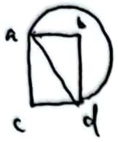
1. K4:



2. assume that every region has ≥ 6 degree.

$\Rightarrow |E(G)| \leq \frac{6}{6-2}(|V(G)|-2)$

$|E(G)| \leq \frac{3}{2}(|V(G)|-2)$ ①

since every vertex in G has degree 3

from hand shaking:

$2|E| = \sum_{v \in V} deg(v) = 3|V(G)|$

$|E| = \frac{3}{2}|V(G)|$, contradicts with ①.

$\Rightarrow$ exist a region that has at most 5 degree

3. edge weight sort:

2, 3, 4, 5, 6, 7.

4 edge min circuit:

a, d, b, c, a: 16.

a b c d: 18

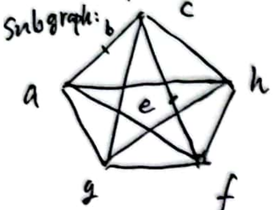
a b d c a: 20

$\Rightarrow$ Tsp solution: a, d, b, c, a, 16.

~~Or~~ a, c, b, d, a, 16.

4. G: non planar:

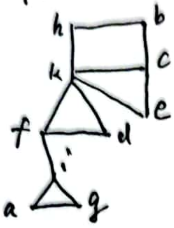
subgraph:



homeomorphic to K5

$\Rightarrow$ nonplanar (kuratowski)

H: planar:



K: nonplannar:

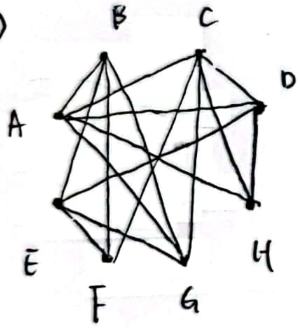
sub graph:

homeomor~~morphic~~phic to K3,3

$\Rightarrow$ nonplanar.

5. take fish type as vertex. 'cannot' relation as edge.

(a)

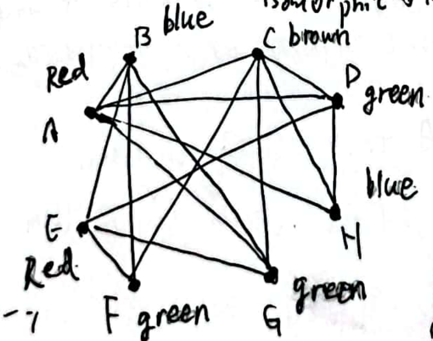


(b) $\max\{deg(v) : v \in V\} = \Delta(G) = 5$

$\chi(G) \leq \Delta(G)+1 = 6$.

$\rightarrow$ subgraph.

$\chi(G) \geq 4$: has subgraph isomorphic to K4.



↑ a 4-coloring solution

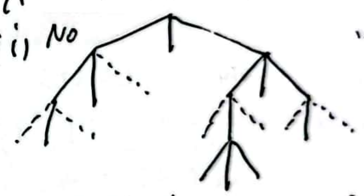
$\Rightarrow \chi(G) = 4$

(c) from (a), (b).

minimal 4 aquarium.

7. (i) Yes, 3-ary

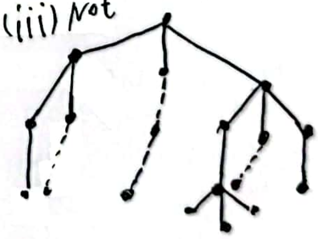
(ii) No



: added edge.

minimal number of edge added: ~~5~~ 7

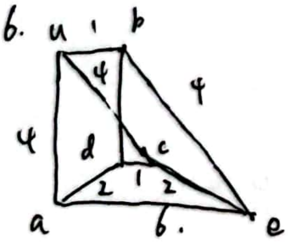
(iii) Not



: added edge.

minimal number: 4

6.



① $S_0 = \phi$ $L_0(u)=0$ $L_0(a)=\infty$ $L_0(b)=\infty$ $L_0(c)=\infty$ $L_0(d)=\infty$ $L_0(e)=\infty$

② $S_1 = \{u\}$. $L_1(u)=0$ $L_1(a)=4$, $L_1(b)=1$ $L_1(c)=4$, $L_1(d)=\infty$ $L_1(e)=\infty$

③ $S_2 = \{u, b\}$ $L_2(a)=4$. $L_2(c)=4$ $L_2(d)=2$ $L_2(e)=5$.

③ $S_3 = \{u, b, d\}$ $L_3(a)=4$. $L_3(c)=3$, $L_3(e)=5$.

④ $S_4 = \{u, b, d, c\}$ $L_4(a)=4$ $L_4(e)=5$.

⑤ $S_5 = \{u, b, d, c, a\}$ $L_5(e)=5$.

⑥ $S_6 = \{u, b, d, c, a, e\}$.

(b) let's use a binary tree. take the 2 children as the input and vertex as the processor's operation (do addition).

let $n = 2^k$

consider a full binary tree with n-1 vertex as our network.

internal vertex number $= \frac{(n-1)-1}{2} = 2^{k-1}-1$.

leaves number $=$ ~~2^{k-1}~~ $(n-1)-(2^{k-1}-1) = 2^{k-1}$

$\Rightarrow$ we can compute 2^{k-1} number at a time.

$\frac{n}{2^{k-1}} = 2$, $2+1=3$, 3 steps.

8. $\Rightarrow$:

① If G is a tree, it contains no circles.

$\Rightarrow$ No edge in G belong to a circle.

if there's ~~≥~~ 2 paths from A to B. there must be a circle.

$\Rightarrow$ Only 1 path ~~...~~ for every vertex pair (A, B) in G

$\Rightarrow$ every edge is a bridge

② Induction methods:

~~suppose~~ G tree with n vetices $\{x_0 \dots x_n\}$.

$n=1$ is trivial

$n=2$ 1 edge ~~contains~~ links 2 vertex. it's a bridge.

suppose $n \geq 3$ $n=k$. the every edge is a bridge.

for $n=k+1$. we add a new vertex to tree in $n=k$.

and this adds only 1 vertex 1 edge, this edge is a bridge and rest ~~#~~ edges, which in $(n=k)$ graph is all bridge by assume.

$\Rightarrow$ Induction: G is a tree $\Rightarrow$ every edge is a bridge.

$\Leftarrow$:

for a connected graph G that every edge is bridge.

let e be an edge in G

let e have ends u and v.

suppose that e is part of a circle $C = u_0 \dots u_m$ in G $(u_i u_{i+2} \cdots u_m u_0 \, u_{i-1})$

let $e = u_i u_{i+1}$. $\Rightarrow$ There is always exist a path from u_i to u_{i+1}

$\Rightarrow$ e is not a bridge, contradicts.

$\Rightarrow$ bridge e can not be in any circle of G

$\Rightarrow$ since every edge is a bridge, G connected ~~undirected~~ simple graph has no circles.

$\Rightarrow$ G is a tree.