Quadratic risk

Exercise 1 (Gaussian model – mean)**:**
The daily power consumption of a company is supposed to have a gaussian distribution. You want to estimate the mean θ through n independent observations; the variance σ^2 is known.

1. Propose an estimator of θ by the method of moments.
2. Is this estimator biased?
3. Compute its quadratic risk.
4. Is the estimator efficient?

Exercise 2 (Poisson model)**:**
The daily number of admissions to a cinema is supposed to be Poisson. You want to estimate the mean θ using n observations.

1. Give the MLE of θ.
2. Is this estimator biased?
3. Compute its quadratic risk.
4. Is the MLE efficient?

Exercise 3 (Poisson model bis)**:**
In the previous exercise, we now want to estimate the probability $g(\theta) = e^{-\theta}$ that there is no admission.

1. Give the MLE of $g(\theta)$ and propose an estimator based on the method of moments.
2. Are these estimators biased?
3. Are these estimators efficient?

Exercise 4 (Translated exponential)**:**
The lifetimes of laptops are supposed to have a translated exponential distribution of the form[1]:

$$P(X > x) = \exp(\theta - x), \quad \forall x \geq \theta,$$

where $\theta > 0$ is unknown. You have n observations.

1. Give the MLE of θ and propose another estimator based on the method of moments.
2. Are these estimators biased?
3. Compute their quadratic risks. Which estimator would you recommend?

[1] Note that this model is *not* regular as the support of X depends on θ.

Exercise 5 (Mixture model)**:**
A proportion θ of smartphones are defective: their lifetime has an exponential distribution with parameter μ while the lifetime of a regular smartphone has an exponential distribution with parameter $\lambda < \mu$. Both parameters λ and μ are known. You observe the lifetime of n smartphones.

1. Propose an estimator based on the method of moments.
2. Compute its quadratic risk.
3. Is this estimator efficient? You may consider the case $\mu \to +\infty$.

Exercise 6 (Gaussian vector)**:**
Some signal is supposed to have a gaussian distribution with unknown mean $\theta \in \mathbb{R}^d$ and known covariance matrix Γ. You want to estimate $g(\theta) = w^T\theta$ for some $w \in \mathbb{R}^d$ through n independent observations $x_1, \dots, x_n$. Consider the estimator:

$$\widehat{g}(x) = w^T\widehat{\theta} \quad \text{with} \quad \widehat{\theta} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

1. Compute its quadratic risk.
2. Is the estimator efficient?