

Homework 2

1. Show that the following k are PD functions:

(a) $k(x, y) = \cos(x - y)$ over $\mathbb{R} \times \mathbb{R}$.

(b) $k(x, y) = (x + y)^{-1}$ over $(0, +\infty) \times (0, +\infty)$.

2. Consider two kernels k_1, k_2 and let $k = k_1 + k_2$. Show that

$$\|f\|_{\mathcal{H}_k}^2 = \min_{f=f_1+f_2} \left\{ \|f_1\|_{\mathcal{H}_{k_1}}^2 + \|f_2\|_{\mathcal{H}_{k_2}}^2 \right\}.$$

3. Suppose k is a bounded kernel with $\sup_x \sqrt{k(x, x)} \leq B$. Let $\mathcal{H}_C = \{h \mid \|h\|_{\mathcal{H}_k} \leq C\}$. Prove that

$$\text{Rad}_n(\mathcal{H}_C) \leq \frac{CB}{\sqrt{n}}.$$

(Hint: use the reproducing property $f(x) = \langle f, k(\cdot, x) \rangle$)