

Blatt 5

Sonntag, 15. Juni 2025

21:52

I. Lane - Emden

1. a) gekoppelte DGL:

$$\frac{dw}{d\xi} = z$$

$$\frac{dz}{d\xi} = -\left(\frac{2}{\xi} z + w^n\right)$$

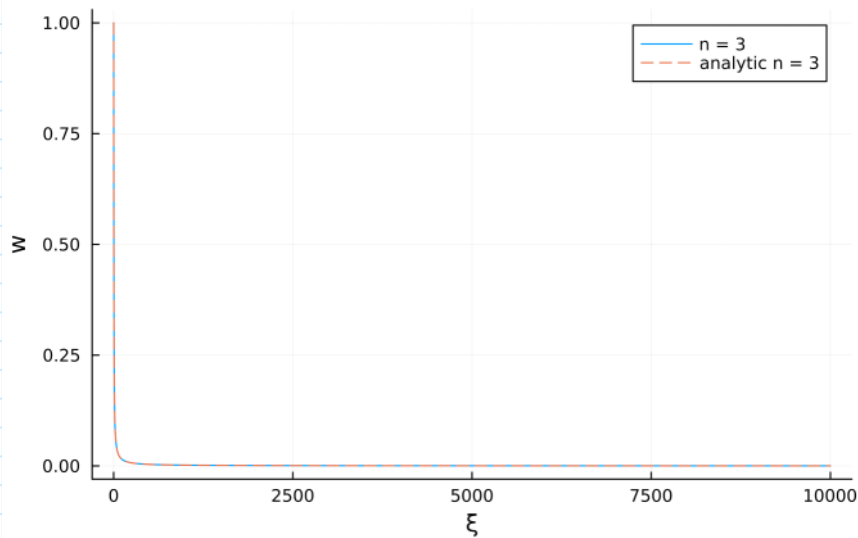
äquivalent zu

$$\frac{d\vec{y}}{d\xi} = f(\xi, \vec{y}, n)$$

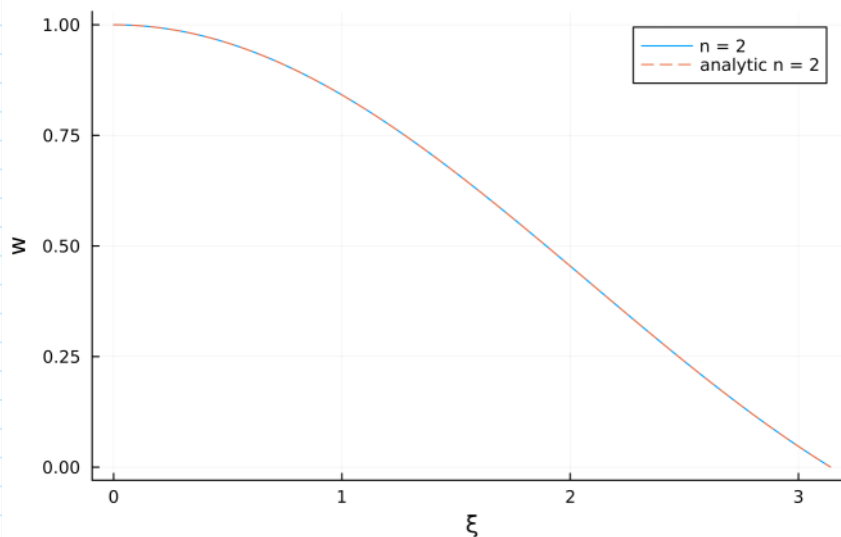
$$\text{mit } \vec{y} = \begin{pmatrix} z \\ -\frac{2}{\xi} z + w^n \end{pmatrix} \Rightarrow \text{RK4-integrierbar}$$

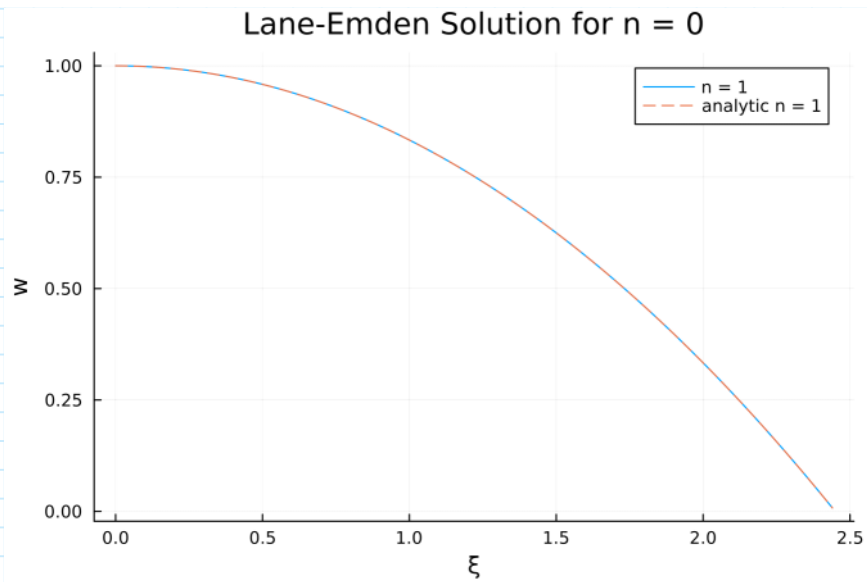
1 b)

Lane-Emden Solution for $n = 5$



Lane-Emden Solution for $n = 1$





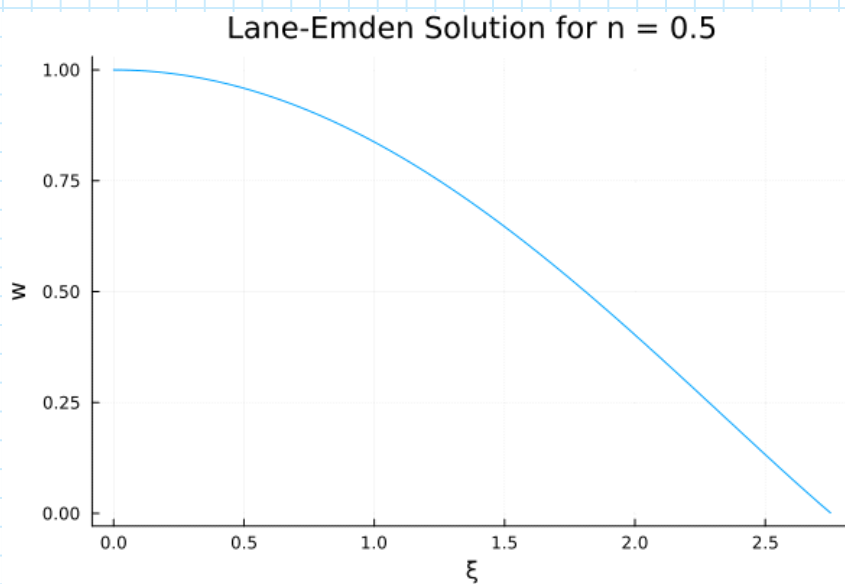
\Rightarrow Stimmt überein

Es wurde $h = 0,01$ genutzt

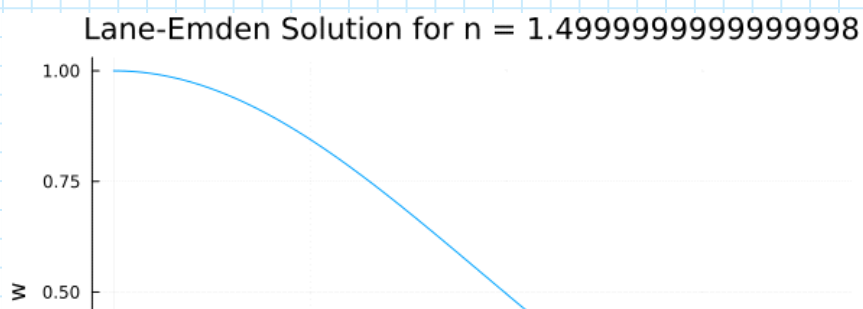
S. Assignment_05 / Plots / comparison-to-analytic

2. a)

Plots:



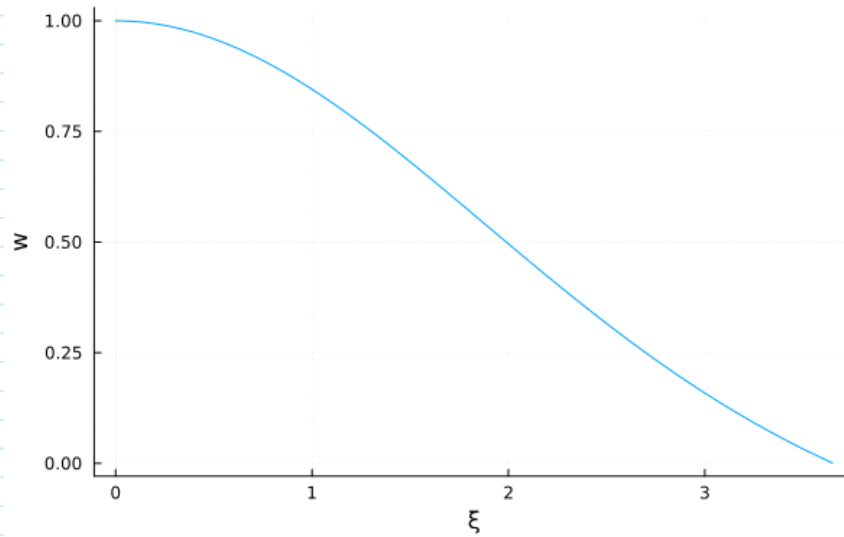
$\Rightarrow \gamma = 3$



$\Rightarrow \gamma = \frac{5}{3}$

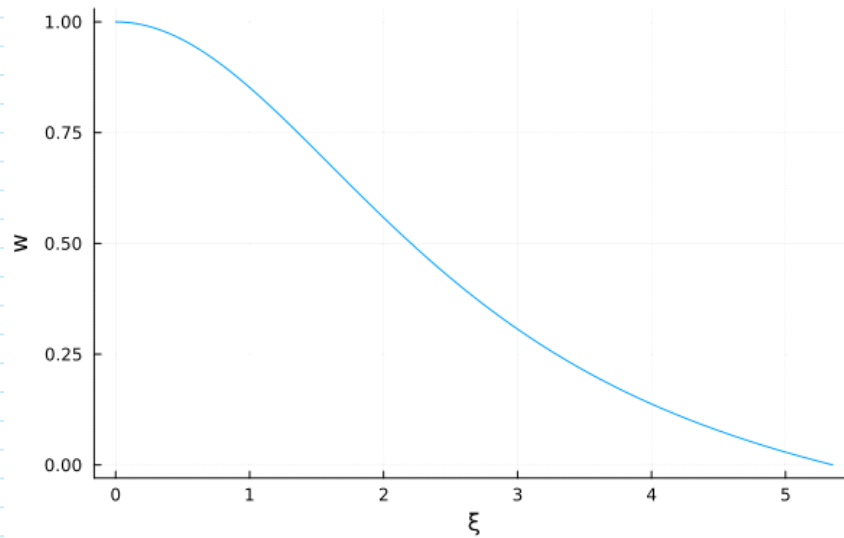
Lane-Emden Solution for $n = 1.4999999999999998$

$$\Rightarrow \rho = \frac{1}{3}$$



Lane-Emden Solution for $n = 2.50000000000000004$

$$\Rightarrow \rho = \frac{7}{5}$$



Command Line output:

```
Solving Lane-Emden with n=0.5...
Solved!
Radius: 2.7500999999999856
total mass: 1.7335110040086898

Solving Lane-Emden with n=1.4999999999999998...
Solved!
Radius: 3.6500999999999664
total mass: 1.9931683581036999

Solving Lane-Emden with n=2.50000000000000004...
Solved!
Radius: 5.350099999999993
total mass: 2.3813780860333544
```

2. b) Zu den Formeln:
 ξ_1 und $\frac{dw}{d\xi} = z$ folgen aus der numerischen Lösung

damit:

$$\frac{p_c}{\bar{p}} = \frac{\xi_1^3}{3 \left[-\xi \frac{dw}{d\xi} \right]_{\xi=\xi_1}}$$

$$\Rightarrow p_c = \frac{p_c}{\bar{p}} \cdot \bar{p}_0$$

aus $r = \alpha \xi$ folgt:
 $\alpha = \frac{r}{\xi}$

damit lässt sich K bestimmen

$$\alpha^2 = \frac{(n+1)K}{4\pi G} \bar{p}^{\frac{1}{n}-1}$$

$$\Rightarrow K = \frac{\alpha^2 \cdot 4\pi G}{(n+1) \bar{p}^{\frac{1}{n}-1}}$$

Es ist außerdem:

$$p = K \rho \Rightarrow p_c = K \rho_c$$

damit kann man nun T_c berechnen:

$$T_c = \frac{p_c M_{\text{MH}}}{k_B \rho_c}$$

Es ergibt sich

```
Critical density: 76.17320573694933 g/cm^3
alpha: 1.0101449906387575e10 cm
K: 3.844659187094361e14 cm^2/s^2
p_c: 1.2414450229752589e17 erg/cm^3
T_c: 1.2248257376708193e7 K
```

Literaturwert: $1,57 \cdot 10^7 \text{ K}$

⇒ gute Übereinstimmung