

Mathematical Analysis

Lecture

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1 Rules

No textbook, so take notes.

Classes are mandatory

2 Requirements

During the classes we will start with a quiz, every practice. To pass the course you need 50% of points from the quizzed. A Quizzes is 15min every quiz is worth 5 points. You get points from your top 10 quizzes.

3 Notation

3.1 Number sets

1. Natural Numbers $N = \{1, 2, 3, \dots\}$
2. Integers $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
3. Rational $\mathbb{Q} = \{\frac{p}{q}; p, q \in \mathbb{Z}, q \neq 0\}$
4. Irrational *ex.* : $\sqrt{2}, \pi, \dots$
5. Real Numbers $\mathbb{R} = \text{Rational} + \text{Irrational}$

3.2 Sets notation

$$(a, b) = \{x \in \mathbb{R} : a < x < b\}$$

$$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

$$(a, \infty) = \{x \in \mathbb{R} : x > a\}$$

$$(a, b) - \text{open interval}$$

$$[a, b] - \text{closed interval}$$

$A \subset B$ A is a subset of B

$x \in A$ x is an element of A, x belongs to A

$x \notin A$ x is not an element of A, x does not belong to A

3.3 Cartesian Product

Given two sets A and B, we can form the set consisting of all ordered pairs of the form (a, b) where $a \in A$ and $b \in B$. This set is called the Cartesian product of A and B and is denoted by $A \times B$

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

If $A = B$, then $A \times A$ is denoted by A^2

3.4 Quantifiers

1. Existential \exists "There exists x such that", "For at least one x"

2. Universal \forall "For all x", "For each x", "For every x"

Example:

$$\exists t > 0 \forall x \in \mathbb{R} \quad x^2 + 4x + 4 > t$$

The statement above is false

The negation of the statement:

$$\forall t > 0 \exists x_0 \in \mathbb{R} \quad x_0^2 + 4x_0 + 4 \leq t$$

4 Functions

A function f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

In our class $A \subset \mathbb{R}$ and $B \subset \mathbb{R}$ The set A is called the domain of the function f

and will be denoted D_f .

The range of the function f is the set of all possible values of $f(x)$ as x varies throughout the domain. The range of f will be denoted by R_f .

The most common method for visualizing a function is its.

If f is a function with domain D_f then its graph is the set of ordered pairs.

$$\{(x, y) \in \mathbb{R}^2 \mid x \in D_f, y = f(x)\}$$

Example:

Min function

$$f(x) = \min\{x, x^2\}$$

$$f(2) = \min\{2, 4\} = 2$$

$$f\left(\frac{1}{2}\right) = \min\left\{\frac{1}{2}, \frac{1}{4}\right\} = \frac{1}{4}$$

Absolute

$$f(x) = |x| = \{x, \text{if } x \geq 0 \text{ or } -x, \text{if } x < 0\}$$

$$f(x) = |x - 2| = \{x - 2, \text{if } x \geq 2 \text{ or } -(x - 2), \text{if } x < 2\}$$

$|x - a|$ represents the distance between x and a