Blatt 6

Sonntag, 15. Juni 2025 22:48

DGL:

$$\ddot{x} - 2\Omega\dot{y} - \Omega^2 x = -\left[\mu_1 \frac{x+\mu_2}{r_1^3} + \mu_2 \frac{x-\mu_1}{r_2^3}\right]$$

$$\ddot{y} + 2\Omega\dot{x} - \Omega^2 y = -\left[\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right] y$$

führe $\vec{u}$ ein:

$$\vec{u} \equiv \begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix} \Rightarrow \frac{d\vec{u}}{dt} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ 2\Omega\dot{y} + \Omega^2 x - \left(\mu_1 \frac{x+\mu_2}{r_1^3} + \mu_2 \frac{x-\mu_1}{r_2^3}\right) \\ -2\Omega\dot{x} - \Omega^2 y - \left[\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right] y \end{pmatrix} \equiv f$$

$$\Rightarrow \frac{d\vec{u}}{dt} = f$$

Jakobi-const:

$$C_J = \Omega^2(x^2+y^2) + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - (\dot{x}^2 + \dot{y}^2)$$

$$\dot{x}_0 = y_0 = 0 \Rightarrow \Omega^2 x_0^2 + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - \dot{y}_0^2 = C_J$$

$$\Leftrightarrow \dot{y}_0 = \sqrt{\Omega^2 x_0^2 + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - C_J}$$

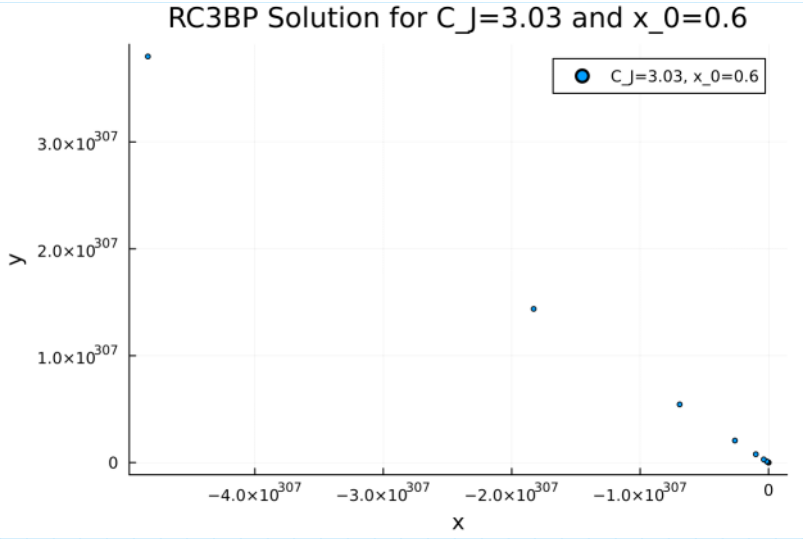
$\Rightarrow$ Startbed: x_0 (Input)

$\dot{y}_0$ (aus x_0, C_J)

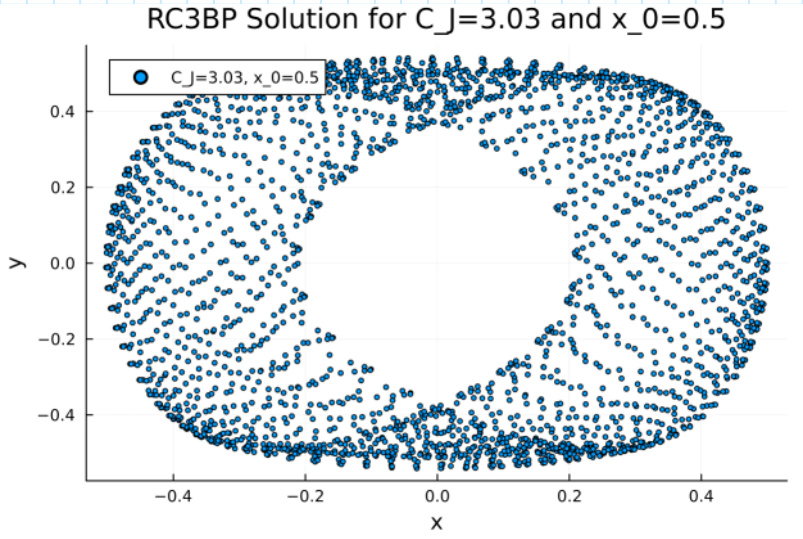
$y_0 = 0$

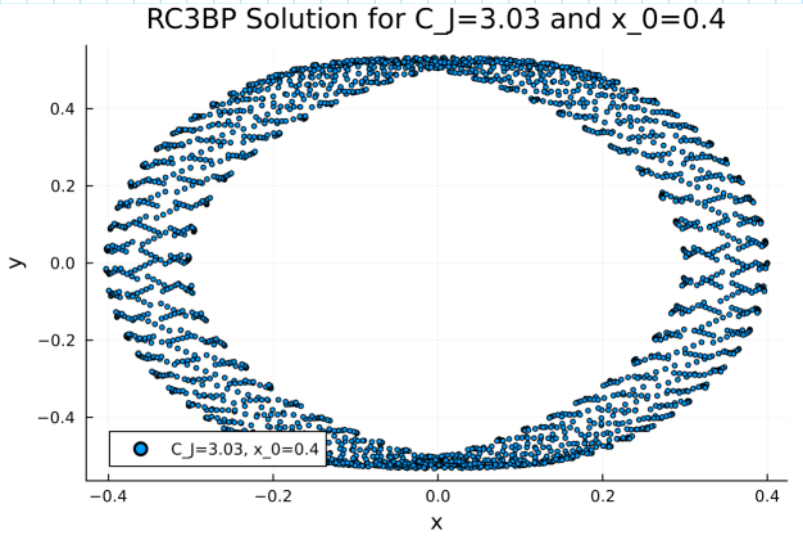
$\dot{x}_0 = 0$

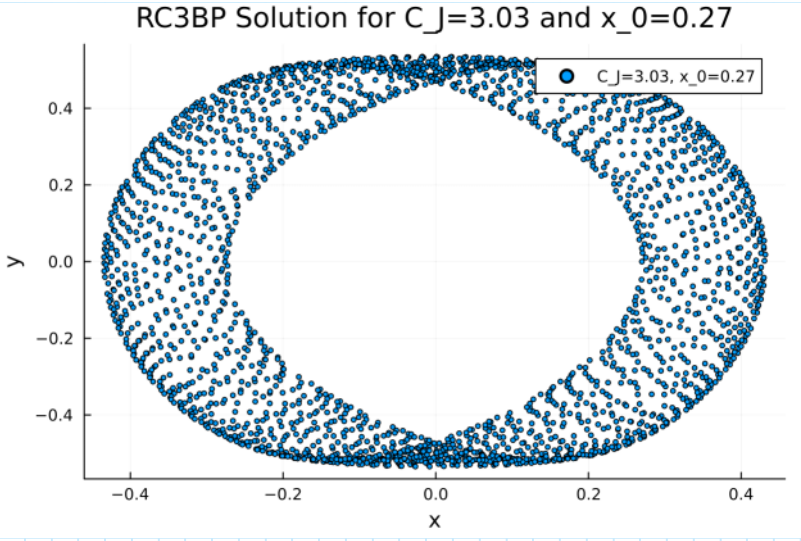
Ergebnisse (Plots):

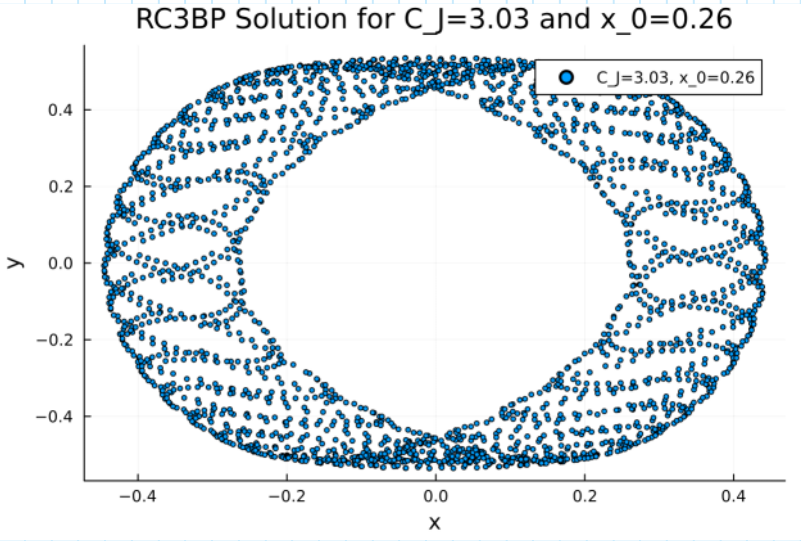

RC3BP Solution for C_J=3.03 and x_0=0.6
C_J=3.03, x_0=0.6
3.0×10^307
2.0×10^307
1.0×10^307
0
y
−4.0×10^307
−3.0×10^307
−2.0×10^307
−1.0×10^307
0
x

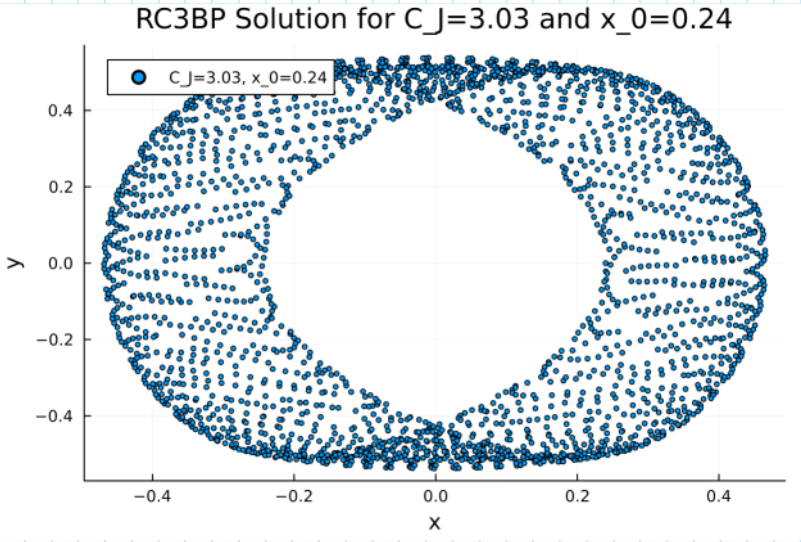
} unstabil!

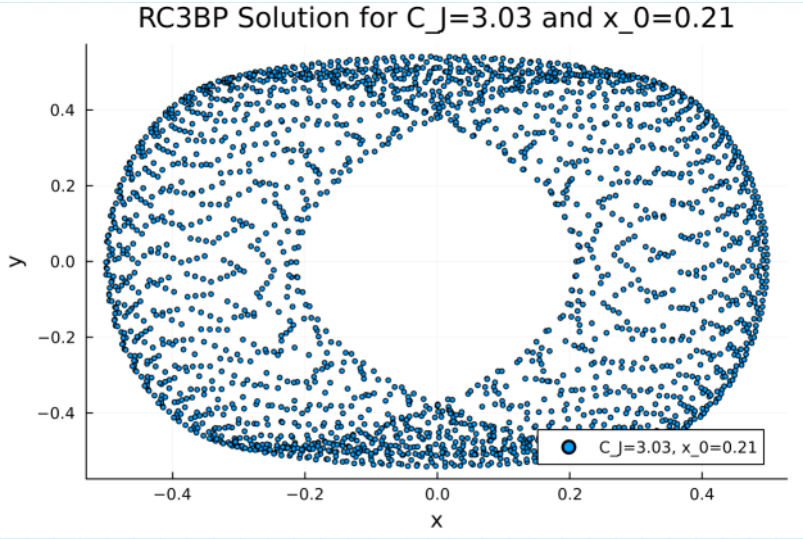

RC3BP Solution for C_J=3.03 and x_0=0.5
C_J=3.03, x_0=0.5
0.4
0.2
0.0
−0.2
−0.4
y
−0.4
−0.2
0.0
0.2
0.4
x

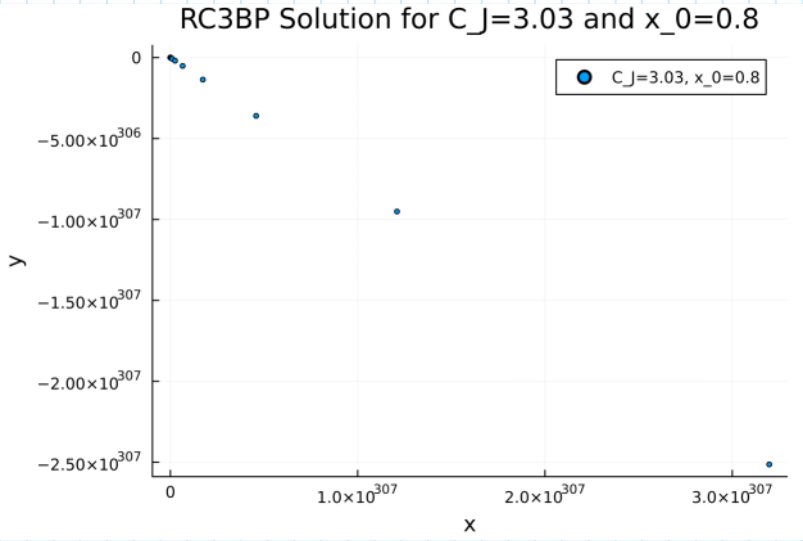

RC3BP Solution for C_J=3.03 and x_0=0.4
0.4
0.2
0.0
−0.2
−0.4
y
C_J=3.03, x_0=0.4
−0.4
−0.2
0.0
0.2
0.4
x


RC3BP Solution for C_J=3.03 and x_0=0.27
C_J=3.03, x_0=0.27
0.4
0.2
0.0
−0.2
−0.4
y
−0.4
−0.2
0.0
0.2
0.4
x


RC3BP Solution for C_J=3.03 and x_0=0.26
C_J=3.03, x_0=0.26
0.4
0.2
0.0
−0.2
−0.4
y
−0.4
−0.2
0.0
0.2
0.4
x


RC3BP Solution for C_J=3.03 and x_0=0.24
C_J=3.03, x_0=0.24
0.4
0.2
0.0
−0.2
−0.4
y
−0.4
−0.2
0.0
0.2
0.4
x


RC3BP Solution for C_J=3.03 and x_0=0.21
C_J=3.03, x_0=0.21
x
y


RC3BP Solution for C_J=3.03 and x_0=0.8
C_J=3.03, x_0=0.8
x
y

} instabil!