Chapter 7
Noise

中科大电子科学与技术系

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教材：模拟CMOS集成电路设计

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7.1 Statistical Characteristics of Noise

Noise limits the minimum signal level that a circuit can process with acceptable quality. Today's analog designers constantly deal with the problem of noise because it trades with power dissipation, speed, and linearity.

Noise is a random process.

construct a "statistical model" for the noise.

R上平均功率
$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} \frac{x^2(t)}{R_L} dt.$$

随机噪声均值=0

X为有效值

To simplify calculations, we write the definition of P_{av} as

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t) dt, \qquad (7.3)$$

系统性能：信噪比

where P_{av} is expressed in V^2 rather than W. The idea is that if we know P_{av} from (7.3), then the actual power delivered to a load R_L can be readily calculated as P_{av}/R_L. In analogy with deterministic signals, we can also define a root-mean-square (rms) voltage for noise as $\sqrt{P_{av}}$ where P_{av} is given by (7.3).

7.1.1 Noise Spectrum

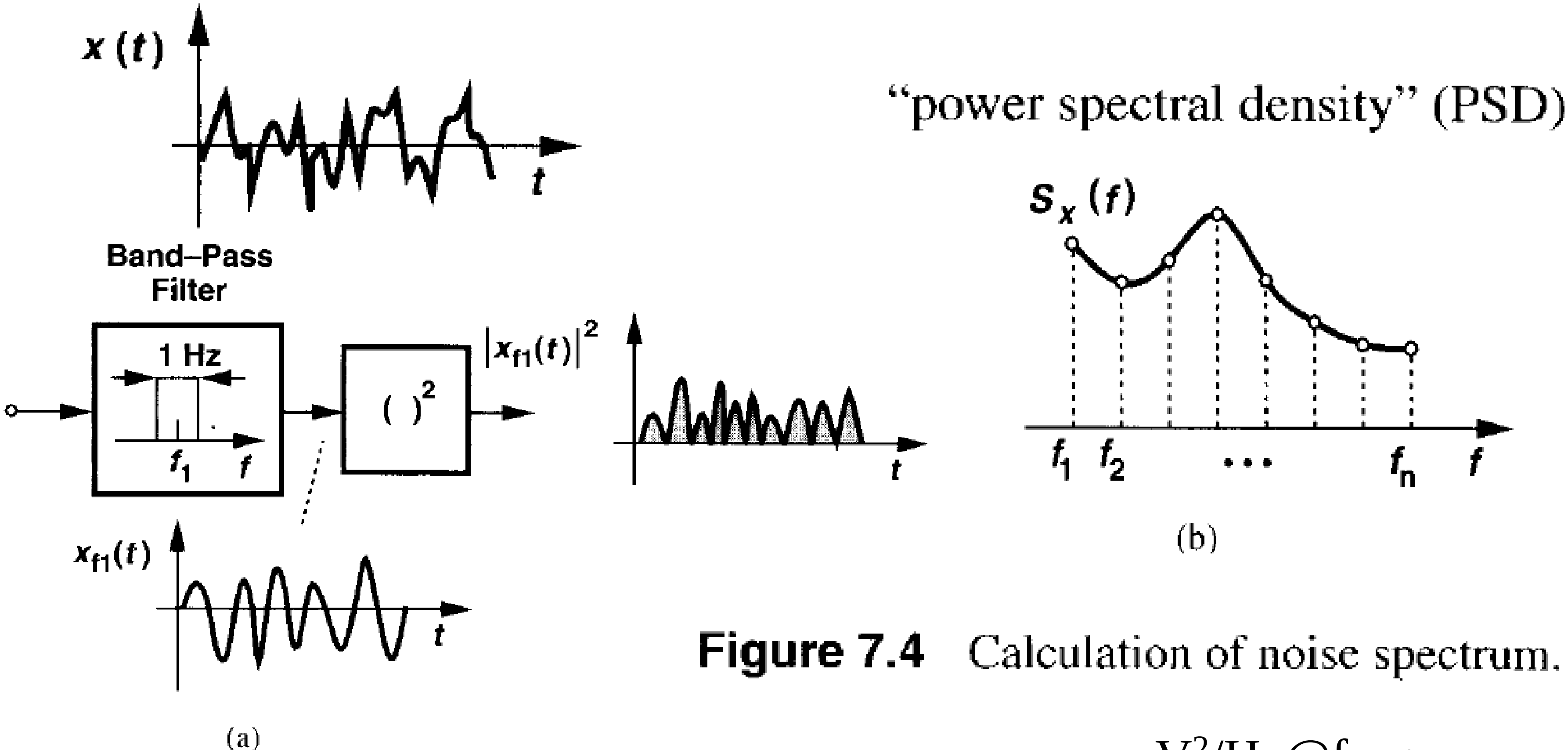


Figure 7.4 Calculation of noise spectrum.

$V^2/Hz@freq$

虽然实际功率谱PSD还应除以R。但是，一般情况下，信号处理中我们仅关注电压。

noise spectrum (cont.)

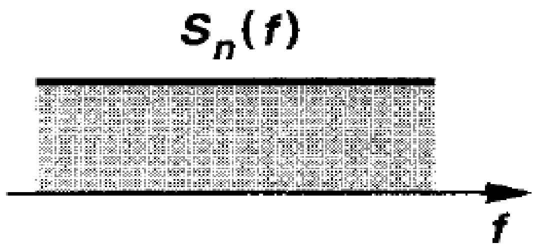


Figure 7.5 White spectrum.

linear time-invariant system

$$S_Y(f) = S_X(f)|H(f)|^2$$

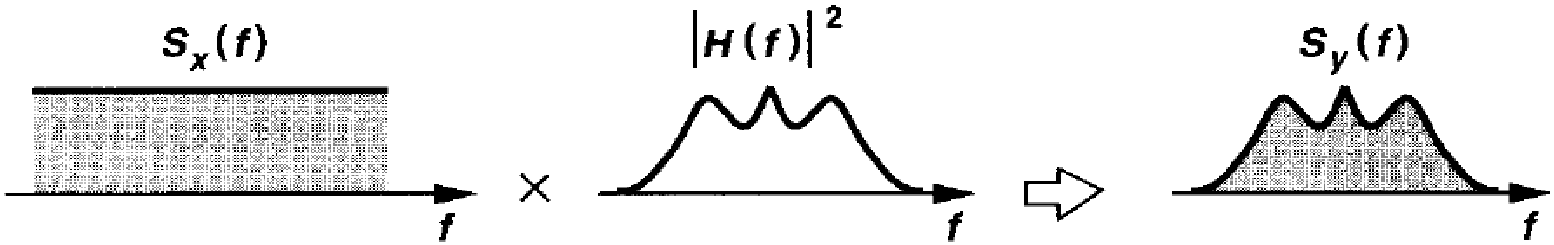


Figure 7.6 Noise shaping by a transfer function.

$S_X(f)$ is an even function of f for real $x(t)$

7.1.2 Amplitude Distribution

"probability density function" (PDF)

"distribution" of the amplitude

$$p_X(x)dx = \text{probability of } x < X < x + dx,$$

where X is the measured value of $x(t)$ at some point in time.

Gaussian PDF $$p_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x-m)^2}{2\sigma^2},$$

where σ and m are the standard deviation and mean of the distribution, respectively.

7.1.3 Correlated and Uncorrelated Sources

the average noise *power* is of interest,

$$P_{av} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}[x_1(t)+x_2(t)]^2dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}x_1^2(t)dt + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}x_2^2(t)dt$$

$$+ \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}2x_1(t)x_2(t)dt$$

$$= P_{av1} + P_{av2} + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}2x_1(t)x_2(t)dt,$$

where P_{av1} and P_{av2} denote the average power of $x_1(t)$ and $x_2(t)$, respectively.

If generated by independent devices, the noise waveforms are "uncorrelated"

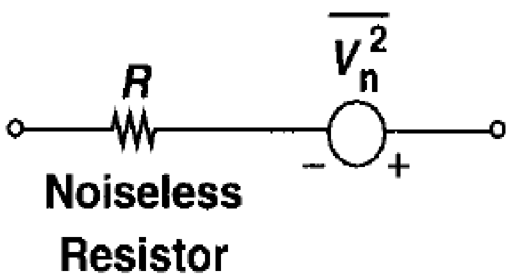
$$P_{av} = P_{av1} + P_{av2}.$$

superposition holds for the *power* of uncorrelated noise sources.

7.2 Types of Noise

7.2.1 Thermal Noise

Resistor Thermal Noise



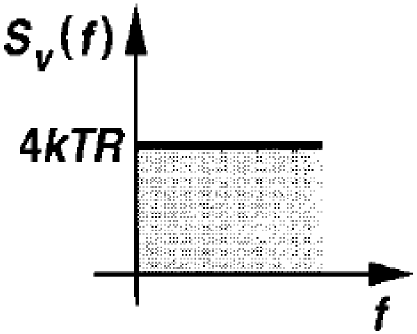


Figure 7.12 Thermal noise of a resistor.

$\overline{V_n^2} = 4kTR$, where the overline indicates averaging.

$$S_v(f) = 4kTR, \quad f \geq 0, \quad \mathrm{V^2/Hz}.$$

where $k = 1.38 \times 10^{-23}$ J/K

T为绝对温度

常温300K下，1欧姆电阻的功率谱密度约为

【$(0.1287nV / \sqrt{Hz})^2$】/欧姆

均方根 = 有效值

> 平均值

乘以R计算功率谱密度。

50欧姆电阻功率谱密度：

$$(0.1287 \times \sqrt{50})^2 \approx (0.91nV / \sqrt{Hz})^2$$

1K欧姆在1Hz带宽上产生4nV（rms）热噪声电压

1K欧姆在1MHz带宽上产生4uV（rms）热噪声电压

例7.1：RC低通电路总噪声

Consider the RC circuit shown in Fig. 7.13. Calculate the noise spectrum and the total noise power in V_{out}.

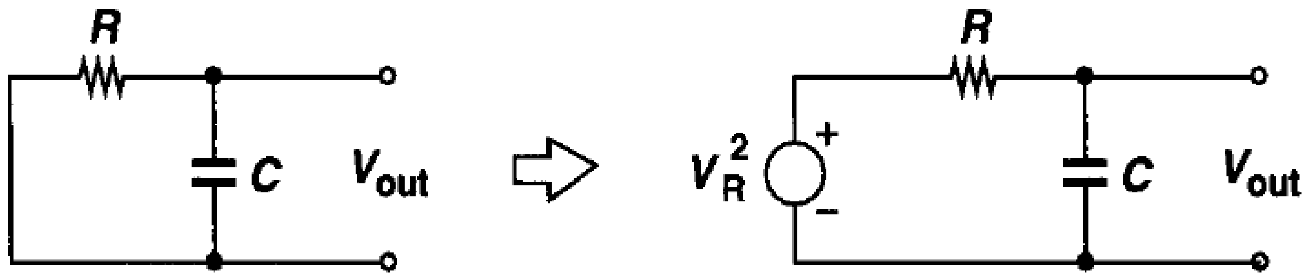


Figure 7.13 Noise generated in a low-pass filter.

$$\frac{V_{out}}{V_R}(s) = \frac{1}{RCs+1}.$$

$$S_{out}(f) = S_R(f)\left|\frac{V_{out}}{V_R}(j\omega)\right|^2 = 4kTR\frac{1}{4\pi^2R^2C^2f^2+1}.$$

$$\int\frac{dx}{x^2+1} = \tan^{-1}x, \qquad P_{n,out} = \int_0^\infty \frac{4kTR}{4\pi^2R^2C^2f^2+1}df = \frac{2kT}{\pi C}\tan^{-1}u\Big|_{u=0}^{u=\infty} = \frac{kT}{C}. \tag{7.19}$$

RC电路常温下(300K)1pf电容总热噪声电压64.3uVrms

一阶RC输出总噪声与R无关：R增大，带宽减小

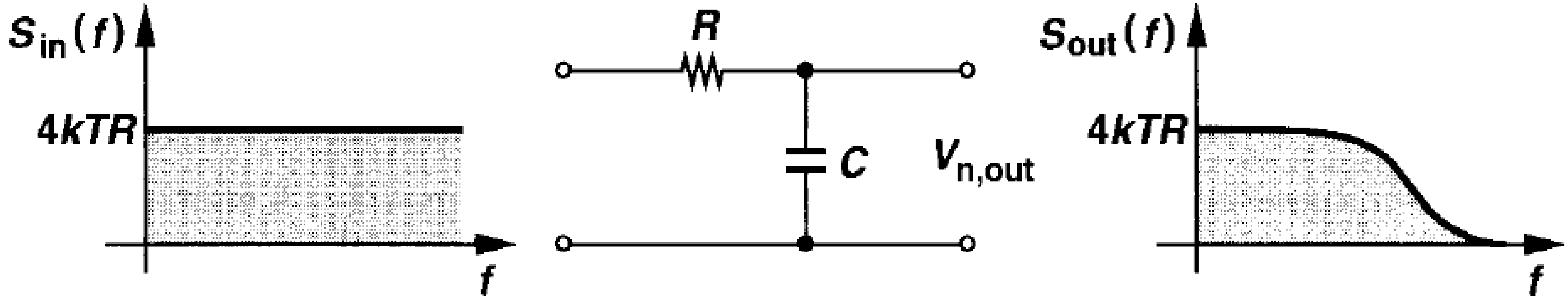


Figure 7.14 Noise spectrum shaping by a low-pass filter.

Note that the unit of kT/C is V^2. We may also consider $\sqrt{kT/C}$ as the total rms noise voltage measured at the output. For example, with a 1-pF capacitor, the total noise voltage is equal to 64.3 μV_{rms}.

Equation (7.19) implies that the total noise at the output of the circuit shown in Fig. 7.13 is independent of the value of R. Intuitively, this is because for larger values of R, the associated noise per unit bandwidth increases while the overall bandwidth of the circuit decreases. The fact that kT/C noise can be decreased by only increasing C (if T is fixed) introduces many difficulties in the design of analog circuits (Chapter 12).

电流源表示噪声

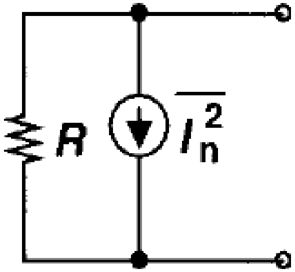


Figure 7.15 Representation of resistor thermal noise by a current source.

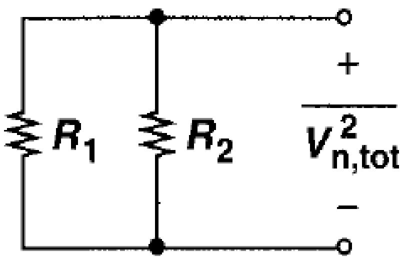
$$\overline{I_n^2} = 4kT/R.$$

Note that $\overline{I_n^2}$ is expressed in A^2/Hz.

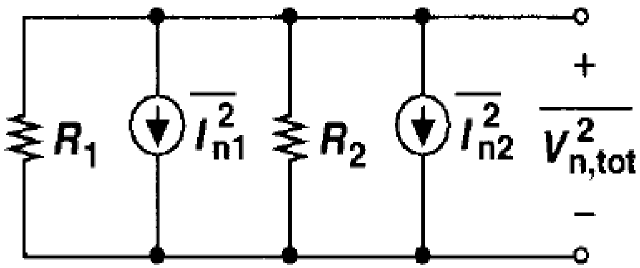
$$\overline{V_n^2}/R^2 = \overline{I_n^2},$$

Example 7.2

Calculate the equivalent noise voltage of two parallel resistors R_1 and R_2



(a)



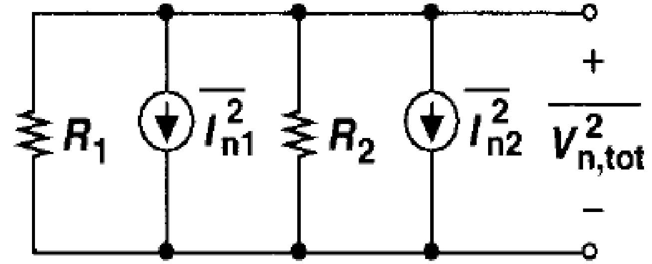
(b)

Figure 7.16

Example 7.2

Solution

$$\overline{I_{n,tot}^2} = \overline{I_{n1}^2} + \overline{I_{n2}^2} \quad = 4kT\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$



(b)

Thus, the equivalent noise voltage is given by

$$\overline{V_{n,tot}^2} = \overline{I_{n,tot}^2}(R_1 \| R_2)^2$$
$$= 4kT(R_1 \| R_2),$$

低温电阻噪声小

as intuitively expected. Note that our notation assumes a 1-Hz bandwidth.

The dependence of thermal noise (and some other types of noise) upon T suggests that low-temperature operation can decrease the noise in analog circuits. This approach becomes more attractive with the observation that the mobility of charge carriers in MOS devices increases at low temperatures

但是，极低温度时迁移率由于载流子冻结而下降！

（1）沟道电流热噪声+（2）1/f（等效为栅极噪声电压）

沟道热噪声“功率”谱: $\overline{I_n^2} = 4kT\gamma g_m$,

这里系数γ与工艺有关（不是体效应系数），

L(= 4um）长沟道时$\gamma = 2/3$.

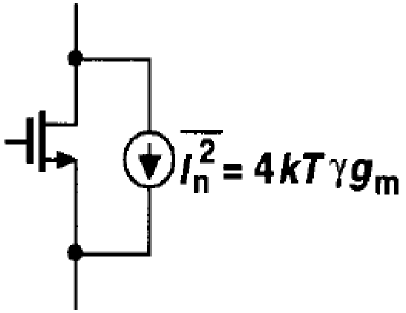


Figure 7.17 Thermal noise of a MOSFET.

$$\overline{I_n^2} = 4kT\gamma g_m = 4kT\gamma \times \mu C_{ox}\frac{W}{L}(V_{GS} - V_{TH})$$

$$= 4kT\gamma \times \text{深线性区}\frac{1}{R_{ds}} = 4kT\gamma \times \text{深线性区}\, g_{ds}$$

$$\text{线性区}\, I_d = \mu C_{ox}[(V_{GS} - V_{TH})V_{ds} - \frac{1}{2}V_{ds}^2]$$

Example 7.3

Find the maximum noise voltage that a single MOSFET can generate.

Solution

the maximum output noise occurs
if the external load is an ideal current source.

The output noise voltage

长沟道热噪声

$$\overline{V_n^2} = \overline{I_n^2} r_O^2 = 4kT\left(\frac{2}{3}g_m\right) r_O^2. \qquad (7.26)$$

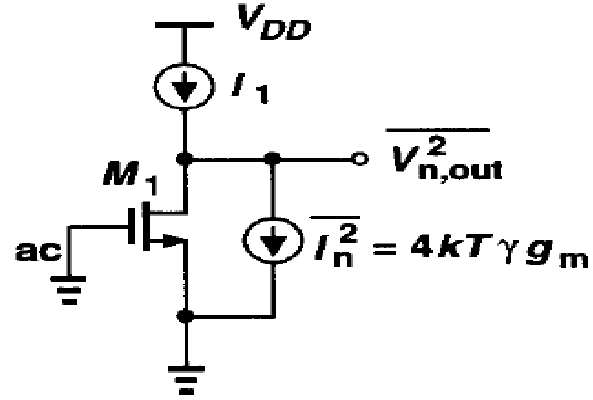


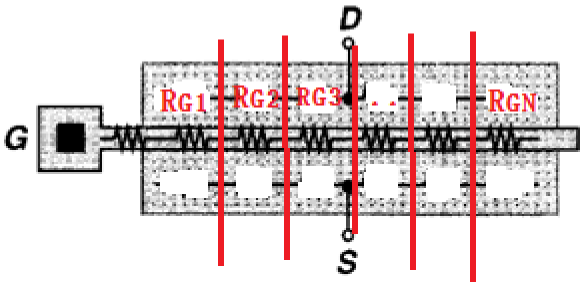
Figure 7.18

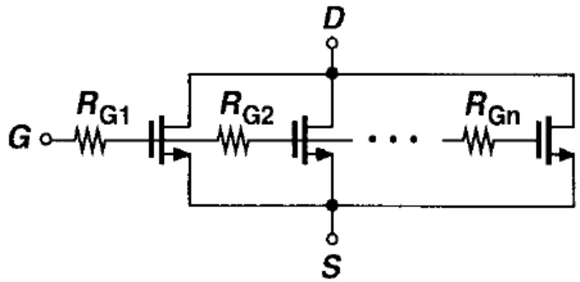
Equation (7.26) suggests that the noise *current* of a MOS transistor decreases if the transconductance drops. For example, if the transistor operates as a constant current source, it is desirable to minimize its transconductance.

Another important conclusion is that the noise measured at the output of the circuit does not depend on where the input terminal is because for output noise calculation, the input is set to zero.[8] For example, the circuit of Fig. 7.18 may be a common-source or a common-gate stage, exhibiting the same output noise.

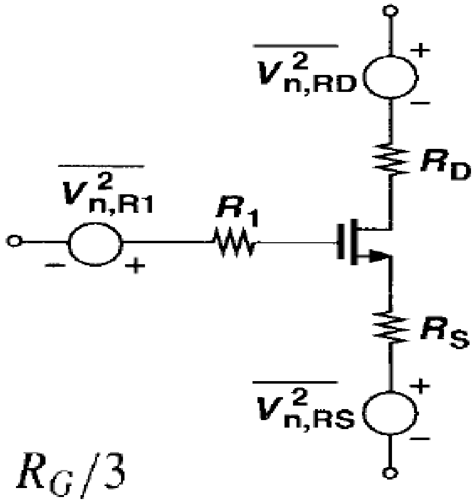
与输入信号无关！

Reduction of gate resistance





$R_{G1} + R_{G2} + \cdots + R_{Gn} = R_G$



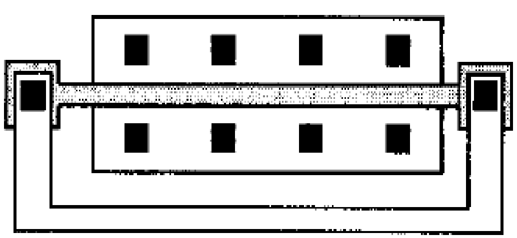
(b)

the lumped resistor in the noise model to be *less* than R_G. $R_1 = R_G/3$

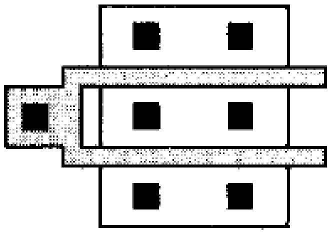
$$\overline{V^2_{n,out}} = 4kT\frac{R_G}{3}(g_m r_O)^2.$$

栅极电阻噪声导致的输出噪声（电压）“功率”谱

减小栅极电阻噪声



reducing R_G by a factor of 4.



7.2.2 Flicker Noise 栅极主要低频噪声

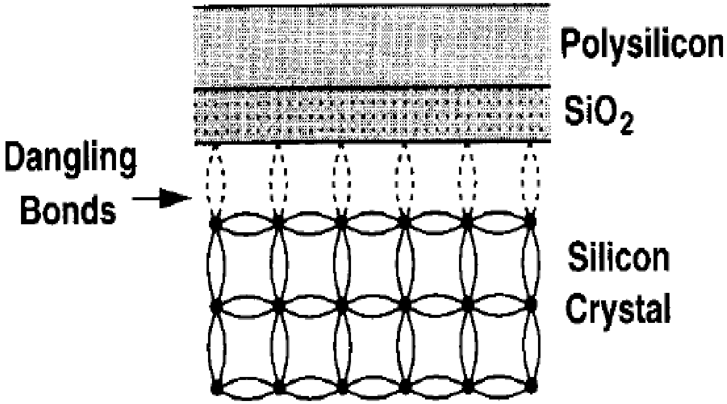


Figure 7.21 Dangling bonds at the oxide-silicon interface.

Unlike thermal noise, the average power of flicker noise cannot be predicted easily. Depending on the "cleanness" of the oxide-silicon interface, flicker noise may assume considerably different values and as such varies from one CMOS technology to another. The flicker noise is more easily modeled as a voltage source in series with the gate and roughly given by

$$\overline{V_{n,1/f}^2} = \frac{K}{C_{ox}WLf} \quad (7.28)$$

assumes a bandwidth of 1 Hz.

建模公式：闪烁噪声电压源与栅极串联

where K is a process-dependent constant on the order of 10^{-25} V^2F.

flicker noise is also called $1/f$ noise. does not depend on the bias current or the temperature.

It is also believed that PMOS devices exhibit less $1/f$ noise than NMOS transistors

Example 7.5

For an NMOS current source, calculate the total thermal and $1/f$ noise in the drain current for a band from 1 kHz to 1 MHz.

Solution

the total thermal noise integrated across the band of interest is

$$\overline{I_{n,th,tot}^2} = 4kT\left(\frac{2}{3}g_m\right)(10^6 - 10^3) \approx 4kT\left(\frac{2}{3}g_m\right) \times 10^6 \text{ A}^2.$$

长沟道，
漏极热噪声
电流"功率"

The total $1/f$ noise is

$$\overline{I_{n,1/f,tot}^2} = \frac{Kg_m^2}{C_{ox}WL}\int_{1\text{ kHz}}^{1\text{ MHz}}\frac{df}{f}$$

drain noise current

$$= \frac{Kg_m^2}{C_{ox}WL}\ln 10^3 = \frac{6.91Kg_m^2}{C_{ox}WL}.$$

闪烁噪声电压源与栅极串联，
漏极输出1/f噪声电流"功率"

1/f （闪烁）噪声转角频率

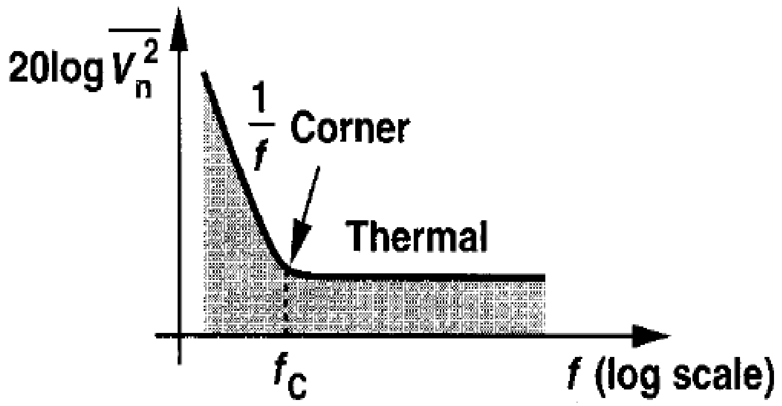


Figure 7.23 Concept of flicker noise corner frequency.

1/f 噪声转角频率：

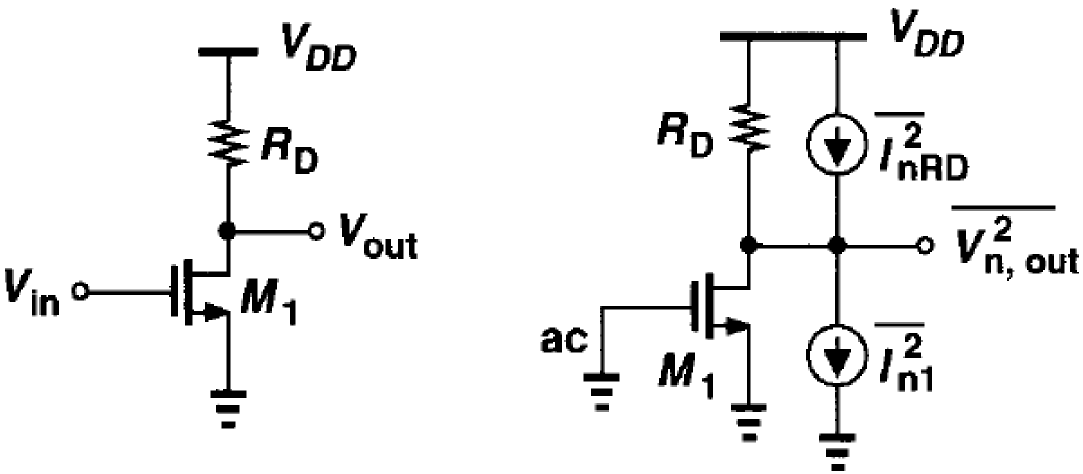
$$4kT\gamma g_m = \frac{K}{C_{ox}WLf_C} g_m^2$$

$$1/f\ 转角频率\ f_C = \frac{K}{C_{ox}WL} g_m \frac{1}{4kT\gamma}$$

在1/f 噪声转角频率之下，MOS热噪声可忽略不计

7.3 Representation of Noise in Circuits

set the input to zero and calculate the total noise at the output



$$\overline{I^2_{n,RD}} = 4kT/R_D$$

r_o本身不产生热噪声

Figure 7.25 (a) CS stage, (b) circuit including noise sources.

Solution

$$\overline{I^2_{n,th}} = 4kT\gamma g_m$$

栅极1/f 噪声电压成为漏极输出噪声电流：

$$\overline{I^2_{n,1/f}} = \frac{K}{C_{ox}WLf} g_m^2 \qquad A^2 / Hz$$

$$\overline{V^2_{n,out}} = (4kT\gamma g_m + \frac{K}{C_{ox}WLf} g_m^2 + \frac{4kT}{R_D})R_D^2 \qquad V^2 / Hz$$

input-referred noise

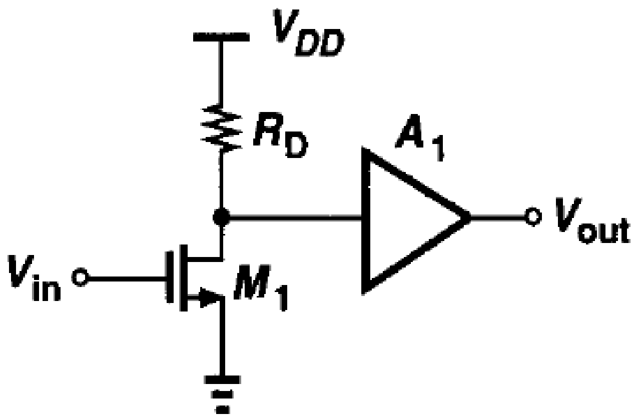
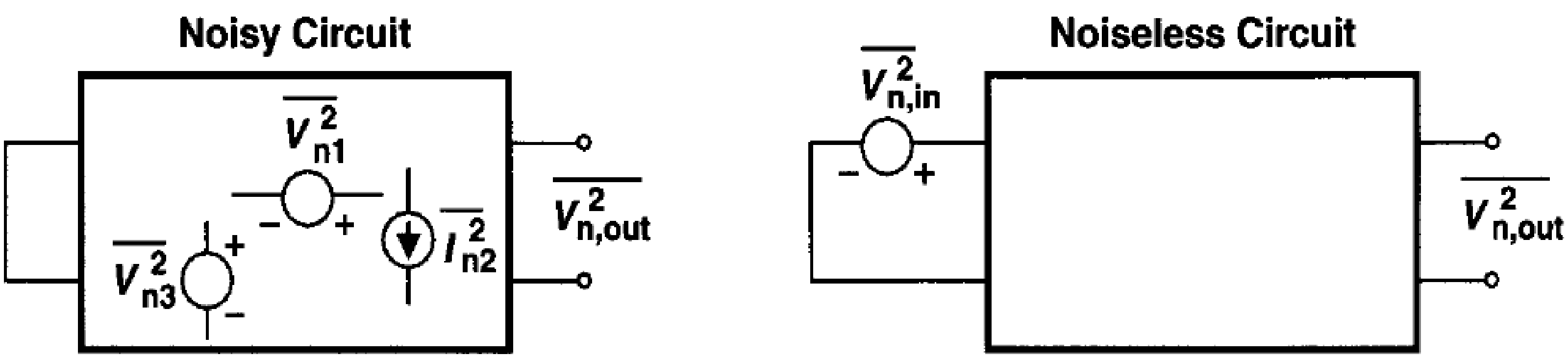


Figure 7.26 Addition of gain stage to a CS stage.

- 性能比较：

输出信噪比下降

$$\overline{V^2_{n,out}} = A_v^2 \overline{V^2_{n,in}},$$

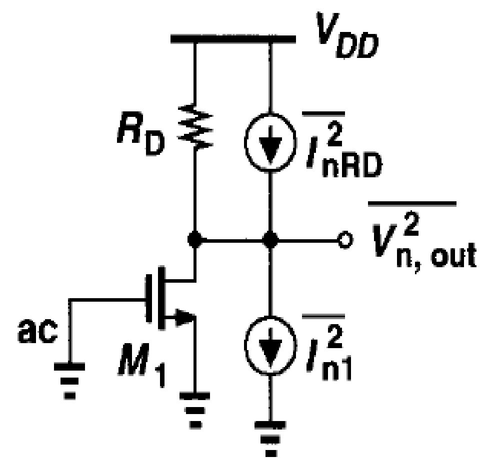


(a) (b)

输入（等效）噪声是计算出来的。不能测量！

Example 7.8

calculate the input-referred noise voltage.



Solution

长沟道，并忽略ro的负载作用

$$\overline{V^2_{n,in}} = \frac{\overline{V^2_{n,out}}}{A_v^2}$$

$$= \left(4kT\frac{2}{3}g_m + \frac{K}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right)R_D^2\frac{1}{g_m^2R_D^2}$$

$$= 4kT\frac{2}{3g_m} + \frac{K}{C_{ox}WL}\cdot\frac{1}{f} + \frac{4kT}{g_m^2R_D}. \qquad (7.41)$$

V^2/Hz

how small an input the circuit can detect with acceptable SNR.

the input-referred noise cannot be *measured* at the input of the circuit.

减小电路内部噪声的方法：减小带宽或温度，增大输入管g_m、输入管MOS尺寸、R_D，低频时增大f

if the circuit has a finite input impedance

计算级联电路噪声时可能不能仅用一个电压源对噪声建模。例：

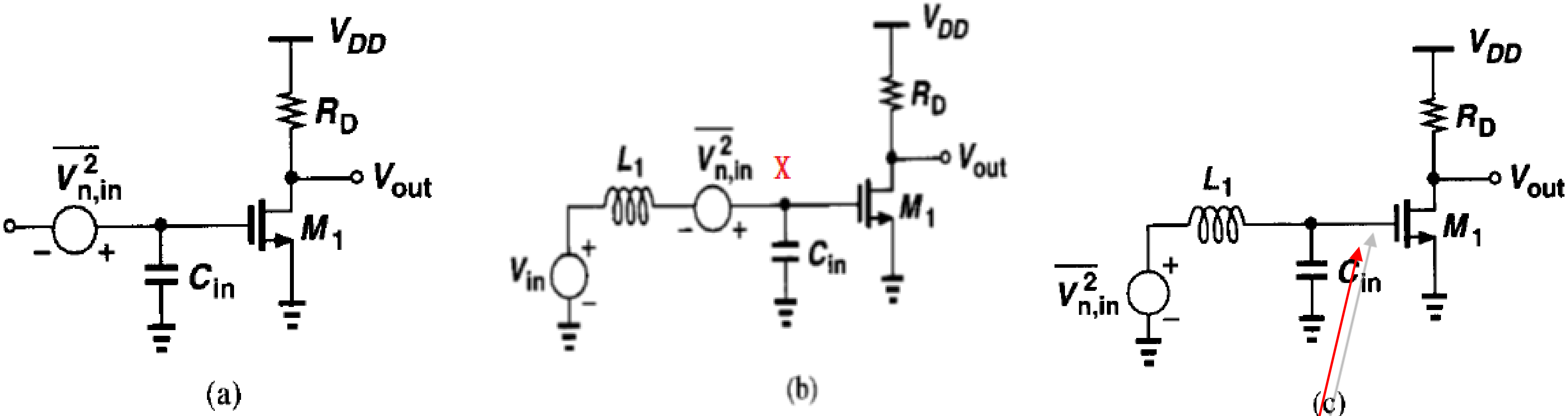


Figure 7.28 CS stage including input capacitance, (b) CS stage stimulated by a finite source impedance, (c) Effect of single noise source.

源阻抗增大则输出噪声变小？
明显错误！

在X点！而不是在Vin处计算右边整个电路的输入参考噪声电压

输入短路求Vn,in， 输入开路求In,in

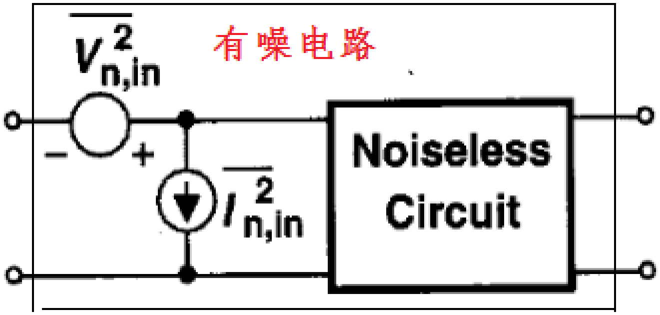
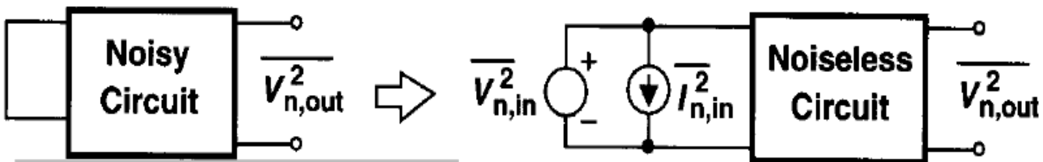


Figure 7.29 Representation of noise by voltage and current sources.

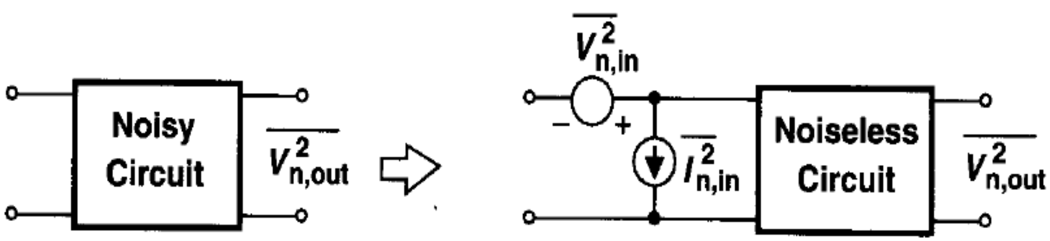
How do we calculate $\overline{V^2_{n,in}}$ and $\overline{I^2_{n,in}}$? Since the model is valid for any source impedance, we consider two extreme cases: zero and infinite source impedances.



内部噪声与输入无关

(a)

$\overline{I^2_{n,in}}$ flows through $\overline{V^2_{n,in}}$ and has no effect on the output.



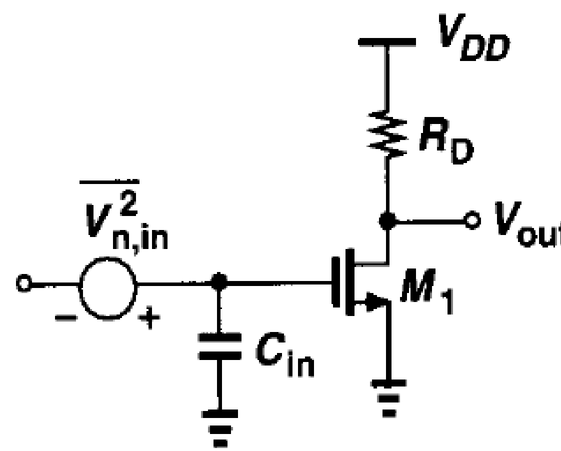
(b)

$\overline{V^2_{n,in}}$和$\overline{I^2_{n,in}}$都能单独代表全部噪声

Figure 7.30 Calculation of input-referred noise (a) voltage, and (b) current.

Example 7.9

Calculate the input-referred noise voltage and current of Fig. 7.28.



Solution

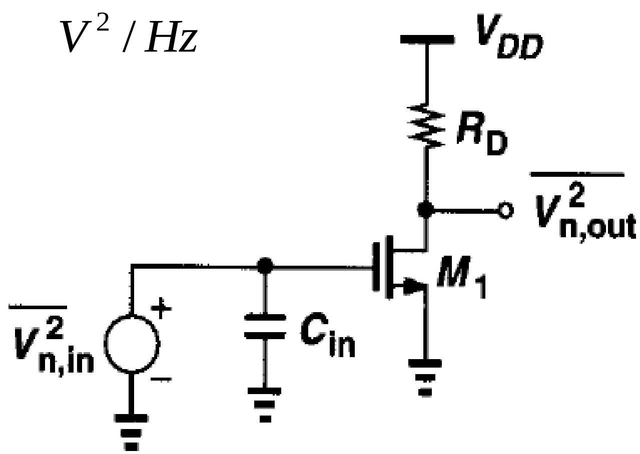
From (7.41), the input-referred noise voltage (excluding $1/f$ noise) is

以后均以长沟道为例

$$\overline{V^2_{n,in}} = 4kT\frac{2}{3g_m} + \frac{4kT}{g_m^2 R_D} \qquad (7.42)$$

V^2/Hz

输入参考噪声电压功率谱



(a)

Figure 7.31

As depicted in Fig. 7.31(a), this voltage generates the same output noise as the actual circuit if the input is shorted.

Example 7.9 续

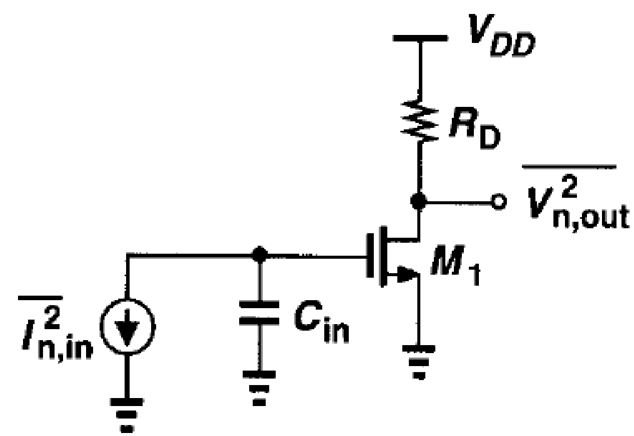
To obtain the input-referred noise current, we open the input and find the output noise in terms of $\overline{I_{n,in}^2}$ [Fig. 7.31(b)]. The noise current flows through C_{in}, generating at the output

$$\overline{V_{n,out}^2} = \overline{I_{n,in}^2}\left(\frac{1}{C_{in}\omega}\right)^2 g_m^2 R_D^2. \qquad (7.43)$$

This value must be equal to the output of the noisy circuit when its input is open:

$$\overline{V_{n,out}^2} = \left(4kT\frac{2}{3}g_m + \frac{4kT}{R_D}\right)R_D^2. \qquad (7.44)$$

V^2/Hz



(b)

From (7.43) and (7.44), it follows that

$$\overline{I_{n,in}^2} = (C_{in}\omega)^2\frac{4kT}{g_m^2}\left(\frac{2}{3}g_m + \frac{1}{R_D}\right). \qquad (7.45)$$

A^2/Hz

假设输入参考噪声电流功率谱

prove that the output noise is correct for any source impedance Z_S.

是否重复计算了噪声？

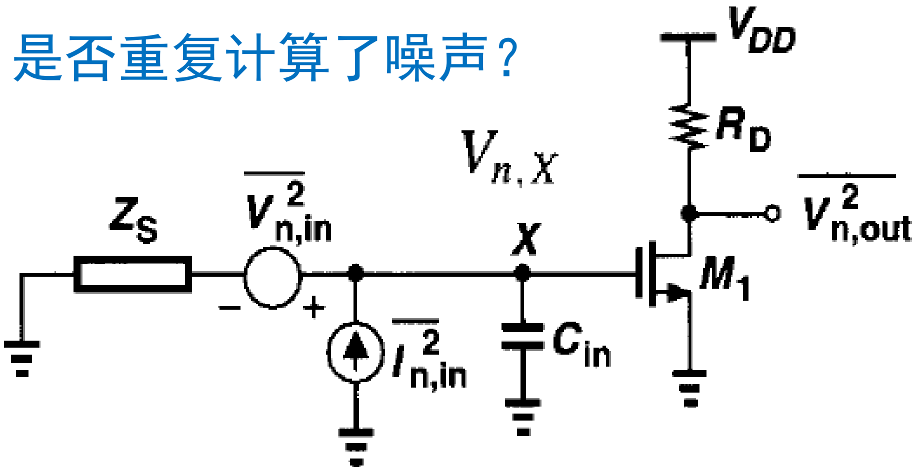


Figure 7.32 CS stage stimulated by a source impedance.

The two sources $\overline{V^2_{n,in}}$ and $\overline{I^2_{n,in}}$ are in general *correlated* because they may represent the same noise mechanisms in the circuit.

Zs应对VnX无影响。电路内部噪声与外部输入无关。

M1管与负载电阻热噪声无关。

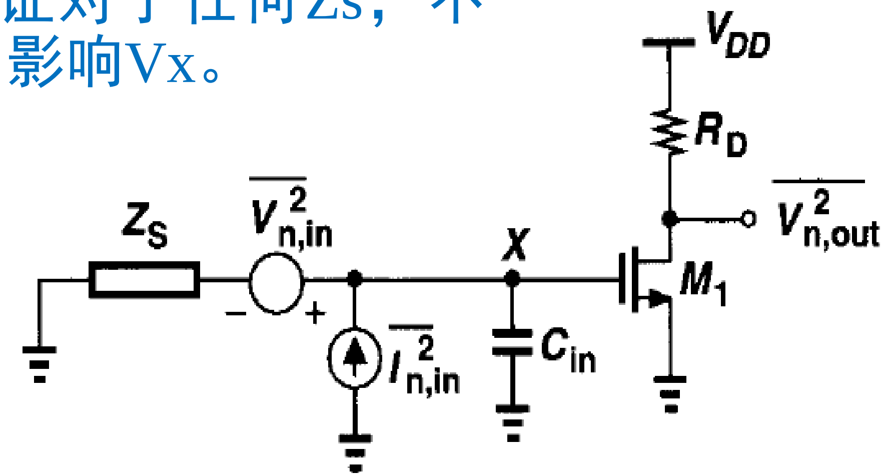
$$\overline{V^2_{n,in}} = 4kT\frac{2}{3g_m} + \frac{4kT}{g_m^2 R_D} \longrightarrow V_{n,in} = V_{n,M1} + \frac{1}{g_m R_D}V_{n,RD}$$

$$\overline{I^2_{n,in}} = (C_{in}\omega)^2\frac{4kT}{g_m^2}\left(\frac{2}{3}g_m + \frac{1}{R_D}\right). \longrightarrow I_{n,in} = C_{in}s V_{n,M1} + \frac{C_{in}s}{g_m R_D}V_{n,RD},$$

where $V_{n,M1}$ denotes 沟道热噪声 and $V_{n,RD}$ the noise voltage of R_D.

输入电压源和电流源相合成的噪声输出

验证对于任何Zs，不影响Vx。



$$V_{n,X} = V_{n,in}\frac{\dfrac{1}{C_{in}s}}{\dfrac{1}{C_{in}s}+Z_S} + I_{n,in}\frac{\dfrac{Z_S}{C_{in}s}}{\dfrac{1}{C_{in}s}+Z_S}$$

$$= \frac{V_{n,in}+I_{n,in}Z_S}{Z_SC_{in}s+1}.$$

$$V_{n,X} = \frac{1}{Z_SC_{in}s+1}\left[V_{n,M1}+\frac{1}{g_mR_D}V_{n,RD}+C_{in}sZ_S\left(V_{n,M1}+\frac{1}{g_mR_D}V_{n,RD}\right)\right]$$

$$= V_{n,M1}+\frac{1}{g_mR_D}V_{n,RD}. \quad = \quad V_{n,in}$$

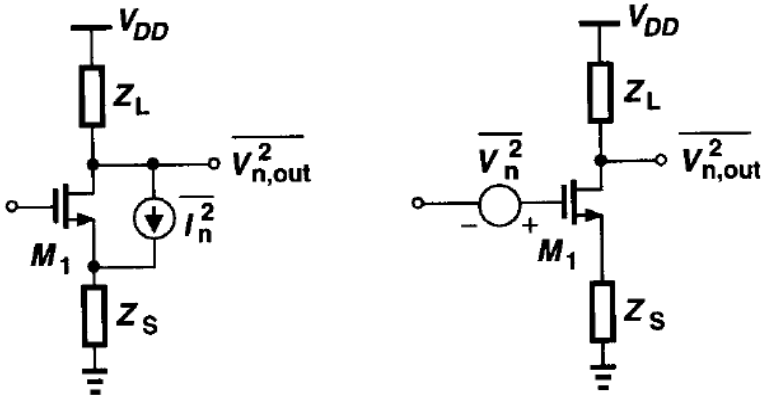
表明：内部噪声与前级输出阻抗无关。

Note that $V_{n,X}$ is independent of Z_S and C_{in}.

$$\overline{V^2_{n,out}} = g_m^2R_D^2\overline{V^2_{n,X}} = 4kT\left(\frac{2}{3}g_m+\frac{1}{R_D}\right)R_D^2,$$

与输入开路时求出的(4.44)相同。因此没有重复计算。

Lemma The circuits shown in Fig. 7.33(a) and (b) are equivalent at low frequencies if $\overline{V_n^2} = \overline{I_n^2}/g_m^2$ and the circuits are driven by a finite impedance.



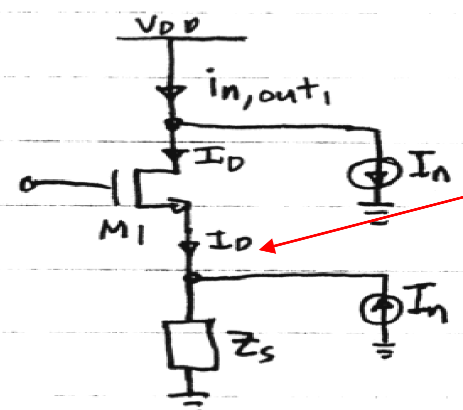
(a) (b)

Figure 7.33 Equivalent CS stages.

何为等效电路？
2电路的短路输出电流、
（负载）开路输出电压、
输出阻抗相同

Proof.

Since the circuits have equal output impedances, examine the output short-circuit currents



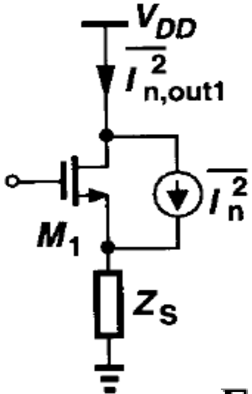
$I_{n,out1} = I_D + I_n$, KCL @ drain

$I_D = \frac{-Z_S}{(\frac{1}{g_m} \parallel r_o) + Z_S} I_n$, current divider @ source

$\therefore I_{n,out1} = \left(\frac{-Z_S}{(1/g_m \parallel r_o) + Z_S} + 1 \right) I_n$

$\therefore I_{n,out1} = \frac{I_n}{Z_S(g_m + \frac{1}{r_o}) + 1}$ //

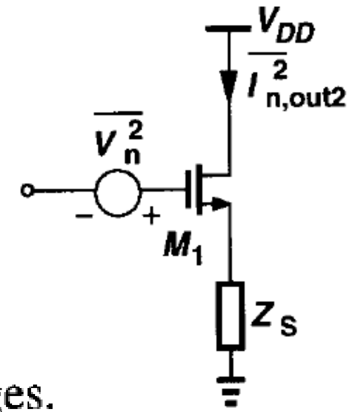
Proof. （续）



(c)

Figure 7.33

Equivalent CS stages.



(d)

回顾

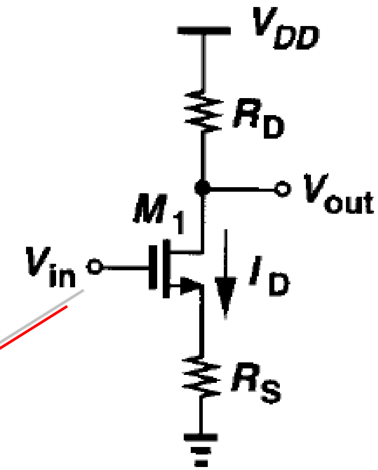


Figure 3.16

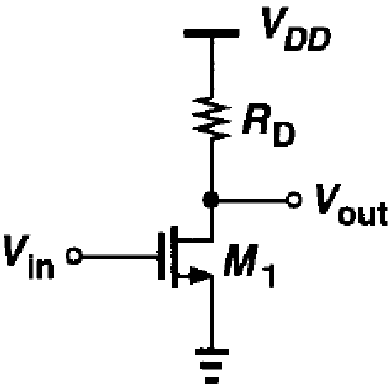
$$I_{n,out1} = \frac{I_n}{Z_s(g_m + 1/r_o) + 1} = \frac{I_n}{Z_s g_m + 1}$$

$$G_m = \frac{I_{out}}{V_{in}} = \frac{g_m r_o}{R_S + [1 + (g_m + g_{mb})R_S]r_o}$$，式（3. 55）

$$= \frac{g_m r_o}{Z_S + [1 + (g_m + g_{mb})Z_S]r_o} = \frac{g_m}{1 + (1/r_o + g_m + g_{mb})Z_S} \approx \frac{g_m}{1 + (1/r_o + g_m)Z_S}$$

$$\therefore\ I_{n,out2} = \frac{g_m V_n}{Z_s(g_m + 1/r_o) + 1} = \frac{g_m V_n}{Z_s g_m + 1} \longrightarrow V_n = I_n / g_m$$

7.4.1 Common-Source Stage

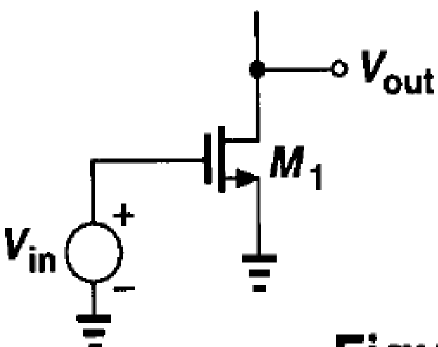


the input-referred noise voltage per unit bandwidth

$$\overline{V_{n,in}^2} = 4kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}. \qquad (7.56)$$

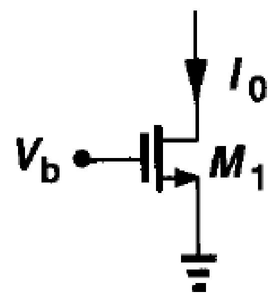
总噪声=热噪声+1/f噪声

How can we reduce the input-referred noise voltage? Equation (7.56) implies that the transconductance of M_1 must be maximized. Thus, the transconductance must be maximized if the transistor is to amplify a voltage signal applied to its gate [Fig. 7.34(a)] whereas it must be minimized if the transistor operates as a current source [Fig. 7.34(b)].



(a)

M1是输入管，gm大噪声小



(b)

M1是电流源，gm小噪声小

$$\overline{I_n^2} = 4kT\gamma g_m. \qquad (7.24)$$

Figure 7.34 Voltage amplification versus current generation.

Example 7.10

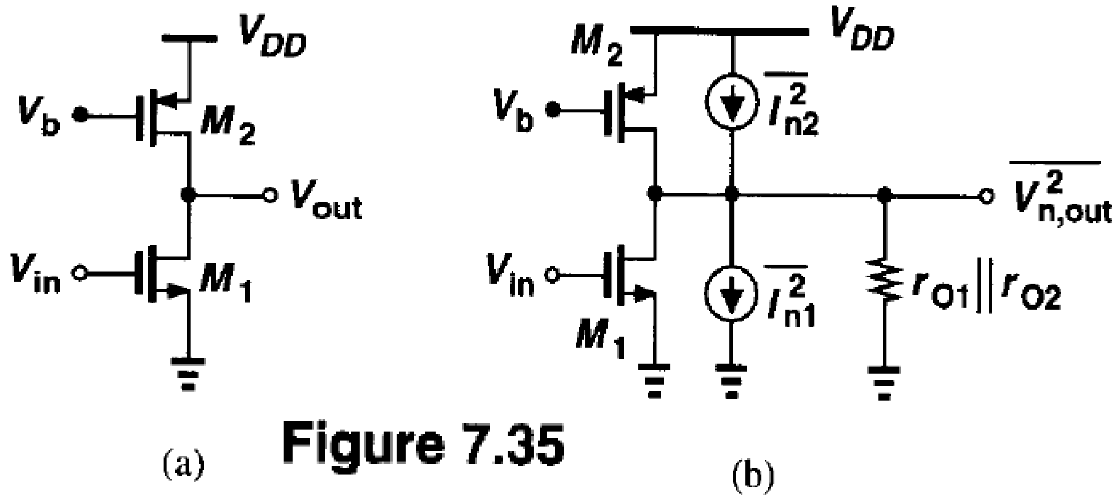


Figure 7.35

the thermal noise of M_1 and M_2 are uncorrelated,

长沟道

热噪声谱

$$\overline{V_{n,out}^2} = 4kT\left(\frac{2}{3}g_{m1} + \frac{2}{3}g_{m2}\right)(r_{O1}\|r_{O2})^2.$$

电流功率
无方向性

Since the voltage gain is equal to $g_{m1}(r_{O1}\|r_{O2})$,
the total noise voltage referred to the gate of M_1 is

输入参考热噪声
电压“功率谱”

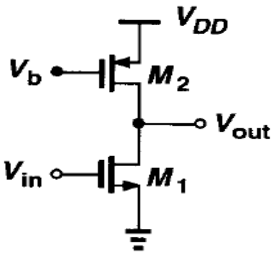
$$\overline{V_n^2} = 4kT\left(\frac{2}{3}g_{m1} + \frac{2}{3}g_{m2}\right)\frac{1}{g_{m1}^2}$$

$$= 4kT\left(\frac{2}{3g_{m1}} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\right).$$

计入输出C_L，计算总热噪声

the total output noise,

$$\overline{V_{n,out,tot}^2} = \int_0^\infty 4kT\left(\frac{2}{3}g_{m1} + \frac{2}{3}g_{m2}\right)(r_{O1}\|r_{O2})^2 \frac{df}{1+(r_{O1}\|r_{O2})^2 C_L^2 (2\pi f)^2}.$$



(a)

$$= \frac{2}{3}(g_{m1}+g_{m2})(r_{O1}\|r_{O2})\frac{kT}{C_L}.$$

长沟道

$$SNR_{out} = \left[\frac{g_{m1}(r_{O1}\|r_{O2})V_m}{\sqrt{2}}\right]^2 \cdot \frac{1}{(2/3)(g_{m1}+g_{m2})(r_{O1}\|r_{O2})(kT/C_L)}$$

$$= \frac{3C_L}{4kT}\cdot\frac{g_{m1}^2(r_{O1}\|r_{O2})}{g_{m1}+g_{m2}}V_m^2.$$

It is also important to observe from (7.56) that the noise contributed by R_D in Fig. 7.25(a) decreases as R_D increases. This is again because the noise voltage due to R_D at the output is proportional to $\sqrt{R_D}$ while the voltage gain of the circuit is proportional to R_D.

Example 7.11

Calculate the input-referred $1/f$ and thermal noise voltage of the circuit depicted in Fig. 7.36(a) assuming M_1 and M_2 are in saturation.

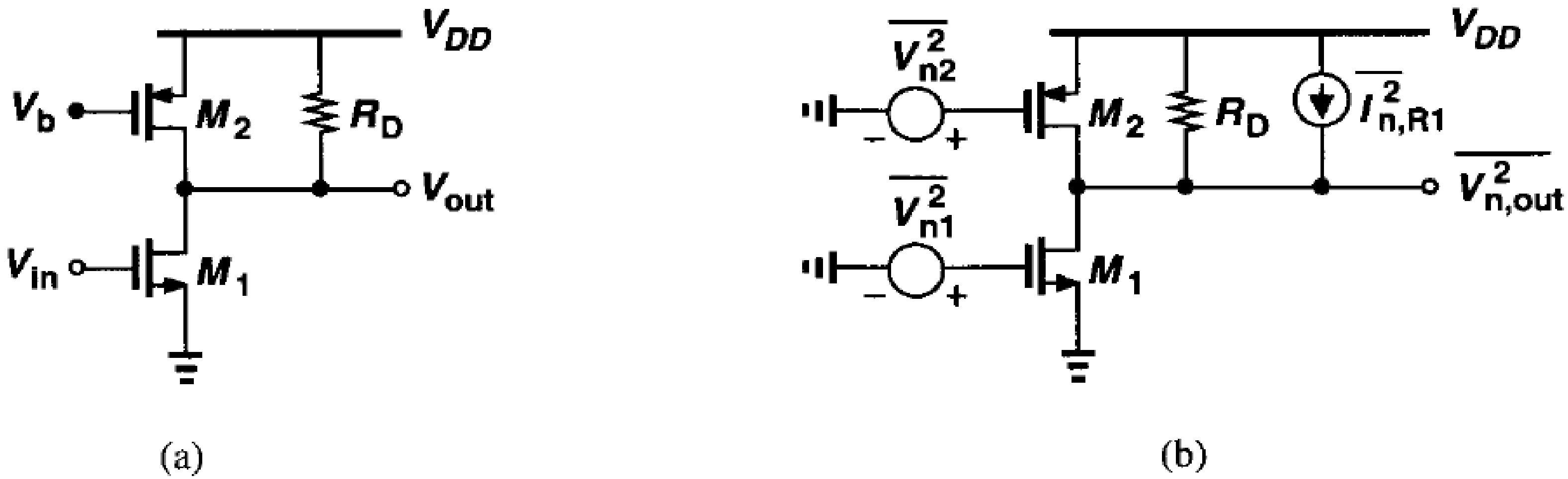


Figure 7.36

Solution

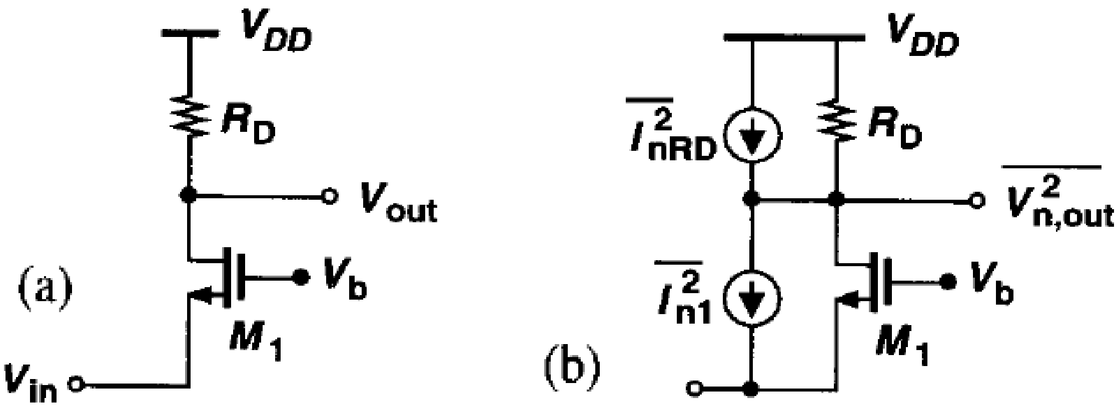
the overall input-referred noise voltage is given by

$$\overline{V_{n,in}^2} = 4kT\frac{2}{3}\left(\frac{g_{m2}}{g_{m1}^2} + \frac{1}{g_{m1}}\right) + \frac{1}{C_{ox}}\left[\frac{K_P g_{m2}^2}{(WL)_2 g_{m1}^2} + \frac{K_N}{(WL)_1}\right]\frac{1}{f} + \frac{4kT}{g_{m1}^2 R_D}, \tag{7.64}$$

where K_P and K_N denote the flicker noise coefficients of PMOS and NMOS devices, respectively.

How do we design a common-source stage for low-noise operation? P187

7.4.2 Common-Gate Stage



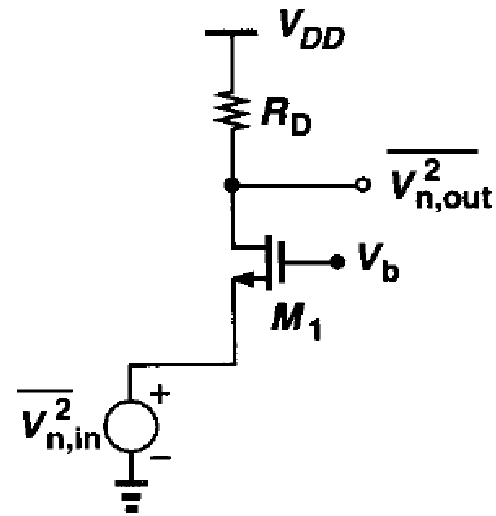


Figure 7.37 (a) CG stage, (b) circuit including noise sources.

低rin

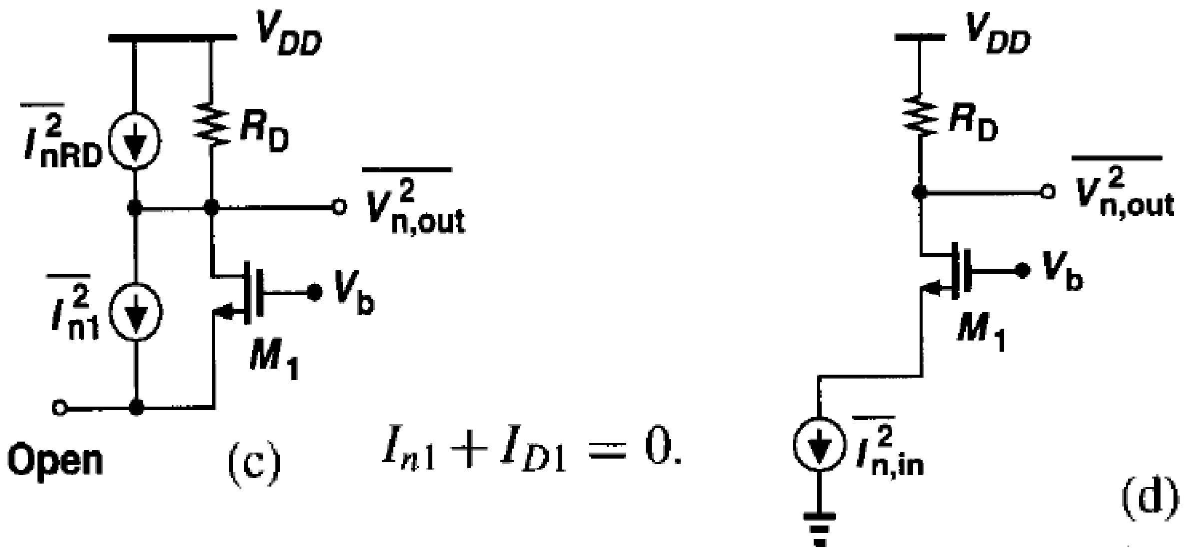
RD上的噪声电压

$$\left(4kT\frac{2}{3}g_m+\frac{4kT}{R_D}\right)R_D^2=\overline{V^2_{n,in}}(g_m+g_{mb})^2R_D^2.$$

输入参考噪声电压“功率”谱密度

$$\overline{V^2_{n,in}}=\frac{4kT(2g_m/3+1/R_D)}{(g_m+g_{mb})^2}.$$

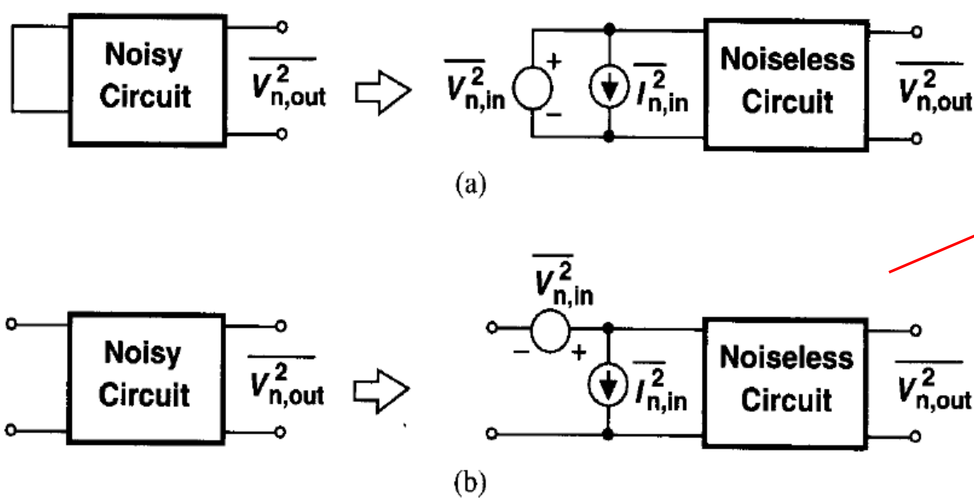
低输入阻抗电路的输入参考噪声电流建模



In,in是等效出来的，并非实际存在

P181

Figure 7.38 Calculation of input-referred noise of a CG stage.



$$\overline{I^2_{n,in}}R_D^2 = 4kTR_D$$

$$\overline{I^2_{n,in}} = \frac{4kT}{R_D}$$

与$M1$沟道热噪声电流无关！

偏置电流源的噪声影响

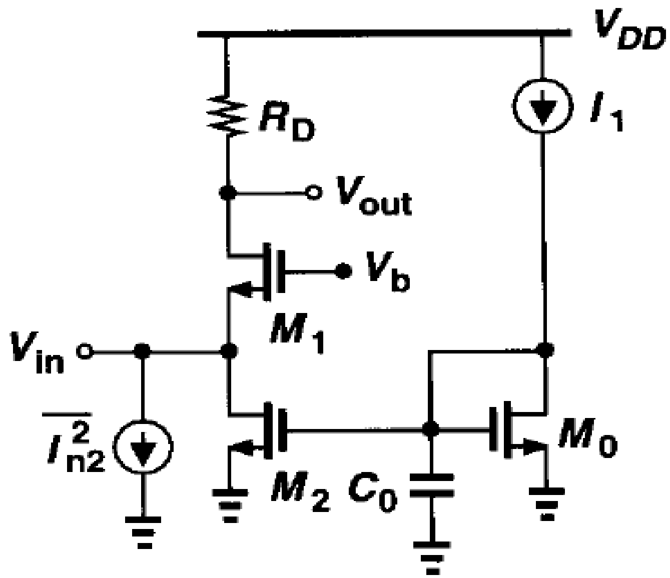


Figure 7.39 Noise contributed by bias current source.

M2的沟道热噪声电流在RD上成为输出噪声电压“功率” $\overline{I_{n2}^2}R_D^2$

CG缺点：输入噪声直接引起负载噪声。

亦即M2的沟道热噪声电流直接成为输入参考噪声电流。

$$\overline{I_{n2}^2} = 4kT\gamma g_{m2} = 4kT\gamma \times \mu C_{ox}\frac{W}{L}(V_{GS2} - V_{TH2})$$

$$= 4kT\gamma \frac{2I_{D2}}{V_{GS2} - V_{TH2}}$$

因此对于给定电流源M2，提高 VOD有利于减小噪声。

Example 7.12

Calculate the input-referred thermal noise voltage and current of the circuit

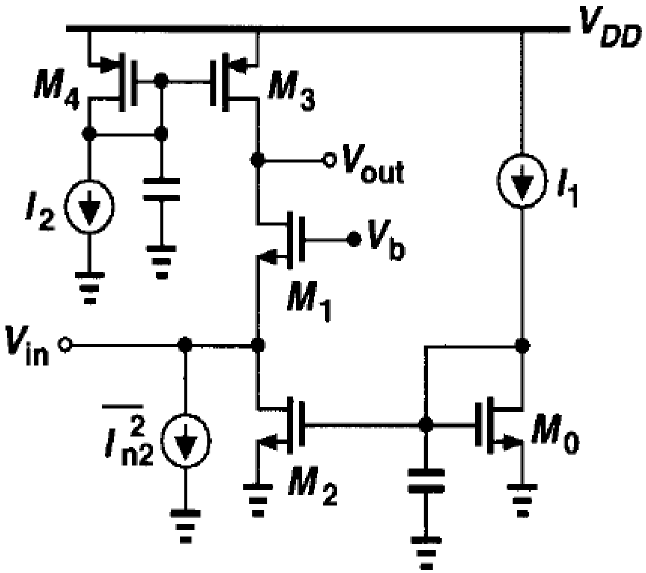


图7.40

（1）计算输入参考热噪声电压“功率”谱：

Vin短路到地

$$\overline{V_{n,out}^2} = 4kT\frac{2}{3}(g_{m1} + g_{m3})(r_{O1}\|r_{O3})^2.$$

$$\overline{V_{n,in}^2}(g_{m1} + g_{mb1})^2(r_{O1}\|r_{O3})^2 = 4kT\frac{2}{3}(g_{m1} + g_{m3})(r_{O1}\|r_{O3})^2,$$

$$\Rightarrow \quad \overline{V_{n,in}^2} = 4kT\frac{2}{3}\frac{(g_{m1} + g_{m3})}{(g_{m1} + g_{mb1})^2}.$$

Example 7.12 续

Calculate the input-referred thermal noise voltage and current of the circuit

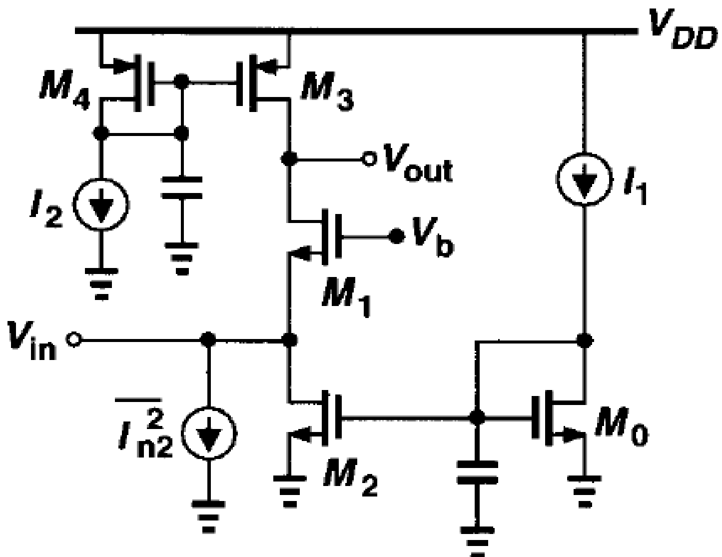
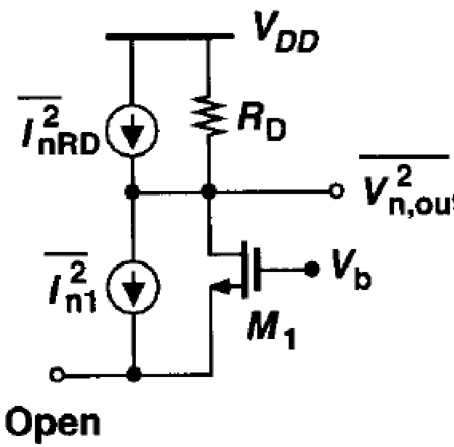


图7.40

（2）计算输入参考热噪声电流“功率”谱：

Vin开路。

图7.38(c)



M1沟道热噪声电流对In,in无贡献！

$$\overline{I_{n,in}^2} = \frac{4kT}{R_D}$$ 与$M1$沟道热噪声电流无关！

$$\overline{I_{n,in}^2} = 4kT\frac{2}{3}(g_{m2} + g_{m3}). \qquad (7.71)$$

$1/f$ noise in a common-gate topology

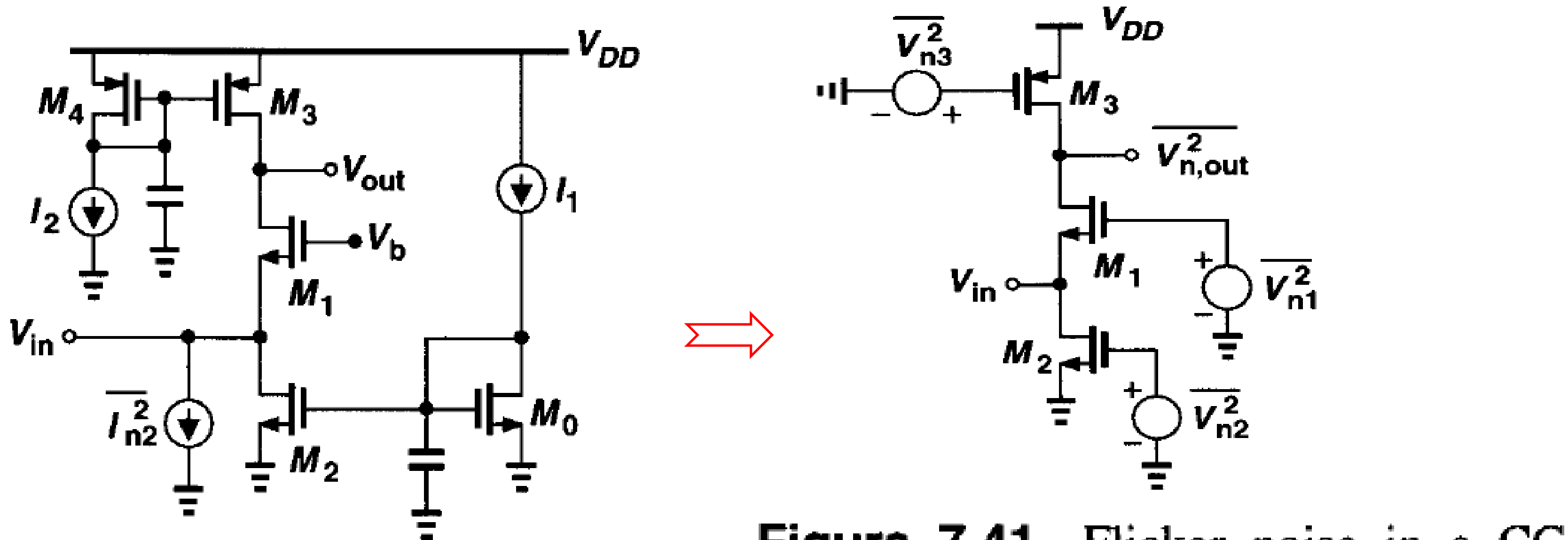


Figure 7.41 Flicker noise in a CG

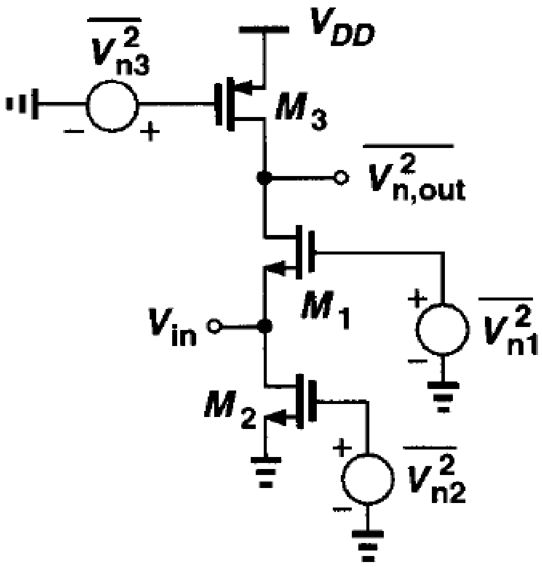
求输入参考噪声电压“功率”谱：Vin短路到地

$$\overline{V_{n,out}^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m1}^2 K_N}{(WL)_1} + \frac{g_{m3}^2 K_P}{(WL)_3}\right](r_{O1} \| r_{O3})^2, \qquad (7.72)$$

$$\Rightarrow \quad \overline{V_{n,in}^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m1}^2 K_N}{(WL)_1} + \frac{g_{m3}^2 K_P}{(WL)_3}\right]\frac{1}{(g_{m1} + g_{mb1})^2}.$$

输入参考1/f噪声电流“功率”谱

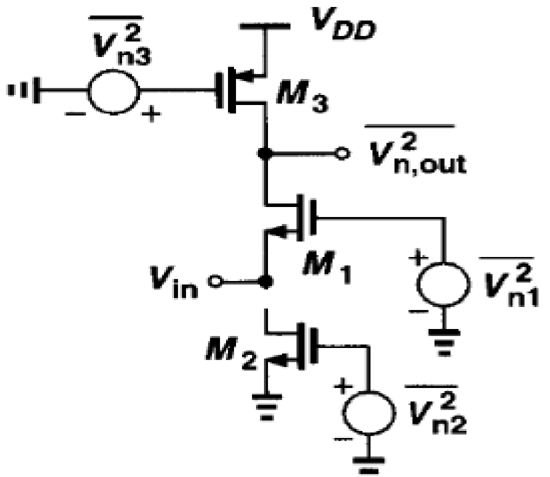
- 输入开路：



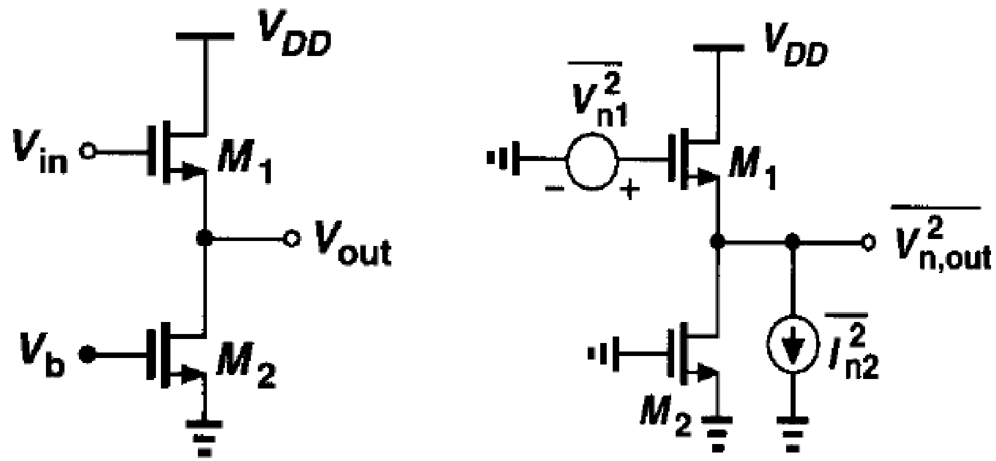
$$\overline{V_{n,out}^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m2}^2 K_N}{(WL)_2} + \frac{g_{m3}^2 K_P}{(WL)_3}\right] R_{out}^2,$$

$$\overline{I_{n,in}^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m2}^2 K_N}{(WL)_2} + \frac{g_{m3}^2 K_P}{(WL)_3}\right]. \quad (7.75)$$

即，总输入参考1/f噪声电流=
M3的1/f噪声电压等效到Vin端电流
+M2的1/f噪声电压跨导到Vin端电流



7.4.3 Source Followers



From Chapter 3,

$$A_v = \frac{\frac{1}{g_{mb1}} \left\| r_{O1} \right\| r_{O2}}{\frac{1}{g_{mb1}} \left\| r_{O1} \right\| r_{O2} + \frac{1}{g_{m1}}}.$$

Figure 7.42 (a) Source follower, (b) circuit including noise sources.

the output noise due to M_2 as

$$\overline{V_{n,out}^2}|_{M2} = \overline{I_{n2}^2}\left(\frac{1}{g_{m1}}\left\|\frac{1}{g_{mb1}}\right\| r_{O1} \| r_{O2}\right)^2.$$

the total input-referred noise voltage is

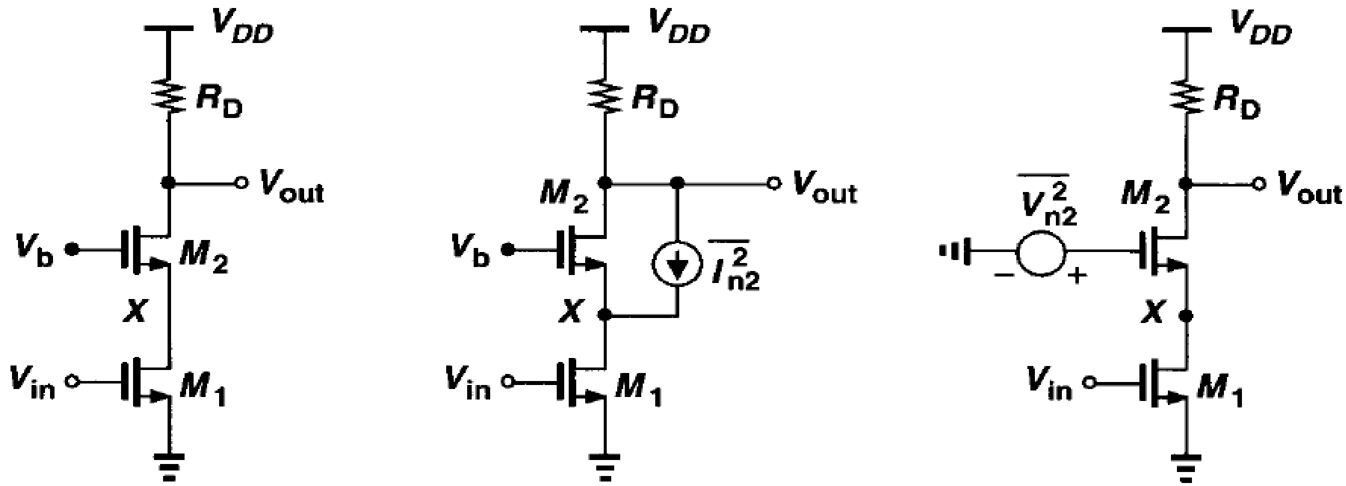
长沟道
热噪声谱

$$\overline{V_{n,in}^2} = \overline{V_{n1}^2} + \frac{\overline{V_{n,out}^2}|_{M2}}{A_v^2} = 4kT\frac{2}{3}\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right). \qquad (7.79).$$

高频时忽略M1的1/f 噪声V_{n1}

两种方法结果一样。
Vn1和In2 可包括1/f噪声。

7.4.4 Cascode Stage



Vn2包括M2
沟道噪声
+1/f

Figure 7.43 (a) Cascode stage, (b) noise of M_2 modeled by a current source, (c) noise of M_2 modeled by a voltage source.

$$\overline{V_{n,in}^2}|_{M1,RD} = 4kT\left(\frac{2}{3g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)$$

where $1/f$ noise of M_1 is ignored.

$$\frac{V_{n,out}}{V_{n2}} \approx \frac{-R_D}{1/g_{m2} + 1/(C_X s)} \quad (3.56)$$

What is the effect of noise of M_2?

this noise contributes negligibly to the output, especially at low frequencies.

the input-referred noise of a cascode stage may rise considerably at high frequencies.

7.5 Noise in Differential Pairs

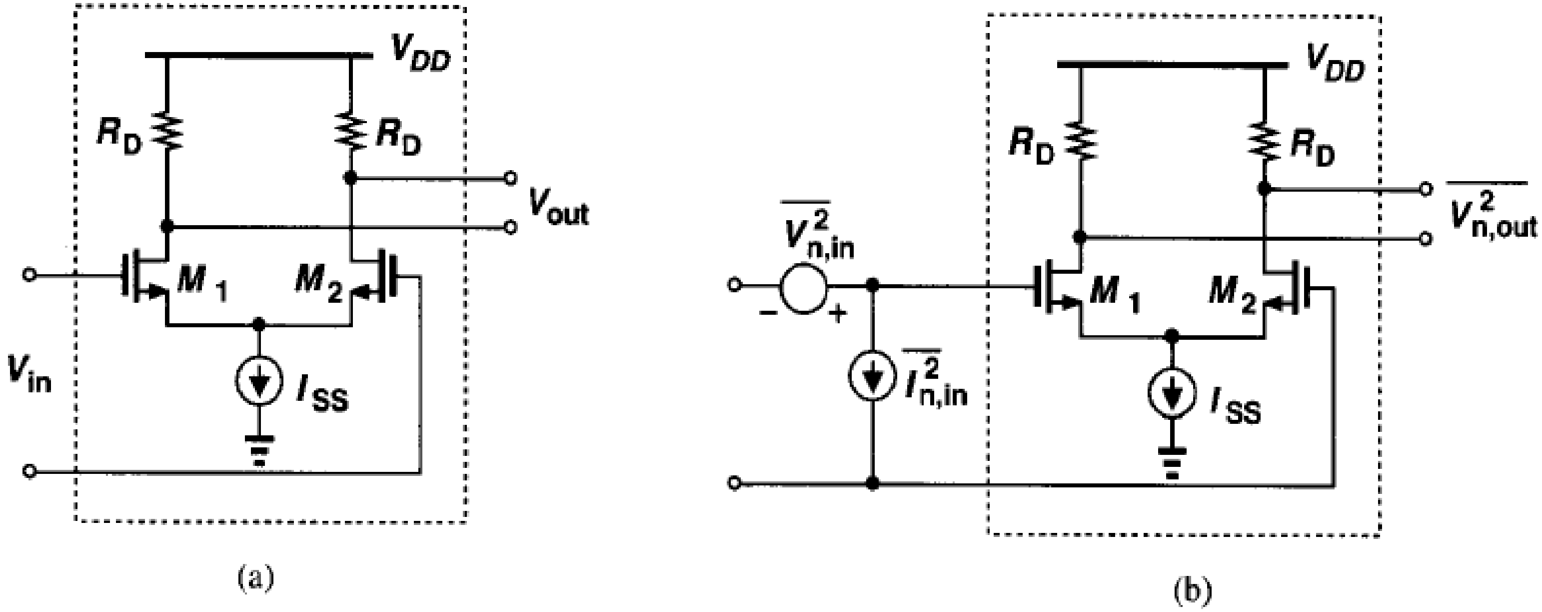


Figure 7.44 (a) Differential pair, (b) circuit including input-referred noise sources.

For low-frequency operation, the magnitude of $\overline{I^2_{n,in}}$ is typically negligible.

差分电路两边器件的噪声不相关，输出（噪声功率）相加！

噪声分析(不能采用半边电路方法)

To calculate the thermal component of $\overline{V_{n,in}^2}$, $R_{D1} = R_{D2} = R_D$,

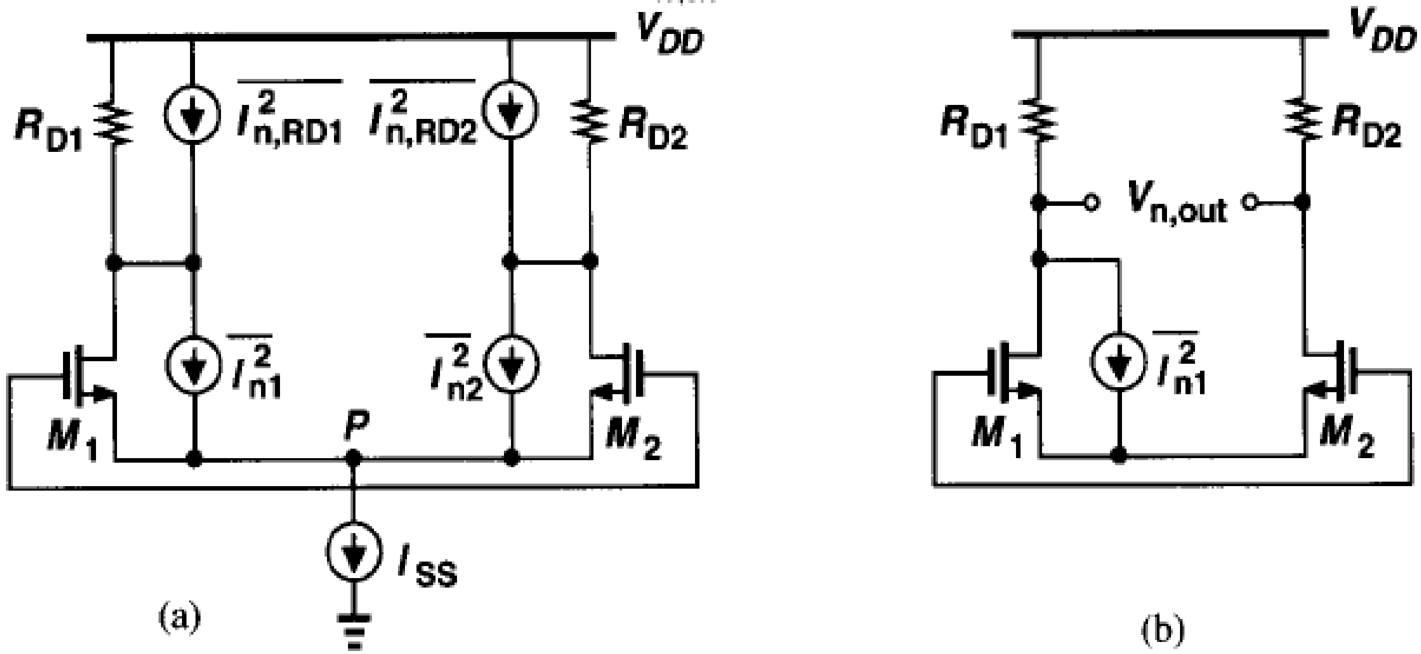


Figure 7.45 Calculation of input-referred noise of a differential pair.

Since I_{n1} and I_{n2} are uncorrelated, node P cannot be considered a virtual ground, making it difficult to use the half-circuit concept.

Thus, we simply derive the effect of each source individually.

MOS热电流噪声的输出电压

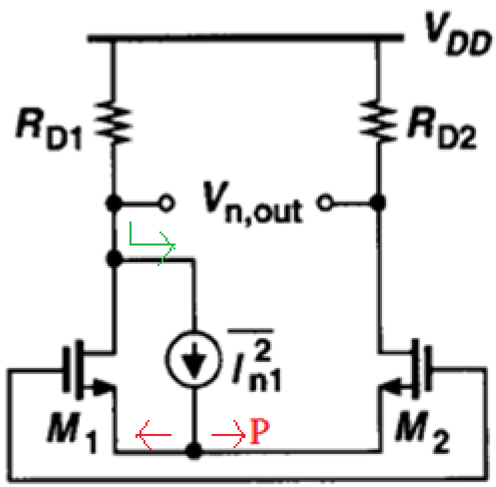


Figure 7.45

设$R_{D1}=R_{D2}=R_D$

$$\overline{I^2_{n1}} = \overline{I^2_{n2}} = \overline{I^2_{n}}$$

neglecting channel-length modulation,

(1) I_{n1}一半流向M_2源极，经$R_{D2}+R_{D1}$流回，

在$R_{D1}+R_{D2}=2R_D$上产生噪声电压功率谱$\overline{I^2_{n1}}R^2_D$;

同样I_{n2}一半流向M_1源极，经$R_{D1}+R_{D2}$流回，

在$2R_D$上产生噪声电压功率谱$\overline{I^2_{n2}}R^2_D$。

(2) 简单方法：设r_o很大，按半边电路计算

（结果正确但概念不对）：

$\overline{I^2_{n1}}$绝大多数流过R_{D1},$\overline{I^2_{n2}}$绝大多数流过R_{D2}。

$$\overline{V^2_{n,out}}\Big|_{M1,M2} = \left(\overline{I^2_{n1}} + \overline{I^2_{n2}}\right)R^2_D.$$

输出端噪声电压只与RD有关

R_D和 MOS的输出热噪声及输入参考噪声

Taking into account the noise of R_{D1} and R_{D2}, we have for the total output noise:

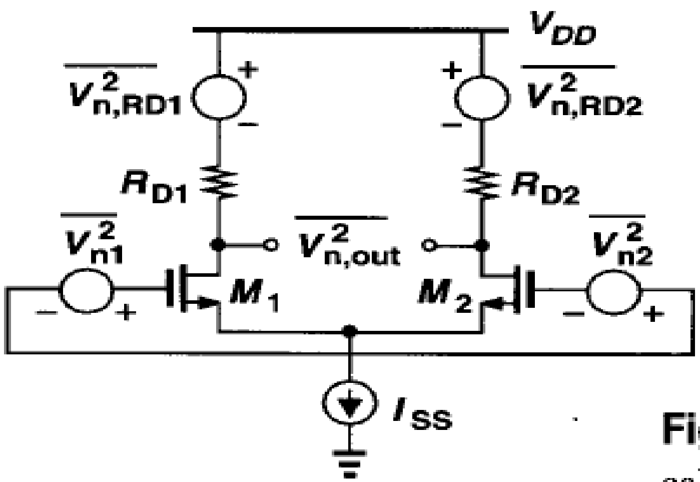
$$\overline{V_{n,out}^2} = \left(\overline{I_{n1}^2} + \overline{I_{n2}^2}\right) R_D^2 + 2(4kTR_D) = 8kT\left(\frac{2}{3}g_m R_D^2 + R_D\right).$$

Dividing the result by the square of the differential gain, $g_m^2 R_D^2$, we have

$$\overline{V_{n,in}^2} = 8kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right). \qquad (7.88)$$

长沟道
热噪声谱

This is simply twice the input noise voltage squared of a common-source stage.

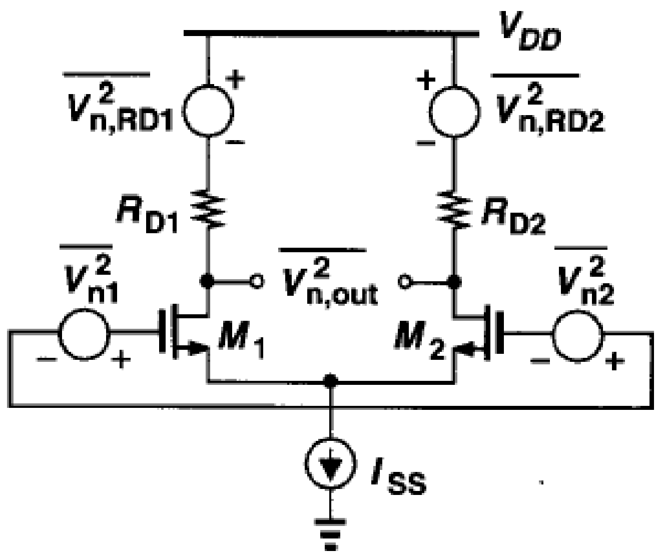


the input-referred noise *voltage* of a differential pair is $\sqrt{2}$ times that of a common-source stage.

噪声是功率相加

Figure 7.46 Alternative method of calculating the input-referred noise.

account for $1/f$ noise of the transistors



rewrite (7.88) as

$$\overline{V^2_{n,in,tot}} = 8kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{2K}{C_{ox}WL}\frac{1}{f}.$$

If the circuit is symmetric, then the noise in I_{SS} divides equally between M_1 and M_2, producing only a common-mode noise voltage at the output.

small differential input, ΔV_{in}, we have

尾电流源噪声影响：
使gm随噪声电流变化

$$\Delta I_{D1} - \Delta I_{D2} = g_m \Delta V_{in}$$

$$= \sqrt{2\mu_n C_{ox}\frac{W}{L}\left(\frac{I_{SS}+I_n}{2}\right)}\Delta V_{in}, \quad (7.91)$$

where I_n denotes the noise in I_{SS} and $I_n \ll I_{SS}$.

尾电流源的噪声影响

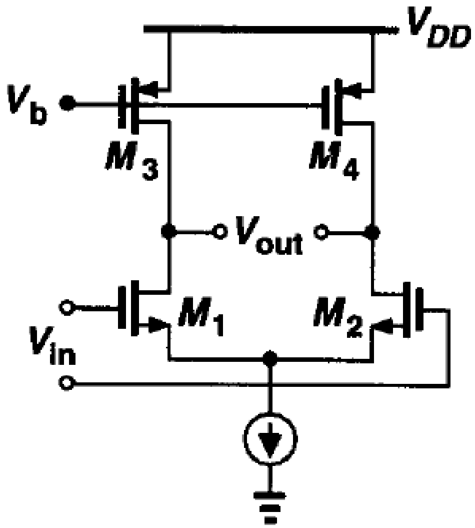
Equation (7.91) can be written as

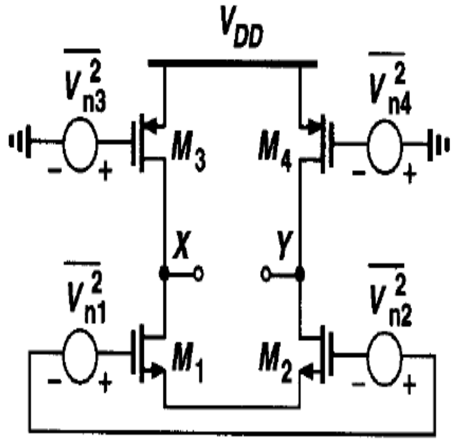
$$\Delta I_{D1} - \Delta I_{D2} \approx \sqrt{2\mu_n C_{ox}\frac{W}{L}\cdot\frac{I_{SS}}{2}}\left(1+\frac{I_n}{2I_{SS}}\right)\Delta V_{in}$$

$$= g_{m0}\left(1+\frac{I_n}{2I_{SS}}\right)\Delta V_{in}, \tag{7.93}$$

where g_{m0} is the transconductance of the noiseless circuit.

Example 7.13 input-referred noise voltage.





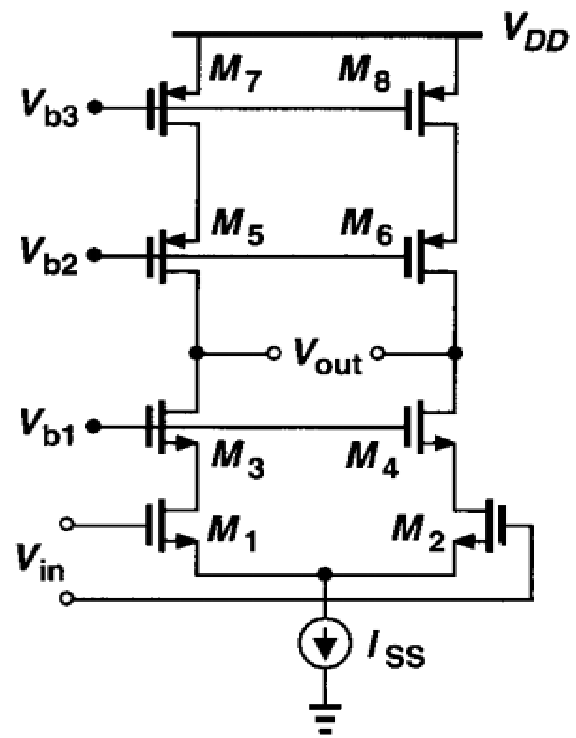
1/f噪声：各自不相关

$$\overline{V_{n,in}^2} = 2\overline{V_{n1}^2} + 2\frac{g_{m3}^2}{g_{m1}^2}\overline{V_{n3}^2},$$

$$\overline{V_{n,in}^2} = 8kT\left(\frac{2}{3g_{m1}} + \frac{2g_{m3}}{3g_{m1}^2}\right) + \frac{2K_N}{C_{ox}(WL)_1 f} + \frac{2K_P}{C_{ox}(WL)_3 f}\frac{g_{m3}^2}{g_{m1}^2}.$$

运放噪声

- 主要来源：输入管、负载电流源；
- cascode管噪声影响小。原因是V_{b1}端和V_{b2}端的噪声（还包括沟道噪声电流折合到栅极电压）在输出端vout的增益很小；
- 与尾电流源基本无关。



输入参考噪声“功率”谱：

增益=$g_{m1}R_{out}$

$$\overline{V_n^2} = 4kT\left(2\frac{2}{3g_{m1,2}} + 2\frac{2g_{m7,8}}{3g_{m1,2}^2}\right) + 2\frac{K_N}{(WL)_{1,2}C_{ox}f} + 2\frac{K_P}{(WL)_{7,8}C_{ox}f}\frac{g_{m7,8}^2}{g_{m1,2}^2}, \qquad (9.45)$$

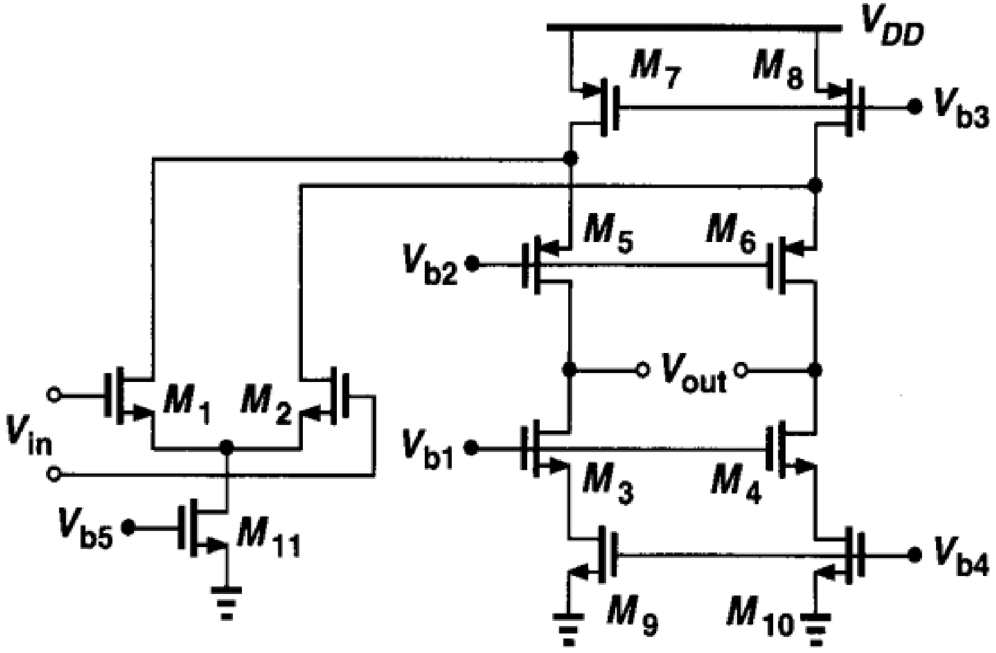
折叠cascade amp 热噪声功率谱

- 增益=$g_{m1}R_{out}$

计算方法提示：

M_7和M_9电流绝大部分流向输出端，乘以Rout得到V_{out}，再除以增益得到输入参考噪声电压功率谱，即=MOS电流功率谱/$g_{m1}{}^2$。

M_5、M_3电压增益很小，可忽略。



输入参考热噪声“功率”谱：

$$\overline{V_{n,int}^2} = 8kT\left(\frac{2}{3g_{m1,2}} + \frac{2}{3}\frac{g_{m7,8}}{g_{m1,2}^2} + \frac{2}{3}\frac{g_{m9,10}}{g_{m1,2}^2}\right)$$

两级放大器输入参考热噪声“功率”谱

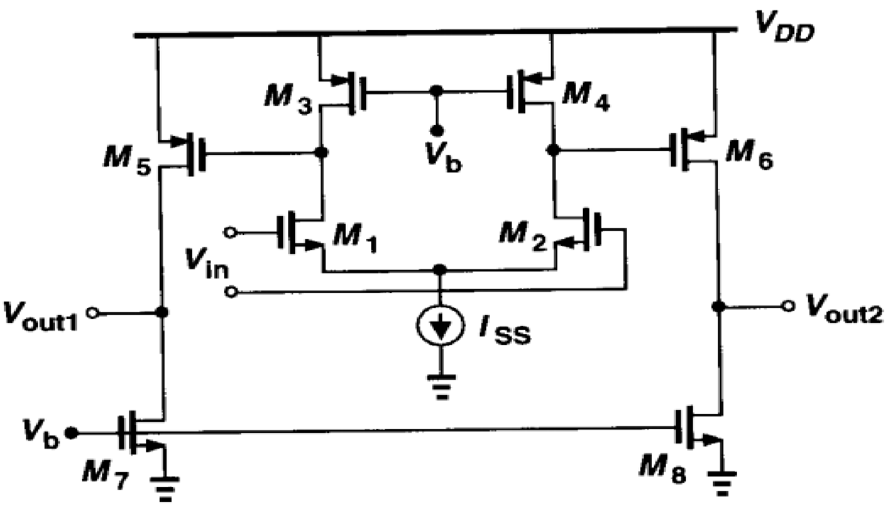


Figure 9.65 Noise in a two-stage op amp.

- 1/f噪声？

$$\overline{V_n^2}\Big|_{M5-8} = 2 \times 4kT\frac{2}{3}(g_{m5}+g_{m7})(r_{O5}\|r_{O7})^2\frac{1}{g_{m1}^2(r_{O1}\|r_{O3})^2 g_{m5}^2(r_{O5}\|r_{O7})^2}$$

$$= \frac{16kT}{3}\frac{g_{m5}+g_{m7}}{g_{m1}^2 g_{m5}^2(r_{O1}\|r_{O3})^2}.$$

$$\overline{V_{n,tot}^2} = \frac{16kT}{3}\frac{1}{g_{m1}^2}\left[g_{m1}+g_{m3}+\frac{g_{m5}+g_{m7}}{g_{m5}^2(r_{O1}\|r_{O3})^2}\right]$$

噪声主要取决于第一级。

第二级噪声折算到输入参考噪声，降低了多少倍？

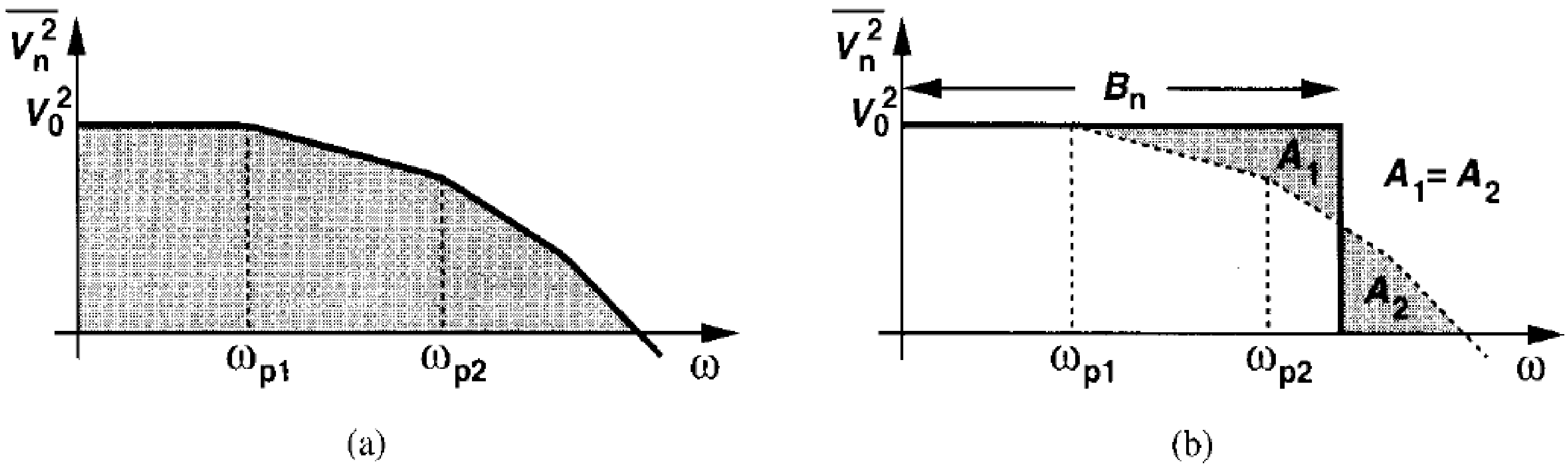


Figure 7.49 (a) Output noise spectrum of a circuit, (b) concept of noise bandwidth.

$$\overline{V_{n,out,tot}^2} = \int_0^{\infty} \overline{V_{n,out}^2} df. = V_0^2 \cdot B_n$$

Called the "noise bandwidth," B_n allows a fair comparison of circuits that exhibit the same low-frequency noise, V_0^2, but different high-frequency transfer functions. As an exercise, the reader can prove that the noise bandwidth of a one-pole system is equal to $\pi/2$ times the pole frequency.