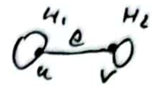
1. $\kappa(G_1)=1$ $\{c\}$
$\lambda(G_1)=3$ $\{(a,b),(a,c),(a,h)\}$.
$\delta(G_1)=3$ like $deg(a)=3$.
$\kappa(G_1)\le\lambda(G_1)\le\delta(G_1)$

$\kappa(G_2)=2$, $\{a,l\}$.
$\lambda(G_2)=2$, $\{(a,b),(k,l)\}$
$\delta(G_2)=3$, $deg(k)=3$, min degree
$\kappa(G_2)\le\lambda(G_2)\le\delta(G_2)$

$\kappa(G_3)=4$ $\{a,b,e,f\}$.
$\lambda(G_3)=4$ $\{(a,d),(a,b),(a,c),(a,f)\}$.
$\delta(G_3)=4$ $deg(a)=4$, min
$\kappa(G_3)\le\lambda(G_3)\le\delta(G_3)$

2. not connected.
components:
$\{A,B,I,F\}$
$\{J,C,E,G\}$.
$\{K,H,D\}$.

3. let's say G is a connected graph with all vertex having even degree.
and the brige $e=(u,v)$



$G-e$ is disconnected. G has 1 component, $G-e$ has 2 component, H_1, H_2
$\Rightarrow G-e=H_1\cup H_2$. let's say u in H_1, v in H_2.
Take H_1, all the other vertex in H_1 has even degree still, and u has odd degree.
$\Rightarrow \sum_{v\in V(H_1)} deg(v)$ is odd, same way $\sum_{v\in G(H_2)} deg(v)$ is odd.
$\Rightarrow \sum_{v\in G} deg(v) = \sum_{v\in H_1} deg(v) + \sum_{v\in GH_2} deg(v) + 1$ is odd + odd + 1 is odd.
but since all degree(v) in G is even, left of the equation should be even $\Rightarrow$ contradicts, so G with all even vertices. should ~~have~~ be bridgeless.

4. multigraph
a) G has Eulergraph (with order ≥ 2)
iff $2 \mid deg(x)$ for every $x\in V(G)$
every vertex in $K_{m,n}$
has degree $=m$ or n.
$\Rightarrow m, n$ should both be een

b). like a). vertex in G
has degree $=m$ or n
G has Eulerpath iff
G has exactly 2 vertex ~~that~~ has odd degree.
this can be:
1° $m=n=1$
2° m is odd and $n=2$
3° n is odd and $m=2$.

5 (a) without lifting the pen: has Euler path.
①: Graph only has 2 vertex has degree $=3$. is odd
other vertices's degree is even
$\Rightarrow$ exist Euler path. ok.

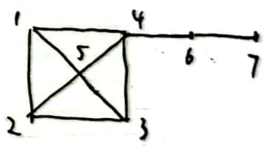
②. Graph has 4 vertex that has degree $=3$ is odd
$\Rightarrow$ without possible method.

③. Graph has 4 vertex that has degree $=3$ is odd
$\Rightarrow$ without possible method.

△ 5(b)(c) see the other page.

6. 1° Suppose there exists. u, v in the graph that, u, v is not connected.
and since u, v have a least degree p, there should be
p vertices connect to u and p different vertices connected to v
(if there's any same, u, v become connected).
$\Rightarrow$ there is $p+p+1+1=2p+2$ vetices, contradict with $2p$.
$\Rightarrow$ any u, v must be connected in graph.

2° Yes we have. if exist u, v not connected
like 1°. $|Neighbour(v)| + |Neighbour(u)| + 2$
$\ge \frac{(n-1)}{2}\times 2+2=n+1>n$, still contradicts.
u, v need to be connected.

7.


both 1, 2, 3 has odd degree $=3$.
$\Rightarrow$ No Euler path
No Euler circuit.

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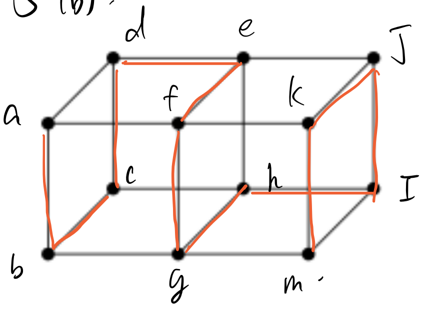
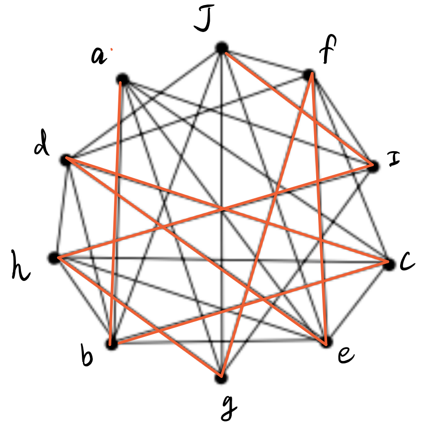
5 (b).



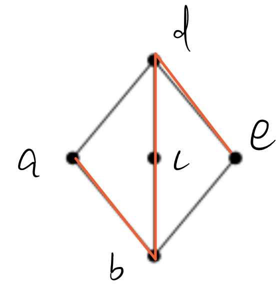
exists.

a, b, c, d, e, f, g, h, I, J, k, m



exists.

a, b, c, d, e, f, g, h, I, J

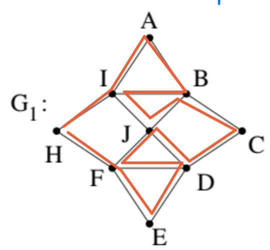


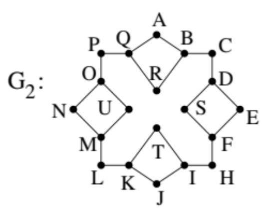
exists.

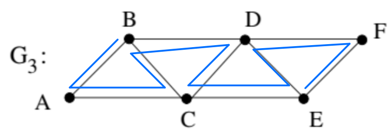
a, b, c, d, e

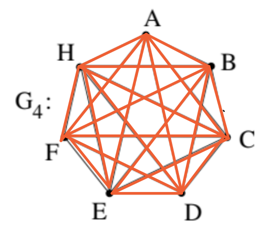
(c) Do the graphs G_1, G_2, G_3 and G_4 below admit any Euler path or Euler circuit? If yes, draw one, otherwise, explain why there is no Euler path nor Euler circuit.

m : Euler path. m : Euler circuit.









G_1:

Euler circuit: H, I, A, B, I, J, B, C, D, J, F, D, E, F, H

Euler path: No exists, since All vertices in G_1 have even degree

G_2:

Euler circuit: NOT have, since O, D, M, F has odd degree=3.

Euler path: NOT have, since O, D, M, F has odd degree=3. 4 odd degree vertex.

G_3:

Euler circuit: NOT have, since B, E. has odd degree=3.

Euler path: B, A, C, B, D, C, E, D, F, E

G_4:

Euler circuit: H, F, A, H, B, D, H, C, B, F, C, D, A, C, E, B, A, E, D, F, E, H

Euler path: NOT have, since all the vertices has even degree=6.

---

8. G is a directed graph.

M^n is not zero matrix

⇒ exist a path with n length

for n length, simple graph path.

There is n+1 vertices on this pth.

And G contains only n vertices

⇒ Exists a circuit on this path (means G contains a circuit).