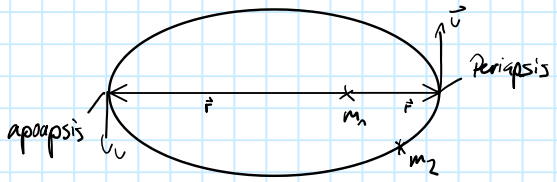
Blatt 2

Mittwoch, 14. Mai 2025 12:39

1)



Suche

$$\varepsilon = \frac{1}{2}\dot{\vec{r}}^2 - \frac{\mu}{r} = \text{const.}$$
$$= \frac{1}{2}v_p^2 - \frac{\mu}{r_p}$$

sub. v_p^2:

In apoapsis & periapsis: $\vec{v}_{a/p} \perp \vec{r}_{a/p}$

$$\Rightarrow h = v_a r_a = v_p r_p$$
$$\Rightarrow v_p = \frac{h}{r_p}$$
$$v_p^2 = \frac{h^2}{r_p^2} \overset{(*)}{=} \frac{\mu a(1-e^2)}{a^2(1-e)^2}$$
$$= \frac{\mu(1+e)}{a(1-e)}$$

(*): $h^2 = \mu a(1-e^2)$

$r_p = a(1-e)$

$$\Rightarrow \varepsilon = \frac{1}{2}\frac{\mu(1+e)}{a(1-e)} - \frac{\mu}{a(1-e)} = \frac{1}{2}\frac{\mu + \mu e - 2\mu}{a(1-e)}$$
$$= \frac{1}{2}\frac{-\mu}{a}\frac{1-e}{1-e} = -\frac{\mu}{2a}$$
$$\Rightarrow \frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$$
$$\Leftrightarrow v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \quad \square$$

(A2) Kepler 3:

$$P = \sqrt{\frac{4\pi^2 a^3}{G\mu}}$$

set $a, G, \underbrace{\mu}_{m_1+m_2} = 1$

$$P = \sqrt{4\pi^2} = 2\pi$$

A3

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \frac{-b-\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}}$$

$$= \frac{b^2-(b^2-4ac)}{2a(-b-\sqrt{b^2-4ac})}$$

$$= \underbrace{\frac{2c}{-b-\sqrt{b^2-4ac}}}$$

no catastrophic cancellation