SI120 Discussion 10
homework 10

1. (10 points) Let $P(x)$ = "x is a person", $L(x, y)$ = "x likes y" and $E(x, y)$ ="$x = y$". Translate the following statements into formulas:

(a) "Every person likes some other person."

(b) "There is a person who is liked by every other person."

1. (a) $\forall x (P(x) \rightarrow \exists y (P(y) \wedge \neg E(x,y) \wedge L(x,y)))$

(b) $\exists x (P(x) \wedge \forall y (P(y) \wedge \neg E(x,y) \rightarrow L(y,x)))$

一般存在∃后面跟∧，任意∀后面跟→。

2. (10 points) Let A be the formula $\forall x\,(\forall y\,((x \neq y) \to \forall z\,((z = x) \vee (z = y))))$

(a) Find a domain $D_1 \neq \emptyset$ such that A is true when x, y, z are taken over D_1.

(b) Find a domain D_2 such that A is false when x, y, z are taken over D_2.

(a) $D_1 = \{0, 1\}$

(b) $D_2 = \{0, 1, 2\}$

3. (10 points) Determine if the following formulas are logically valid, satisfiable or unsatisfiable.

(a) $(\exists x P(x) \leftrightarrow \exists x Q(x)) \rightarrow \exists x(P(x) \leftrightarrow Q(x))$

(b) $\exists x(\mathbf{T} \lor P(x) \rightarrow \mathbf{F})$

(c) $\forall x(P(x) \lor \neg\exists y(Q(y) \land \neg Q(y)))$

3. (a) satisfiable

$D: x \in N$

T: $P(x)$: x is an even number

$Q(x)$: $2 | x$

F: $P(x)$: $2 | x$ $\quad Q(x)$: $2 \nmid x$

$\exists x P(x) = T$

$\exists x Q(x) = T$ $\Rightarrow \exists x P(x) \leftrightarrow \exists x Q(x)$

$\exists x (P(x) \leftrightarrow Q(x)) = F$

(b) unsatisfiable

$\exists x (T \lor P(x) \rightarrow F) = \exists x (T \rightarrow F) = \exists x (\neg T \lor F) = \exists x F$

(c) logically valid

$\forall x (P(x) \lor \neg \exists y (Q(y) \land \neg Q(y))) = \forall x (P(x) \lor \neg \exists y F)$

$= \forall x (P(x) \lor \forall y T) = \forall x (\forall y T)$

4. (20 points) Show the following statements with interpretations of the formulas.

(a) $\forall x(P(x) \vee Q(x))$ and $\forall xP(x) \vee \forall xQ(x)$ are not logically equivalent.

(b) $\exists x(P(x) \wedge Q(x))$ and $\exists xP(x) \wedge \exists xQ(x)$ are not logically equivalent.

4. (a) $x \in \mathbb{R}$ $\quad P(x): x \geq 0 \quad Q(x): x < 0$

$\forall x \ (P(x) \vee Q(x)) = T. \quad \forall x P(x) \vee \forall x Q(x) = F$

(b) $x \in \mathbb{R}$ $\quad P(x): x \geq 0 \quad Q(x): x < 0$

$\exists x P(x) \wedge \exists x Q(x) = T. \quad \exists x (P(x) \wedge Q(x)) = F$

5. (10 points) Show that $\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$.

5, $\Rightarrow$: Let x_0 satisfy $P(x_0) \vee Q(x_0)$,

Then $P(x_0)$ is right, or $Q(x_0)$ is right

When $P(x_0) = T$, $\exists x P(x) = T$

when $Q(x_0) = T$. $\exists x Q(x) = T$

So $\exists x P(x) \vee \exists x Q(x) = T$

$\Leftarrow$: when $\exists x P(x)$, Let x_1 satisfy $P(x_1)$,

so x_1 satisfies $P(x_1) \vee Q(x_1)$,

so $\exists x(P(x) \vee Q(x)) = T$

when $\exists x Q(x)$, same

6. (20 points) Show that $\forall x\left(P(x) \rightarrow Q(x)\right) \Rightarrow \forall x P(x) \rightarrow \forall x\, Q(x)$.

$\forall x \ P(x) \rightarrow Q(x) = \forall x \neg P(x) \vee Q(x))$

if $\forall x P(x) \rightarrow \forall x Q(x) = F$.

i.e. $\neg(\forall x P(x)) \vee (\forall x Q(x)) = F$.

i.e. $\forall x P(x) = T$ and $\forall x Q(x) = F$

for x_0, $P(x_0) = T$, $Q(x_0) = F$

$\Rightarrow$ $\neg P(x_0) \vee Q(x_0) = F$. controdict

7. (20 points) Show that $\exists x P(x) \wedge \forall x Q(x) \Rightarrow \exists x(P(x) \wedge Q(x))$.

suppose x_0 satisfy $P(x_0)$

as $\forall x Q(x)$, $Q(x_0)=T$

so $P(x_0) \wedge Q(x_0)=T$

so $\exists x(P(x) \wedge Q(x))$