What is a number system?

Background

A number system is simply a way to write or represent numbers. Number systems represent numbers using a combination of symbols and rules to follow. Number systems also allow us to perform mathematical operations on these numbers like addition and subtraction. Different number systems have different techniques for performing these operations.

Tally marks are one example of a number system. When using tally marks to represent a number, we draw one vertical tally mark to represent a value of 1. Four of these tally marks represent the fact that we have four copies of this value of 1, corresponding to 4 in our decimal system. Oftentimes, to represent the value of 5 a horizontal line is drawn across four tally marks. This creates an effect where we can use groups of fives with a number of single tallies less than five to represent any number. Tally marks are a unary (base-one) number system. This means that to represent a number X, we need to have X individual tally marks.

We most commonly use the decimal number system in our everyday lives, which is an example of a denary (base-ten) number system. But why do different number systems exist in our modern world, and why do we need to bother converting between them? Lets try and answer these questions by going through a couple of examples.

Denary (decimal)

A unary system only has one symbol (e.g. a tally mark) however a denary (decimal) system has ten symbols (e.g. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9). Let's refer to the space taken up by a symbol as a position. You can only have one symbol in any given position at a time. Using our decimal system, we can represent up to ten different numbers using a single position (that position could have nothing or anything between 0-9), however, in a unary system, each position can only represent a single number (that position could have nothing or a tally mark). Note that in our decimal system, 10 isn't a single symbol, it is instead a combination of a 1 and a 0 symbol in two subsequent positions.

In both of these number systems, when we have reached the symbol with the highest value in a certain position (a single tally for unary or a 9 symbol in decimal), we move up to the next position. In unary, we place another tally mark (I -> II). In decimal, we place a 1 in front of the 9 and replace the 9 with a 0 as a placeholder (9 -> 10). Each position in the unary system represents a multiple of 1 (as there is one possible value), whereas each position in the decimal system represents a multiple of 10 (as there are ten possible values in each position). Let's consider the decimal number 428:

Name:	Thousands' position	Hundreds' position	Tens' position	Ones' position
Decimal value:	10^3	10^2	10^1	10^0
Decimal representation:	1000	100	10	1
		4	2	8

A 4 symbol in the hundreds' position represents the fact that we have four copies of 100, a 2 symbol in the tens' position represents the fact that we have two copies of 10 and an 8 symbol in the ones' position represents the fact that we have eight copies of 1.

4*100 + 2*10 + 8*1 = 428

So, why do we use a decimal system in our everyday lives? The fact that our number system uses ten symbols is relatively arbitrary. We could remove one of the 0-9 symbols from our numbers system and instead count in multiples of nine and get on just fine. It might be hard to get used to counting and performing calculations with a base nine number system but that is only because we are already used to base ten. A young child who hasn't learnt to count yet wouldn't have much more difficulty learning base nine as their first number system. The reason that we use base ten is likely due to the fact that most humans have ten fingers in total, and many early humans would have relied on their fingers to represent digits when counting.

Binary

Although humans commonly use our decimal 0-9 number system to perform our everyday maths, computers use a binary number system. In this binary system, there are only two possible symbols: 0 and 1.

The binary system works very similarly to our decimal system. It also uses positions that increase to the left. However, rather than each position going up to the symbol 9, each position only goes up to the symbol 1. Rather than each position increasing by a multiple of ten, each position increases by a multiple of two. Let's consider the binary number 101:

Name:	Eights' position	Fours' position	Twos' position	Ones' position
Decimal value:	2^3	2^2	2^1	2^0

Decimal representation:	8	4	2	1
Example:		1	0	1

A 1 symbol in the fours' position represents the fact that we have one copy of 4, a 0 symbol in the twos' position represents the fact that we have no copies of 2 and a 1 symbol in the ones' position represents the fact that we have eight copies of 1.

1*4 + 0*2 + 1*1 = 5

We can see how the decimal number 101 is the same as the decimal number 5.

Why do computers use a binary system? Computers are comprised of billions of tiny pieces of electronics called transistors. Each transistor can either be in an ON state or an OFF state. We can represent these two states using a 1 symbol for ON and a 0 symbol for OFF. Computers use large quantities of these transistors to store information based on the unique combination of ONs and OFFs. Because all information stored on computers uses this binary ON/OFF system, it also makes sense to represent and perform calculations on this information using a binary 0/1 number system for consistency.

Even more

Unary, binary and denary number systems are some of the most common, but there is no reason why we can't use a base 7 or a base 500 number system. In fact, base 16 (hexadecimal) is another common number system that is used to represent colours. Here is a table containing the symbols used for some common number systems:

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9						
Binary	0	1														
Tally	I															

Importance of conversions

So, why do we need to convert between different number systems, and where might a situation like this occur?

Well, let's return to our decimal and binary examples. Using binary is a hard limitation of modern computers. Computers are designed to represent information in one of two states and only this way (at least for all mainstream modern-day computers). However, humans use of a decimal number system predates computing. While most humans can understand the physical quantities that decimal numbers represent and how to perform calculations using them, most humans do not have the same understanding of binary. To help humans understand what a binary number represents, we need to first convert it into a different number system that they do understand (decimal).

Computers have to use binary, but humans don't have to use a decimal number system. So then, why don't all humans just start using binary so we don't have to do any conversions? Here are a few reasons:

1. Numbers take up a lot more space when written in binary compared to decimal. For example, the decimal number 2022 is written as 11111100110 in binary.
2. Humans already have a lot of history using a base ten number system. Trying to shift everyone on the planet to start using a different number system would be next to impossible as so many of our modern systems, like currency, are built upon a decimal system. Base ten makes sense for humans, with most of us having ten fingers on our hands.
3. There are more number systems used today than just binary. For example, we often use a hexadecimal number system to represent colours in our modern world.

The fact of the matter is that different number systems are ideal for different purposes. While having one universal number system would alleviate the need for difficult conversions, this theoretical "jack of all trades" number system would likely be less efficient or even incompatible for some specific applications than a custom-made alternative.