Algebra and Analytic Geometry lecture

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9th October 2020

Contents

1 Sets of numbers

- Natural Numbers $N = \{1, 2, 3, 4, ...\}$
- Integers $Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Rational Numbers $Q = \{\frac{p}{q}; q, p \subset Z, q \neq 0\}$
- Real Numbers R*Any distance from 0 on the number line*

2 Operation law on numbers

1. Commutative law
   $a + b = b + a$
   $ab = ba$
2. Associative law
   $(a + b) + c = a + (b + c)$ $(ab)c = a(cb)$ $a(b + c) = ab + bc$

For $\{N, Z, Q, R\}$ all the laws listed above are true

3 Divisibility

$A|B$ if there is a $c \subset N$

4 Prime Numbers

A natural number $\neq 1$ is called a prime if it has only two divisors, namely 1 and itself.
$Examples : 2, 3, 5, 7, ...$

4.1 Theorem

Every natural number can be uniquely (up to orders of factors) described as a product of prime numbers.
$Example : 24 = 2 * 2 * 2 * 3$

5 Principle of Mathematical Induction

Law for proving statements
If $p_1, p_2, ..., p_k$ are statements and:

1. p_1 is true
2. if p_k is true, then p_{k+1} is also true

Example:

6 Binomial formula

Newtons formula for expanding powers of sums
$(x+y)^1 = x+y$
$(x+y)^2 = x^2+2xy+y^2$
$(x+y)^3 = x^3+3x^2y+3xy^2+x^2$
$(x+y)^4 =$ *To difficult to remember or to expand*

6.1 Theorem

$(a+b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{n}a^{n-n}b^n$

This can be simplified to:
$\sum\limits_{k=0}^{n} \binom{n}{k}a^{n-k}b^k$

6.2 Find a given coefficient of a given equation

Question: Find the coefficient of x^4 in $(2x - \frac{1}{x})^6$

$(2x - \frac{1}{x})^6 = (2x + \frac{-1}{x})^6$

$\sum\limits_{k=0}^{6} \binom{6}{k}(2x)^{6-k}(\frac{-1}{x})^k$
To calculate 'k' to substitute it into the equation we have to have only one x so:
$2^{6-k}x^{6-k} - x^{-k} -> x^{6-k-k} -> x^{6-2k}$
Based on that and the fact that we want to get the coefficient of x^4 we do the following:

$6-2k=4$
$k=1$
So we substitute and we get:

$\binom{6}{1}2x^{6-1} * (\frac{-1}{x})^1 = \frac{6!}{5!}2x^5 * \frac{-1}{x} = 6 * 2x^5 * \frac{-1}{x} = -12x^4$

Therefore the answer is -12

7 Complex numbers

Def: A complex number is a pair of real numbers.
Example: (2, 3) they are traditionally denoted by z, u, w .. .
The set of all complex numbers will be denoted by $\mathbb{C}$ so:
$\mathbb{C} = \{(x,y) : x,y \in \mathbb{R}\}$
if z = (x, y) $(x,y) \in \mathbb{R}$, then x is called the real part of z and denoted as R_ez and y is the imaginary part.

Geometric interpretation:
A complex number z=(x, y) can be viewed as a point on a Cartesian plane. Such plane will be called the complex plane.

As Vectors on the plane with a initial point at the origin of the plane

7.1 Operations on complex numbers

1. Addition:
   f $z_1 = (x_1, y_1)$, $z_2 = (x_2, y_2) \in \mathbb{C}$, then their sum is defined to be $z_1 + z_2 = (x_1 + x_2, y_1 + y_2)$
   $(1, 0) + (b, 0) = (a + b, 0)$

2. Multiplication:
   if $z_1 = (x_1, y_1), z_2 = (x_1, y_1)$ then their product is defined to be the number
   $z_1 z_2 = (x_1 x_2 - y_1 y_2,\ x_1 y_2 + x_2 y_1)$
   Example: $(1, 2) * (3, -1) = (1 * 3 - 2(-1),\ 1(-1), 3 * 2) = (5, 5)$
   Properties: Commutative and Associative
   $(a, 0)(b, 0) = (ab, 0)$
   $(0, 1)(0, 1) = (-1, 0)$
   $z = (x, y) = (x, 0) + (0, y) = (x, 0) + (0, 1)(y, 0) = x + iy$

3. Better multiplication:
   $(3 + 2i)(1 - 2i) = (3 + 2i)1 + (3 + 2i)(2i) = 3 + 2i + 6i - 4 = -1 + 8i$

4. Inverse:
   $z = x + iy$
   $w = \frac{x}{x^2+y^2} = i\frac{-y}{x^2+y^2}$
   $w = \frac{1}{z}$

5. Conjugate:
   Properties:
   $\overline{z} = x + i(-y) = x - iy$

   (a) $\overline{z + w} = \overline{z} + \overline{w}$
   (b) $\overline{z * w} = \overline{z} * \overline{w}$
   (c) $\overline{z^n} = \overline{z}^n$

6. Modulus:
   if $z = x + iy$ the modulus is $|z| = \sqrt{x^2 + y^2}$