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Tutorial on filter banks

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1 CQF filter bank

A two-channel filter bank is defined by the diagram in Figure 1.

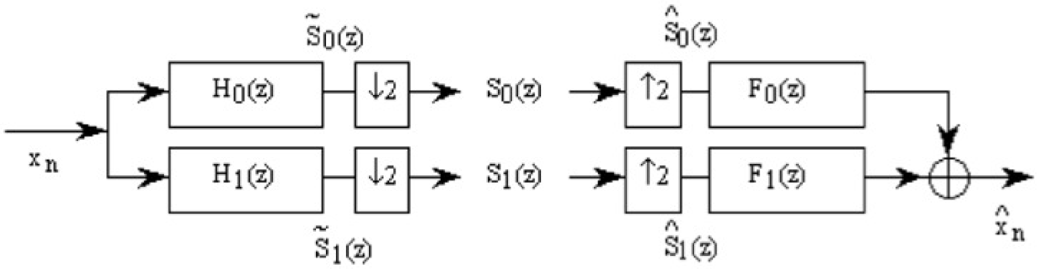

Figure 1: General diagram of a two-channel filter bank.

1. Express $\hat{X}(z)$ as a function of $X(z)$.
2. Deduce that the aliasing cancellation (AC) conditions of a 2-channel filter bank are $F_0(z) = H_1(-z)$ and $F_1(z) = -H_0(-z)$, and that its transfer function (TF) is $T(z) = \frac{1}{2}(H_0(z)F_0(z) + H_1(z)F_1(z))$.
3. Now we assume that H_0 and H_1 are *conjugate quadrature filters* (CQF): $H_1(z) = -z^{-(N-1)}\widetilde{H}_0(-z)$ where $\widetilde{H}_0(z) = H_0^*(\frac{1}{z})$ and N is even. Prove that equations (AC) and (CQF) imply that $\forall k \in \{0, 1\}$, $F_k(z) = z^{-(N-1)}\widetilde{H}_k(z)$: we say that the analysis and synthesis filters are *paraconjugate* (PC).
4. Finally we assume that $H_0(z)$ is a *symmetric power* (SP) filter: $\widetilde{H}_0(z)H_0(z) + \widetilde{H}_0(-z)H_0(-z) = 2c$. Prove that equations (TF), (CQF), (PC) et (SP) imply that $T(z) = cz^{-(N-1)}$: the CQF filter bank guarantees perfect reconstruction.

2 Transmultiplexer

We implement the transmultiplexer represented in Figure 2 by means of the filters defined in Exercise 1.

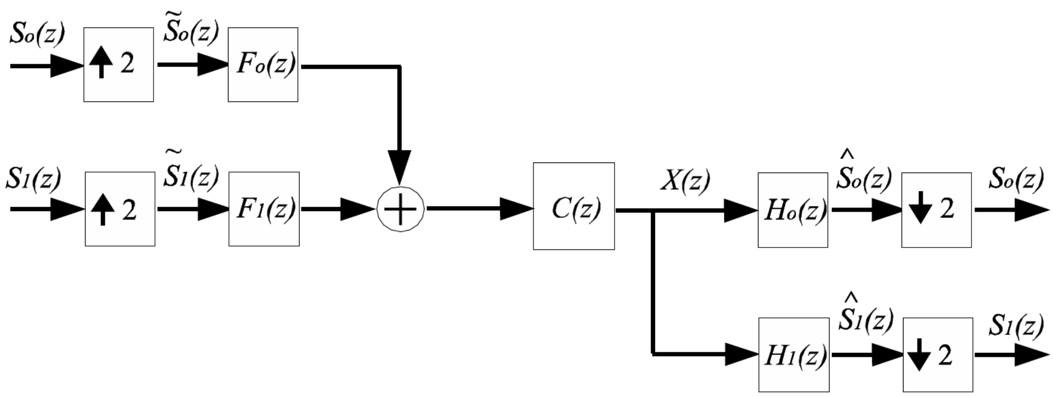

Figure 2: Transmultiplexer.

1. Prove that the transmultiplexer guarantees perfect reconstruction at the output when $C(z) = z^{-1}$.

2. From now on, filter $C(z)$ will represent the transfer function of a transmission channel between the encoder and the decoder. It is uniformly equal to 1 if the channel is transparent, but in general, the transmission channel is imperfect, and its transfer function $C(z)$ is not constant. In order to simplify, let us assume that $C(z)$ is of the form $1 - \alpha z^{-1}$. In order to keep the perfect reconstruction property at the output of the transmultiplexer, we will have to introduce just after $C(z)$ a causal filter $D(z)$ such that $C(z)D(z) = dz^{-n_0}$, where n_0 is an odd number.

   (a) How to choose $D(z)$ if $\alpha = 0.9$?

   (b) What problem do we encounter if $\alpha = 1.2$? Propose an approximate solution.

3. If the channel transfer function $C(z)$ is unknown, propose a method for estimating it from the output signals, by choosing appropriate filter $D(z)$ and input signals s_0 and s_1.