1. (a) $\forall x(P(x) \to \exists y(P(y) \land L(x,y)))$

(b) $\exists x(P(x) \land \forall y(P(y) \land \neg E(x,y) \to L(y,x)))$

2. (a) $D_1 = \{1\} \neq \phi$, $x,y,z \in D_1$, $x=y=z=1$

$\forall x(\forall y(x \neq y) \to \forall z((z=x) \lor (z=y)))$

$= \forall x(\forall y(F \to T)) = \forall x\ (T \lor T) = T$

(b). $\neg A = \neg(\forall x(\forall y((x \neq y) \to \forall z((z=x) \lor (z=y)))))$

$= \exists x(\exists y((x \neq y) \land \exists z\ \neg(z=x) \land \neg(z=y))))$

$D_2 = \{1,2,3\}$. when $x=1$, $y=2$, $z=3$.

$\neg A = \exists x(\exists y(T \land T) = T$, $A = F$.

4. (a) Interpretation: $P(x) = $ "$x=1$" $Q(x) = $ "$x=0$"

$x \in$ Domain: $\{0,1\}$.

$\exists x(P(x) \lor Q(x)) = T$ for $x \in D$ ($x=0, x=1$).

$\forall x\, P(x) \lor \forall x\, Q(x) = F$

when $\forall x P(x)\big|_{x=0} \lor \forall x Q(x)\big|_{x=1} = F \lor F = F$.

$\Rightarrow$ not logically equivalent

(b) Interpretation:

$P(x) = $ "$x=1$" $Q(x) = $ "$x=0$" $x \in$ Domain: $\{0,1\}$.

$\exists x(P(x) \land Q(x)) = F$ for $x \in \{0,1\}$.

$(\exists x P(x)\big|_{x=1}) \land (\exists x Q(x)\big|_{x=0})$ is true.

$\Rightarrow$ not logically equivalent.

5. ① $\exists x(P(x) \lor Q(x)) \to \exists x(P(x)) \lor \exists x Q(x)$

suppose left is T in interpretation I.

$\Rightarrow$ exist x P(x) is T or Q(x) is T.

$\Rightarrow$ exist x, P(x) is T or exist x, Q(x) is T.

$\Rightarrow$ $\exists x P(x) \lor \exists x Q(x)$ is T in I.

② $\exists x\, P(x) \lor \exists x Q(x) \to \exists x(P(x) \cup Q(x))$

Suppose left is T in interpretation I.

$\Rightarrow$ exist x. P(x) is T or exist x, Q(x) is T.

$\Rightarrow$ exist x, $P(x) \lor Q(x)$ is T.

$\Rightarrow$ $\exists x(P(x) \lor Q(x))$ is T in I.

from ①② $\exists x(P(x) \lor Q(x)) \equiv \exists x P(x) \cup \exists x Q(x)$.

3. (a) $D = R$

$P(x) = $ "$x=1$" $Q(x) = $ "$x=1$", formula is T

$P(x) = $ "$x=1$" $Q(x) = $ "$x \neq 1$". $\exists x(P(x) \leftrightarrow Q(x))$ is false.

$\Rightarrow$ satisfiable formula is F

(b) $\exists x(T \lor P(x) \to F)$

$\equiv \exists x((F \land \neg P(x)) \lor F) = F$, unsatisfiable.

(c) $\forall x(P(x) \lor \neg\exists y(Q(y) \land \neg Q(y)))$

$\equiv \forall x(P(x) \lor \neg F) \equiv T$. logically valid.

6. $\forall x(P(x) \to Q(x)) \Rightarrow \forall x P(x) \to \forall x Q(x)$.

suppose left is T for all interpretation I.

$\Rightarrow$ $P(x) \to Q(x)$ is T for all $x \in D$.

$\Rightarrow$ $\neg P(x) \cup Q(x)$ is T for all $x \in D$.

$\Rightarrow$ $\neg P(x)$ is T or $Q(x)$ is T.

$\Rightarrow$ $\forall x \neg P(x) \cup Q(x)$ is T for all I.

$\Rightarrow$ $\forall x P(x) \to \forall x Q(x)$ is T for all I.

7. $\exists x\, P(x) \land \forall x Q(x) \Rightarrow \exists x(P(x) \land Q(x))$

suppose left is T for interpretation I.

$\Rightarrow$ $\exists x\, P(x)$ is T, $\forall x Q(x)$ is T in I.

$\Rightarrow$ exist x that P(x) is T, Q(x) is T

$\Rightarrow$ $\exists x(P(x) \land Q(x))$ is T in I

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