Converting from decimal to binary

Let's consider the three number systems below. Each of the symbols used by that number system is shown here for your reference:

Base thirteen	0	1	2	3	4	5	6	7	8	9	A	B	C
Base ten	0	1	2	3	4	5	6	7	8	9			
Base three	0	1	2										

Decimal to another base

Each number system has a maximum value that can be represented at each digit. For example, a binary number system only has two symbols at each position (0, 1), so the maximum decimal value that can be represented at a single position is 1 (the answer is 1 instead of 2 because we need to reserve one symbol to represent 0). A hexadecimal number system has sixteen symbols at each position (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F), so the maximum decimal value that can be represented at a single position is 15.

The maximum value at a single position of a decimal number system is 9. Therefore, the value 9 can be represented with a single position in the decimal number system. However, if we use a base 7 number system where the maximum value that can be represented with a single position is 6, we will need to use two positions to represent the value 9. The next question becomes, "How many positions do we need?"

We can answer this question using some division. Let's start by putting the maximum number in the first position of our base 7 number system. This would give us the value 6.

Position 3	Position 2	Position 1
$7^2 = 49$	$7^1 = 7$	$7^0 = 1$
		6

Using the table above, 6 ones gives us a total decimal value of 6. We are short of our original number 9 by 3. What do we do? Well, if we can't represent the number using a single position, we must start using the second. What happens if we move our 6 to the second position and leave a 0 as a placeholder? Well, we end up with this:

Position 3	Position 2	Position 1

$7^2 = 49$	$7^1 = 7$	$7^0 = 1$
	6	**0**

6 sevens give us a total of 42. We have shot past our target! Remember how we count using the decimal system. When we add 1 to a position that already has the highest possible symbol (9), we move to the next highest position but reset our count to 1. Let's try that with our base 7 number system. Let's move our 6 one position to the left but reset its value to 1.

Position 3	Position 2	Position 1
$7^2 = 49$	$7^1 = 7$	$7^0 = 1$
	1	**0**

1 seven gives us a total of 7. We have increased our value of 6 by one; however, we are still 2 short of 9! The next step is easy though. We know that we can represent any single value between 0 and 6 using the far right column, so let's add 2 to that like so:

Position 3	Position 2	Position 1
$7^2 = 49$	$7^1 = 7$	$7^0 = 1$
	1	**2**

1 seven gives us a total of 7, and 2 ones gives a total of 2. Add them together to get our original total of 9!

What does this mean? Well, we can do this process much quicker by asking ourselves the following question: "how many 9s can fit in the number 7?" The answer is 1 with a remainder of 2.

1 * 7 + **2** * 1 = **9**

What about converting the decimal number 60 to base 7?

1. How many 7s fit into 60? 8 with a remainder of **4**.

We now know that our answer is going to involve 8 copies of 7 with **4** left-over. We will put these 4 singles into the far right position. We can't put the 8 into the second position as the largest number we can place in any position using base 7 is 6. We, therefore, ask ourselves the following question:

2. How many 7s fit into 8? 1 with a remainder of **1**.

Let's put this remainder in the second position.

3. How many 7s fit into 1? 0 with a remainder of **1**.

We put this final value into the third position. We don't have any whole number results left, so we can stop. And thus, the number 60 in base 10 is the same as the number **114** in base 7.

We can use this approach to convert any decimal number into any other base. This is what the algorithm that we derived above looks like:

1. **Divide the number by the value of the new base (the value of base ten is 10 and the value of base two is 2 etc.). Note the *whole number* result and the *remainder*.**
2. **Use the *remainder* from step 1 as the rightmost (least significant) digit of the number in the new base.**
3. **Divide the *whole number* result from the last calculation by the value of the new base. Note the *whole number* result and the *remainder*.**
4. **Assign the *remainder* from step 3 as the digit in the next highest position of the number in the new base.**
5. **If the *whole number* result of the last calculation is zero, you have successfully converted the number. If not, return to step 3 and keep going.**

The following examples showcase how to use this method with a variety of conversions:

Base ten number 8 to base 3:

Calculation	Result (Base 10)	Remainder (Base 10)	Answer (Base 3)
8/3	2	2	2
2/3	0	2	**22**

Base ten number 9 to base 3:

Calculation	Result (Base 10)	Remainder (Base 10)	Answer (Base 3)
9/3	3	0	0
3/3	1	0	00
1/3	0	1	**100**

Base ten number 16 to base 3:

Calculation	Result (Base 10)	Remainder (Base 10)	Answer (Base 3)
16/3	5	1	1
5/3	1	2	21
1/3	0	1	**121**

Base ten number 8 to base 13:

Calculation	Result (Base 10)	Remainder (Base 10)	Answer (Base 13)
8/13	0	8	**8**

Base ten number 13 to base 13:

Calculation	Result (Base 10)	Remainder (Base 10)	Answer (Base 13)
13/13	1	0	0
1/13	0	1	**10**

Base ten number 23 to base 13:

Calculation	Result (Base 10)	Remainder (Base 10)	Answer (Base 13)
23/13	1	10	A
1/13	0	1	**1A**

Note that the decimal value of 10 corresponds to the symbol A in the base 13 system.

The same algorithm above can be used to convert to any number system from base 10. Return to your learning journal and complete the research questions for the third subsystem.