Algebra and Analytic Geometry Lecture

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16th October 2020

1 Complex numbers

Def: A complex number is a pair of real numbers.
Example: (2, 3) they are traditionally denoted by z, u, w .. .
The set of all complex numbers will be denoted by $\mathbb{C}$ so:
$\mathbb{C} = \{(x, y) : x, y \in \mathbb{R}\}$
if z = (x, y) $(x, y) \in \mathbb{R}$, then x is called the real part of z and denoted as $R_e z$ and y is the imaginary part.

Geometric interpretation:
A complex number z=(x, y) can be viewed as a point on a Cartesian plane. Such plane will be called the complex plane.

As Vectors on the plane with a initial point at the origin of the plane

2 Operations on complex numbers

1. Addition:
   f $z_1 = (x_1, y_1)$, $z_2 = (x_2, y_2) \in \mathbb{C}$, then their sum is defined to be $z_1 + z_2 = (x_1 + x_2, y_1 + y_2)$
   $(1, 0) + (b, 0) = (a + b, 0)$

2. Multiplication:
   if $z_1 = (x_1, y_1), z_2 = (x_1, y_1)$ then their product is defined to be the number
   $z_1 z_2 = (x_1 x_2 - y_1 y_2,\ x_1 y_2 + x_2 y_1)$
   Example: $(1,2) * (3,-1) = (1 * 3 - 2(-1),\ 1(-1), 3 * 2) = (5,5)$
   Properties: Commutative and Associative
   $(a,0)(b,0) = (ab,0)$
   $(0,1)(0,1) = (-1,0)$
   $z = (x,y) = (x,0) + (0,y) = (x,0) + (0,1)(y,0) = x + iy$

3. Better multiplication:
   $(3+2i)(1-2i) = (3+2i)1 + (3+2i)(2i) = 3 + 2i + 6i - 4 = -1 + 8i$

4. Inverse:
   $z = x + iy$
   $w = \frac{x}{x^2+y^2} = i\frac{-y}{x^2+y^2}$
   $w = \frac{1}{z}$

5. Conjugate:
   Properties:
   $\overline{z} = x + i(-y) = x - iy$

   (a) $\overline{z+w} = \overline{z} + \overline{w}$

   (b) $\overline{z * w} = \overline{z} * \overline{w}$

   (c) $\overline{z^n} = \overline{z}^n$

6. Modulus:
   if z=x+iy the modulus is $|z| = \sqrt{x^2+y^2}$