Elementary logic.

Definition 1 *A* ***statement*** *(assertion, proposition, closed sentence) is a (declarative) sentence that may be either true or false.*

Operations on statements (compound statements):
The truth table of a compound statement is the table describing logical value of the compound statement in terms of logical values of the atomic statements. Usually the value "true" is represented by 1 and 0 is used for "false".

Definition 2 *The* ***negation*** *of a statement p is the statement „not p", denoted by* $\neg p$*. It is true if and only if p is false.*

Truth table of the negation :

p	$\neg p$
0	1
1	0

Definition 3 *The* ***disjunction*** *of two statements p and q is the statement "p or q", denoted by* $p \vee q$*. It is false if and only if both p and q are false.*

Truth table of the disjunction:

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Definition 4 *The* ***conjunction*** *of two statements p and q is the statement "p and q", denoted by* $p \wedge q$*. It is true if and only if both p and q are true.*

Truth table of the conjunction:

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Definition 5 *The* ***implication*** *of two statements p and q is the statement "if p then q", denoted by* $p \Rightarrow q$*. It is false if and only if p is true and q is false.*

Truth table of the implication:

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Definition 6 *The* ***equivalence*** *(or biconditional) of two statements p and q is the statement "p if and only if q", denoted by $p \Leftrightarrow q$. It is true if and only if both p and q are either true or false.*

Truth table of the equivalence:

p	q	$p \Leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Definition 7 *A* ***tautology*** *(law of logic) is a statement which is always true.*

Quantifiers.

Open sentence is a statement depending on at least one parameter.

If $P(x)$ is an open sentence depending on a parameter $x \in X$ then we can create two new statements with the use of so called quantifiers:

1. Universal quantifier:

Statement $\bigwedge_{x \in X} P(x)$ reads "for every $x \in X$ $P(x)$" and it is true if and only if for every element $x \in X$ the statement $P(x)$ is true.

Instead of $\bigwedge$ sometimes symbol $\forall$ is being used.

1. Existential quantifier:

Statement $\bigvee_{x \in X} P(x)$ reads "there is $x \in X$ such that $P(x)$" and it is true if and only if there exists (at least one) element $x_o \in X$ such that the statement $P(x_o)$ is true.

Instead of $\bigvee$ sometimes symbol $\exists$ is being used.

The two most important tautologies involving quantifiers are so called generalized De Morgan's Laws:

$$\neg[\bigwedge_{x \in X} P(x)] \Leftrightarrow \bigvee_{x \in X} [\neg P(x)];$$

$$\neg[\bigvee_{x \in X} P(x)] \Leftrightarrow \bigwedge_{x \in X} [\neg P(x)].$$