Algebra and Analytic Geometry lecture

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1 Sets of numbers

- Natural Numbers $N = \{1, 2, 3, 4, ...\}$
- Integers $Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Rational Numbers $Q = \{\frac{p}{q}; q, p \subset Z, q \neq 0\}$
- Real Numbers *RAny distance from 0 on the number line*

2 Operation law on numbers

1. Commutative law
   $a + b = b + a$
   $ab = ba$
2. Associative law
   $(a + b) + c = a + (b + c)$ $(ab)c = a(cb)$ $a(b + c) = ab + bc$

For $\{N, Z, Q, R\}$ all the laws listed above are true

3 Divisibility

$A|B$ if there is a $c \subset N$

4 Prime Numbers

A natural number $\neq 1$ is called a prime if it has only two divisors, namely 1 and itself.
Examples : $2, 3, 5, 7, ...$

4.1 Theorem

Every natural number can be uniquely (up to orders of factors) described as a product of prime numbers.
Example : $24 = 2 * 2 * 2 * 3$

5 Principle of Mathematical Induction

Law for proving statements
If $p_1, p_2, ..., p_k$ are statements and:

1. p_1 is true
2. if p_k is true, then p_{k+1} is also true

Example:

6 Binomial formula

Newtons formula for expanding powers of sums
$(x+y)^1 = x + y$
$(x+y)^2 = x^2 + 2xy + y^2$
$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + x^2$
$(x+y)^4 =$ *To difficult to remember or to expand*

6.1 Theorem

$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{n}a^{n-n}b^n$

This can be simplified to:
$\sum_{k=0}^{n} \binom{n}{k}a^{n-k}b^k$

6.2 Find a given coefficient of a given equation

Question: Find the coefficient of x^4 in $(2x - \frac{1}{x})^6$

$(2x - \frac{1}{x})^6 = (2x + \frac{-1}{x})^6$

$\sum_{k=0}^{6} \binom{6}{k}(2x)^{6-k}(\frac{-1}{x})^k$
To calculate 'k' to substitute it into the equation we have to have only one x so:
$2^{6-k}x^{6-k} - x^{-k} -> x^{6-k-k} -> x^{6-2k}$
Based on that and the fact that we want to get the coefficient of x^4 we do the following:

$6 - 2k = 4$
$k = 1$
So we substitute and we get:

$\binom{6}{1}2x^{6-1} * (\frac{-1}{x})^1 = \frac{6!}{5!}2x^5 * \frac{-1}{x} = 6 * 2x^5 * \frac{-1}{x} = -12x^4$

Therefore the answer is -12