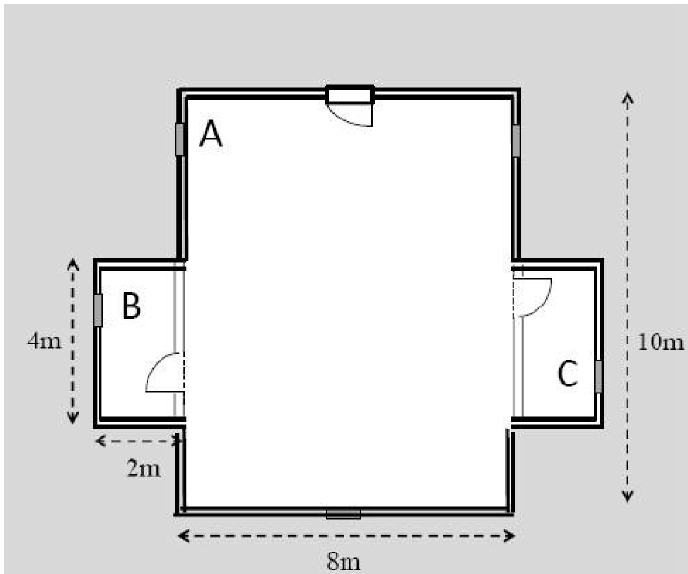
Temperature control of a three-rooms building

Consider the system illustrated in the following figure, consisting of a three-rooms building.



Model

$$
\begin{aligned}
c\rho V_A \dot{T}_A &= s_r u(T_B - T_A) + s_r u(T_C - T_A) + s_A U(T_E - T_A) + q_A \\
c\rho V_{B,C} \dot{T}_B &= s_r u(T_A - T_B) + s_{B,C} U(T_E - T_B) + q_B \\
c\rho V_{B,C} \dot{T}_C &= s_r u(T_A - T_C) + s_{B,C} U(T_E - T_C) + q_C
\end{aligned}
$$

c: specific heat of the air; V_i, $i = A, B, C$: Volume of room i; s_r: wall surface between A and B (and C); s_A: wall surface between A and the environment; $s_{B,C}$: wall surface between B (and C) and the environment; u, U: transmittances; q_i, $i = A, B, C$: inputs (heating power).

- Equilibrium condition: $T_E = 0^\circ$ C, $T_A = T_B = T_C = \bar{T} = 20^\circ$, $q_A = \bar{q}_A = s_A \cdot U\bar{T}$ W, $q_B = q_C = \bar{q}_B = \bar{q}_C = s_{B,C} \cdot u\bar{T}$ W.
- We define: $\delta T_A = T_A - \bar{T}$, $\delta T_B = T_B - \bar{T}$, $\delta T_C = T_C - \bar{T}$, $\delta q_A = q_A - \bar{q}_A$, $\delta q_B = q_B - \bar{q}_B$, and $\delta q_C = q_C - \bar{q}_C$.

Around the given equilibrium condition:

$$\begin{bmatrix} \delta \dot{T}_B \\ \delta \dot{T}_A \\ \delta \dot{T}_C \end{bmatrix} = \begin{bmatrix} -(\Gamma + \gamma_r) & \Gamma & 0 \\ \gamma & -(2\gamma + \gamma_A) & \gamma \\ 0 & \Gamma & -(\Gamma + \gamma_r) \end{bmatrix} \begin{bmatrix} \delta T_B \\ \delta T_A \\ \delta T_C \end{bmatrix} + \begin{bmatrix} \frac{1}{c\rho V_{B,C}} & 0 & 0 \\ 0 & \frac{1}{c\rho V_A} & 0 \\ 0 & 0 & \frac{1}{c\rho V_{B,C}} \end{bmatrix} \begin{bmatrix} \delta q_B \\ \delta q_A \\ \delta q_C \end{bmatrix}$$

$\gamma = \frac{s_r u}{c\rho V_A}$, $\Gamma = \frac{s_r u}{c\rho V_{B,C}}$, $\gamma_A = \frac{s_A U}{c\rho V_A}$, and $\gamma_r = \frac{s_{B,C} U}{c\rho V_{B,C}}$.

Consider the latter linear model and define:

$$u_1 = \delta q_A$$

$$u_2 = \delta q_B$$

$$u_3 = \delta q_C$$

$$x = [\delta T_A, \delta T_B, \delta T_C]^T$$

We can finally write the decomposed model as

$$\dot{x} = Ax + B_1 u_1 + B_2 u_2 + B_3 u_3$$

$$y_1 = [1 \quad 0 \quad 0]x$$

$$y_2 = [0 \quad 1 \quad 0]x$$

$$y_3 = [0 \quad 0 \quad 1]x$$

where

$$A = \begin{bmatrix} -(\Gamma + \gamma_r) & \Gamma & 0 \\ \gamma & -(2\gamma + \gamma_A) & \gamma \\ 0 & \Gamma & -(\Gamma + \gamma_r) \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Problem:

1. Generate the system matrices (both continuous-time and discrete-time, the latter with a sampling time selected compatibly with the continuous-time dynamics). Perform the following analysis:
    a. Compute the eigenvalues and the spectral abscissa of the (continuous-time) system. Is it open-loop asymptotically stable?
    b. Compute the eigenvalues and the spectral radius of the (discrete-time) system. Is it open-loop asymptotically stable?
2. For different "control structures" (i.e., centralized, decentralized, and different distributed schemes) perform the following actions
    a. Compute the continuous-time fixed modes (if necessary, adjust the rounding tolerance in the dedicated function)
    b. Compute the discrete-time fixed modes (if necessary, adjust the rounding tolerance in the dedicated function)
    c. Compute, if possible, the corresponding stabilizing CONTINUOUS-TIME control gain using the LMIs
    d. Compute, if possible, the corresponding stabilizing DISCRETE-TIME control gain using the LMIs
    e. Analyze the properties of the so-obtained closed-loop systems (e.g., stability, eigenvalues) and compute the closed-loop system trajectories (generated both in continuous-time and in discrete-time) of the room temperatures starting from a common random initial condition.