1. $A = (P \vee Q \to R \wedge S) \wedge (S \vee W \to U) \wedge P$; $B = U \vee V$

$$A \wedge \neg B = \big((\neg P \wedge \neg Q) \vee (R \wedge S)\big) \wedge \big((\neg S \wedge \neg W) \vee U\big) \wedge P \wedge (\neg U) \wedge (\neg V)$$

$$= \big((\neg P \wedge \neg Q \wedge P) \vee (R \wedge S \wedge P)\big) \wedge \big((\neg S \wedge \neg W \wedge \neg U) \vee (\neg U \wedge U)\big) \wedge (\neg V)$$

$$= (R \wedge \underset{\sim}{S} \wedge P) \wedge (\underset{\sim}{\neg S} \wedge \neg W \wedge \neg U) \wedge \neg V$$

$$= F$$

$\Rightarrow A \Rightarrow B$.

2. p: it rains
q: it is foggy
r: sailing race will be held
s: lifesaving demonstration will go on.
t: trophy will be awarded.

Premises: $\neg p \vee \neg q \to r \wedge s$, $r \to t$, $\neg t$

conclusion: p.

$A = (\neg p \vee \neg q \to r \wedge s) \wedge (r \to t) \wedge (\neg t)$ $B = p$.

$$A \wedge \neg B = \big((p \wedge q) \vee (r \wedge s)\big) \wedge (\neg r \vee t) \wedge (\neg t) \wedge \neg p$$

$$= \big((p \wedge q \wedge \neg p) \vee (r \wedge s \wedge \neg p)\big) \wedge (\neg r \wedge \neg t)$$

$$= r \wedge s \wedge \neg p \wedge \neg r \wedge \neg t$$

$$= F$$

$\Rightarrow A \Rightarrow B$.

argument 1: $p_1 \cdots p_n, q$

3. premises: $A = p_1 \wedge \cdots \cdots p_n \wedge q$.

Conclusion: $B = r$

$A \wedge \neg B = p_1 \wedge \cdots \cdots p_n \wedge q \wedge \neg r$

is argument 1 valid, $A \wedge \neg B \equiv F$

argument 2:

premises: $p_1 \cdots p_n$.

$A' = p_1 \wedge \cdots \cdots \wedge p_n$.

coclusion: $q \to r$.

$B' = q \to r$

$A' \wedge \neg B' = p_1 \wedge \cdots \wedge p_n \wedge \neg(\neg q \vee r)$

$= p_1 \wedge \cdots \wedge p_n \wedge q \wedge \neg r = A \wedge \neg B = F$

argument 2 valid if 1 valid.

1. arguments method: premises: $P \wedge (P \wedge Q \to R \wedge S) \wedge (S \vee W \to U)$

conclusion: $U \vee V$.

(1) P: premise
(2) $P \vee Q$: addition on (1)
(3) $P \vee Q \to R \wedge S$: Premise
(4). $R \wedge S$: Modus ~~tollens~~ ponens. on (2), (3)
(5) S: simplification on (4).
(6). $S \vee W$: addition on (5).
(7). $S \vee W \to V$: premise
(8) U: modus ponons on (6), (7).
(9) $U \vee V$: addition on (8)

2. argument method:

(1) $\neg t$: premise
(2) $r \to t$: premise
(3) $\neg r$: Modus tollens on (1), (2)
(4). $\neg r \vee \neg s$: addition on (3)
(5) $\neg p \vee \neg q \to r \wedge s$: premise.
(6) $p \wedge q$ Modus tollens on (4), (5)
(7) p: simplification on (6).

4. p: math is hard
q: leibiz like math.
r: SINO is easy.

premises: $p \vee \neg q$, $r \to \neg p$.

(a): $q \to \neg r$. True.

$(p \vee \neg q) \wedge (r \to \neg p)$

rule of replacement

$\Rightarrow (p \vee \neg q) \wedge (p \to \neg r)$.

$\equiv (p \wedge (p \to \neg r)) \vee (\neg q \wedge (p \to \neg r))$

$\equiv (\neg r \vee \neg q) \wedge (\neg r \vee (p \to \neg r))$

$\Rightarrow \neg r \vee \neg q \equiv q \to \neg r$.

simplification.

4.(b) $\neg r \to \neg q$ False.

$r = F$, $p = T$, $q = T$ satisfy premises.

but $\neg r \to \neg q$ is False.

(c) $\neg p \vee \neg r$ True.

$(p \vee \neg q) \wedge (r \to \neg p)$

$\equiv (p \vee \neg q) \wedge (\neg r \vee \neg p)$

$\Rightarrow \neg q \vee \neg r$

(Resolution).

(d). $\neg q \to (\neg r \vee \neg p)$ True.

$(p \vee \neg q) \wedge (r \to \neg p)$

$\equiv (p \vee \neg q) \wedge (\neg r \vee \neg p) \equiv \neg r \vee p$.

$\Rightarrow q \vee (\neg r \vee \neg p) \Rightarrow \neg q \to (\neg r \vee \neg p)$.

rule of addition

logic equivalence