Confidence intervals

Exercise 1 (Gaussian model – unknown mean, known variance)**:**
Some signal is supposed to be Gaussian with unknown mean θ and variance 1. You observe n i.i.d. samples.

1. Propose a symmetric confidence interval of level 90% on θ.
2. Give an upper confidence bound of level 90% on θ.
3. In both cases, give the result for an empirical mean of 2 over 25 samples.

Exercise 2 (Gaussian model – unknown mean and variance)**:**
The daily power consumption of a company is supposed to be Gaussian. You observe an average daily power consumption of 100 units over 10 days, with a standard deviation of 20.

1. Provide a confidence interval of level 95% on the daily average power consumption.
2. Give the result of Student's test at level 5% for the null hypothesis that the mean is equal to 90 units.

Exercise 3 (Gaussian model – known mean, unknown variance)**:**
A company sells steel balls of diameter 1cm. The actual diameter is Gaussian with mean 1cm and unknown variance θ. You observe a standard deviation of 0.1mm over 50 balls.

1. Propose a confidence interval of level 99% on θ, so that the probability that the upper bound is incorrect is equal to the probability that the lower bound is incorrect.
2. Give an upper confidence bound of level 99% on θ.

Exercise 4 (Poisson model)**:**
The daily number of emails you receive is supposed to be Poisson distributed with parameter θ. You've received 200 emails in 10 days. Give an upper confidence bound of level 95% on θ using a Gaussian approximation.

Exercise 5 (Uniform model)**:**
The lifetime of an electronic device has a uniform distribution over $[0, \theta]$, for some unknown parameter $\theta > 0$. You observe n i.i.d. samples.

1. Propose a lower confidence bound of level $1 - \alpha$ on θ based on the maximum lifetime of these n devices.
2. Give the result for $\alpha = 10\%$ and $n = 5$ devices with maximum lifetime equal to 3 years.

Exercise 6 (Bernoulli model)**:**
Let θ be the fraction of electric cars in Paris. You count 20 electric cars out of 100. Using a Gaussian approximation, give for θ, each at level 99%:

1. an upper confidence bound,
2. a lower confidence bound,
3. a confidence interval.