Converting from binary to decimal

Let's consider the three number systems below. Each of the symbols used by that number system is shown here for your reference:

Base thirteen	0	1	2	3	4	5	6	7	8	9	A	B	C
Base ten	0	1	2	3	4	5	6	7	8	9			
Base three	0	1	2										

From any base to decimal

In the base ten (decimal) number system, each position in a number represents a "value" multiplier of 10, as demonstrated by the following table:

Name:	Thousands' position	Hundreds' position	Tens' position	Ones' position
Decimal value:	10^3	10^2	10^1	10^0
Decimal representation:	1000	100	10	1

This means that if we have a 4 in the thousands' position, we know that we have four copies of 1000. Other number systems work the same way, except their "value" multipliers are different. While a base ten number system has a multiplier of 10, a base three number system has a multiplier of 3 and a base 13 number system has a multiplier of 13 and so on.

If we want to work out the decimal value of a number in any system, we can simply multiply the decimal equivalent of the number in each position with the value of that position, repeat for each position, and then add up the result.

This produces a simple algorithm:

1. **Set the current total to 0.**
2. **For each position in the current number:**
    a. **Calculate the decimal value of that position.**
    b. **Find the decimal equivalent of the number in that position.**
    c. **Multiply the result of (a) and (b)**
    d. **Add the result from (c) to the current total**

At the end of this algorithm, once every position in the current number has been parsed, the current total will now be equal to the decimal equivalent of the original number.

Let's look at a quick example where we convert the base three number 1201 into the decimal equivalent:

1. Let's set the current total (sum) to be equal to 0
2. Let's start with the first (far right) position:
    a. Because this is a base three number system, there are three possible numbers that could occupy this position. Because it is the first position, we find its value to be 3^0 = 1
    b. The number in this position is 1
    c. 1 * 1 = 1
    d. Sum = 0 + 1 = 1
3. Let's now move to the next position:
    a. 3^1 = 3
    b. The number in this position is 0
    c. 3 * 0 = 0
    d. Sum = 1 + 0 = 1
4. And the next:
    a. 3^2 = 9
    b. The number in this position is 2
    c. 9 * 2 = 18
    d. Sum = 1 + 18 = 19
5. And the final (far left) position:
    a. 3^3 = 27
    b. The number in this position is 1
    c. 27 * 1 = 27
    d. Sum = 19 + 27 = 46

The decimal equivalent of the number 1201 is therefore 46.

For another example, let's convert the base thirteen number 1B into the decimal equivalent:

6. Let's set the current total (sum) to be equal to 0
7. Let's start with the first (far right) position:

    a. Because this is a base thirteen number system, there are thirteen possible numbers that could occupy this position. Because it is the first position, we find its value to be $13^0 = 1$
    b. The number in this position is B. The decimal equivalent of A is 10 and the decimal equivalent of B is 11
    c. $1 * 11 = 11$
    d. Sum = $0 + 11 = 11$
8. Let’s now move to the second and final position:
    a. $13^1 = 13$
    b. The number in this position is 1
    c. $1 * 13 = 13$
    d. Sum = $11 + 13 = 24$

The decimal equivalent of the number 1B is therefore 24.

The same algorithm above can be used to convert any number system to base 10, including binary and hexadecimal. Return to your learning journal and complete the research questions for the second subsystem.