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360° Varactor Linear Phase Modulator

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Abstract—**Theory is presented which 1) derives the circuit impedance requirements to match the nonlinearity of the varactor reactance-versus-voltage curve to the tangent θ curve to obtain 180° linear phase modulation from one diode; 2) gives the value and position of a resistor to make insertion loss invariant with phase; and 3) derives the circuit requirements for combining two 180° diode phase modulators in an admittance adding network to obtain 360° phase modulation. Experiments are disclosed using series tuning at 1 GHz providing 360° phase modulation within ± 3.0 percent of linearity, and using shunt tuning at 5 GHz providing 360° phase modulation within ± 3.3 percent of linearity. A discussion is given of the application of the modulators to the serrodyne function.**

I. INTRODUCTION

A CONTINUOUS phase modulator can be made by placing a varactor diode on one terminal of a circulator [1]. Power in the first port of the circulator is reflected by the diode on the second port and emerges from the third port with a phase and amplitude dictated by the reflection coefficient of the diode. As the reverse voltage applied to the varactor is varied, the magnitude of the reflection coefficient remains high and the phase changes. The modulator requires very little modulation power and responds quickly to changes in modulation voltage. This type of phase modulator typically has three problems. It has a nonlinear voltage-phase relationship, an insertion loss that varies with phase, and less than 180° modulation. For many applications such performance is inadequate. The phase should vary linearly with voltage, the insertion loss should be constant, and the modulator should be able to provide 360° modulation.

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In comparison with digital phase modulators this type of phase modulator has the disadvantage of not working at higher RF power levels. However if the three problems can be solved it has the following advantages: no analog-to-digital converter is required; only two diodes are required, making the device smaller and less expensive to fabricate; and the diodes are operated at reverse bias, having very fast response times and requiring voltage but no current. These advantages may be of significant importance in remote applications, such as satellites, where high efficiency is required, in automatic control systems where simplicity is needed, and in frequency translators where high speed is needed.

The phase modulators under consideration here were principally intended for performing frequency translation. By applying a sawtooth waveform to a linear 360° phase modulator it is possible to generate power at a frequency displaced from the input frequency by the sawtooth frequency. It is common practice to perform frequency translation with a traveling wave tube or a single sideband modulator using mixer diodes. More recently, frequency translation has been performed using digital phase modulators [2]. The varactor linear phase modulator has advantages over the TWT in that it is much smaller, requires less total power, and has a longer life. The varactor linear phase modulator has advantages over the single sideband modulator using mixer diodes in that it has a theoretical conversion loss approaching zero and the unwanted-sideband suppression is independent of incident power level below 10 mW. The varactor linear phase modulator has advantages over digital phase modulators in this application in that it is smaller, faster, more efficient, and less expensive to fabricate.

There is another way to make continuous varactor phase modulators other than using the reflection properties of the diode when connected to a circulator. This is by periodically loading the transmission line with varactors [3]. The periodically loaded transmission line technique requires many diodes and several wavelengths of transmission line for 360° modulation. Because of its size and inefficient use of diodes, it will not be further considered here. The reflection-type phase modulator can be made small and close to ideal in performance when the varactors are mounted in a circuit designed to solve the three problems associated with it: linearizing the voltage-phase relationship, increasing the phase modulation to 360°, and making the insertion loss constant.

Several efforts have been made to solve these problems, but no effort has been made to solve all three in one circuit. Searing [4] demonstrated that a series inductance could be used to increase the phase modulation of one diode to 180°. Gardner and Hawke [5] obtained linearity and constant insertion loss at 9 GHz using a varactor (or pair of diodes) in waveguide with an adjustable short circuit behind it and a slide screw tuner in front of it, but they had no control over how much maximum modulation each unit provided, obtaining about 110° for one diode pair. Kim *et al.* [6] obtained 280° phase modulation within 0.5 percent of linearity at S-band using four diodes, three circulators, and sixteen transmission line sections.

The work described here solves simultaneously all three problems associated with the reflection phase modulator. It 1) takes a different approach to deriving the linearizing circuit which results in a minimum of circuitry, 2) presents a practical method for maintaining insertion loss constant, and 3) derives admittance relationships for combining varactor circuits to obtain 360° phase modulation from two 180° phase modulators without the use of additional circulators.

II. Theory

The circuit necessary for obtaining 360° phase modulation within ± 1.4 percent of linearity and with constant insertion loss is shown in Fig. 1. The varactor diodes are mounted with their junction ends adjacent to the $\lambda_g/4$ line section. This mounting minimizes the stray capacitance normally associated with a varactor diode. Additional inductance which is needed for linearizing the voltage-phase relationship is shown outside the varactor diode. In practice this can be a length of the diode lead which is shorted to RF ground some distance from the diode glass package. The characteristic impedance of the circulator is $Z_0/2$. In addition to the components shown in Fig. 1 diode biasing circuits will be required. The voltage may be applied to the varactors through a bias T in the $Z_0/2$ line section or through baseband filters which present RF short circuits at the ends of L. Other dc open and short circuits will be needed for biasing the diodes; however, these, when well designed, will not interact with the linearizing circuit. To obtain linearity, C must be selected to match Z_0, or Z_0 must be adjusted to accommodate C, and L must be properly adjusted. R_p is dictated by R_s and Z_0 to provide constant insertion loss as phase is varied.

A. Linearity

When a movable short circuit is placed on port 2 of a circulator all of the power into port 1 emerges from port 3. Power into port 1 is directed to port 2 by circulator action. The incident wave in the line connected to port 2 is all reflected by the short circuit reentering the circulator to emerge from port 3. Moving the short circuit to a position 1 cm further away from the circulator will cause the incident wave to travel an additional 1 cm before being reflected and the reflected wave to travel on additional 1 cm, giving a total additional path length of 2 cm. When the short circuit is moved $\lambda/4$ (90°) the reflected wave will have traveled an additional $\lambda/2$ (180°), therefore the phase delay at the output of the circulator is twice the phase corresponding to the position of the short circuit.

Consider the impedance of a short circuit terminating a low-loss transmission line of characteristic impedance Z_0 and length l. This impedance is a reactance represented by

$$\frac{X}{Z_0} = \tan(2\pi l/\lambda). \tag{1}$$

The reactance seen from port 2 of the circulator is a tangent function of the argument $(2\pi/\lambda)l$ and the output phase is directly proportional to l. Therefore if the transmission line and moveable short circuit are replaced by any normalized reactance which is a tangent function of some argument then the change in output phase will be equal to twice the change in the argument [6]. A simple method for obtaining a tangent function proportional to applied voltage is to match the nonlinearity of a varactor reactance directly to that of a tangent curve [7]. (Because of the analytical approach taken in [7] the full potential of this simple circuit was not realized.)

$$\frac{X}{Z_0} = \tan(kV) \tag{1a}$$

$$\begin{aligned} \Gamma = |\Gamma| e^{j\phi} &= \frac{j\tan(kV) - 1}{j\tan(kV) + 1} \\ &= \frac{\tan^2(kV) - 1 + 2j\tan(kV)}{\tan^2(kV) + 1} \end{aligned} \tag{1b}$$

$$\begin{aligned} \phi &= \tan^{-1}\left[\frac{2\tan(kV)}{\tan^2(kV) - 1}\right] \\ &= \tan^{-1}[-\tan(2kV)] = -2kV \end{aligned} \tag{1c}$$

From [8] the capacitance of the varactor junction is

$$C = C_{\min} V_N^{-\gamma} \tag{2}$$

in which $C_{\min}$ is the minimum capacitance to be used (corresponding to the maximum applied voltage $V_{\max}$), γ is 1/2 for abrupt junction varactors and 1/3 for diffused junction varactors, and V_N is the normalized voltage given by

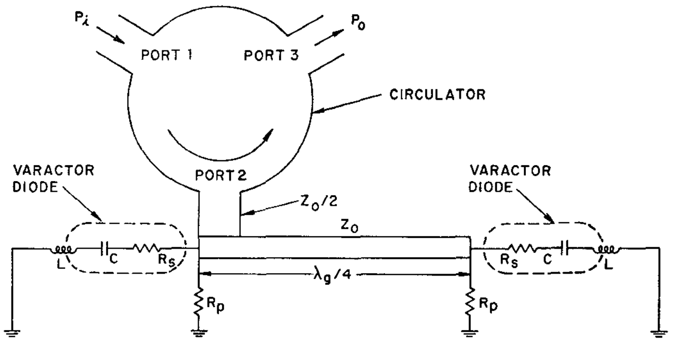

Fig. 1. Circuit for 360° phase modulator.

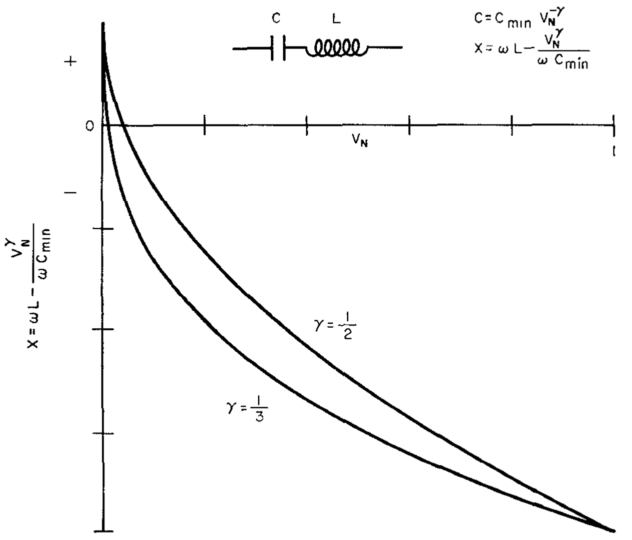

Fig. 2. Varactor reactance.

$$V_N = \frac{V - \phi}{V_{\max} - \phi} \tag{3}$$

in which ϕ is the contact potential and V is the applied voltage. This capacitance produces the reactance shown in Fig. 2. Changing the series inductance L moves the curves up or down. V_N is a linear function of V so that changing $V_{\max}$ and ϕ does not change the shapes of the curves. The normalized reactance of the varactor diode and series inductance is thus

$$\frac{X}{Z_0} = \frac{\omega L}{Z_0} - \frac{V_N^{\gamma}}{\omega C_{\min} Z_0} = A - BV_N^{\gamma}. \tag{4}$$

In Fig. 3 the reactance curve for $\gamma = \frac{1}{2}$ has been adjusted to closely match the tangent curve over a 90° range (which corresponds to 180° phase modulation). It is assumed that R_s is negligible. To match the curves it is necessary that $A = 2.35$ and $B = 2.78$. As voltage is increased from 0 to $V_{\max}$, the varactor circuit impedance goes from inductive through zero ohms to capacitive which corresponds to a short circuit being moved toward the circulator. The microwave pathlength to the output port of the circulator is being effectively shortened; therefore, increasing the varactor voltage advances the RF phase.

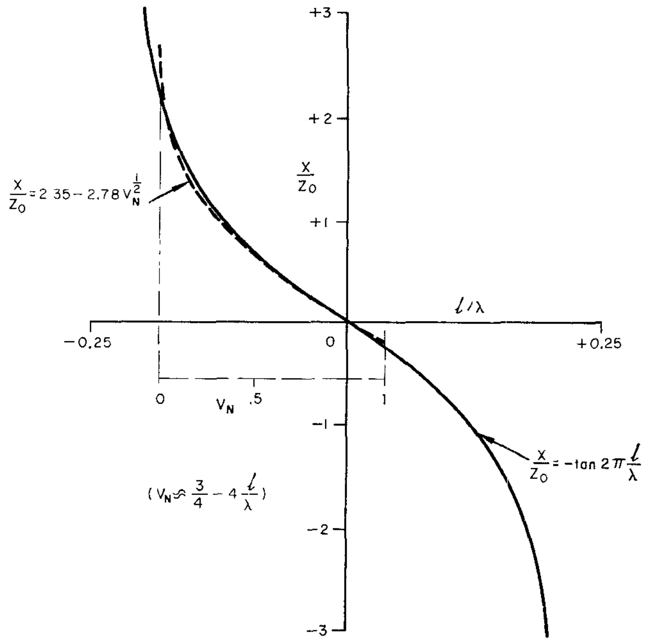

Fig. 3. Varactor reactance matched to tangent function.

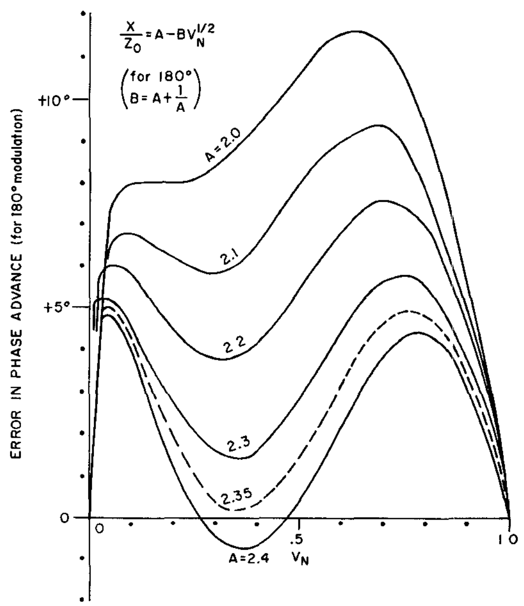

Fig. 4. Phase error for 180° modulation ($\gamma = \frac{1}{2}$).

As V_N is varied from 0 to 1, the phase should vary from 0° to 180°. The difference between the calculated phase and this exact proportionality is the error which is shown in Fig. 4 for various values of diode impedance. For each A there is a B which provides 180° modulation given by the formula in the figure. The condition is that the reactance of the circuit at $V_N = 1$ be the complex conjugate of its reactance at $V_N = 0$ see [9]). This condition produces reactance extremes which are halfway around the Smith chart from each other, which in turn provides the equivalent of a quarter-wavelength travel of short position.

Note that for $A = 2.35$ both maximums of the error curve

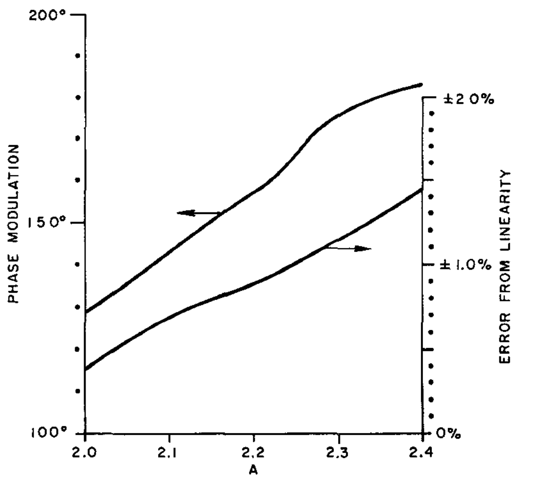

Fig. 5. Phase modulation and error for one varactor ($\gamma=\frac{1}{2}$).

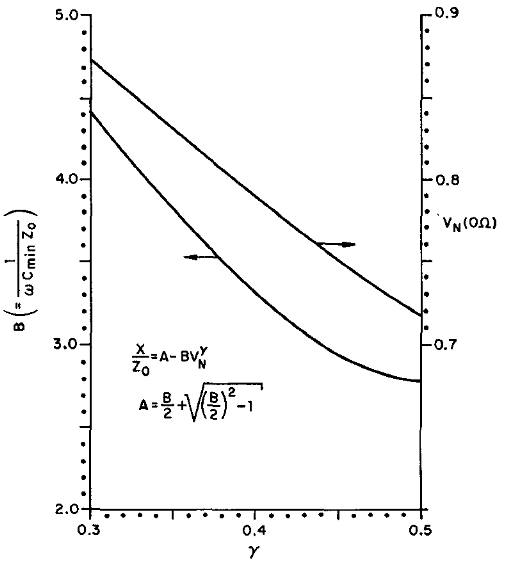

Fig. 6. B needed to match varactor reactance curve to tangent curve for 180° modulation; and V_N for which the diode and inductor should be 0 ohm.

reach 5° which provides phase linearity within $\pm 2.5°/180° = \pm 1.4$ percent of full modulation. For less than 180° modulation per diode even greater linearity is possible. For example; for $A=2.0$, the phase is within $\pm 0.5°$ of linearity for V_N ranging from 0.04 to 0.73. Total modulation is 128.8° within ± 0.39 percent of linearity. The phase modulation for minimum error and the error accompanying it are shown in Fig. 5. Extrapolating the curve to an error of ± 0.25 percent at $A=1.96$ shows that one diode should be able to provide 123° phase modulation. By combining four of these diodes it should be possible to get close to 500° phase modulation within ± 0.25 percent of linearity. Kim *et al.* [6] obtained $280° \pm 0.25$ percent using four varactors, however they avoided the low-voltage region in order to keep their maximum power high.

As γ changes, the A and B required for 180° phase modulation and for minimum deviation from linearity vary as shown in Fig. 6. Here B is used on the graph because it relates directly to C_{min} and Z_0 [see (4)]. In order to keep nonlinearities down, it is necessary to operate slightly away from $V_N=0$ when γ becomes low. For example at $\gamma=\frac{1}{3}$ it was necessary to operate between $V_N=0.01$ and $V_N=1.07$. (Note that when V_{max} can be exceeded giving $V_N>1$ then V_{max} is no longer a maximum voltage but a convenient reference for normalizing the voltage.) The error was $\pm 2.6°$ for 180° modulation, providing ± 1.4 percent error from linearity. Working between limits of V_N other than 0 and 1 returns the degree of freedom to the analysis removed by requiring that $B=A+1/A$ for 180° phase shift for $-\phi \leq V \leq V_{max}$. The curves of Fig. 4 and 5 thus become a convenient aid to closely evaluating the errors from nonlinearity for all phase modulations. Less error cannot be achieved with this simple circuit, and attempts to increase the linearity with additional circuitry have failed. If higher precision is required, more diodes have to be used to reduce the phase modulation per diode.

B. Constant Insertion Loss

The impedance of a varactor diode ($\gamma=0.5$) put in a circuit to satisfy linearity requirements is shown in Fig. 7. Note that because of R_s, chosen here to satisfy

$$\frac{R_s}{Z_0} = 0.2,$$

the magnitude of the reflection coefficient varies from 0.667 at -32 volts (where the curve would cross the real axis) to 0.855 at $+0.8$ volt. The corresponding insertion losses are 3.52 dB and 1.36 dB, which provides 2.44 dB $\pm$ 1.08 dB for 180° phase modulation. This ± 1.08-dB amplitude modulation may be reduced by putting a resistance in parallel with the diode [9]. The normalized impedance points of the diode circuit fall on a circle having its center at A. The object is to move this center to the middle of the Smith chart; then the reflection coefficient will be constant. This can be done by finding the reference plane for which the center of the circle corresponding to the normalized admittance points of the diode circuit falls on the real axis at a normalized value less than one. For this diode circuit that plane is the same as the input plane, because the center of the circle for normalized admittance falls at A'. Adding $G/Y_0=0.2$ causes the center to move to B. When the plane for the parallel resistor is the input plane of the varactor, the resistor must satisfy

$$\frac{G}{Y_0} \approx \frac{R_s}{Z_0} \tag{5}$$

$$R_p \approx Z_0^2/R_s. \tag{6}$$

When the plane is removed from the diode, the resistance can be calculated from

$$R_p = \rho_{min} Z_0 \tag{7}$$

in which ρ_{min} is the minimum VSWR of the diode circuit before any resistance is added. R_p is then placed in the plane of the minimum position of ρ_{min}. For Fig. 7 the insertion loss becomes constant at 3.5 dB.

This may seem like a high insertion loss but it is high because of the high R_s/Z_0 assumed to demonstrate the procedure for leveling insertion loss. This diode satisfies

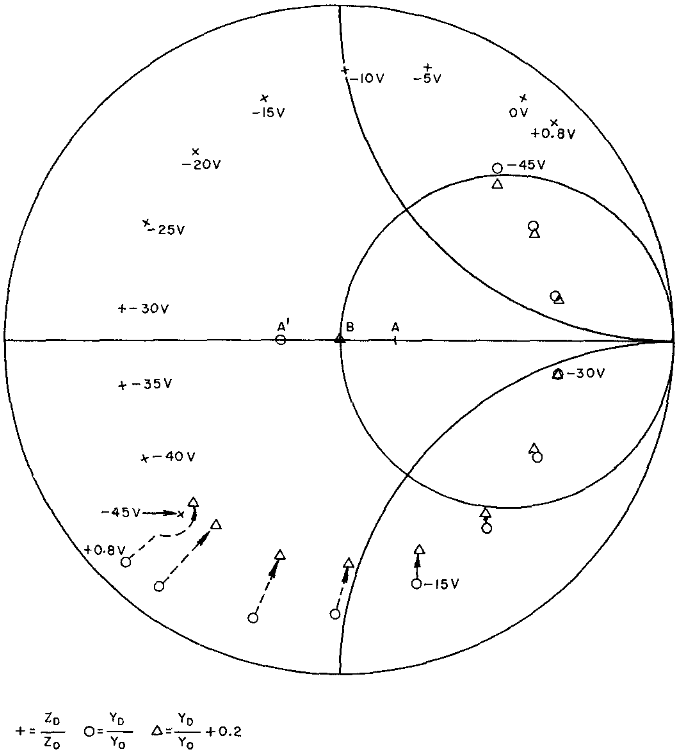

Fig. 7. The addition of R_p to make the insertion loss of a one-diode 180° modulator invariant with phase.

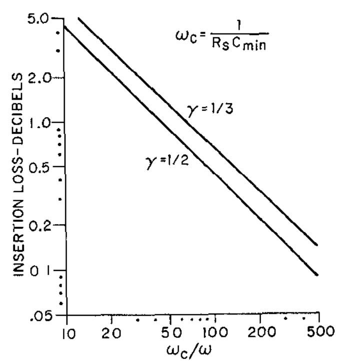

Fig. 8. Insertion loss of one-varactor 180° modulator.

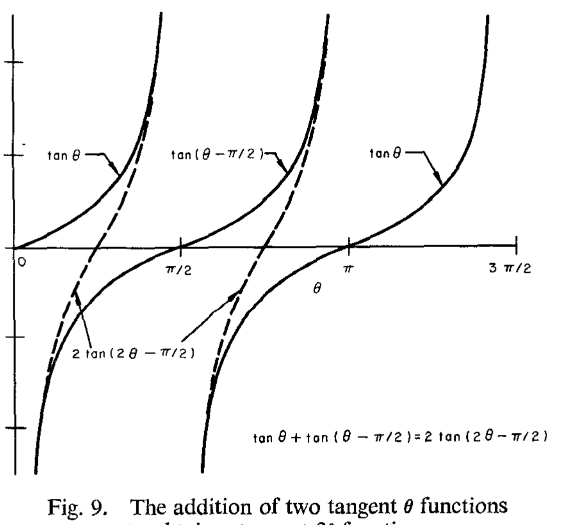

Fig. 9. The addition of two tangent θ functions to obtain a tangent 2θ function.

$$B = 2.78 = \frac{1}{\omega C_{\min} Z_0} \tag{8}$$

$$\frac{R_s}{Z_0} = 0.2 \tag{9}$$

and

$$\omega_c = \frac{1}{R_s C_{\min}}, \tag{10}$$

where ω_c is the cutoff frequency of the varactor at $V_{\max}$. Combining these gives $\omega_c/\omega = 13.9$. In general the insertion loss δ of the phase modulator (for 180° modulation) is given by

$$\delta = 20 \log \frac{\left(\frac{\omega B}{\omega_c}\right)^2 + \left(\frac{\omega B}{\omega_c}\right) + 1}{\left(\frac{\omega B}{\omega_c}\right)^2 - \left(\frac{\omega B}{\omega_c}\right) + 1} \approx 50 \frac{\omega}{\omega_c} \tag{11}$$

in which B is taken from Fig. 6. This insertion loss is shown in Fig. 8. For $\gamma = \frac{1}{2}$, 1 dB is obtained for an ω_c/ω of 48. Most manufacturers specify ω_c at -6 volts rather than at $V_{\max}$ as used here; therefore the requirement on ω_c is lower than the manufacturer's specification. For example, a 180° phase modulator operating up to 40 volts with $\gamma = \frac{1}{2}$ would have to have $\omega_c(-6\text{ V})/\omega = 20$ for 1-dB loss. By increasing ω_c/ω, the insertion loss can be made to approach zero.

White [10] made a calculation of minimum insertion loss to be expected from varactor reflection-type phase modulators. For 180° and with the voltage being driven to $-\phi$, this figure is $\delta = 27\omega/\omega_c$. This figure, based on the assumption that each diode provides less than 45° modulation, is about half the insertion loss calculated here for 180° per diode.

C. 360° *Modulation*

Two 180° modulators on two circulators in series could provide 360° modulation. However, a simpler method is to add the admittances of the diodes in the proper phase and attach the circuit to one circulator. The trigonometric identity for adding the admittances is shown in Fig. 9. Since the varactor impedances are tangent functions, then their admittances are also:

$$Y_0 \tan \theta + Y_0 \tan (\theta - \pi/2) = 2 Y_0 \tan (2\theta - \pi/2). \tag{12}$$

The identity shows that adding two tangent functions which are 90° out of phase produces a double tangent of the double angle function. Two diodes each providing 180° modulation can thus be placed in parallel to provide 360° modulation when they are 90° out of RF phase and work into twice the admittance that each is made to work into separately. This means that for two diodes to work on a 50-ohm circulator, they must be designed to work separately into 100 ohms, and one of them must be a quarter wavelength (at $Z_0 = 100$ ohms) farther from their common point than the other.

When diodes are combined in this fashion, their insertion losses approximately add; thus for 360° modulation, the insertion loss figures of Fig. 8 must be doubled.

It is useful to note that junction capacitance in picofarads at 6 volts (C_6) for varactors designed to provide 360° phase modulation should have $C_6=1.39/F$ for $\gamma=\frac{1}{2}$ and $C_6=0.72/F$ for $\gamma=\frac{1}{3}$. F is the operating frequency in GHz. It is assumed each diode will work into 100 ohms, and $V_{\max}$ is 40 volts.

III. Series Tuning Design Technique—1 GHz

Varactors in glass cartridges are most easily tuned using a series inductance as described in Section II-A. Varactors mounted in pill packages are more easily shunt-mounted in stripline, making it more desirable to tune the varactor for linear phase modulation with a shunting susceptance.

The results obtained with a series-tuned MA/4325C1 diode at 1 GHz are shown in Fig. 10. This diode had $C_{\min}$ (40 volts)$=0.63$ pF, $C_c=0.4$ pF, $\gamma=0.54$. The theory predicted $Z_0=91.6$ ohms and $X_T=AZ_0=+21.4$ ohms. The values which provided the results shown were $Z_0=16.3$ ohms and $X_T=+80$ ohms, which would imply that $C_{\min}=1.92$ pF and $\gamma=0.27$. Factors contributing to these differences were that C_c was not tuned out, the voltage was varied from 0 to 40 volts for 180° instead of from $-\phi$ to 40 volts, and the varactor capacitance does not follow the power law exactly over the entire reverse bias range. Since the theory gives only a coarse guide to circuit requirements, a design technique is needed to simplify construction.

The two factors which dictate the linearizing circuit are $C_{\min}$ and γ, which are measured as in the Appendix. When C_c is not to be tuned out, $C_{\min}$ can be taken as the total capacitance, and γ can be determined by the slope of the curve of Fig. 21 for the upper half of $V-\phi$ (20 to 40 volts in Fig. 21). This gives a good approximation over most of the voltage range. The normalized voltage when the diode and circuit have an impedance of 0 ohm, V_N (0 ohm), is independent of the characteristic impedance of the transmission line in which it is being measured. This V_N (0 ohm) is also not strongly changed by γ as illustrated in Fig. 6. Using the γ from the above measurement, V_N is set to the value specified in Fig. 6, and ωL is adjusted until the input impedance is 0 ohm. To make a 180° phase modulator, a quarter-wavelength transformer is placed in front of the diode and its Z_0 is adjusted until linearity is obtained. Referring to Fig. 10, the data points will fall too high above the straight line of exact proportionality (from 0° and 0 volt to 180° and 40 volts) for small values of voltage (0 to 10 volts) when the characteristic impedance of the quarter-wavelength line is too high, and below it when it is too low. Furthermore, the error at low voltages will not be strongly influenced by ωL, so that the procedure may be summarized as follows: 1) ωL is made close to correct using V_N (0 ohm), 2) Z_0 is made close to correct minimizing the deviation from linearity at low voltages, 3) finally, fine adjustment of ωL brings the phase deviation at $V_{\max}$ near zero. When ωL is too small, more than 180° modulation is obtained, and when too large, less than 180° modulation is obtained.

When 360° phase modulation is needed, fine adjustments must be made in Z_{01} and Z_{02} shown in Fig. 11. For diode A working in Z_{0A} and diode B working in Z_{0B},

$$Z_{02} = \sqrt{Z_{0A}Z_{0B}} \tag{13}$$

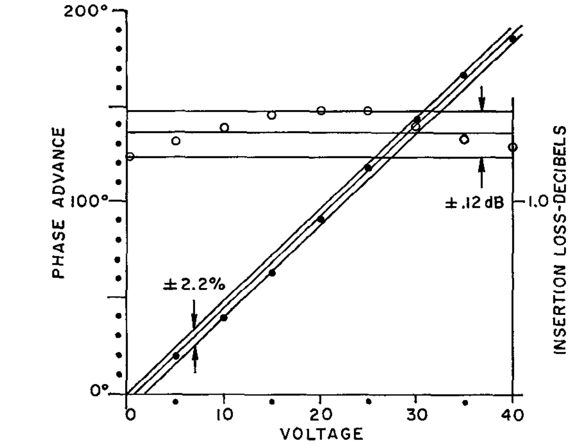

Fig. 10. Phase modulation and insertion loss of the MA4325C1 at 1 GHz (deduced from slotted-line measurements).

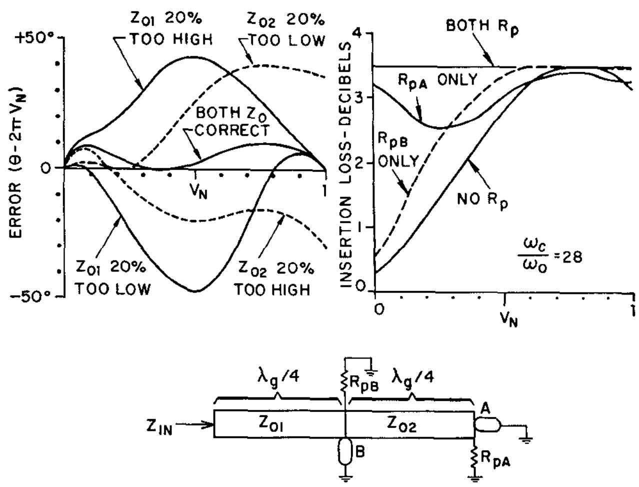

Fig. 11. Corrections for combining two varactors to obtain 360° modulation.

and

$$Z_{01} = \sqrt{\tfrac{1}{2}Z_0Z_{0B}} \tag{14}$$

where Z_0 is the characteristic impedance of the circulator to be used with the circuit.

The procedure for eliminating the variation in insertion loss is to select R_{pA} so that the insertion loss (or VSWR) is the same at 0° as it is at 360°. Then R_{pB} may be selected to further reduce the variation when it is required. R_{pB} may be calculated using the lowest VSWR ρ_L and highest VSWR ρ_H looking into the circuit:

$$R_{pB} = \frac{Z_{01}^2}{Z_0}\left(\frac{\rho_L\rho_H}{\rho_H-\rho_L}\right). \tag{15}$$

The resistor is put in the plane of the minimum of ρ_L.

The experimental results using two MA4325C1 varactors are shown in Figs. 12 and 13. For applications requiring 360° modulation over a fixed voltage range, the bandwidth is not very broad. But when the voltage range can be modified to suit the frequency, linearity over 360° can be obtained with this modulator from 0.9 to 1.0 GHz. For power below 10

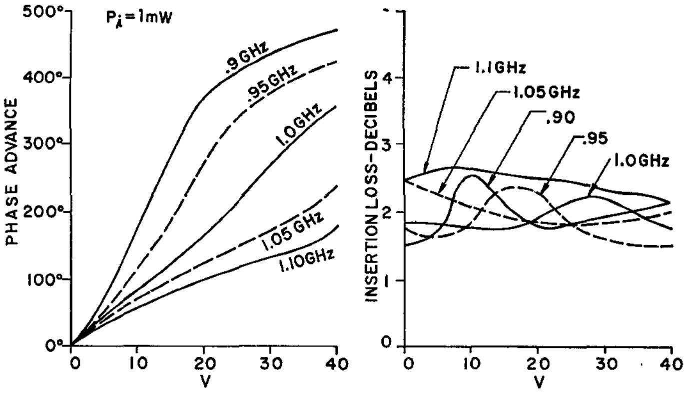

Fig. 12. Phase modulation and insertion loss of two MA4325Cl diodes as frequency is varied (deduced from slotted-line measurements).

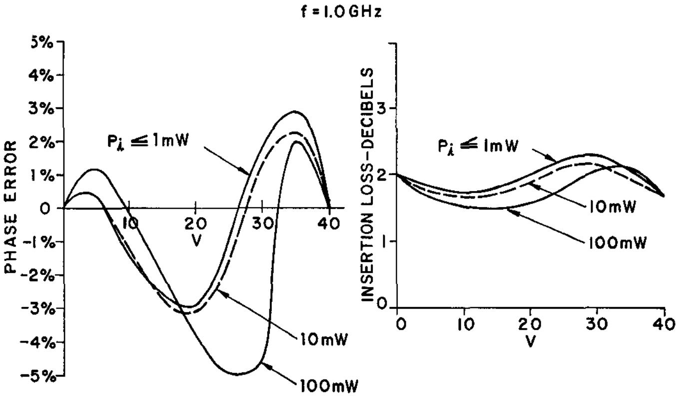

Fig. 13. Phase error and insertion loss of two MA4325Cl diodes as power is varied (deduced from slotted-line measurements).

mW, the phase is within ± 3 percent of linearity, and the insertion loss is 1.94 ± 0.26 dB. Finer adjustments in the circuit may have reduced the nonlinearity to a figure closer to the theoretical ± 1.4 percent, however the deviation of the varactor capacitance from the power law adds to the nonlinearity. This deviation occurs at the higher voltages, providing less capacitance charge with voltage than prescribed by the power law. At 100 mW, the nonlinearity goes to ± 3.5 percent, and the insertion loss variation goes to ± 0.28 dB.

IV. Shunt-Tuning Design Technique—5 GHz

At higher frequencies, the inductance of varactors in glass packages becomes greater than that required for linearity, and C_c becomes even more troublesome than it was at 1 GHz. The effective inductance can be reduced by using a capacitor behind the diode (shorted stub in which $0.25 < l/\lambda < 0.50$). A better varactor package for higher frequencies is the pill package which has a low inductance. It is most easily tuned in shunt. The curves shown in Fig. 14 give the design parameters for obtaining linearity by shunt tuning when $\gamma = \frac{1}{2}$ and $\gamma = \frac{1}{3}$. When $k = 0.848$, $B_T/Y_0 = 0$, $\gamma = \frac{1}{2}$, the conditions are the same as series tuning. Note that in this circuit arrangement, C_c is included in B_T, and the consequent deviations from theory are much less. Curves for other values of γ may be derived by requiring B_T/Y_0 to satisfy 180° modulation for

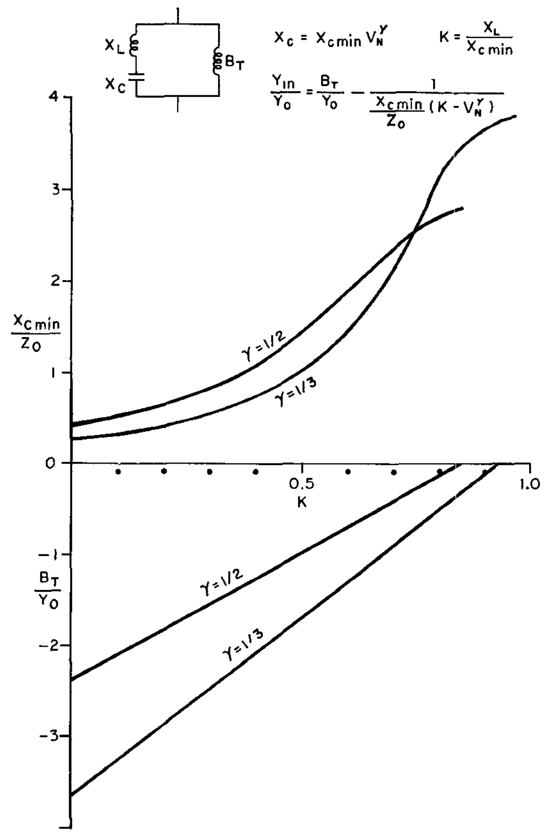

Fig. 14. Circuit parameter requirements for shunt tuning a varactor for linear 180° modulation.

V_N going from 0 to 1 (similar to the relationship between B and A for series tuning). $X_{c\min}/Z_0$ is then adjusted to make the phase error 0° to +1° (out of 90° depending on γ) at $V_N = 0.4$. As before, adjusting Z_0 varies the deviation from linearity at low voltages, and adjusting B_T affects the deviation near $V_N = 1$. The V_N (0 ohm) criterion, however, no longer applies.

Since B_T must be inductive without shunting out the modulation voltage, an open-circuited stub $\lambda/4 < l < \lambda/2$ is required. This stub may have any characteristic admittance, however one characteristic admittance makes the stub length least critical in the range of interest. This is derived as follows:

$$B_T = Y_0 \tan 2\pi l/\lambda$$

$$\frac{\partial B_T}{\partial l} = \frac{2\pi}{\lambda} Y_0 \left[\left(\frac{B_T}{Y_0}\right)^2 + 1\right] \tag{16}$$

$$\frac{\partial}{\partial Y_0}\left(\frac{\partial B_T}{\partial l}\right)_{B_T} = -\frac{2\pi}{\lambda}\frac{B_T^2}{Y_0^2} + \frac{2\pi}{\lambda} = 0 \tag{17}$$

$$Y_0 = |B_T|. \tag{18}$$

In making a 360° phase modulator the effects of having B_T incorrect are shown in Fig. 15. This figure indicates that a straightforward tuning procedure is to tune B_{TA} first to obtain 360° modulation at $V_N = 1$. (This amounts to setting the slotted line probe at the minimum position for $V_N = 0$ and adjusting B_{TA} to obtain the same minimum with V_N set to 1.)

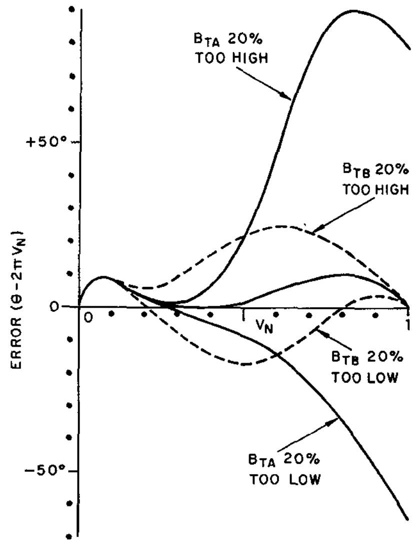

Fig. 15. Corrections for adjusting the tuning inductors for two shunt-tuned varactors.

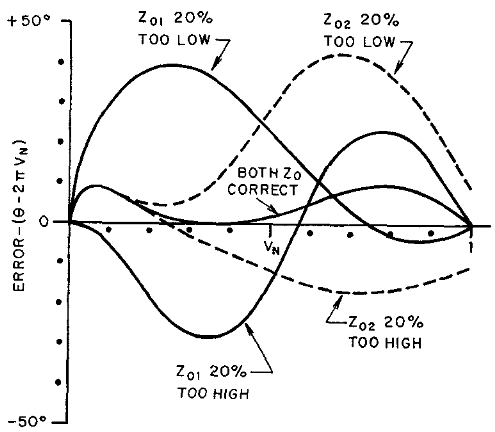

Fig. 16. Corrections for adjusting the characteristic impedances of two shunt-tuned varactors.

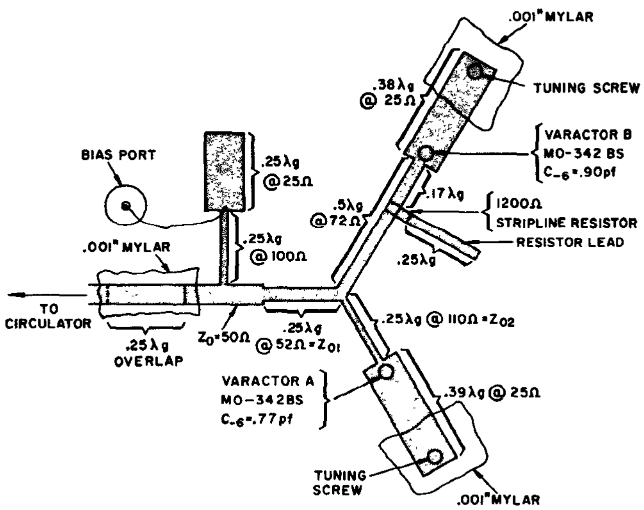

Fig. 17. Circuit for 5-GHz 360° phase modulator using two shunt-tuned MO-342BS diodes.

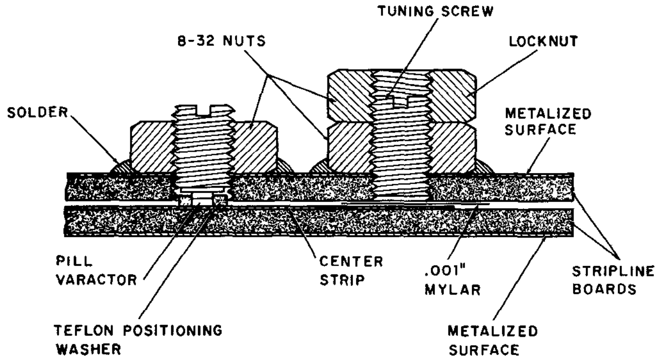

Fig. 18. Diode mount and fine tuner for a shunt-tuned pill varactor.

The next step is to adjust B_{TB} so that the error at $V_N=0.5$ is close to zero. (This is done by moving the slotted line probe $\lambda/4$ from the $V_N=0$ minimum position and adjusting for a minimum at $V_N=0.5$.)

Errors caused by Z_{01} and Z_{02} are shown in Fig. 16. The tuning procedure used above will shift all curves to pass through the axis at $V_N=0.5$ and $V_N=1.0$, and cause Z_{02} low and Z_{01} high to look the same, and Z_{02} high and Z_{01} low to be the same. Thus, either Z_{01} or Z_{02} can be adjusted to bring the error to a minimum.

The circuit used for making a 360° phase modulator at 5 GHz using pill varactors is shown in Fig. 17. For shunt diodes the plane of the resistor for making the insertion loss constant is toward the generator from the diode circuit. This makes it necessary to have a half-wavelength line between diode B and the common point of the varactor circuits. As before, the diode the greatest distance from the admittance adding point makes the greater contribution to insertion-loss variation. The characteristic impedance of the half-wavelength line may have any value. It should be low enough to provide sufficient width for the metal part of the stripline resistor to be substantially included in it (metal sticking out contributes a capacitive obstacle in conjunction with the resistance and would disturb phase linearity). The characteristic impedance of this half-wavelength line section may be adjusted to change the effective value of R_p.

All center strips are 0.001-inch shim brass cut to within ± 0.0005 inch on a precision sheet metal shear. They are soldered together using a miniature 3-watt soldering loop to facilitate circuit modifications.

The details of the diode mount and tuning screw are shown in Fig. 18. A short Teflon collar is placed in the varactor hole to position the diode precisely. The tuning screw is placed near the end of the open-circuited line, which is a high electric-field point. The tuning screw makes the effective length of the open-circuit section variable (and thus B_T). Mylar (0.001-inch) is put below the tuning screw to prevent it from shorting out the bias voltage when it is close to the center strip.

The performance of this modulator at 5 GHz is shown in Fig. 19. The phase was within ± 3.3 percent of linearity, and the insertion loss was 4.3 ± 1.2 dB. Since the device was in-

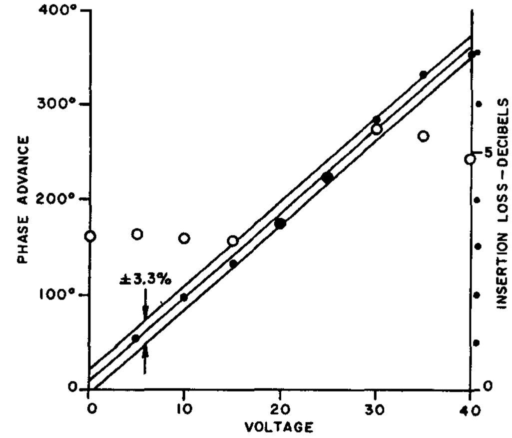

Fig. 19. Phase modulation and insertion loss of the 5-GHz modulator (deduced from slotted-line measurements).

tended for performing the serrodyne function and only 20-dB suppression of carrier and sidebands was required, the insertion-loss variation was not further reduced. The values of Z_0 and B_T for these varactors were within ± 25 percent of their theoretical values based on their measured values of $C_{\min}$ and γ. (See Appendix I.) This shunt tuning circuit should be equally adaptable for making phase modulators in waveguide.

V. Application to Serrodyne

By applying a sawtooth waveform to a linear 360° phase modulator, one RF cycle can be wrapped up or unwrapped for each cycle of the sawtooth. The output frequency will be correspondingly raised above or lowered below the input frequency. Ideally all of the power would emerge at the one translated frequency. However there are five factors contributing to output power at the carrier and sideband frequencies: 1) nonlinearity of phase modulation, 2) circulator leakage, 3) amplitude modulation, 4) flyback time of the sawtooth, and 5) over- or under-modulation.

Fig. 20 shows how much these factors contribute to the output power at the carrier and sideband frequencies. For the figure power incident at f_0 is to be converted to the $+1$ sideband which corresponds to the sum frequency of the sawtooth and the input carrier. The sawtooth is sweeping from $V_N=0$ to $V_N=1$. All other output frequencies are undesired sidebands. The curves apply equally for down conversion, in which case all of the signs on the sideband labels are reversed.

Nonlinearity of phase modulation contributes to sidebands according to

$$S_{+2,f_0} = 20 \log J_1(2\pi\epsilon) \tag{19}$$

in which J_1 is a Bessel function of the first order and ϵ is the magnitude of the phase deviation (here assumed to be a single sine wave over 360°). Figs. 4, 11, 13, 15, and 16 show $1\frac{1}{2}$ cycles of error, so the sidebands would be represented approximately by their Fourier components. For a nonlinearity of ± 2 percent, the $\epsilon=0.02$.

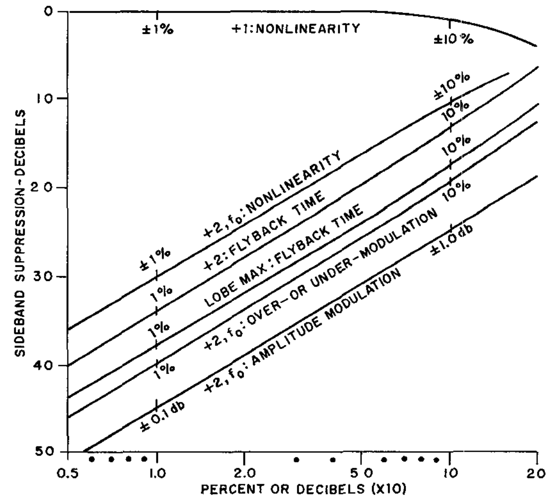

Fig. 20. Contributions to carrier and sideband output for a serrodyne modulator.

Nonlinearity is the major contributing factor to the sidebands. Improved suppression could be obtained by using more diodes (less phase modulation per diode) or by shaping the sawtooth to compensate for the nonlinearity of the modulator.

The *directivity* of the circulator makes a contribution to the sidebands. Assuming that the circulator is connected to perfect, external, matched loads on all ports, then the finite directivity η of the circulator is caused by a mismatch Γ_M on the nonisolated port as seen from the ferrite,

$$\eta = 20 \log |\Gamma_M|. \tag{20}$$

This directivity makes a direct contribution to the carrier because that small amount of incident power is not modulated. An indirect contribution is made to the $+2$ sideband and carrier due to nonlinearities introduced by the mismatch Γ_M.

The error ϵ in phase of a reflected wave Γ_R when observed through a discontinuity having a reflection coefficient Γ_M is given by

$$\epsilon = \pm \sin^{-1} |\Gamma_M| \cdot |\Gamma_R| \text{ (reference [11]).} \tag{21}$$

The contribution to phase nonlinearity, because the phase modulator doubles impedance phase, is 2ϵ. Γ_R is close to unity in a low-loss phase modulator. A circulator with 20-dB isolation will contribute $\pm 11.5°$ phase error, which is ± 3.2 percent. According to Fig. 20, this will generate sidebands 20 dB down at $+2$ and f_0 form nonlinearity. The circulator thus contributes directly to the carrier due to directivity and to the $+2$ sideband and carrier, an equal amount due to the discontinuity causing the limited directivity. Depending on the phase of the power reflected from the discontinuity of f_0 it can either add to or cancel the power generated by the consequent phase nonlinearity, giving in this case suppression of the carrier as low as 17 dB. Therefore, it is important that the directivity of the circulator be high or be tuned out.

For *amplitude modulation* the sideband suppression is given by

$$S_{+2,f_0} = 20 \log (2/m) \tag{22}$$

in which m is the modulation index,

$$\epsilon(\pm\, dB) = 10 \log \frac{1+m}{1-m} \cdot \tag{23}$$

For insertion loss varying from 1.2 dB to 1.4 dB, ϵ ($\pm$dB) $=0.1$.

Over- or under-modulation causes the output waveform to have a slight sawtooth phase modulation on it. Taken from Cumming [12, eq. 19], the primary sidebands are given by

$$S_{+2,f_0} = 20 \log \left[\frac{\pi(1-\delta)}{\sin (\pi\delta)} \right]. \tag{24}$$

Ten-percent overmodulation produces $\delta = 0.1$.

The *flyback time* contributes to a manifold of spectral lines which has a few high-magnitude lines immediately above the desired output rapidly decaying in magnitude and a lobe below the desired output (in the opposite direction as translation). From Cumming [13]

$$S_n = 20 \log \left\{ \frac{[1 - n(1-F)]\pi[1+nF]}{\sin \pi[1+nF]} \right\} . \tag{25}$$

A flyback time of 10 percent gives $F=0.1$. The $+2$ sideband suppression is given by

$$S_{+2} = 20 \log \left[\frac{\pi(1-F)}{2 \sin (\pi F)} \right] . \tag{26}$$

The maximum of the lobe is centered at $n=-1-1/2F$ and its suppression is given by

$$S_{\text{lobe max}} = 20 \log \left[\frac{\pi^2(1-F)}{4 \sin (\pi F)} \right]. \tag{27}$$

It is not difficult to make the AM sidebands low. It was found possible to adjust the over- or under-modulation to partially cancel the sidebands contributed by nonlinearity. Both sidebands occur at the same frequencies and the over- or under-modulation sidebands can be at one of two opposite phases at any continuously variable magnitude.

Both modulators have given at least 20-dB sideband suppression when driven with a sawtooth giving $\pm$3.3-percent additional nonlinearity and about 10-percent flyback time. The flyback time would dictate a $+2$ sideband down 13 dB and other sidebands down 18 dB. The total nonlinearity on the 1-GHz modulator would dictate 14-dB-down sidebands. The sidebands due to AM at 1 GHz should be down about 40 dB. When the voltage was adjusted for 0 to 40 volts, the $+2$ sideband was 14 dB below the desired output, and the f_0 sideband was 15 dB down. By adjusting the modulation from 2 to 36 volts, all sidebands were 21 dB below the desired output. This demonstrates that 15-percent under-modulation generated sidebands 15 dB below the desired output, improving sideband suppression by 7 dB, and that the effective flyback time must have been about 7 percent. The device could be turned off and on and still have the same sideband suppression. The suppression was not changed as incident power was varied up to 10 mW. Similar results were obtained with the 5-GHz modulator.

VI. Conclusion

This paper has shown how the nonlinearity of a varactor reactance curve can be matched directly to a tangent function to provide phase modulation within a few percent of linearity. A technique for eliminating AM has been given, and the necessary insertion loss has been calculated based on operating frequency, diode cutoff frequency, and the diode nonlinearity characteristic γ. The necessary relationship for combining two 180° phase modulator diodes to obtain 360° phase modulation without the use of additional circulators has been derived.

Detailed techniques for building these modulators have been given which should make their construction a more straightforward and efficient process.

Appendix

Diode Measurement

All work was performed in stripline because of the ease with which structures can be modified. The diode should be measured in the same mount that will be used in the design because the mount influences the measured parameters of the diode. The usual diode measurement precautions should be observed: use a slotted line with a low residual VSWR on the measurement end; put the bias T on the generator end of the slotted line; remove the detector from the probe and put a low-pass filter between the probe and detector (to prevent harmonics generated by the diode from filling in the minimums of high VSWR's and making them appear lower); use a low-VSWR in-line transition to stripline. For the phase modulator the power should be maintained at the same approximate level at which it will be operated, and the applied voltage should be measured with a precision voltmeter. All other stripline precautions should also be observed: all center conductor joints soldered, and eyelets or screws within less than a quarter wavelength of the strip center and close enough together to prevent parallel-plate mode propagation (less than 1/8 wavelength apart).

A reference plane is needed to calculate the minimum shifts for the impedance points. An accurate method for obtaining a reference plane is to solder shim brass to the face of the end launch coax-to-stripline adaptor in the plane where the the stripline board usually begins. Then, to measure the diode, it is placed in the board a short, accurately measured distance from that plane. Using this technique to measure open and short circuit elements it was determined that a short circuit comprised of a center strip which suddenly becomes wide enough to have the suppressor screws run through it has an inductance of 0.77 nH (two 1/16-inch Teflon-fiberglass boards sandwiching a 0.088-by 0.001-inch shim brass center conductor). An open circuit (completely surrounded by the Teflon-fiberglass) had a capacitance of 0.051 pF. This capacitance can be calculated approximately using [13] which gives 0.046 pF. Since the corners of the end

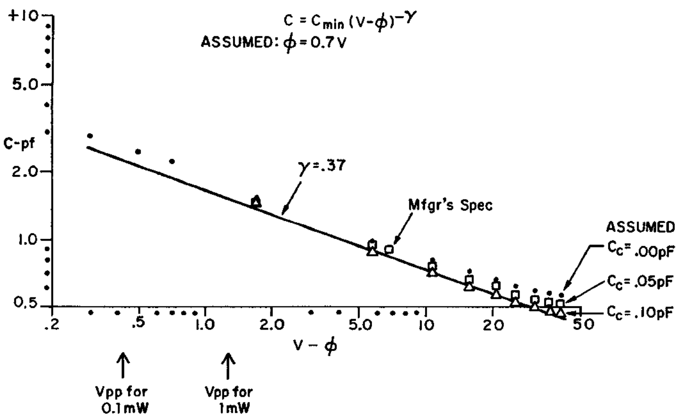

Fig. 21. Varactor capacitance MO-342BS.

of the center strip contribute additional capacitance, the theoretical capacitance is probably close to 0.051 pF. When the reference plane is not near the transition to stripline, an open circuit is the recommended reference, causing less error than a short circuit, especially if the correction for end capacitance is made.

In measuring a diode in a glass package which in the application will have a short circuit behind it, the best procedure is to use an accurate method to derive the input plane of the diode and to use the same short circuit behind the diode that will be used in the application.

The distributed capacitance of the diode whisker inductance can be prevented from distorting the measured impedance by mounting the diode so that the junction end is closest to the input plane. The diode inductance will include the short section of diode lead in glass going to the semiconductor chip as well as the whisker inductance; and the cartridge capacitance will include the distributed capacitance to the ground planes of this lead section. The diode junction is assumed to have no reactance at heavy forward bias. The forward bias reactance of the diode is then subtracted from the reactances measured at other voltages, and the capacitance calculated from them can be plotted as shown in Fig. 21. Adjusting ϕ bends the left portion of the curve and adjusting C_c, the cartridge capacitance, bends the right portion of the curve. C_c can be subtracted directly because the inductance between it and the junction is very small. Normally the measurement power for varactors is maintained very low by putting the power in the probe of the slotted line and putting the filter and detector on the bias T end of the slotted line. When this is done, C_c and ϕ can be adjusted to obtain a straight line, and accurately characterize the varactor. For phase modulators the measurements should be made at the power level at which the modulator will be operating. The peak-to-peak voltages for an open-circuited 50-ohm transmission line are shown below the voltage axis of Fig. 21. When these voltages are of the same order of magnitude or exceed the bias voltage, the capacitance is nonlinear during the RF cycle and produces an average value greater than its small signal value. Thus it is not meaningful to adjust too carefully to obtain a striaght line in the low-voltage region. Most of the interest for the phase modulator lies in the high-voltage region, thus the curve should be closely matched in this range by adjusting C_c. The data shown in Fig. 21 is of a MO-342BS varactor taken at 5 GHz.

Note added in proof: Calculations by S. J. Gordon indicate that the model of the phase modulator shown in Fig. 11 should provide 10 percent bandwidth with ± 3 percent of deviation from linearity when diode capacitances are such that $Z_{01} = 18.3\ \Omega$ and $Z_{02} = 30.0\ \Omega$ for $\gamma = \frac{1}{2}$.

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