

# Vibing Math (Geometry Problem)

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## 1 Question

Consider an isosceles triangle  $ABC$  in which  $AB = BC = 10$  units. Let  $P_1, P_2, P_3, \dots, P_{60}$  be 60 points on  $BC$ . Then,

$$\sum_{i=1}^{60} (AP_i^2 + P_iB \times P_iC) = ?$$

## 2 Solution

By Stewart's Theorem,

$$(AB^2 \times P_iC) + (AC^2 \times P_iB) = BC(AP_i^2 + P_iB \times P_iC)$$

Now,  $AB = 10$  and  $AC = 10$ , so above expression becomes,

$$100(P_iB + P_iC) = BC(AP_i^2 + P_iB \times P_iC)$$

But here,  $P_iB + P_iC = BC$ , hence the above equation becomes

$$100(\cancel{P_iB + P_iC}) = \cancel{BC}(AP_i^2 + P_iB \times P_iC)$$

$$100 = (AP_i^2 + P_iB \times P_iC)$$

This is true for all  $i$  in  $P_i$ . Hence, the answer is

$$\sum_{i=1}^{60} (AP_i^2 + P_iB \times P_iC) = 60 \times 100$$

$$\boxed{= 6,000}$$