Mathematical Analysis
Lecture

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1 Rules

No textbook, so take notes.
Classes are mandatory

2 Requirements

During the classes we will start with a quiz, every practice. To pass the course you need 50% of points from the quizzed. A Quizes is 15min every quiz is worth 5 points. You get points from your top 10 quizes.

3 Notation

3.1 Number sets

1. Natural Numbers $N = \{1, 2, 3, ...\}$
2. Integers $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$
3. Rational $\mathbb{Q} = \{\frac{p}{q}; p, q \in Z, q \neq 0\}$
4. Irrational $ex. : \sqrt{2}, \pi, ...$
5. Real Numbers $\mathbb{R} = Rational + Rational$

3.2 Sets notation

$(a,b) = x \in R : a < x < b$
$(a,b) = x \in R : a \leq x \leq b$
$(a,\infty) = x \in R : x > a$

$(a,b) - openinterval$
$[a,b] - closedinterval$

$A \subset B$ A is a subset of B
$x \in A$ X is an element of A, x belongs to A
$x \notin A$ X is not an element of A, x does not belong to A

3.3 Cartesian Product

Given two sets A and B, we can form the set consisting of all ordered pairs of the form (a,b) where $a \in A$ and $b \in B$. This set is called the Cartesian product of A and B and is denoted by AxB
$AxB\{(a,b) : a \in A, \in B\}$
If $A = B$, then AxA is denoted by A^2

3.4 Quantifiers

1. Existential $\exists$ "There exists x such that", "For at least one x"

2. Universal $\forall$ "For all x", "For each x", "For every x"

Example:
$\exists\, t > 0\ \forall\, x \in \mathbb{R}\ x^2 + 4x + 4 > t$
The statement above is false
The negation of the statement:
$\forall\, t > 0\ \exists\, x_0 \in \mathbb{R}\ x_0^2 + 4x_0 + 4 \leq t$

4 Functions

A function f is a rule that assigns to each element x in a set A exactly one element , called f(x), in a set B.
In our class $A \subset \mathbb{R}$ and $B \subset \mathbb{R}$ The set A is called the domain of the function f

and will be denoted D_f.
The range of the function f is the set of all possible values of f(x) as x varies throughout the domain. The range of f will be denoted by R_f.
The most common method for visualizing a function is its.
If f is a function with domain D_f then its graph is the set of ordered pairs.
$\{(x, y) \in \mathbb{R}^2 \; x \in D_f, y = f(x)\}$

Example:
Min function

$$\begin{gathered} f(x) = min\{x, x^2\} \\ f(2) = min\{2, 4\} = 2 \\ f(\frac{1}{2}) = min\{\frac{1}{2}, \frac{1}{4}\} = \frac{1}{4} \end{gathered} \tag{1}$$

Absolute

$$\begin{gathered} f(x) = |x| = \{x, if x \geq 0 \; or - x, if x - 2 < 0\} \\ f(x) = |x - 2| = \{x - 2, if x \geq 0 \; or - (x - 2), if x - 2 < 0\} \end{gathered} \tag{2}$$

$|x - a|$ represents the distance between x and a