
Sage Reference Manual

Release 9.1

The Sage Development Team

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This manual contains documentation for (almost) all of Sage's features, each illustrated with examples that are systematically tested with each release. A thematic index is available below.

USER INTERFACE

- Command Line Interface (REPL)
- For the Jupyter notebook interface, visit [its documentation](#).
- For the legacy notebook interface, which is no longer actively maintained, visit the [source repository](#).

GRAPHICS

- 2D Graphics
- 3D Graphics

3.1 Parents and Categories

- Parents and Elements
- Coercion
- Categories

3.2 Basic Rings and Fields

- Integers and Rational Numbers
- Real and Complex Numbers
- Polynomials
- Power Series and Laurent Series
- Finite Rings and Fields
- p-Adic Numbers
- Quaternion Algebras

3.3 Linear Algebra

- Matrices and Spaces of Matrices
- Vectors and Modules
- Tensors on Free Modules of Finite Rank

3.4 Calculus and Analysis

- Symbolic Calculus
- Mathematical Constants
- Elementary and Special Functions
- Asymptotic Expansions

- Numerical Optimization

3.5 Probability and Statistics

- Probability
- Statistics
- Quantitative Finance

3.6 Mathematical Structures

- Sets
- Monoids
- Groups
- Semirings
- Rings
- Algebras

3.7 Discrete Mathematics

- Combinatorics
- Graph Theory
- Quivers
- Matroid Theory
- Discrete Dynamics
- Coding Theory
- Cryptography
- Game Theory
- Symbolic Logic
- SAT solvers

3.8 Geometry and Topology

- Euclidean Spaces and Vector Calculus
- Combinatorial and Discrete Geometry
- Cell Complexes and their Homology
- Manifolds and Differential Geometry
- Hyperbolic Geometry

- Parametrized Surfaces
- Knot Theory

3.9 Number Fields, Function Fields, and Valuations

- Number Fields
- Function Fields
- Discrete Valuations

3.10 Number Theory

- Diophantine approximation
- Quadratic Forms
- L-Functions
- Arithmetic Subgroups of $SL_2(\mathbb{Z})$
- General Hecke Algebras and Hecke Modules
- Modular Symbols
- Modular Forms
- Modular Forms for Hecke Triangle Groups
- Modular Abelian Varieties
- Miscellaneous Modular-Form-Related Modules

3.11 Algebraic and Arithmetic Geometry

- Schemes
- Plane, Elliptic and Hyperelliptic Curves

3.12 Miscellaneous

- Databases
- Games

PROGRAMMING

- Data Structures
- Utilities
- Test Framework
- Parallel Computing

4.1 Interfaces

- Interpreter Interfaces
- C/C++ Library Interfaces
- Python technicalities

GENERAL INFORMATION

- References
- History and License
- genindex
- modindex
- search

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