

# Messy Integral (@Vibingmath)

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## 1 Question

$$\int_1^e \frac{x - x \ln x + 1}{x(x+1)^2 + x \ln^2 x} dx$$

## 2 Solution

Let us take substitution  $\ln x = t$  so that  $dx = e^t dt$ , thus our integral becomes

$$\int_0^1 \frac{(e^t - te^t + 1)}{e^{t^2} [(e^t + 1)^2 + t^2]} e^t dt$$

Now dividing both numerator and denominator by  $t^2$  we get,

$$\int_0^1 \frac{\frac{e^t}{t^2} + \frac{1}{t^2} - \frac{e^t}{t}}{\left(\frac{e^t}{t} + \frac{1}{t}\right)^2 + 1} dt$$

Now taking a final substitution

$$\frac{e^t}{t} + \frac{1}{t} = u$$

so that

$$\left(\frac{e^t}{t} - \frac{e^t}{t^2} - \frac{1}{t^2}\right) dt = du$$

The integral simplifies as:

$$\int_{e+1}^{\infty} \frac{1}{1+u^2} du$$

Plugging in the limits, the answer is

$$\boxed{= \tan^{-1} \left( \frac{1}{1+e} \right)}$$