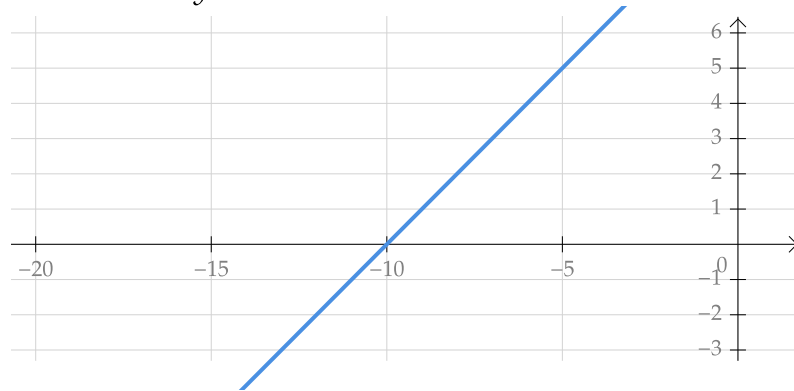


## Quadratics - Lesson 2 - Study Notes

What is an x-intercept:

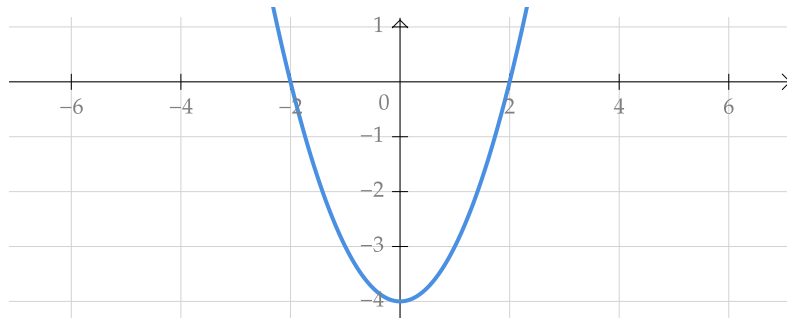
An x-intercept would be the point that any graphed function crosses the x-axis. In a linear function as  $ax + b$ , it would only intercept the x axis once. In a quadratic function in the form  $ax^2 + bx + c$ , it can cross the x axis twice.

Linear Function (This function is  $y = x + 10$ :



The X-Intercept in this case can be seen as -10.

Quadratic Function (This function is  $y = x^2 - 4$ ):



The x-intercepts of this function are -2 and 2.

The output of the function has to be 0 for it to be an x-intercept, since the x-intercept is at  $y = 0$ .

For the first function,  $y = x + 10$ , since  $y$  has to be 0, it can be rewritten as  $x + 10 = 0$ . Subtracting both sides by 10, you get  $x = -10$ , which proves that the x-intercept is -10.

For the second function,  $y = x^2 - 4$ , where  $y = 0$ . It can be rewritten as  $x^2 - 4 = 0$ . Adding 4

to both sides, it can be written as  $x^2 = 4$ . Square rooting both sides, you get  $x = 2$ , and  $-2$ .

The problem arises where there's a  $bx$  term.

With the function  $x^2 + 6x + 12 = y$ . Set it to  $x^2 + 6x + 12 = 0$ . Attempt to rearrange.  $x^2 + 6x = -12$ . Factor out the  $x$ .  $x(x + 6) = -12$ . Where does one go from here? This is why there is a need for a formula to solve any quadratic equation out, which is the quadratic formula.

### Deriving the Quadratic Formula:

The quadratic formula starts off with any possible quadratic,  $ax^2 + bx + c$ .

As we've established, for it to find the  $x$ -intercepts, we need to set it equal to 0.

$$ax^2 + bx + c = 0$$

*Multiply both sides by  $4a$ .*

*Since this is for both sides, it does not alter the value of  $x$ .*

$$4a(ax^2 + bx + c) = 4a(0)$$

$$\implies 4a^2x^2 + 4abx + 4ac = 0$$

*Subtract  $4ac$  from both sides.*

$$\implies 4a^2x^2 + 4abx = -4ac$$

*Add  $b^2$  to both sides. Since this is being added to both sides, it's the equivalent of adding 0 to the overall equation.*

$$\implies 4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

*The left hand side can now be factored.*

$$\implies (2ax + b)^2 = b^2 - 4ac$$

*Take the square root of both sides.*

$$\implies 2ax + b = \pm\sqrt{b^2 - 4ac}.$$

*Notice, the plus minus sign is because it is a square root. Positive and negative forms of the same number squared are equivalent. This also relates to how a parabola has two  $x$ -intercepts.*

*Now, rearranged for  $x$ .*

$$\implies 2ax = -b \pm \sqrt{b^2 - 4ac}$$

*Divided both sides by  $2a$ .*

$$\implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Hence, the  $x$  – intercepts of any quadratic function can be calculated using the final quadratic formula :*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$