Quadratics - Lesson 3 - Study Notes - The Discriminant

What is the Discriminant:

In a quadratic expression, as $ax^2 + bx + c$ where $a \neq 0$, and the quadratic formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the discriminant is what is under the square root. Hence, the disriminant is $b^2 - 4ac$.

Recalling some rules on square roots.

1. Any number greater than zero will have two distinct roots.

Example: $\sqrt{9} = 3, -3$. This because any negative number multiplied by itself is positive, hence:

$(-a)^2 = a^2$

2. The square root of zero is zero. Zero has one distinct root, which is itself.

Example: $\sqrt{0} = 0$

3. Any negative number will never have any real roots. A negative number has no distinct real roots.

Example: $\sqrt{-4}$, no real values satisfy this expression.

Returning to quadratics, the discriminant, which is $b^2 - 4ac$ is under a square root in the quadratic formula. Therefore, if the discriminant is positive, we've established that it will have 2 distinct roots. If the discriminant is equals to 0, it will have one distinct root, which is 0. If it is negative, then it will have no distinct real roots.

Linking it back to a parabola, which is a curve that can have either 2 x-intercepts, 1 x-intercept, or none. If the discriminant is positive, then the quadratic formula used will have to be done with two seperate values, which means that it will have two x-intercepts. If the discriminant is equal to zero, then it will only have one x-intercept. If the discriminant is negative, the parabola will not touch the x-intercept, nor will it go through it.

These three examples will clarify.

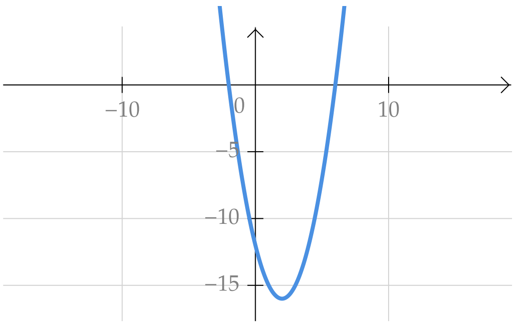
1. $x^2 - 4x - 12$
2. $x^2 - 8x + 16$
3. $x^2 - 6x + 12$

With the discriminant being equal to $b^2 - 4ac$, the discriminant for every expression would be:

1. $(-4)^2 - 4(1)(-12) \;=\; 16 \;+48 \;= 64$
2. $(-8)^2 - 4(1)(16) \;=\; 64 - 64 = 0$
3. $(-6)^2 - 4(1)(12) \;= 36 - 48 = \;-12$

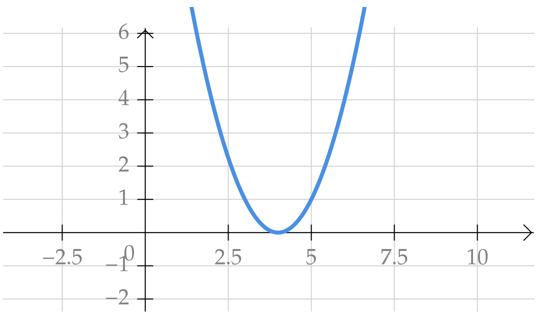
The discriminant of the first expression is positive, being 64. This means that it will have two x-intercepts, and so two distinct roots. The discriminant of the second expression is 0, meaning it will only have one x-intercept. The third expression has a negative discriminant, meaning it will have no x-intercepts, and no real distinct roots.

This can be confirmed graphically.
For the first expression, this is its graph:



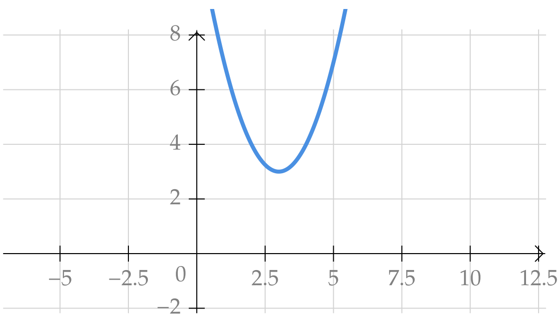
It can be seen that it crosses the x-axis twice.

For the second expression, this is its graph:



It can be seen that it only touches the x-axis once.

For the third expression, this is its graph:



It can be seen that it does not cross the x-axis at all.

In summary:

$b^2 - 4ac > 0$, 2 distinct real roots.

$b^2 - 4ac = 0$, 1 distinct real root.

$b^2 - 4ac < 0$, 0 distinct real roots.