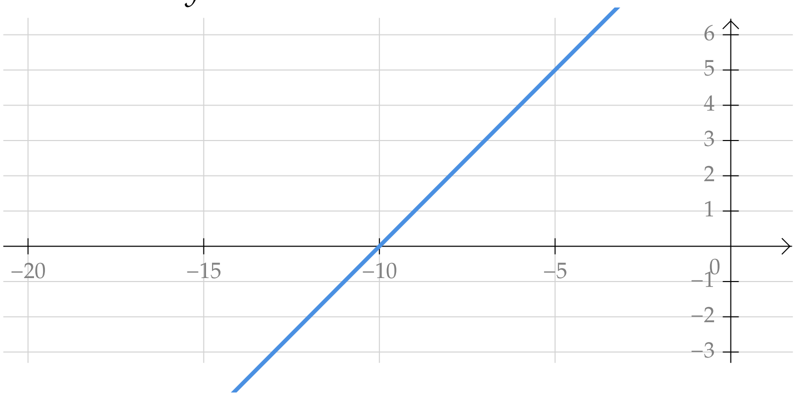
Quadratics - Lesson 2 - Study Notes - The x-intercept and the Quadratic Formula

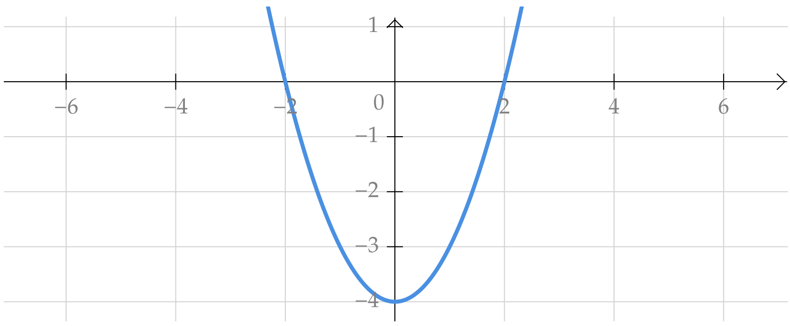
What is an x-intercept:
An x-intercept would be the point that any graphed function crosses the x-axis. In a linear function as $ax + b$, it would only intercept the x axis once. In a quadratic function in the form $ax^2 + bx + c$, it can cross the x axis twice.

Linear Function (This function is $y = x + 10$:



The X-Intercept in this case can be seen as -10.

Quadratic Function (This function is $y = x^2 - 4$):



The x-intercepts of this function are -2 and 2.

The output of the function has to be 0 for it to be an x-intercept, since the x-intercept is at
y = 0.

For the first function, y = x + 10, since y has to be 0, it can be rewritten as x + 10 = 0. Subtracting both sides by 10, you get x = -10, which proves that the x-intercept is -10.

For the second function, y = x^2 - 4, where y = 0. It can be rewritten as x^2 - 4 = 0. Adding 4 to both sides, it can be written as x^2 = 4. Square rooting both sides, you get x = 2, and -2.

The problem arises where theres a bx term.
With the function $x^2 + 6x + 12 = y$. Set it to $x^2 + 6x + 12 = 0$. Attempt to rearrange. $x^2 + 6x = -12$. Factor out the x. $x(x + 6) = -12$. Where does one go from here? This is why there is a need for a formula to solve any quadratic equation out, which is the quadratic formula.

Deriving the Quadratic Formula:

The quadratic formula is starts off with any possible quadratic, $ax^2 + bx + c$.
As we've established, for it to find the x-intercepts, we need to set it equals to 0.

$$ax^2 + bx + c = 0$$

Multiply both sides by $4a$.
Since this is for both sides, it does not alter the value of x.

$$4a\left(ax^2 + bx + c\right) = 4a(0)$$

$$\implies 4a^2x^2 + 4abx + 4ac = 0$$

Subtract $4ac$ from both sides.

$$\implies 4a^2x^2 + 4abx = -4ac$$

Add b^2 to both sides. Since this is being added to both sides, it's the equivalent of adding 0 to the overall equation.

$$\implies 4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

The left hand side can now be factored.

$$\implies (2ax + b)^2 = b^2 - 4ac$$

Take the square root of both sides.

$$\implies 2ax + b = \pm\sqrt{b^2 - 4ac}.$$

Notice, the plus minus sign is because it is a square root. Positive and negative forms of the same number squared are equivalent. This also relates to how a parabola has two x – intercepts.

Now, rearranged for x.

$$\implies 2ax = -b \pm\sqrt{b^2 - 4ac}$$

Divided both sides by $2a$.

$$\implies x \;=\; \frac{-b \;\pm \sqrt{b^2 - 4ac}}{2a}$$

Hence, the $x-$*intercepts of any quadratic function can be calculated using the final quadratic formula* :

$$x = \frac{-b \;\pm \sqrt{b^2 - 4ac}}{2a}$$

Here is an example of actually using the quadratic formula:

$x^2 + 7x + 10 = 0$

$a \;=\; 1$
$b = 7$
$c = 10$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(10)}}{2(1)}$$

$$\implies \frac{-7 \pm \sqrt{49 - 40)}}{2}$$

$$\implies \frac{-7 \pm \sqrt{9}}{2}$$

$$x_1 \;=\; \frac{-7+3}{2} = \frac{-4}{2} \;=\; -2$$

$$x_2 = \frac{-7-3}{2} = \frac{-10}{2} \;=\; -5$$

The two x-intercepts are x = -2, -5