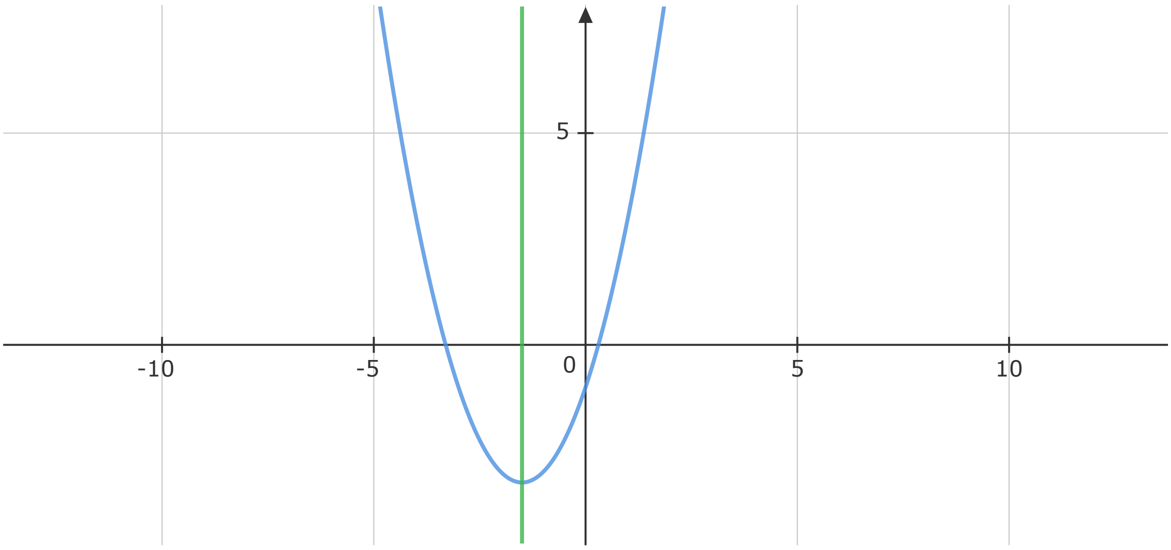
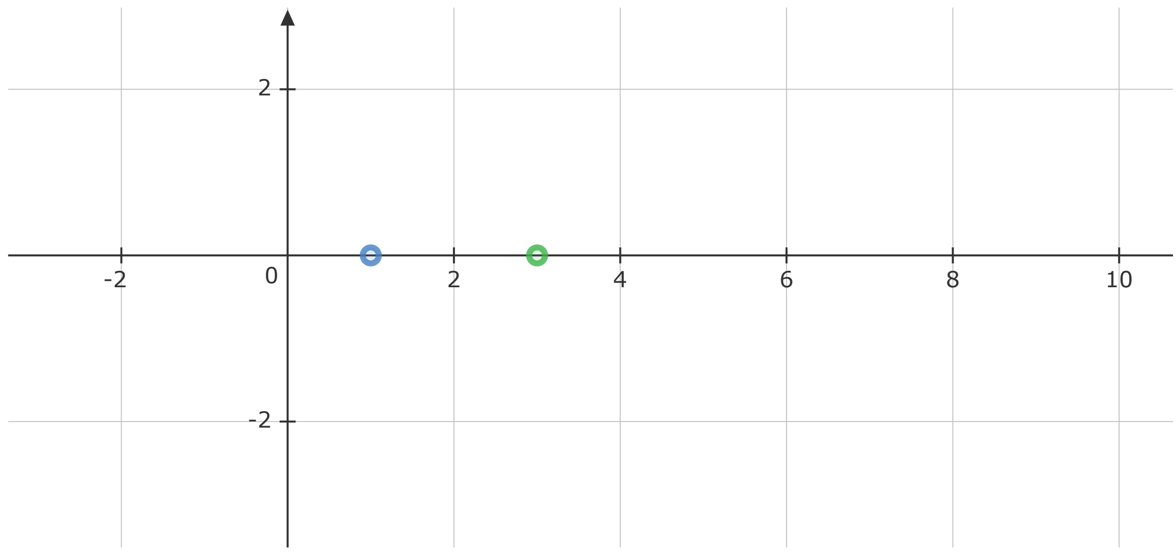
Quadratics - Lesson 4 - Study Notes - The Axis of Symmetry, and the Vertex

What is the axis of symmetry?
The axis of symmetry is a vertical line in form x = c, where c is a constant, that bisects a parabola. In Layman's terms, it's a vertical line that splits the big curve in half. Finding this line is important to find the vertex, or turning point of a parabola, but the axis of symmetry will be spoken about further first.
Here's what it looks like:



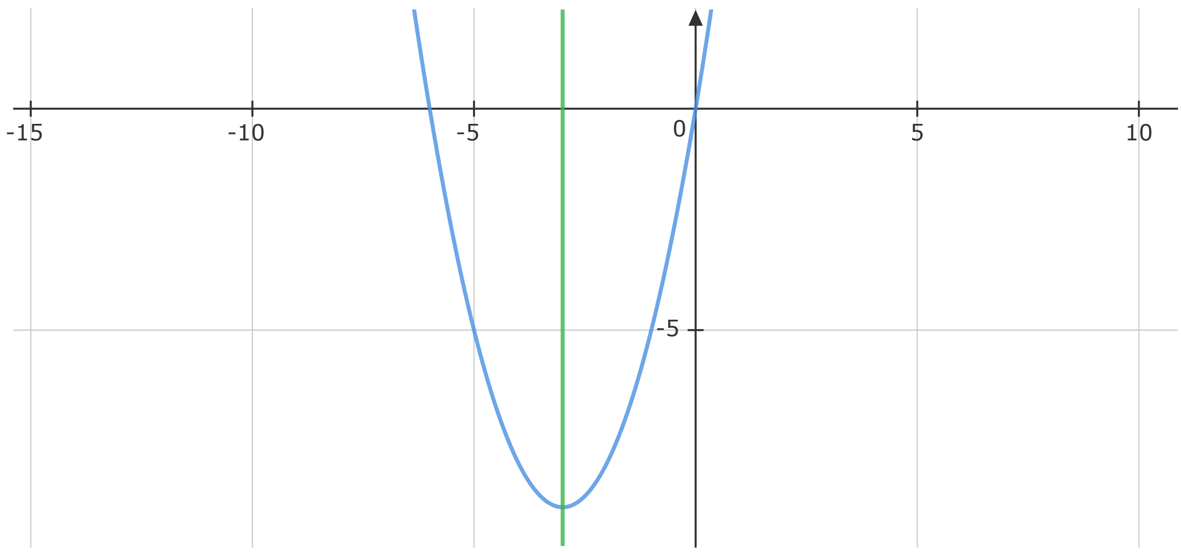

The green line is the axis of symmetry, where it splits the parabola straight in half.
Firstly, let's think about the middle of two points on the x-axis:




There are two points on the graph above. One on 1,0 and one on 3,0. What is the middle of two points? From intuition, you can clearly tell that the middle of those two points is 2,0. So we can establish that, if two points are on the y-value of 0, the midpoint between them is:

$$x_3 = \frac{x_1 + x_2}{2}, \textit{ where } x_3 \textit{ is the midpoint, and } x_1, x_2 \textit{ are the two other points.}$$

This helps us with the axis of symmetry, because we know from seeing a parabola:




Since the axis of symmetry is the midpoint of any two points on the same y-value, the easiest two points to find would be the x intercepts.

The quadratic formula for both x-intercepts as we've studied earlier is:

$$x_1,\ x_2 \ = \ \frac{-b\ \pm\sqrt{b^2-4ac}}{2a},\ where\ ax^2+bx+c,\ and\ x_1,\ x_2\ are\ the\ x-intercepts,\ assuming\ the\ parabola\ has\ two\ x-intercepts.$$

Hence, the equation for each x-intercept is:

$$x_1 \ = \ \frac{-b\ +\sqrt{b^2-4ac}}{2a}$$

$$x_2 = \ \frac{-b\ -\sqrt{b^2-4ac}}{2a}$$

To find the midpoint between these points, we apply the formula we talked about above:

$$Midpoint \ = \frac{x_1+x_2}{2}$$

Inserting each variable into the equation looks like:

$$x \ = \ \frac{\frac{-b\ +\sqrt{b^2-4ac}}{2a}+\frac{-b\ -\sqrt{b^2-4ac}}{2a}}{2}$$

Simplifying:

$$\Longrightarrow x = \left(\frac{-b\ +\sqrt{b^2-4ac}}{2a}+\frac{-b\ -\sqrt{b^2-4ac}}{2a}\right) * \frac{1}{2}$$

$$\Longrightarrow x = \left(\frac{-b\ +\sqrt{b^2-4ac}-b\ -\sqrt{b^2-4ac}}{2a}\right) * \frac{1}{2}$$

$$\Longrightarrow x = \left(\frac{-2b}{2a}\right) * \frac{1}{2}$$

$$\Longrightarrow x = \frac{-2b}{2a*2}$$

$$\implies x = \frac{-b}{2a}$$

Hence, the formula to get the axis of symmetry from $ax^2 + bx + c$, is $x = \dfrac{-b}{2a}$.

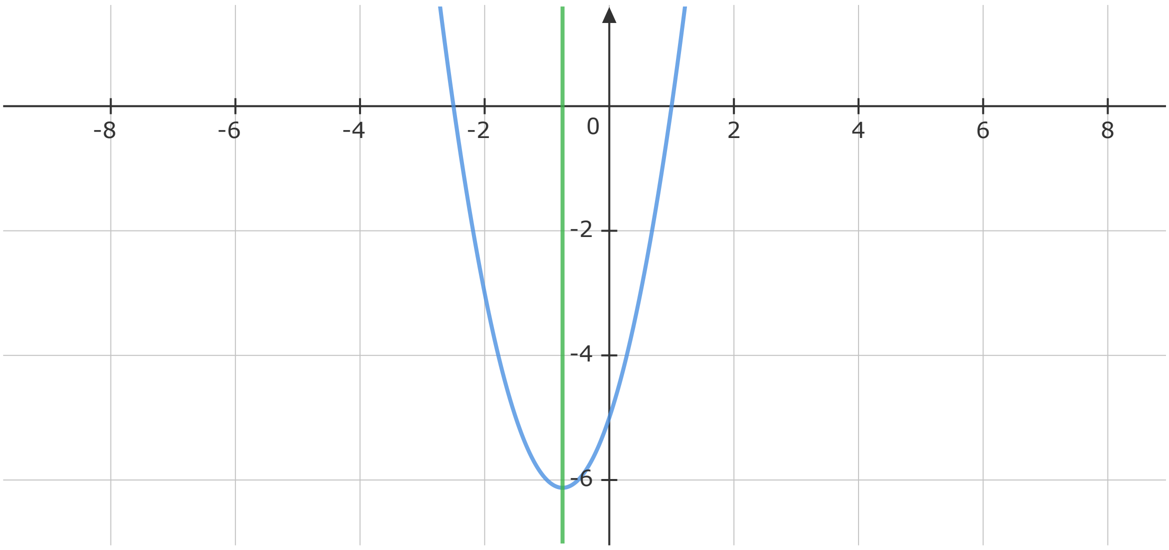
Here are a few examples.

What is the axis of symmetry for the function: $f(x) = 2x^2 + 3x - 5$.

Knowing the formula: $x = \dfrac{-b}{2a}$, plugging those values in:

$x = \dfrac{-3}{4} = -0.75.$

Graphing the function, we can test this result out:

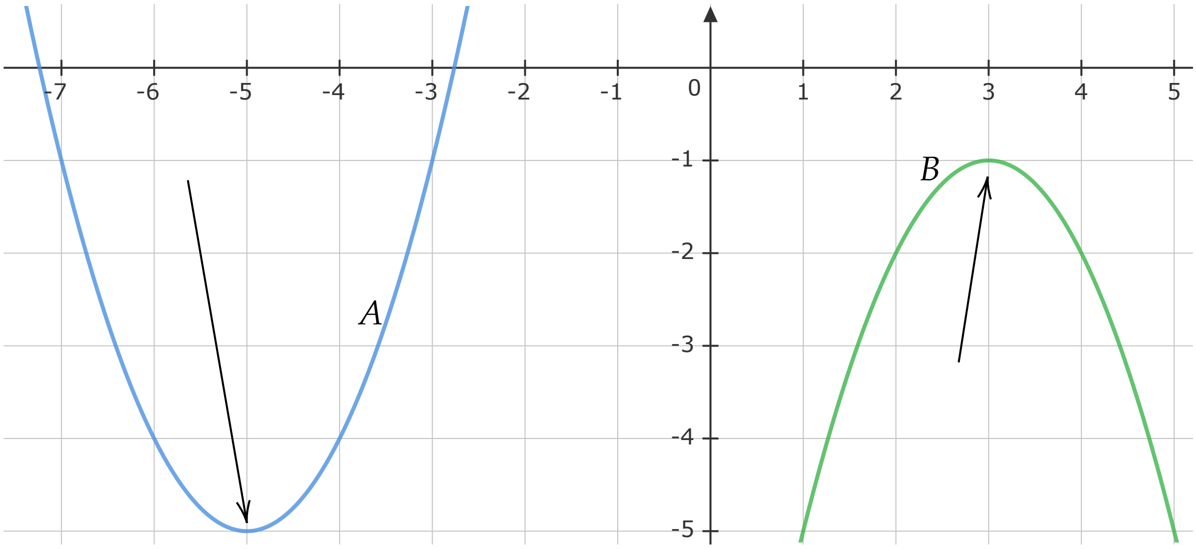



So we can see that this formula works.

What is a turning point/vertex?

The turning point of a parabola, is just literally the turning point. It is where the parabola turns, and where it has its highest/lowest point.

Look at the following parabolas:




Parabola A has its vertex at coordinates (-5, - 5) and Parabola B has its vertex at coordinates
(3,-1).

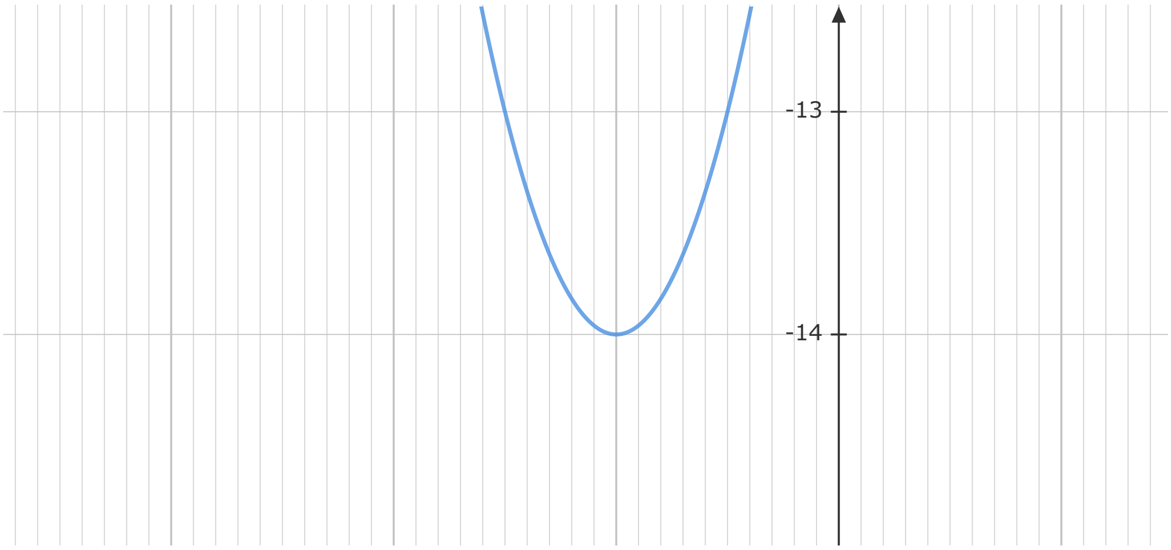
The axis of symmetry relates to this because the x-coordinate that the formula for the axis of symmetry gives, is the same x-coordinate that the turning point is on. Hence, the value of the axis of symmetry should be inputed into the function, giving the y-coordinate point.

**Sample question and answer:**
What are the coordinates for the vertex of the function: $f(x) \ = \ 4x^2 \ + \ 8x \ - 10$?

To solve this question, you first have to find the x-value for the vertex. To do this, you use the
formula for the axis of symmetry: $x \ = \ \frac{-b}{2a}$.
Inputting the values: $x \ = \ \frac{-8}{8} \ = \ -1$.
So the x-coordinate is -1.
Plugging -1 into the original function, $f(x) \ = \ 4x^2 \ + \ 9x \ - 10$.
$f(-1) \ = \ 4(-1)^2 + 8(-1) \ - 10$.

$\Longrightarrow f(-1) = \ 4 \ - 8 \ - \ 10 \ = \ - 14$
Hence, the y-coordinate is y = 6.

So the coordinates of the vertex of the function $f(x) \ = \ 4x^2 \ + \ 8x \ - 10$ are: (-1, -14).
This can be proven graphically:




From the graph, we know that it the coordinates of the vertex are (-1,-14).

Hence, we can derive the following conclusion:

$$\textit{The coordinates of the vertex of function } f(x) \textit{ are}: \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$