Transmission-Line Transformers

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Abstract—**The radio-frequency transformers described in this paper consist of matched transmission lines of equal length and characteristic impedance. The lines are connected according to rules given in the text. These transformers exhibit a very broad frequency response which can be readily estimated; two methods of analysis are presented. The computations agree well with test results.**

I. Introduction

A. Background

IN 1944, Guanella [1] reported a transformer of novel design which consisted of two interconnected transmission lines. The transmission lines were coiled in order to suppress undesirable current modes. Later, when ferrites became available, the transmission lines were wrapped on ferrite cores. Such a transformer has been in use for some time in all television receivers, where it provides an impedance match between the 300-Ω antenna and the 75-Ω tuner input. The device has a wide frequency response, yet its construction is very simple: four wires wrapped on a ferrite core. The wires form two 150-Ω transmission lines that are connected in series at the 300-Ω end, and in parallel at the 75-Ω end. In 1959, Ruthroff [2] reported new transmission line circuits implemented with ferrite toroids.[1]

Innovative companies quickly adopted and perfected the new ideas. During 1964–1965, a generation of ultra-wide-band components [3], sometimes referred to as "ferrite-loaded broad-band devices," emerged on the market. In this paper, Guanella's circuit is generalized and the design of arbitrarily complex transmission-line transformers (TLT's), is presented.

B. The Model

The TLT consists of two-wire lines, most frequently implemented as twisted-wire pairs. The wire diameter, the insulation thickness, and, to some extent, the twisting pitch determine the characteristic impedance z_o. Transmission lines relative to ground are unintentional and undesirable. Throughout this paper it is assumed that 1) all transmission lines are of equal length l, 2) the lines are connected at their ends only, and 3) they are not coupled magnetically or capacitively.

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[1] A 4 : 1 impedance transformer discussed in detail by Ruthroff [2, fig. 3 and Appendix A] does not fit into our classification of transmission-line transformers.

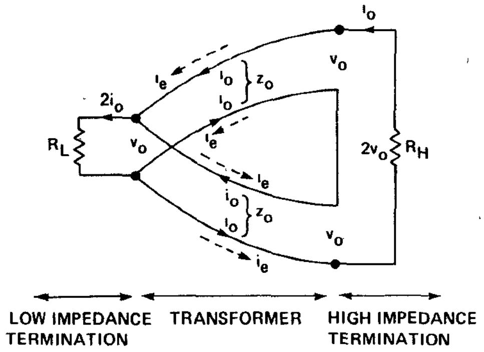

Fig. 1. Current modes in a transmission-line transformer (TLT).

C. Current Modes in a TLT

It is convenient to decompose the currents which circulate in a TLT into odd-mode currents i_o, and even-mode currents i_e, as illustrated in Fig. 1. The odd-mode currents couple with both the generator and the load and provide the desirable transformation ratio ($2i_o/i_o = 2:1$ in Fig. 1). The even-mode currents usually couple with only one termination, shunting the other; in rare cases, they couple with both the generator and the load. In the former instance they restrict the passband of the transformer and must be suppressed. The even-mode currents can be suppressed by increasing the even-mode impedance, which is accomplished by winding the lines on ferrite cores. The odd-mode currents of a tightly twisted transmission line generate a negligible external magnetic field which does not couple with the ferrite. The even-mode currents generate a strong magnetic field which is greatly affected by the ferrite.

II. Design

A. Design Approach

Since the transformation properties of the TLT's are determined by the odd-mode currents, the even-mode currents are ignored during the design procedure. The transformers so designed are then analyzed taking both modes into account.

B. Construction Algorithm

The number of transmission lines which comprise a transformer is termed the *order* of the TLT. An order-m transformer is a two-terminal pair device which consists of m connected lines. A TLT of order-$(m+1)$ is obtained by

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connecting a new transmission line to the *terminals* of the order-m transformer, in parallel at one end and in series at the other. Parallel (or serial) connection at both ends is excluded because it does not alter the transformation ratio.

A transformer of order-1 is a single transmission line, whose current and voltage are denoted i_o and v_o, respectively. An order-2 TLT is formed when a second line is connected in series at one end and in parallel at the other end. As a result of the serial connection between the two lines, the current in the second line equals the current in the first line i_o; as a result of the parallel connection the voltage across the second line equals the voltage across the first line v_o. The terminal parameters of the order-2 TLT are v_o, $2i_o$ at one end and $2v_o$, i_o at the other end (Fig. 1). In general, the terminal parameters of a TLT are Lv_o, Hi_o at one end and Hv_o, Li_o at the other end, where L and H are integer numbers.

C. Synthesis Procedure

Transformers of $r:1$ voltage ratio have the simplest configuration; they consist of r transmission lines, all of them connected in series at one end and in parallel at the other.

The synthesis procedure for an arbitrary integer voltage ratio follows. An $H:L$ $(H>L)$ voltage ratio transformer is decomposed into an $(H-L):L$ ratio TLT and a transmission line which is connected in series with the $H-L$ side and in parallel with the L side. The procedure is repeated until a $1:1$ transformer is reached.

For example, suppose that a $2.5:1$ *impedance* transformer is desired. This ratio may be approximated by $5:3$, $8:5$, $11:7$, etc., *voltage* ratios. The simplest transformer is selected and is decomposed into $(5-3):3=2:3$, $(3-2):2=1:2$, and $(2-1):1=1:1$ *voltage* ratio transformers. Fig. 2 shows the circuit diagram and illustrates the details of TLT operation. The voltage (current) transformation ratios of the first five orders of TLT's are shown in Fig. 3.

D. Characteristic Impedance

Let z_o denote the characteristic impedance of the bottom line in Fig. 1. Since the voltage and current of the top line are identical with those of the bottom line, the characteristic impedance of the top line must also be z_o. The above observation applies in general; all transmission lines in a TLT must have identical characteristic impedance z_o. The proof is straightforward. Consider a TLT with terminal parameters Lv_o, Hi_o at one end and Hv_o, Li_o at the other end. Suppose that a new line is connected, at one end in parallel with Hv_o, at the other end in series with Hi_o. Its characteristic impedance, again, must be $Hv_o/Hi_o=z_o$.

The value of z_o is determined from the specified terminal impedances, $R_L=z_0L/H$ and $R_H=z_0H/L$. For example, if $L/H=3:5$ and R_L is specified as 50 Ω, $z_o=83$ Ω. Note that such a TLT will exhibit the best frequency response between $R_L=50$ Ω and $R_H=139$ Ω, because only then will the constituent 83-Ω lines be properly terminated.

Before the transformer is constructed, the characteristic impedance of the transmission lines must be adjusted to the design value z_o. This can be done by trial and error

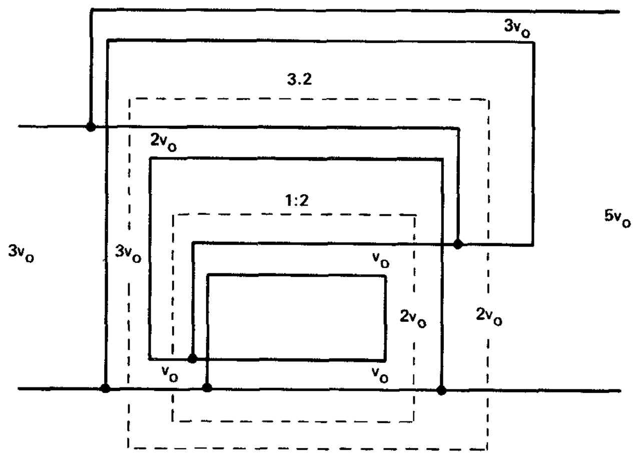

Fig. 2. A 5:3 voltage ratio transformer.

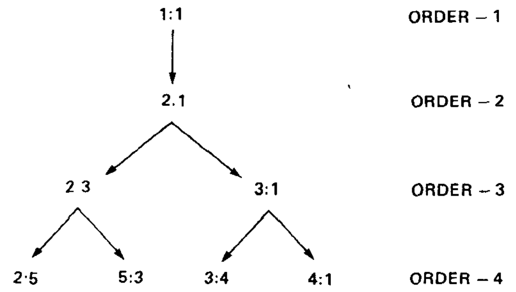

Fig. 3. The voltage transformation ratios of the first four orders of TLT's.

using wires of different diameters and insulation thicknesses. A time-domain reflectometer (TDR) is a suitable instrument for controlling the z_o value attained.

III. Analysis

For the sake of clarity, the analysis methods outlined below are illustrated with simple transformers. However, they can be applied readily to transformers of arbitrary complexity.

A. Method 1

This method employs the transformer's low-frequency equivalent circuit diagram.

First, two basic transmission line structures are analyzed. Fig. 4 shows an *in-phase* $1:1$ voltage transformer. The even-mode currents couple with both the generator and the load, and, consequently, the frequency response is unlimited. Fig. 5 shows a *reversing* $1:-1$ voltage transformer. The even-mode currents circulate in a shunting short-circuited stub. For low frequencies, the stub can be replaced with the wire inductance L_w. If ferrite is used, it increases this inductance to L_p and introduces losses. For convenience, these losses are represented with a parallel resistance R_p. In the equivalent circuit shown in Fig. 5(c), the parallel combination L_p–R_p, designated as Z_p, determines the voltage insertion loss

$$\left(1+\frac{R_0}{2R_p}\right)\left(1+\frac{\dfrac{R_0R_p}{R_0+2R_p}}{j\omega L_p}\right). \qquad (1)$$

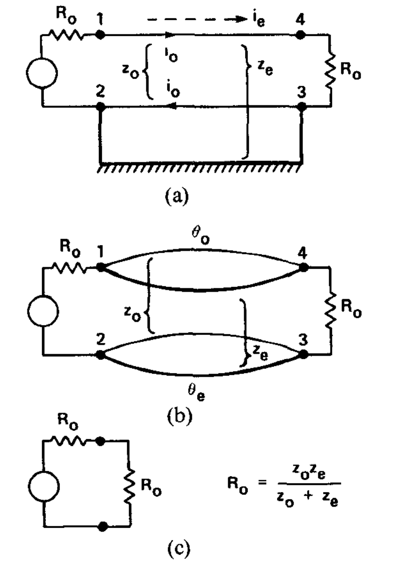

Fig. 4. An *in-phase* 1:1 voltage transformer. (a) Circuit diagram. (b) Equivalent circuit. (c) Low-frequency equivalent circuit.

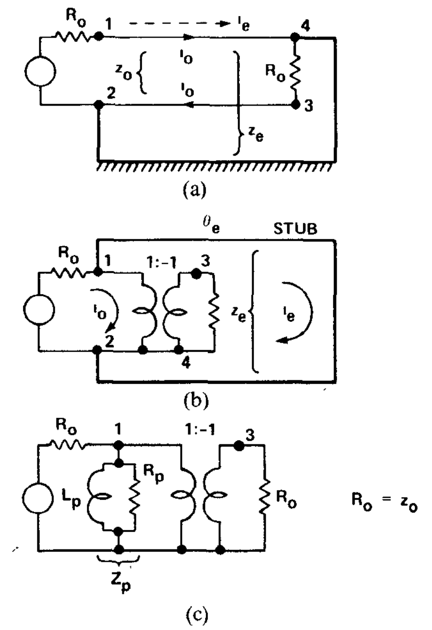

Fig. 5. A *reversing* 1: −1 voltage transformer. (a) Circuit diagram. (b) Equivalent circuit. (c) Low-frequency equivalent circuit.

L_p and R_p are the equivalent parallel inductance and loss resistance of one wire wound on ferrite core. They can be measured conveniently with an R–X Meter (Boonton Radio, Type 250-A) from 0.5 to 250 MHz.

Similar analyses can be applied to higher order TLT's. The TLT shown in Fig. 6 consists of an *in-phase* transformer 1-2-3-4 and a *reversing* transformer 1-2-4-8. Only the latter affects the frequency response. The shunting impedance Z_2 is determined by the F_2 ferrite core.

Another order-2 TLT, shown in Fig. 7, consists of two *reversing* transformers. Both Z_1 (determined by F_1) and Z_2 (determined by F_2) affect the frequency response. The voltage insertion loss is calculated with (1), if appropriate impedance transformations are made in accordance with the equivalent circuit diagrams.

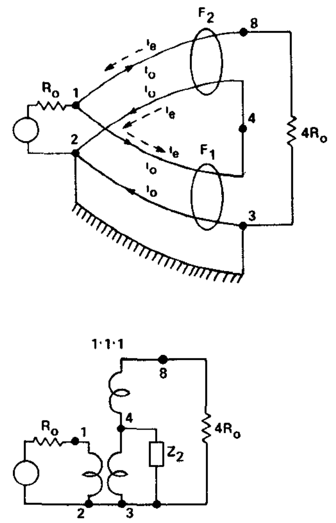

Fig. 6. An order-2 transformer and its low-frequency equivalent circuit.

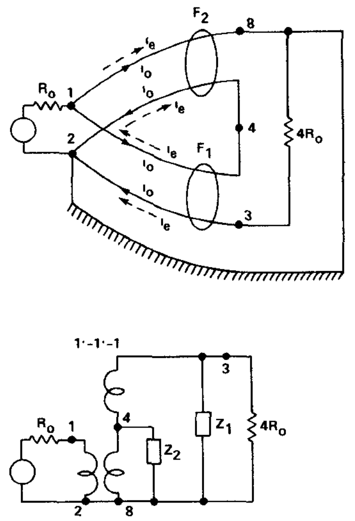

Fig. 7. An order-2 transformer and its low-frequency equivalent circuit.

B. Method 2

This method uses the admittance matrix of the TLT. The insertion loss is calculated from it by familiar matrix manipulations.

Each transmission line (two conductors running parallel to the ground plane) can be represented with a 4×4 admittance matrix [4]. (The ground plane coincides with the reference node.) Let matrix Y_1 represent transmission line 1, Y_2—line 2, etc. In general, $Y_1 \neq Y_2$, etc., because some transmission lines may use ferrite and some may not.

The admittance matrix of the transformer can be derived from $Y_1, Y_2, \cdots$ in two steps. First, the transmission lines are considered disconnected. The assembly of disconnected lines is represented by the following matrix:

$$\begin{bmatrix} Y_1 & 0 & 0 & \cdot \\ 0 & Y_2 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}. \tag{2}$$

Then, transmission-line terminals are connected in accor-

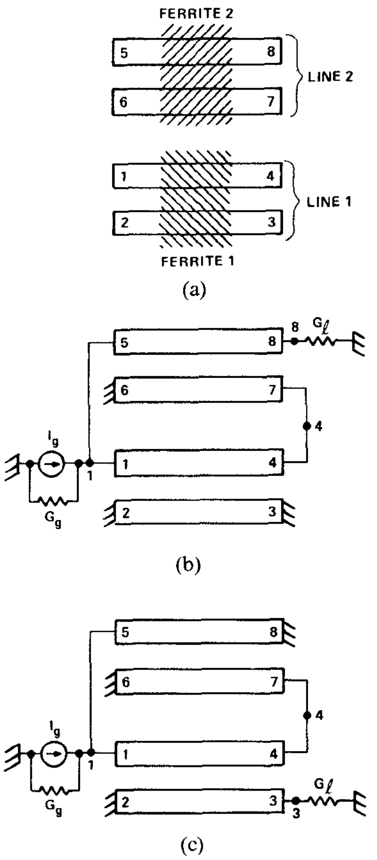

Fig. 8. Two order-2 transformers, (b) and (c), obtained by appropriate connections between disconnected transmission lines, (a). This represents a top view of the transformers. The lines run parallel to the ground plane which coincides with the surface of the drawing.

dance with the desired TLT configuration and the admittance matrix is modified correspondingly.

1) When node j is merged with node i, row j and column j are eliminated and row i and column i are modified [5], $Y_{ik}(\text{new}) = Y_{ik}(\text{old}) + Y_{jk}(\text{old})$, $Y_{ki}(\text{new}) = Y_{ki}(\text{old}) + Y_{kj}(\text{old})$, for all $k \neq i$ and $Y_{ii}(\text{new}) = Y_{ii}(\text{old}) + Y_{jj}(\text{old}) + Y_{ij}(\text{old}) + Y_{ji}(\text{old})$.

2) When a node is grounded, the corresponding row and column are eliminated.

The admittance matrix of the transformer shown in Fig. 8(b) is

$$\begin{array}{c} \\ 1 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 4 & 8 \end{array} \\ \begin{bmatrix} Y_{11}+Y_{55}+G_g & Y_{14}+Y_{57} & Y_{58} \\ Y_{41}+Y_{75} & Y_{44}+Y_{77} & Y_{78} \\ Y_{85} & Y_{87} & Y_{88}+G_l \end{bmatrix} \end{array}. \tag{3}$$

The voltage insertion loss is $\Delta/(2\Delta_{18}\sqrt{G_g G_l})$, where Δ is the determinant and Δ_{18} is a cofactor of the above matrix.

The admittance matrix of the transformer shown in Fig. 8(c) is

$$\begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} Y_{11}+Y_{55}+G_g & Y_{13} & Y_{14}+Y_{57} \\ Y_{31} & Y_{33}+G_l & Y_{34} \\ Y_{41}+Y_{75} & Y_{43} & Y_{44}+Y_{77} \end{bmatrix} \end{array}. \tag{4}$$

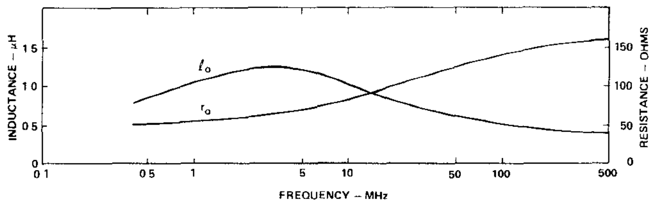

Fig. 9. Measured parallel equivalent inductance and resistance per bead.

The voltage insertion loss is $\Delta/(2\Delta_{13}\sqrt{G_g G_l})$, where Δ and Δ_{13} apply to the above matrix. The admittance matrix of TLT's can also be written by inspection. The admittance matrix elements are[2]

For line 1

$$\begin{aligned} Y_{11} &= Y_{22} = Y_{33} = Y_{44} = Y_{0o1}\coth\gamma_{o1}l + Y_{0e1}\coth\gamma_{e1}l \\ Y_{12} &= Y_{21} = Y_{34} = Y_{43} = -Y_{0o1}\coth\gamma_{o1}l + Y_{0e1}\coth\gamma_{e1}l \\ Y_{13} &= Y_{31} = Y_{24} = Y_{42} = Y_{0o1}\operatorname{csch}\gamma_{o1}l - Y_{0e1}\operatorname{csch}\gamma_{e1}l \\ Y_{14} &= Y_{41} = Y_{23} = Y_{32} = -Y_{0o1}\operatorname{csch}\gamma_{o1}l - Y_{0e1}\operatorname{csch}\gamma_{e1}l. \end{aligned} \tag{5}$$

For line 2

$$Y_{55} = Y_{66} = Y_{77} = Y_{88} = Y_{0o2}\coth\gamma_{o2}l + Y_{0e2}\coth\gamma_{e2}l, \text{ etc.} \tag{6}$$

Hyperbolic functions are used because of losses caused by the ferrite. The odd-mode and even-mode propagation constants are

$$\gamma_o = j\omega\sqrt{L'_w(C'_a + 2C'_{ab})} \quad \text{and} \quad \gamma_e = \sqrt{(R'_s + j\omega L'_s)j\omega C'_a}. \tag{7}$$

The odd-mode and even-mode characteristic admittances are

$$Y_{0o} = \sqrt{(C'_a + 2C'_{ab})/L'_w} \quad \text{and} \quad Y_{0e} = \sqrt{j\omega C'_a/(R'_s + j\omega L'_s)}. \tag{8}$$

All primed quantities are per unit length. L'_w is wire inductance without ferrite. L'_s and R'_s are series equivalent wire inductance and loss resistance in the presence of ferrite; they depend on the frequency. C'_a is capacitance of one wire to ground and C'_{ab} is capacitance between the transmission line wires.

Method 2 is based on the assumption that the transmission lines to ground are homogeneous. However, this condition is not required for TLT operation. Although the method is rigorous, it requires a ferrite characterization (L'_s and R'_s) usually unavailable above 250 MHz.

C. Ferrite Core

TLT's use ferrite core in two forms, beads and toroids [3]. In the first case, the transmission line is threaded through n_B beads. The equivalent parallel inductance and loss resistance are linearly proportional to n_B: $L_p = n_B l_0$ and $R_p = n_B r_0$, where l_0 and r_0 are inductance and resis-

[2]Sign errors appear in the $Y_{12} = \cdots$ and $Y_{14} = \cdots$ parameters of the Jones and Bolljahn paper, [4].

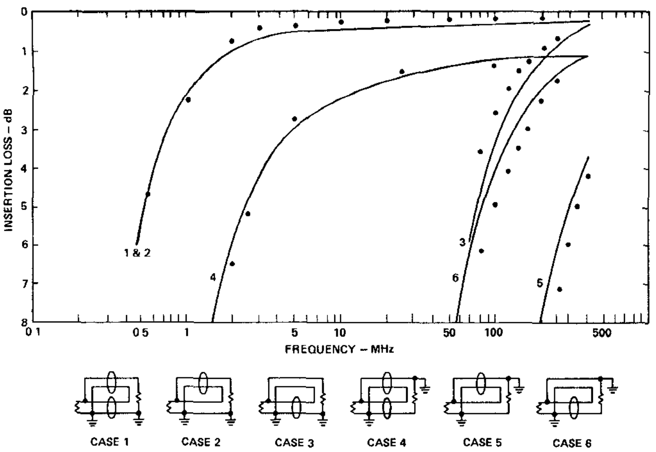

Fig. 10. Calculated (solid line) and measured (dots) insertion loss of an order-2 transformer for various ferrite and grounding combinations, as indicated by the six cases.

tance per bead. Measured values of l_0 and r_0 for a particular ferrite bead are shown in Fig. 9.

If a toroid core is used, the transmission line is wound n_t turns around it. In this case, $L_p = n_t^2 l_0$ and $R_p = n_t^2 r_0$.

Several transmission lines of a transformer may be wound on a single toroid, as demonstrated by Ruthroff [2]. Correct winding sense must be observed, so that the magnetic flux generated by each even-mode current flows in the same direction in the core.

The high-frequency cutoff of TLT's is inversely proportional to the length of the transmission lines. Geometric considerations indicate that for a given L_p, the bead designs require the same wire length as the toroid designs. However, toroid designs are more compact.

D. Insertion-Loss Measurements

In order to assess the feasibility of method 1, an order-2 transformer was tested from 0.5 to 500 MHz.

The transformer operated between 50- and 200-Ω terminal impedances, thus requiring 100-Ω transmission lines. Such a characteristic impedance was obtained with teflon-insulated wire twisted to 30 turns/130 mm. The wire used had a 0.255-mm conductor diameter (size 30) and a 0.483-mm overall diameter. The transmission lines length was 38 mm and the wire inductance L_w was 35 nH. The transmission lines were threaded through six ferrite beads. The inductance and parallel loss resistance per bead are shown in Fig. 9.

The measured and calculated insertion losses for various ferrite and grounding combinations of the transformer are shown in Fig. 10. Good agreement between them indicates that the analytical method is adequate.

Several conclusions can be drawn from this figure: 1) not all transmission lines require ferrite (cf. cases 1 and 2); and 2) grounding of a TLT is not a trivial matter (cases 4, 5, and 6 are clearly inferior to their counterpart cases 1, 2, and 3). For a wide frequency response, grounding must create *in-phase* transformers. Those transmission lines which are *in-phase* transformers do not need ferrite.

IV. Summary

TLT's consist of lines of equal length and characteristic impedance. The characteristic impedance value depends on the specified terminal impedances. The number of transmission lines and their interconnections depend on the required transformation ratio. Any integer voltage ratio may be obtained.

The frequency response is very wide. The low-frequency response is affected by undesirable even-mode currents and can be extended by using ferrite cores on some of the transmission lines. The choice of grounding points is important. In theory, the high-frequency response depends primarily on the length of the transmission lines, because at these frequencies ferrite no longer exhibits magnetic properties. In practice, transmission-line discontinuities (stray reactances) at line junctions are the determining factor.

Several methods can be used for analyzing these transformers. Adequate agreement with measured performance can be obtained with a simple model.

References

[1] G. Guanella, "Nouveau transformateur d'adaptation pour haute fréquence," *Rev. Brown Boveri*, pp. 327–329, Sept. 1944.
[2] C. L. Ruthroff, "Some broad-band transformers," *Proc. IRE*, vol. 47, pp. 1337–1342, Aug. 1959.
[3] Cover Feature, "Two ultra-broadband hybrids," *Microwaves*, pp. 106–107, Mar. 1965.
[4] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 75–81, Apr. 1956.
[5] J. Shekel, "Voltage reference node—Its transformation in nodal analysis," *Wireless Eng.*, vol. 31, pp. 6–19, Jan. 1954.