R^2$  is the one corresponding to (4)). All three reported  $R^2$ 's are correlations squared, or, if you prefer, from second-round regressions.

## xtreg and associated commands

## Example 1: Between-effects model

Using nlswork.dta described in [XT] xt, we will model ln\_wage in terms of completed years of schooling (grade), current age and age squared, current years worked (experience) and experience squared, current years of tenure on the current job and tenure squared, whether black (race = 2), whether residing in an area not designated a standard metropolitan statistical area (SMSA), and whether residing in the South.

```
. use https://www.stata-press.com/data/r17/nlswork
(National Longitudinal Survey of Young Women, 14-24 years old in 1968)
```

To obtain the between-effects estimates, we use xtreg, be. nlswork.dta has previously been xtset idcode year because that is what is true of the data, but for running xtreg, it would have been sufficient to have xtset idcode by itself.

<pre>. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure &gt; c.tenure#c.tenure 2.race not_smsa south, be</pre>							
Between regres		ion on grou	p means)	Number	of obs = of groups =	28,091 4,697	
1	s. Idcode				0 1	4,037	
R-squared: Within =	= 0 1501			Obs per	group: min =	1	
Between =					avg =	6.0	
Overall =					max =	15	
				F(10,46	86) =	450.23	
sd(u_i + avg(e	e i.)) = .3036	114		Prob >		0.0000	
24(4_1 4,6)					-	0.0000	
ln_wage	Coefficient	Std. err.	t	P> t	[95% conf.	interval]	
grade	.0607602	.0020006	30.37	0.000	.0568382	.0646822	
age	.0323158	.0087251	3.70	0.000	.0152105	.0494211	
J							
c.age#c.age	0005997	.0001429	-4.20	0.000	0008799	0003194	
ttl_exp	.0138853	.0056749	2.45	0.014	.0027598	.0250108	
<pre>c.ttl_exp# c.ttl_exp</pre>	.0007342	.0003267	2.25	0.025	.0000936	.0013747	
C.CCI_exp	.0007342	.0003207	2.20	0.020	.0000330	.0013747	
tenure	.0698419	.0060729	11.50	0.000	.0579361	.0817476	
c.tenure#							
c.tenure	0028756	.0004098	-7.02	0.000	0036789	0020722	
race							
Black	0564167	.0105131	-5.37	0.000	0770272	0358061	
not_smsa	1860406	.0112495	-16.54	0.000	2080949	1639862	
south	0993378 .3339113	.010136	-9.80	0.000	1192091	0794665	
_cons	.3339113	.1210434	2.76	0.006	.0966093	.5712133	

The between-effects regression is estimated on person-averages, so the "n = 4697" result is relevant. xtreg, be reports the "number of observations" and group-size information: describe in [XT] xt showed that we have 28,534 "observations"—person-years, really—of data. If we take the subsample that has no missing values in  $ln_wage$ , grade, ..., south leaves us with 28,091 observations on person-years, reflecting 4,697 persons, each observed for an average of 6.0 years.

For goodness of fit, the  $R^2$  between is directly relevant; our  $R^2$  is 0.4900. If, however, we use these estimates to predict the within model, we have an  $R^2$  of 0.1591. If we use these estimates to fit the overall data, our  $R^2$  is 0.3695.

The F statistic tests that the coefficients on the regressors grade, age, ..., south are all jointly zero. Our model is significant.

The root mean squared error of the fitted regression, which is an estimate of the standard deviation of  $\nu_i + \overline{\epsilon}_i$ , is 0.3036.

For our coefficients, each year of schooling increases hourly wages by 6.1%; age increases wages up to age 26.9 and thereafter decreases them (because the quadratic  $ax^2 + bx + c$  turns over at x = -b/2a, which for our age and c.age#c.age coefficients is  $0.0323158/(2\times0.0005997)\approx 26.9$ ); total experience increases wages at an increasing rate (which is surprising and bothersome); tenure on the current job increases wages up to a tenure of 12.1 years and thereafter decreases them; wages of blacks are, these things held constant, (approximately) 5.6% below that of nonblacks (approximately because 2.race is an indicator variable); residing in a non-SMSA (rural area) reduces wages by 18.6%; and residing in the South reduces wages by 9.9%.

## Example 2: Fixed-effects model

To fit the same model with the fixed-effects estimator, we specify the fe option.

```
. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure
> c.tenure#c.tenure 2.race not_smsa south, fe
note: grade omitted because of collinearity.
note: 2.race omitted because of collinearity.
Fixed-effects (within) regression
                                                   Number of obs
                                                                            28,091
Group variable: idcode
                                                   Number of groups =
                                                                             4,697
R-squared:
                                                   Obs per group:
     Within = 0.1727
                                                                  min =
                                                                                  1
     Between = 0.3505
                                                                               6.0
                                                                  avg =
     Overall = 0.2625
                                                                  max =
                                                                                 15
                                                   F(8,23386)
                                                                            610.12
corr(u_i, Xb) = 0.1936
                                                   Prob > F
                                                                            0.0000
     ln_wage
                Coefficient
                             Std. err.
                                                   P>|t|
                                                              [95% conf. interval]
                                             t.
       grade
                         0
                             (omitted)
                  .0359987
                              .0033864
                                                   0.000
                                          10.63
                                                              .0293611
                                                                          .0426362
         age
                  -.000723
                              .0000533
                                         -13.58
                                                   0.000
                                                            -.0008274
                                                                         -.0006186
 c.age#c.age
                  .0334668
                              .0029653
                                          11.29
                                                   0.000
                                                              .0276545
                                                                            .039279
     ttl_exp
   c.ttl_exp#
                  .0002163
                              .0001277
                                           1.69
                                                   0.090
                                                            -.0000341
                                                                          .0004666
   c.ttl_exp
      tenure
                  .0357539
                              .0018487
                                          19.34
                                                   0.000
                                                              .0321303
                                                                          .0393775
    c.tenure#
                                                            -.0022151
    c.tenure
                 -.0019701
                               .000125
                                         -15.76
                                                   0.000
                                                                         -.0017251
        race
                             (omitted)
      Black
                         0
    not_smsa
                 -.0890108
                              .0095316
                                          -9.34
                                                   0.000
                                                            -.1076933
                                                                         -.0703282
                 -.0606309
                              .0109319
                                          -5.55
                                                   0.000
                                                            -.0820582
                                                                         -.0392036
       south
                   1.03732
                                          21.36
                                                   0.000
                              .0485546
                                                              .9421496
                                                                           1.13249
       _cons
                 .35562203
     sigma_u
                 .29068923
     sigma_e
                 .59946283
                              (fraction of variance due to u_i)
         rho
```

F test that all  $u_i=0$ : F(4696, 23386) = 6.65 Prob > F = 0.0000

The observation summary at the top is the same as for the between-effects model, although this time it is the "Number of obs" that is relevant.

Our three  $R^2$ 's are not too different from those reported previously; the  $R^2$  within is slightly higher (0.1727 versus 0.1591), and the  $R^2$  between is a little lower (0.3505 versus 0.4900), as expected, because the between estimator maximizes  $R^2$  between and the within estimator  $R^2$  within. In terms of overall fit, these estimates are somewhat worse (0.2625 versus 0.3695).

If the unobserved time-invariant component  $\nu$  is not correlated with the regressors, estimates from the fixed-effects model are consistent but inefficient relative to estimates from the random-effects model. In this case, the interpretation of sigma\_u in the coefficient table is the same for the fixed-effects and random-effects models. However, sigma\_u is a nuisance parameter when  $\nu$  is correlated with the covariates.

Here both grade and 2.race were omitted from the model because they do not vary over time. Because grade and 2.race are time invariant, our estimate  $u_i$  is an estimate of  $\nu_i$  plus the effects of grade and 2.race, so our estimate of the standard deviation is based on the variation in  $\nu_i$ , grade, and 2.race. On the other hand, had 2.race and grade been omitted merely because they were collinear with the other regressors in our model,  $u_i$  would be an estimate of  $\nu_i$ , and 0.355622 would be an estimate of  $\sigma_{\nu}$ . (xtsum and xttab allow you to determine whether a variable is time invariant; see [XT] xtsum and [XT] xttab.)

Regardless of the status of  $u_i$ , our estimate of the standard deviation of  $\epsilon_{it}$  is valid (and, in fact, is the estimate that would be used by the random-effects estimator to produce its results).

Our estimate of the correlation of  $u_i$  with  $\mathbf{x}_{it}$  suffers from the problem of what  $u_i$  measures. We find correlation but cannot say whether this is correlation of  $\nu_i$  with  $\mathbf{x}_{it}$  or merely correlation of grade and 2.race with  $\mathbf{x}_{it}$ . In any case, the fixed-effects estimator is robust to such a correlation, and the other estimates it produces are unbiased.

So, although this estimator produces no estimates of the effects of grade and 2.race, it does predict that age has a positive effect on wages up to age 24.9 years (compared with 26.9 years estimated by the between estimator); that total experience still increases wages at an increasing rate (which is still bothersome); that tenure increases wages up to 9.1 years (compared with 12.1); that living in a non-SMSA reduces wages by 8.9% (compared with a more drastic 18.6%); and that living in the South reduces wages by 6.1% (as compared with 9.9%).

4

### Example 3: Fixed-effects models with robust standard errors

If we suspect that there is heteroskedasticity or within-panel serial correlation in the idiosyncratic error term  $\epsilon_{it}$ , we could specify the vce(robust) option:

```
. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure
> c.tenure#c.tenure 2.race not_smsa south, fe vce(robust)
note: grade omitted because of collinearity.
note: 2.race omitted because of collinearity.
Fixed-effects (within) regression
                                                 Number of obs
                                                                         28,091
Group variable: idcode
                                                Number of groups =
                                                                         4,697
R-squared:
                                                 Obs per group:
     Within = 0.1727
                                                               min =
                                                                              1
     Between = 0.3505
                                                               avg =
                                                                            6.0
     Overall = 0.2625
                                                               max =
                                                                             15
                                                 F(8,4696)
                                                                         273.86
corr(u_i, Xb) = 0.1936
                                                Prob > F
                                                                   =
                                                                         0.0000
                              (Std. err. adjusted for 4,697 clusters in idcode)
```

ln_wage	Coefficient	Robust std. err.	t	P> t	[95% conf.	interval]
grade age	0.0359987	(omitted) .0052407	6.87	0.000	.0257243	.046273
c.age#c.age	000723	.0000845	-8.56	0.000	0008887	0005573
ttl_exp	.0334668	.004069	8.22	0.000	.0254896	.0414439
<pre>c.ttl_exp# c.ttl_exp</pre>	.0002163	.0001763	1.23	0.220	0001294	.0005619
tenure	.0357539	.0024683	14.49	0.000	.0309148	.040593
c.tenure# c.tenure	0019701	.0001696	-11.62	0.000	0023026	0016376
race						
Black not_smsa	0 0890108	(omitted) .0137629	-6.47	0.000	1159926	062029
south	0606309	.0163366	-3.71	0.000	0926583	0286035
_cons	1.03732	.0739644	14.02	0.000	.8923149	1.182325
sigma_u sigma_e rho	.35562203 .29068923 .59946283	(fraction	of varia	nce due 1	to u_i)	

Although the estimated coefficients are the same with and without the vce(robust) option, the robust estimator produced larger standard errors and a p-value for c.ttl\_exp#c.ttl\_exp above the conventional 10%. The F test of  $\nu_i = 0$  is suppressed because it is too difficult to compute the robust form of the statistic when there are more than a few panels.

#### □ Technical note

The robust standard errors reported above are identical to those obtained by clustering on the panel variable idcode. Clustering on the panel variable produces an estimator of the VCE that is robust to cross-sectional heteroskedasticity and within-panel (serial) correlation that is asymptotically equivalent to that proposed by Arellano (1987). Although the example above applies the fixed-effects estimator, the robust and cluster-robust VCE estimators are also available for the random-effects estimator. Wooldridge (2020) and Arellano (2003) discuss these robust and cluster-robust VCE estimators for the fixed-effects and random-effects estimators. More details are available in *Methods and formulas*.

## Example 4: Random-effects model

Refitting our log-wage model with the random-effects estimator, we obtain

```
. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure
> c.tenure#c.tenure 2.race not_smsa south, re theta
Random-effects GLS regression
                                                    Number of obs
                                                                             28,091
Group variable: idcode
                                                   Number of groups
                                                                              4,697
R-squared:
                                                    Obs per group:
     Within = 0.1715
                                                                   min =
                                                                                   1
     Between = 0.4784
                                                                                 6.0
                                                                   avg =
     Overall = 0.3708
                                                                   max =
                                                                                  15
                                                    Wald chi2(10)
                                                                            9244.74
corr(u_i, X) = 0 (assumed)
                                                   Prob > chi2
                                                                             0.0000
                     theta
  min
            5%
                     median
                                    95%
                                              max
0.2520
         0.2520
                     0.5499
                                 0.7016
                                           0.7206
                Coefficient
     ln_wage
                              Std. err.
                                              z
                                                   P>|z|
                                                               [95% conf. interval]
                  .0646499
                              .0017812
                                           36.30
                                                    0.000
                                                               .0611589
                                                                            .0681409
       grade
                  .0368059
                              .0031195
                                           11.80
                                                    0.000
                                                               .0306918
                                                                            .0429201
         age
 c.age#c.age
                 -.0007133
                                .00005
                                          -14.27
                                                    0.000
                                                             -.0008113
                                                                          -.0006153
                  .0290208
                               .002422
                                           11.98
                                                    0.000
                                                               .0242739
                                                                            .0337678
     ttl_exp
   c.ttl_exp#
                                            2.62
   c.ttl_exp
                  .0003049
                              .0001162
                                                    0.009
                                                                .000077
                                                                            .0005327
                  .0392519
      tenure
                              .0017554
                                           22.36
                                                   0.000
                                                               .0358113
                                                                            .0426925
    c.tenure#
                 -.0020035
                              .0001193
                                          -16.80
                                                    0.000
                                                             -.0022373
                                                                          -.0017697
    c.tenure
        race
                  -.053053
                              .0099926
                                           -5.31
                                                    0.000
                                                             -.0726381
                                                                          -.0334679
      Black
                 -.1308252
                              .0071751
                                          -18.23
                                                    0.000
                                                             -.1448881
                                                                          -.1167622
    not_smsa
                 -.0868922
                              .0073032
                                          -11.90
                                                    0.000
                                                             -.1012062
                                                                          -.0725781
       south
       _cons
                  .2387207
                               .049469
                                            4.83
                                                    0.000
                                                               .1417633
                                                                            .3356781
                 .25790526
     sigma_u
     sigma_e
                 .29068923
                 .44045273
                              (fraction of variance due to u_i)
         rho
```

According to the  $R^2$ 's, this estimator performs worse within than the within fixed-effects estimator and worse between than the between estimator, as it must, and slightly better overall.

We estimate that  $\sigma_{\nu}$  is 0.2579 and  $\sigma_{\epsilon}$  is 0.2907 and, by assertion, assume that the correlation of  $\nu$  and  $\mathbf{x}$  is zero.

All that is known about the random-effects estimator is its asymptotic properties, so rather than reporting an F statistic for overall significance, xtreg, re reports a  $\chi^2$ . Taken jointly, our coefficients are significant.

xtreg, re also reports a summary of the distribution of  $\theta_i$ , an ingredient in the estimation of (4).  $\theta$  is not a constant here because we observe women for unequal periods.

We estimate that schooling has a rate of return of 6.5% (compared with 6.1% between and no estimate within); that the increase of wages with age turns around at 25.8 years (compared with 26.9 between and 24.9 within); that total experience yet again increases wages increasingly; that the effect of job tenure turns around at 9.8 years (compared with 12.1 between and 9.1 within); that being black reduces wages by 5.3% (compared with 5.6% between and no estimate within); that living in a non-SMSA reduces wages 13.1% (compared with 18.6% between and 8.9% within); and that living in the South reduces wages 8.7% (compared with 9.9% between and 6.1% within).

## Example 5: Random-effects model fit using ML

We could also have fit this random-effects model with the maximum likelihood estimator:

```
. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure
> c.tenure#c.tenure 2.race not_smsa south, mle
Fitting constant-only model:
Iteration 0: log likelihood = -12663.954
Iteration 1: \log likelihood = -12649.756
Iteration 2: log likelihood = -12649.614
Iteration 3: log likelihood = -12649.614
Fitting full model:
Iteration 0: log likelihood = -8922.145
Iteration 1:
             log likelihood = -8853.6409
             log likelihood = -8853.4255
Iteration 2:
```

Random-effects ML regression Group variable: idcode

Iteration 3: log likelihood = -8853.4254

Random effects u\_i ~ Gaussian

Number of obs = 28,091Number of groups = 4,697

Obs per group: min =

1 avg = 6.0 max = 15

= 7592.38 LR chi2(10) Prob > chi2 = 0.0000

Prob >= chibar2 = 0.000

Log likelihood = -8853.4254

ln_wage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
grade	.0646093	.0017372	37.19	0.000	.0612044	.0680142
age	.0368531	.0031226	11.80	0.000	.030733	.0429732
c.age#c.age	0007132	.0000501	-14.24	0.000	0008113	000615
ttl_exp	.0288196	.0024143	11.94	0.000	.0240877	.0335515
<pre>c.ttl_exp# c.ttl_exp</pre>	.000309	.0001163	2.66	0.008	.0000811	.0005369
C.CCI_exp	.000303	.0001103	2.00	0.000	.0000011	.0003303
tenure	.0394371	.0017604	22.40	0.000	.0359868	.0428875
c.tenure#						
c.tenure	0020052	.0001195	-16.77	0.000	0022395	0017709
race						
Black	0533394	.0097338	-5.48	0.000	0724172	0342615
not_smsa	1323433	.0071322	-18.56	0.000	1463221	1183644
south	0875599	.0072143	-12.14	0.000	1016998	0734201
_cons	. 2390837	.0491902	4.86	0.000	.1426727	.3354947
/sigma_u	. 2485556	.0035017			.2417863	.2555144
/sigma_e	.2918458	.001352			.289208	.2945076
rho	.4204033	.0074828			.4057959	.4351212

The estimates are nearly the same as those produced by xtreg, re—the GLS estimator. For instance, xtreg, re estimated the coefficient on grade to be 0.0646499, xtreg, mle estimated 0.0646093, and the ratio is 0.0646499/0.0646093 = 1.001 to three decimal places. Similarly, the standard errors are nearly equal: 0.0017811/0.0017372 = 1.025. Below we compare all 11 coefficients:

	Coefficient ratio			SE ratio		
Estimator	mean	min.	max.	mean	min.	max.
xtreg, mle (ML) xtreg, re (GLS)	1. .997	1. .987	1. 1.007	1. 1.006	1. .997	

1

## Example 6: Population-averaged model

We could also have fit this model with the population-averaged estimator:

```
. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure
```

Iteration 1: tolerance = .0310561 Iteration 2: tolerance = .00074898 Iteration 3: tolerance = .0000147 Iteration 4: tolerance = 2.880e-07

GEE population-averaged model Group variable: idcode

Family: Gaussian Link: Identity

Correlation: exchangeable

Scale parameter = .1436709

Number	of	obs	=	28,091
Number	of	groups	=	4,697
Obs per	gı	coup:		

min = 1 avg = 6.0 max = 15

Wald chi2(10) = 9598.89Prob > chi2 = 0.0000

ln_wage	Coefficient	Std. err.	z	P> z	[95% conf	. interval]
grade	.0645427	.0016829	38.35	0.000	.0612442	.0678412
age	.036932	.0031509	11.72	0.000	.0307564	.0431076
c.age#c.age	0007129	.0000506	-14.10	0.000	0008121	0006138
ttl_exp	.0284878	.0024169	11.79	0.000	.0237508	.0332248
c.ttl_exp#						
c.ttl_exp	.0003158	.0001172	2.69	0.007	.000086	.0005456
tenure	.0397468	.0017779	22.36	0.000	.0362621	.0432315
c.tenure#						
c.tenure	002008	.0001209	-16.61	0.000	0022449	0017711
race						
Black	0538314	.0094086	-5.72	0.000	072272	0353909
not_smsa	1347788	.0070543	-19.11	0.000	1486049	1209526
south	0885969	.0071132	-12.46	0.000	1025386	0746552
_cons	.2396286	.0491465	4.88	0.000	.1433034	.3359539

<sup>&</sup>gt; c.tenure#c.tenure 2.race not\_smsa south, pa

These results differ from those produced by xtreg, re and xtreg, mle. Coefficients are larger and standard errors smaller, xtreg, pa is simply another way to run the xtgee command. That is, we would have obtained the same output had we typed

. xtgee ln\_w grade age c.age#c.age ttl\_exp c.ttl\_exp#c.ttl\_exp > tenure c.tenure#c.tenure 2.race not\_smsa south (output omitted because it is the same as above)

See [XT] xtgee. In the language of xtgee, the random-effects model corresponds to an exchangeable correlation structure and identity link, and xtgee also allows other correlation structures. Let's stay with the random-effects model, however. xtgee will also produce robust estimates of variance, and we refit this model that way by typing

. xtgee ln\_w grade age c.age#c.age ttl\_exp c.ttl\_exp#c.ttl\_exp > tenure c.tenure#c.tenure 2.race not\_smsa south, vce(robust) (output omitted, coefficients the same, standard errors different)

In the previous example, we presented a table comparing xtreg, re with xtreg, mle. Below we add the results from the estimates shown and the ones we did with xtgee, vce(robust):

		Coefficient ratio				SE ratio		
Estimator		mean	min.	max.	mear	n min.	max.	
xtreg, mle	(ML)	1.	1.	1.	1.	1.	1.	
xtreg, re	(GLS)	.997	.987	1.007	1.00	.997	1.027	
xtreg, pa	(PA)	1.060	.847	1.317	.85	.626	.986	
<pre>xtgee, vce(robust)</pre>	(PA)	1.060	.847	1.317	1.30	5 .957	1.545	

So, which are right? This is a real dataset, and we do not know. However, in example 2 in [XT] xtreg postestimation, we will present evidence that the assumptions underlying the xtreg, re and xtreg, mle results are not met.

1

### Stored results

Scalars

xtreg, re stores the following in e():

```
e(N)
                                 number of observations
       e(N_g)
                                 number of groups
       e(df_m)
                                 model degrees of freedom
       e(g_min)
                                 smallest group size
       e(g_avg)
                                 average group size
       e(g_max)
                                 largest group size
       e(Tcon)
                                 1 if T is constant
       e(sigma)
                                 ancillary parameter (gamma, lnormal)
                                 panel-level standard deviation
       e(sigma_u)
       e(sigma_e)
                                 standard deviation of \epsilon_{it}
                                 R^2 for within model
       e(r2_w)
       e(r2_o)
                                 R^2 for overall model
                                 R^2 for between model
       e(r2_b)
       e(N_clust)
                                 number of clusters
       e(chi2)
                                 \chi^2
                                 p-value for model test
       e(p)
       e(rho)
       e(thta_min)
                                 minimum \theta
       e(thta_5)
                                 \theta, 5th percentile
       e(thta_50)
                                 \theta, 50th percentile
       e(thta_95)
                                 \theta, 95th percentile
       e(thta_max)
                                 maximum \theta
                                 root mean squared error of GLS regression
       e(rmse)
                                 harmonic mean of group sizes
       e(Tbar)
       e(rank)
                                 rank of e(V)
   Macros
       e(cmd)
                                 xtreg
       e(cmdline)
                                 command as typed
       e(depvar)
                                 name of dependent variable
       e(ivar)
                                 variable denoting groups
       e(model)
                                 name of cluster variable
       e(clustvar)
                                 Wald; type of model \chi^2 test
       e(chi2type)
       e(vce)
                                 vcetype specified in vce()
                                 title used to label Std. err.
       e(vcetype)
       e(sa)
                                 sa, if specified
       e(properties)
       e(predict)
                                 program used to implement predict
       e(marginsnotok)
                                 predictions disallowed by margins
       e(asbalanced)
                                 factor variables fvset as asbalanced
       e(asobserved)
                                 factor variables fyset as asobserved
   Matrices
                                 coefficient vector
       e(b)
                                 coefficient vector for fixed-effects model
       e(bf)
       e(theta)
                                 variance-covariance matrix of the estimators
       e(V)
       e(VCEf)
                                 VCE for fixed-effects model
   Functions
       e(sample)
                                 marks estimation sample
In addition to the above, the following is stored in r():
   Matrices
                                 matrix containing the coefficients with their standard errors, test statistics, p-values,
       r(table)
                                     and confidence intervals
Note that results stored in r() are updated when the command is replayed and will be replaced when
```

any r-class command is run after the estimation command.

```
xtreg, be stores the following in e():
```

```
Scalars
                              number of observations
    e(N)
                              number of groups
    e(N_g)
    e(mss)
                              model sum of squares
                              model degrees of freedom
    e(df_m)
                              residual sum of squares
    e(rss)
    e(df_r)
                              residual degrees of freedom
    e(11)
                              log likelihood
    e(11_0)
                              log likelihood, constant-only model
    e(g_min)
                              smallest group size
    e(g_avg)
                              average group size
    e(g_max)
                              largest group size
                              1 if T is constant
    e(Tcon)
                              R^2
    e(r2)
                              adjusted \mathbb{R}^2
    e(r2_a)
                              R^{2} for within model
    e(r2_w)
                              \mathbb{R}^2 for overall model
    e(r2_o)
                              R^2 for between model
    e(r2_b)
    e(F)
                              p-value for model test
    e(p)
    e(rmse)
                              root mean squared error
                              harmonic mean of group sizes
    e(Tbar)
    e(rank)
                              rank of e(V)
Macros
    e(cmd)
                              xtreg
    e(cmdline)
                              command as typed
    e(depvar)
                              name of dependent variable
    e(ivar)
                              variable denoting groups
    e(model)
    e(typ)
                              WLS, if wls specified
                              title in estimation output
    e(title)
    e(vce)
                              vcetype specified in vce()
    e(properties)
    e(predict)
                              program used to implement predict
    e(marginsok)
                              predictions allowed by margins
    e(marginsnotok)
                              predictions disallowed by margins
    e(asbalanced)
                              factor variables fyset as asbalanced
                              factor variables fyset as asobserved
    e(asobserved)
Matrices
    e(b)
                              coefficient vector
    e(V)
                              variance-covariance matrix of the estimators
Functions
    e(sample)
                              marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
r(table) matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

#### xtreg, fe stores the following in e():

```
Scalars
                              number of observations
    e(N)
    e(N_g)
                              number of groups
    e(mss)
                              model sum of squares
    e(df_m)
                              model degrees of freedom
    e(rss)
                              residual sum of squares
    e(df_r)
                              residual degrees of freedom
                              total sum of squares
    e(tss)
                              smallest group size
    e(g_min)
                              average group size
    e(g_avg)
    e(g_max)
                              largest group size
    e(Tcon)
                              1 if T is constant
                              ancillary parameter (gamma, lnormal)
    e(sigma)
    e(corr)
                              corr(u_i, Xb)
                              panel-level standard deviation
    e(sigma_u)
    e(sigma_e)
                              standard deviation of \epsilon_{it}
                              R^2
    e(r2)
                              adjusted R^2
    e(r2_a)
    e(r2_w)
                              R^2 for within model
                              R^2 for overall model
    e(r2_o)
                              R^2 for between model
    e(r2_b)
    e(11)
                              log likelihood
                              log likelihood, constant-only model
    e(11_0)
    e(N_clust)
                              number of clusters
    e(rho)
                              F statistic
    e(F)
    e(F_f)
                              F statistic for test of u_i=0
                              p-value for model test
    e(p)
    e(p_f)
                              p-value for test of u_i=0
    e(df_a)
                              degrees of freedom for absorbed effect
    e(df_b)
                              numerator degrees of freedom for F statistic
    e(rmse)
                              root mean squared error
    e(Tbar)
                              harmonic mean of group sizes
    e(rank)
                              rank of e(V)
Macros
    e(cmd)
                              xtreg
    e(cmdline)
                              command as typed
    e(depvar)
                              name of dependent variable
                              variable denoting groups
    e(ivar)
    e(model)
                              fе
    e(wtype)
                              weight type
    e(wexp)
                              weight expression
                              name of cluster variable
    e(clustvar)
    e(vce)
                              vcetype specified in vce()
    e(vcetype)
                              title used to label Std. err.
    e(properties)
    e(predict)
                              program used to implement predict
    e(marginsnotok)
                              predictions disallowed by margins
    e(asbalanced)
                              factor variables fyset as asbalanced
    e(asobserved)
                              factor variables fyset as asobserved
Matrices
                              coefficient vector
    e(b)
    e(V)
                              variance-covariance matrix of the estimators
    e(V_modelbased)
                              model-based variance
Functions
    e(sample)
                              marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
r(table) matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

```
xtreg, mle stores the following in e():
```

```
e(N)
                               number of observations
                               number of groups
    e(N_g)
                               model degrees of freedom
    e(df_m)
                              smallest group size
    e(g_min)
                              average group size
    e(g_avg)
    e(g_max)
                              largest group size
                              panel-level standard deviation
    e(sigma_u)
    e(sigma_e)
                              standard deviation of \epsilon_{it}
                              log likelihood
    e(11)
    e(11_0)
                              log likelihood, constant-only model
    e(11_c)
                               log likelihood, comparison model
    e(N_clust)
                              number of clusters
    e(chi2)
                              \chi^2 for comparison test
    e(chi2_c)
    e(p)
                              p-value for model test
    e(rho)
    e(rank)
                              rank of e(V)
Macros
    e(cmd)
                              xtreg
    e(cmdline)
                              command as typed
                              name of dependent variable
    e(depvar)
    e(ivar)
                              variable denoting groups
    e(model)
                              ml
    e(wtype)
                               weight type
    e(wexp)
                               weight expression
                              title in estimation output
    e(title)
                              name of cluster variable
    e(clustvar)
                              vcetype specified in vce()
    e(vce)
                              title used to label Std. err.
    e(vcetype)
                              Wald or LR; type of model \chi^2 test Wald or LR; type of model \chi^2 test corresponding to e(chi2_c)
    e(chi2type)
    e(chi2_ct)
                              Gaussian: the distribution of the RE
    e(distrib)
    e(properties)
    e(predict)
                              program used to implement predict
    e(marginsnotok)
                              predictions disallowed by margins
    e(asbalanced)
                              factor variables fyset as asbalanced
                              factor variables fyset as asobserved
    e(asobserved)
Matrices
    e(b)
                              coefficient vector
    e(V)
                              variance-covariance matrix of the estimators
Functions
    e(sample)
                              marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
r(table) matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Scalars

r(table)

```
xtreg, pa stores the following in e():
```

```
e(N)
                                 number of observations
       e(N_g)
                                 number of groups
       e(df_m)
                                 model degrees of freedom
                                 \chi^2
       e(chi2)
       e(p)
                                 p-value for model test
                                 degrees of freedom for Pearson \chi^2
       e(df_pear)
                                 \chi^{2} test of deviance
       e(chi2_dev)
                                 \chi^2 test of deviance dispersion
       e(chi2_dis)
       e(deviance)
                                 deviance
       e(dispers)
                                 deviance dispersion
       e(phi)
                                 scale parameter
       e(g_min)
                                 smallest group size
       e(g_avg)
                                 average group size
                                 largest group size
       e(g_max)
       e(rank)
                                 rank of e(V)
       e(tol)
                                 target tolerance
                                 achieved tolerance
       e(dif)
       e(rc)
                                 return code
   Macros
       e(cmd)
                                 xtgee
       e(cmd2)
       e(cmdline)
                                 command as typed
       e(depvar)
                                 name of dependent variable
       e(ivar)
                                 variable denoting groups
       e(tvar)
                                 variable denoting time within groups
       e(model)
                                рa
       e(family)
                                 Gaussian
       e(link)
                                 identity; link function
       e(corr)
                                 correlation structure
       e(scale)
                                 x2, dev, phi, or #; scale parameter
       e(wtype)
                                 weight type
                                 weight expression
       e(wexp)
       e(offset)
                                 linear offset variable
       e(chi2type)
                                 Wald; type of model \chi^2 test
       e(vce)
                                 vcetype specified in vce()
       e(vcetype)
                                 title used to label Std. err.
       e(rgf)
                                 rgf, if rgf specified
                                 nmp, if specified
       e(nmp)
       e(properties)
                                 b V
       e(predict)
                                 program used to implement predict
                                 predictions disallowed by margins
       e(marginsnotok)
       e(asbalanced)
                                 factor variables fyset as asbalanced
       e(asobserved)
                                 factor variables fyset as asobserved
   Matrices
       e(b)
                                 coefficient vector
       e(R)
                                 estimated working correlation matrix
                                 variance-covariance matrix of the estimators
       e(V)
                                 model-based variance
       e(V_modelbased)
   Functions
       e(sample)
                                 marks estimation sample
In addition to the above, the following is stored in r():
   Matrices
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

and confidence intervals

matrix containing the coefficients with their standard errors, test statistics, p-values,

### Methods and formulas

The model to be fit is

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it}$$

for i = 1, ..., n and, for each i, t = 1, ..., T, of which  $T_i$  periods are actually observed.

Methods and formulas are presented under the following headings:

xtreg, fe xtreg, be xtreg, re xtreg, mle xtreg, pa

## xtreg, fe

xtreg, fe produces estimates by running OLS on

$$(y_{it} - \overline{y}_i + \overline{\overline{y}}) = \alpha + (\mathbf{x}_{it} - \overline{\mathbf{x}}_i + \overline{\overline{\mathbf{x}}})\boldsymbol{\beta} + (\epsilon_{it} - \overline{\epsilon}_i + \overline{\nu}) + \overline{\overline{\epsilon}}$$

where  $\overline{y}_i = \sum_{t=1}^{T_i} y_{it}/T_i$ , and similarly,  $\overline{\overline{y}} = \sum_i \sum_t y_{it}/(nT_i)$ . The conventional covariance matrix of the estimators is adjusted for the extra n-1 estimated means, so results are the same as using OLS on (1) to estimate  $\nu_i$  directly. Specifying vce(robust) or vce(cluster clustvar) causes the Huber/White/sandwich VCE estimator to be calculated for the coefficients estimated in this regression. See [P] \_robust, particularly Introduction and Methods and formulas. Wooldridge (2020) and Arellano (2003) discuss this application of the Huber/White/sandwich VCE estimator. As discussed by Wooldridge (2020), Stock and Watson (2008), and Arellano (2003), specifying vce(robust) is equivalent to specifying vce(cluster panelvar), where panelvar is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in  $\epsilon_{it}$ .

The cluster-robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation induced by the within transform.

From the estimates  $\widehat{\alpha}$  and  $\widehat{\beta}$ , estimates  $u_i$  of  $\nu_i$  are obtained as  $u_i = \overline{y}_i - \widehat{\alpha} - \overline{\mathbf{x}}_i \widehat{\beta}$ . Reported from the calculated  $u_i$  are its standard deviation and its correlation with  $\overline{\mathbf{x}}_i \hat{\boldsymbol{\beta}}$ . Reported as the standard deviation of  $e_{it}$  is the regression's estimated root mean squared error, s, which is adjusted (as previously stated) for the n-1 estimated means.

Reported as  $R^2$  within is the  $R^2$  from the mean-deviated regression.

Reported as  $R^2$  between is  $\operatorname{corr}(\overline{\mathbf{x}}_i\widehat{\boldsymbol{\beta}}, \overline{y}_i)^2$ .

Reported as  $R^2$  overall is  $\operatorname{corr}(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$ .

### xtreg, be

xtreg, be fits the following model:

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta} + \nu_i + \overline{\epsilon}_i$$

Estimation is via OLS unless  $T_i$  is not constant and the wls option is specified. Otherwise, the estimation is performed via WLS. The estimates and conventional VCE are obtained from regress for both cases, but for WLS, [aweight= $T_i$ ] is specified.

Reported as  $\mathbb{R}^2$  between is the  $\mathbb{R}^2$  from the fitted regression.

Reported as  $R^2$  within is  $\operatorname{corr}\{(\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}, y_{it} - \overline{y}_i\}^2$ .

Reported as  $R^2$  overall is  $\operatorname{corr}(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$ .

### xtreg, re

The key to the random-effects estimator is the GLS transform. Given estimates of the idiosyncratic component,  $\hat{\sigma}_e^2$ , and the individual component,  $\hat{\sigma}_u^2$ , the GLS transform of a variable z for the random-effects model is

$$z_{it}^* = z_{it} - \widehat{\theta}_i \overline{z}_i$$

where  $\overline{z}_i = 1/T_i \sum_{t=1}^{T_i} z_{it}$  and

$$\widehat{\theta}_i = 1 - \sqrt{\frac{\widehat{\sigma}_e^2}{T_i \widehat{\sigma}_u^2 + \widehat{\sigma}_e^2}}$$

Given an estimate of  $\widehat{\theta}_i$ , one transforms the dependent and independent variables, and then the coefficient estimates and the conventional variance-covariance matrix come from an OLS regression of  $y_{it}^*$  on  $\mathbf{x}_{it}^*$  and the transformed constant  $1-\widehat{\theta}_i$ . Specifying vce(robust) or vce(cluster clustvar) causes the Huber/White/sandwich VCE estimator to be calculated for the coefficients estimated in this regression. See [P] \_robust; in particular, see Introduction and Methods and formulas. Wooldridge (2020) and Arellano (2003) discuss this application of the Huber/White/sandwich VCE estimator. As discussed by Wooldridge (2020), Stock and Watson (2008), and Arellano (2003), specifying vce(robust) is equivalent to specifying vce(cluster panelvar), where panelvar is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in  $\epsilon_{it}$ .

The cluster-robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation that is generally induced by the random-effects transform when there is heteroskedasticity or within-panel serial correlation in the idiosyncratic errors.

Stata has two implementations of the Swamy-Arora method for estimating the variance components. They produce the same results in balanced panels and share the same estimator of  $\sigma_e^2$ . However, the two methods differ in their estimator of  $\sigma_u^2$  in unbalanced panels. We call the first  $\hat{\sigma}_{u\overline{T}}^2$  and the second  $\hat{\sigma}_{uSA}^2$ . Both estimators are consistent; however,  $\hat{\sigma}_{uSA}^2$  has a more elaborate adjustment for small samples than  $\hat{\sigma}_{u\overline{T}}^2$ . (See Baltagi [2013], Baltagi and Chang [1994], and Swamy and Arora [1972] for derivations of these methods.)

Both methods use the same function of within residuals to estimate the idiosyncratic error component  $\sigma_e$ . Specifically,

$$\widehat{\sigma}_e^2 = \frac{\sum_{i=1}^n \sum_{t=1}^{T_i} e_{it}^2}{N - n - K + 1}$$

where

$$e_{it} = (y_{it} - \overline{y}_i + \overline{\overline{y}}) - \widehat{\alpha}_w - (\mathbf{x}_{it} - \overline{\mathbf{x}}_i + \overline{\overline{\mathbf{x}}})\widehat{\boldsymbol{\beta}}_w$$

and  $\widehat{\alpha}_w$  and  $\widehat{\beta}_w$  are the within estimates of the coefficients and  $N = \sum_{i=1}^n T_i$ . After passing the within residuals through the within transform, only the idiosyncratic errors are left.

The default method for estimating  $\sigma_u^2$  is

$$\widehat{\sigma}_{u\overline{T}}^2 = \max\left\{0, \frac{\mathrm{SSR}_b}{n-K} - \frac{\widehat{\sigma}_e^2}{\overline{T}}\right\}$$

where

$$SSR_b = \sum_{i=1}^{n} \left( \overline{y}_i - \widehat{\alpha}_b - \overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}}_b \right)^2$$

 $\widehat{\alpha}_b$  and  $\widehat{\beta}_b$  are coefficient estimates from the between regression and  $\overline{T}$  is the harmonic mean of  $T_i$ :

$$\overline{T} = \frac{n}{\sum_{i=1}^{n} \frac{1}{T_i}}$$

This estimator is consistent for  $\sigma_u^2$  and is computationally less expensive than the second method. The sum of squared residuals from the between model estimate a function of both the idiosyncratic component and the individual component. Using our estimator of  $\sigma_e^2$ , we can remove the idiosyncratic component, leaving only the desired individual component.

The second method is the Swamy-Arora method for unbalanced panels derived by Baltagi and Chang (1994), which has a more precise small-sample adjustment. Using this method,

$$\widehat{\sigma}_{\mathrm{uSA}}^2 = \, \max \left\{ 0, \frac{\mathrm{SSR}_b^* - (n-K) \widehat{\sigma}_e^2}{N - c_{\mathrm{tr}}} \right\}$$

where

$$\begin{aligned} \text{SSR}_b^* &= \sum_{i=1}^n T_i \left( \overline{y}_i - \widehat{\alpha}_b - \overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}}_b \right)^2 \\ c_{\text{tr}} &= \text{trace} \left\{ (\mathbf{X}' \mathbf{P} \mathbf{X})^{-1} \mathbf{X}' \mathbf{Z} \mathbf{Z}' \mathbf{X} \right\} \\ \mathbf{P} &= \text{diag} \left\{ \left( \frac{1}{T_i} \right) \iota_{T_i} \iota'_{T_i} \right\} \\ \mathbf{Z} &= \text{diag} \left[ \iota_{T_i} \right] \end{aligned}$$

**X** is the  $N \times K$  matrix of covariates, including the constant, and  $\iota_{T_i}$  is a  $T_i \times 1$  vector of ones.

The estimated coefficients  $(\widehat{\alpha}, \widehat{\beta})$  and their estimated covariance matrix  $\widehat{\mathbf{V}}$  are reported together with the previously calculated quantities  $\widehat{\sigma}_e$  and  $\widehat{\sigma}_u$ . The standard deviation of  $\nu_i + e_{it}$  is calculated as  $\sqrt{\widehat{\sigma}_e^2 + \widehat{\sigma}_u^2}$ .

Reported as  $R^2$  between is  $\operatorname{corr}(\overline{\mathbf{x}}_i\widehat{\boldsymbol{\beta}}, \overline{y}_i)^2$ .

Reported as  $R^2$  within is  $\operatorname{corr}\{(\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}, y_{it} - \overline{y}_i\}^2$ .

Reported as  $R^2$  overall is  $\operatorname{corr}(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$ .

## xtreg, mle

The log likelihood for the ith unit is

$$l_{i} = -\frac{1}{2} \left( \frac{1}{\sigma_{e}^{2}} \left[ \sum_{t=1}^{T_{i}} (y_{it} - \mathbf{x}_{it}\boldsymbol{\beta})^{2} - \frac{\sigma_{u}^{2}}{T_{i}\sigma_{u}^{2} + \sigma_{e}^{2}} \left\{ \sum_{t=1}^{T_{i}} (y_{it} - \mathbf{x}_{it}\boldsymbol{\beta}) \right\}^{2} \right] + \ln \left( T_{i} \frac{\sigma_{u}^{2}}{\sigma_{e}^{2}} + 1 \right) + T_{i} \ln(2\pi\sigma_{e}^{2}) \right)$$

The mle and re options yield essentially the same results, except when total  $N = \sum_i T_i$  is small (200 or less) and the data are unbalanced.

Similarly to xtreg, fe and xtreg, re, specifying vce(robust) or vce(cluster *clustvar*) causes the Huber/White/sandwich VCE estimator to be calculated for the estimated parameters in this regression.

Specifying vce(robust) is equivalent to specifying vce(cluster panelvar), where panelvar is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in it.

The cluster-robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation that is generally induced by the random-effects transform when there is heteroskedasticity or within-panel serial correlation in the idiosyncratic errors.

# xtreg, pa

See [XT] **xtgee** for details on the methods and formulas used to calculate the population-averaged model using a generalized estimating equations approach.

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### Also see

- [XT] **xtreg postestimation** Postestimation tools for xtreg
- [XT] **xteregress** Extended random-effects linear regression
- [XT] **xtgee** Fit population-averaged panel-data models by using GEE
- [XT] xtgls Fit panel-data models by using GLS
- [XT] **xtheckman** Random-effects regression with sample selection
- [XT] xtivreg Instrumental variables and two-stage least squares for panel-data models
- [XT] **xtregar** Fixed- and random-effects linear models with an AR(1) disturbance
- [XT] **xtset** Declare data to be panel data
- [BAYES] bayes: xtreg Bayesian random-effects linear model
- [ME] mixed Multilevel mixed-effects linear regression
- [MI] Estimation Estimation commands for use with mi estimate
- [R] areg Linear regression with a large dummy-variable set
- [R] regress Linear regression
- [SP] spxtregress Spatial autoregressive models for panel data
- [TS] **forecast** Econometric model forecasting
- [TS] **prais** Prais-Winsten and Cochrane-Orcutt regression
- [U] 20 Estimation and postestimation commands