Module 1
Discrete Mathematics

Sets, Functions, Relations, Induction, Strong Induction. Set theory in python-creating sets & Modifying sets in Python. Python set operations, Built-in Functions with Set. Python frozen set.

Sets

A **Set** is a well-defined collection of distinct objects.

Examples

1. Collection of natural numbers.
2. Collection of alphabets in English.
3. $A = \{1,2,3,4, \dots\}$
4. $B = \{a, b, c, \dots\}$

Non-examples

1. Collection of 5 world-renowned Mathematicians.
2. Collection of beautiful flowers in a garden.
3. Collection of easy words in the English language.

Members/ Elements:

The objects in a set are called **Members/ Elements**.

Note:

- Sets are usually denoted by capital letters $A, B, C, X, Y, Z, \ldots$
- Elements are usually denoted by small letters $a, b, c, x, y, z, \ldots$
- While defining a set, there should be no ambiguity, whether a given object belongs to the set or does not belong to the set.

Set Membership:

If $\boldsymbol{a}$ is an element of a set $\boldsymbol{A}$, we say that $\boldsymbol{a}$ belongs to $\boldsymbol{A}$. The Greek symbol ∈(epsilon) is used to denote the phrase **belongs to**. Thus, we write $\boldsymbol{a} \in \boldsymbol{A}$.

If $\boldsymbol{b}$ is not an element of a set $\boldsymbol{A}$, we write $b \notin A$ and read $\boldsymbol{b}$ does not belong to $\boldsymbol{A}$.

Cardinality:

The number of elements in a set $\boldsymbol{A}$ is called cardinality of $\boldsymbol{A}$, denoted as $|\boldsymbol{A}|$ or $\boldsymbol{n}(\boldsymbol{A})$ or **o(A)**.

Example: If $A = \{1,2,3\}$ then $|A| = 3$.

Representation of Sets:

There are two methods of representing a set

1. Roaster form or tabular form.
2. Rule Method or Set builder form.

1. **Roaster or tabular form:**

In this method, all the elements of the set are listed. The elements are separated by commas and are enclosed within curly braces.

Example:

1. $A = \{1,2,3,6,7,14,21,42\}$.
2. $B = \{a, e, i, o, u\}$.
3. $C = \{1,3,5, \ldots\}$.
4. $D = \{1,2,3,4, \ldots\}$

Note:

- In this form, the order in which the elements are listed, is immaterial.
- In this form, the elements are not generally repeated.

2. Rule Method or Set builder form:

In this method, all the elements of the set posses a single common property which is not possessed by any elements outside the set.

Example:

$A = \{x : x \text{ is a natural number which divides } 42\}.$

$B = \{y : y \text{ is a vowel in the English alphabet}\}.$

$C = \{z : z \text{ is an odd natural number}\}.$

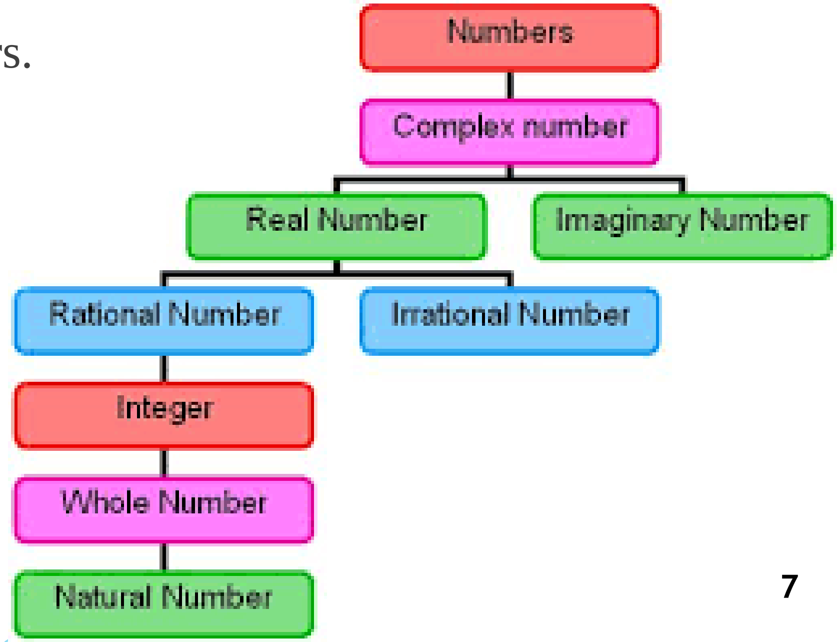
$D = \{n : n \in \mathbb{N}\}$

Note:

In Rule method set can also be written as $\{x/P(x)\}$, where x is the element of set and $P(x)$ is the property satisfied by the element.

Some important set notation:

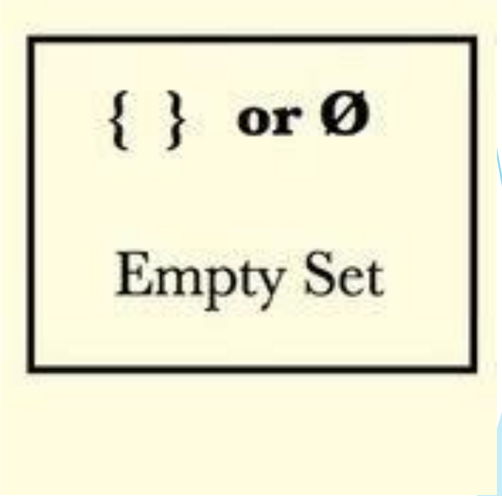
- $\mathbb{N}$ = Set of natural numbers.
- $\mathbb{Z}$ = Set of integers.
- $\mathbb{Q}$ = Set of rational numbers.
- $\mathbb{Q}'$ = Set of irrational numbers
- $\mathbb{R}$ = Set of real numbers.
- $\mathbb{C}$ = Set of complex numbers.
- $\mathbb{Z}^+$ = Set of positive integers.
- $\mathbb{Z}^*$ = Set of non-zero integers.
- $\mathbb{Q}^+$ = Set of positive rational numbers.
- $\mathbb{Q}^*$ = Set of non-zero rational numbers.
- $\mathbb{R}^+$ = Set of positive real numbers.
- $\mathbb{R}^*$ = Set of non-zero real numbers.



Types of Sets:

- **Empty Set:**

A set which does not contain any element is called the **empty set** or the **null set** or the **void set**. The empty set is denoted by the symbol $\emptyset$ or $\{\}$. The cardinality of an empty set is 0.



Example:

i. $A = \{x: 1 < x < 2, x \in \mathbb{N}\}$ is an empty sent, because there is no natural number between 1 and 2.

ii. $B = \{\ \}$.

Types of Sets:

- **Singleton set:**

A set consisting of only one element is called a singleton set.

The cardinality of a singleton set is 1.

Example:

i. $A = \{2\}$.

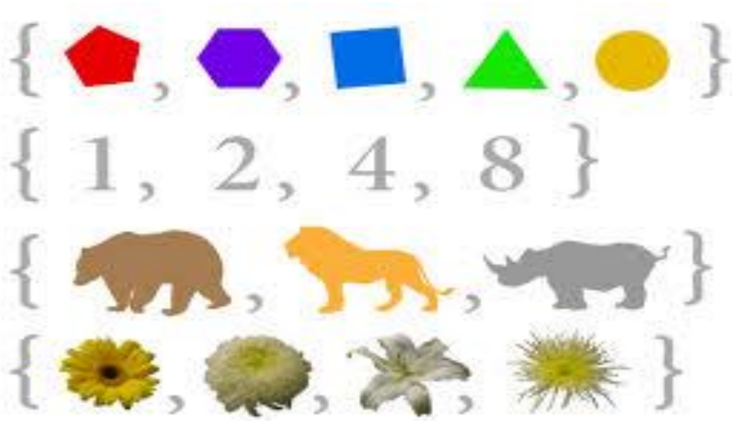
ii. $B = \ Set\ of\ even\ prime\ number.$

iii. $C = Director\ of\ SET$-JU.

Types of Sets:

- **Finite Set:**

A set which is empty or consists of a definite number of elements is called a finite set. The cardinality of a finite set is the number of elements in the set (without repetition of elements).



Example:

i. $A = \{2\}$.

ii. $B = \emptyset$.

iii. $C = \{a, e, i, o, u\}$.

iv. The Solution set S, of the equation $x^2 - 16 = 0$.

Types of Sets:

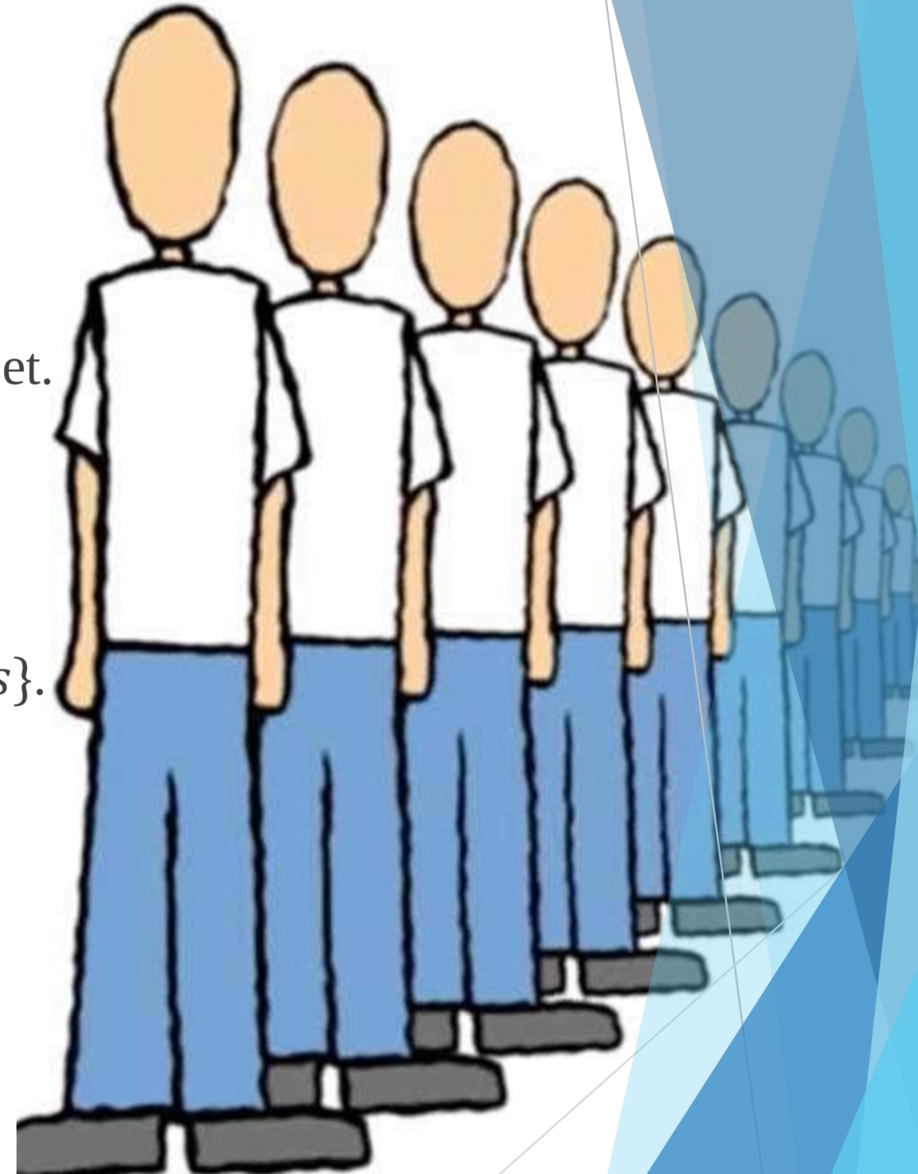
- **Infinite Set:**

If the number of elements in a set is infinite, then the set is called infinite set. The cardinality of an infinite set is ∞.

Example:

i. $A = \{Stares\ in\ the\ Sky\}.$

ii. $\mathbb{N} = \{Set\ of\ all\ natural\ numbers\}.$



Integers(Z)

{...,-2,-1,0,1,2,...}

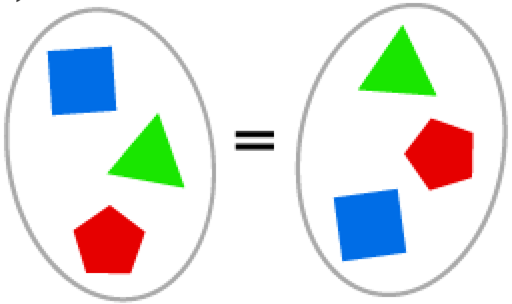
Types of Sets:

- **Equal Sets and Unequal Sets:**

Two sets A and B are said to be equal if they have exactly the same elements and we write $A = B$. Otherwise, the sets are said to be unequal and we write $A \neq B$. The cardinality of two equal sets is equal.

Example:

i. Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Then A and P are equal, since 2, 3 and 5 are the only prime factors of 30 and these are less than 6. i.e., $A = \{2,3,5\}$ and $P = \{2,3,5\}$.



ii. Let $A = \{1,2,3,4\}$ and $B = \{3,4,1,2\}$ then $A = B$.

iii. Let $A = \{1,2,3\}$ and $B = \{3,4,1,2\}$ then $A \neq B$.

Types of Sets:

Note:

A set does not change if one or more elements of the set are repeated. For example, the sets $A = \{1, 2, 3\}$ and $B = \{2, 2, 1, 3, 3\}$ are equal, since each element of A is in B and vice-versa. That is why we generally do not repeat any element in describing a Set.

- **Equivalence sets:**

The finite sets are said to equivalence set if both of them having same Number of elements. i.e., cardinality of both sets must be equal.

Example:

Let $A = \{1,2,3,4\}$ and $B = \{3,4,1,2\}$ then $A = B$.

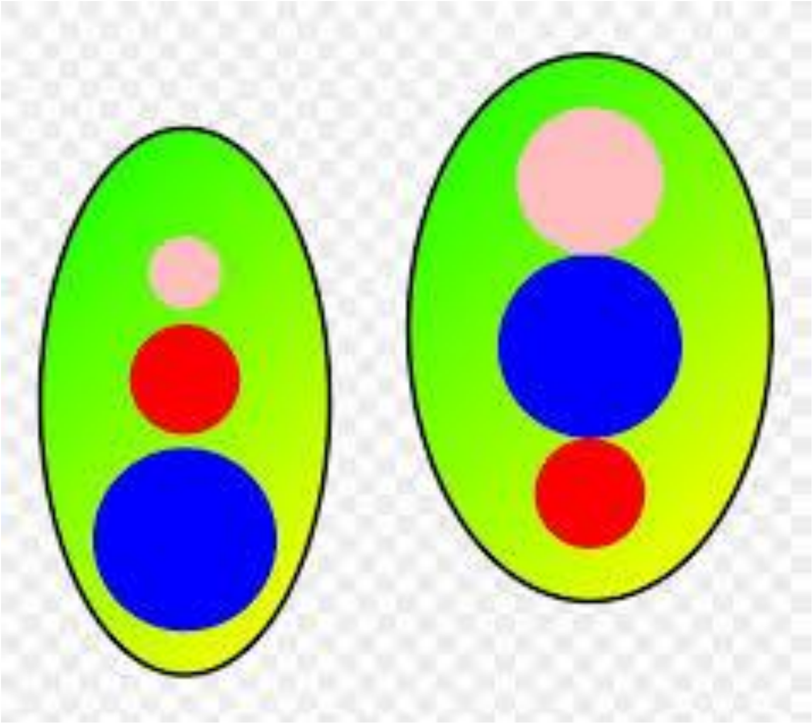
Types of Sets:

- **Disjoint sets:**

If no single elements are common in two given sets then the sets are called Disjoint sets.

Example:

Let $A = \{1,2,3,4\}$ and $B = \{a, e, i, o, u\}$.



Types of Sets:

- **Sub Set:**

A set A is said to be a subset of a set B if every element of A is also an element of B. It is denoted by $A \subset B$. In other word, $A \subset B$ if whenever $a \in A$, then $a \in B$.

If set A contains an element which is not in set B then A is not a subset of B and is denoted by $A \not\subset B$.

The symbol $\subset$ stands for "is a subset of" or "is contained in"

Example:

- Let $A = \{a, b, c, d, e, f\}, B = \{a, d, f\}, C = \{d, e, f, g, h\}$.

  Then $B \subset A$,

- while $B \not\subset C$.

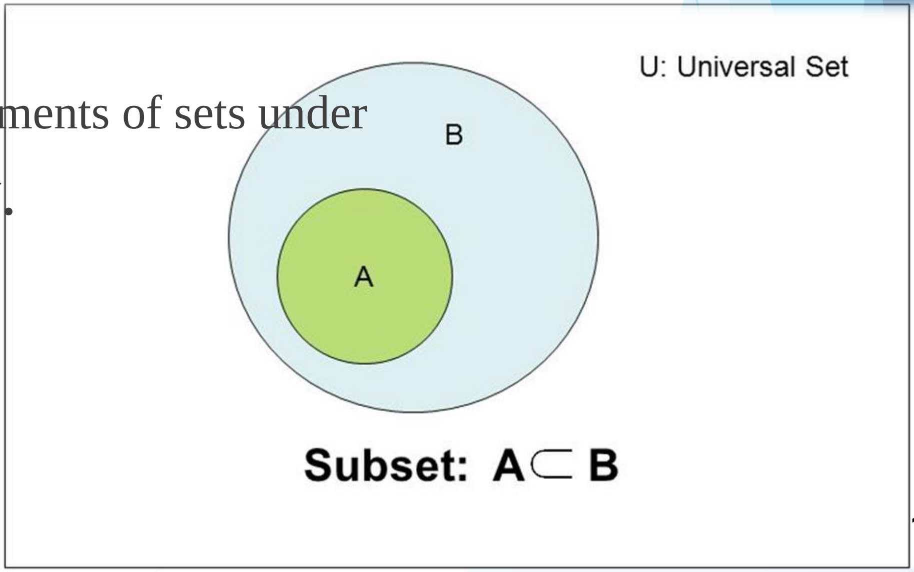
Types of Sets:

- **Example:**
- $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$ then A is the proper subset of B .

  - A subset which contains all the elements of the original set is called an **improper subset.** And is denoted by $\subseteq$
  - If $A \subseteq B$ and $B \subseteq A$ the $A = B$.

- **Universal Set:**

- A universal set is set, which has all the elements of sets under consideration. It is generally denoted by ***U***.



Types of Sets:

- **Power set:**

The collection of all subsets of a set A is called the power set of A. It is denoted by $P(A)$. If the number of elements in $A = n$, i.e., $|A| = n$, then the number of elements in $P(A) = 2^n$.

$\{a,b,c\}$

$$\left\{ \begin{array}{l} \{\}, \\ \{a\}, \{b\}, \{c\}, \\ \{a,b\}, \{a,c\}, \{b,c\}, \\ \{a,b,c\} \end{array} \right\}$$

Example:

- If $A = \{1,2\}$ then $P(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$. Here $|A| = 2$, thus $|P(A)| = 2^2 = 4$.

Operation in Sets:

- **Union of Sets:**

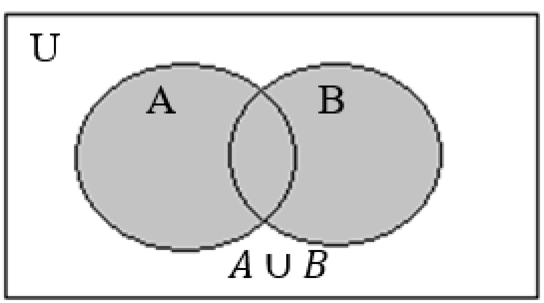
Let A and B be any two sets. The union of A and B is the set which consists of all the elements of either A or B or both and is denoted by $A \cup B$ and read as A union B. Symbolically we write $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

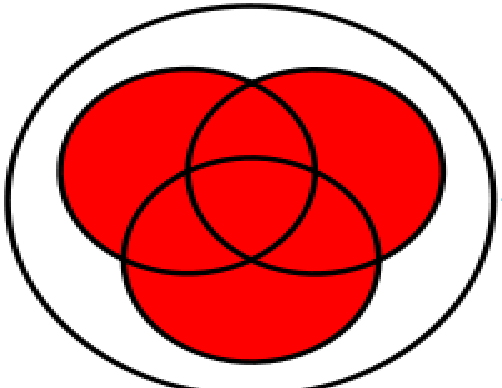
- **Example:**

If $A = \{1,2,3\}$ & $B = \{2,3,4\}$ then $A \cup B = \{1,2,3,4\}$.

Note: $A \subseteq A \cup B, B \subseteq A \cup B$.

Ven Diagram:





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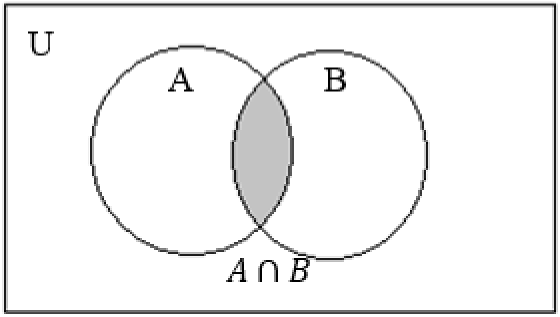
Operation in Sets:

- **Intersection of Sets:**

Let A and B be any two sets. The Intersection of A and B is the set which consists of all the elements belongs to both A and B and is denoted by $A \cap B$ and read as A intersection B. Symbolically we write $A \cap B = \{x : x \in A \; and \; x \in B\}$.

Ven Diagram:



Example:

If $A = \{1,2,3\}$ & $B = \{2,3,4\}$ then $A \cap B = \{2,3\}$

Note:

- $A \cap B \subseteq A, A \cap B \subseteq B$.
- Two sets A & B are said to be disjoint whenever $A \cap B$ is a null set.

Operation in Sets:

- **Difference of Sets or Relative complement of Sets:**

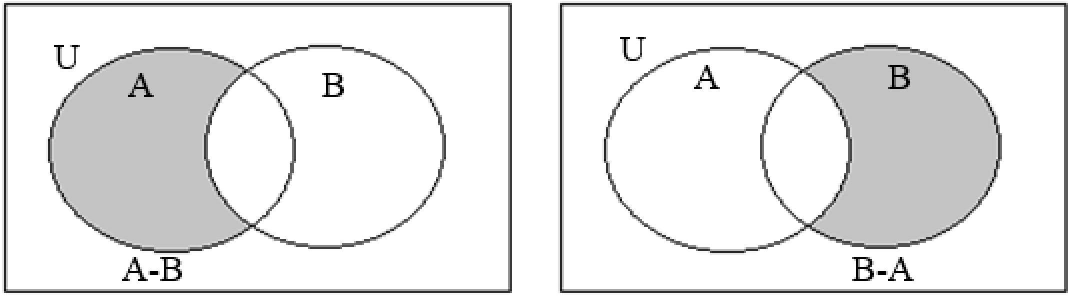
The difference of two sets A and B, denoted by $A - B$ is defined as set of elements which belong to A but not to B.

We write $A - B = \{x : x \in A \; and \; x \notin B\} \; or \; B - A = \{x : x \in B \; and \; x \notin A\}$.

Example:

If $A = \{1,2,3\}$ & $B = \{2,3,4\}$ then $A - B = \{1\}$ and $B - A = \{4\}$

Ven Diagram:



Operation in Sets:

Compliment of a Set:

Let U be a universal set and A be any subset of U, then the compliment of A is the set consists of the elements belongs to U but not A. It is denoted by A' or $\bar{A}$.

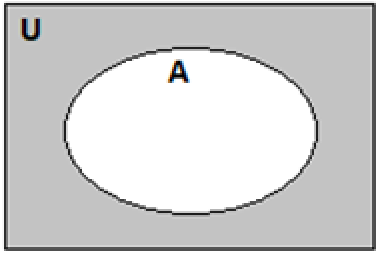
Symbolically we write $\bar{A} = U - A = \{x : x \in U \; and \; x \notin A\}$.

Example:

If $U = \{1,2,3,4,5\}$ & $A = \{2,3,4\}$ then $\bar{A} = U - A = \{1,5\}$.

Note: $\bar{U} = \emptyset$; $\bar{\emptyset} = U$; $A \subset U$; $\bar{A} \subset U$; $\bar{A}$ and A are disjoint set

Ven Diagram:



Functions

A function in mathematical relationship among the inputs (i.e. the domain) and their outputs (known as the codomain) where each input has exactly one output, and the output can be traced back to its input.

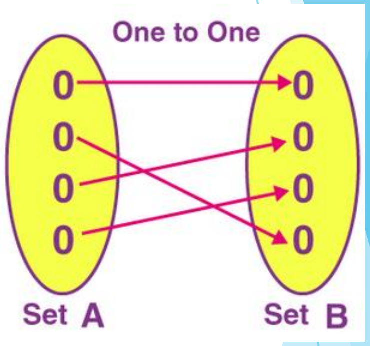
An example of a simple function is $f(x) = x^2$. In this function, the function $f(x)$ takes the value of "x" and then squares it. For instance, if $x = 3$, then $f(3) = 9$. A few more examples of functions are: $f(x) = \sin x$, $f(x) = x^2 + 3$, $f(x) = 1/x$, $f(x) = 2x + 3$, etc.

Injective Function

One-to-One function/Injective function define that each element of one set, say Set (A) is mapped with a unique element of another set, say, Set (B).

(or)

An injective function (injection) or one-to-one function is a function that maps distinct elements of its domain to distinct elements of its codomain.



Formally, it is stated as, if $f(x) = f(y)$ implies $x = y$, then f is one-to-one mapped, or f is 1-1.

And equivalently, if $x \neq y$, then $f(x) \neq f(y)$.

Examples:

1. The function $g(x) = x - 4$ is a one to one function since it produces a different answer for every input. Also, the function $g(x) = x^2$ is NOT a one to one function since it produces 4 as the answer when the inputs are 2 and -2.

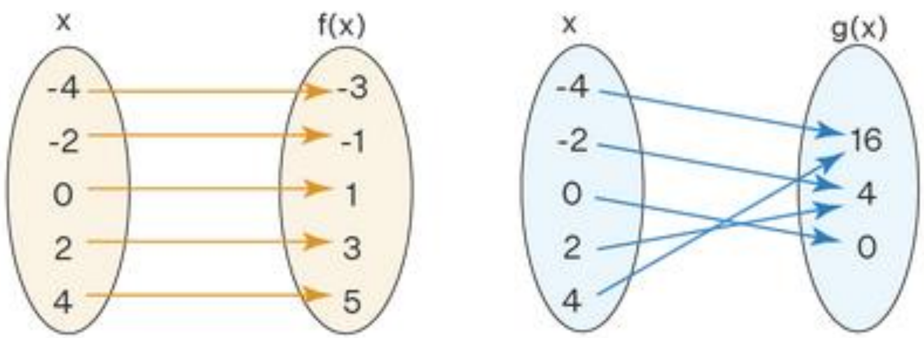


Fig (a) Fig (b)

- In Fig(a), for each x value, there is only one unique value of *f(x)* and thus, *f(x)* is one to one function.
- In Fig (b), different values of x, 2, and -2 are mapped with a common g(x) value 4 and (also, the different x values -4 and 4 are mapped to a common value 16). Thus, *g(x)* is a function that is not a one to one function.

Applications:

1. One person has one passport, and the passport can only be used by one person.
2. One person has one ID number, and the ID number is unique to one person.
3. A company creates only one product, and that product is only made by that company.
4. Each employee has a specific phone extension, which can only reach one employee.
5. Humans have unique fingerprints, which only belong to that human.
6. Each name corresponds to only one phone number, and each phone number links only to the correct name.

Surjective Function

Consider two sets, Set A and Set B, which consist of elements. If for every element of B, there is at least one or more than one element matching with A, then the function is said to be **onto function or surjective function**.

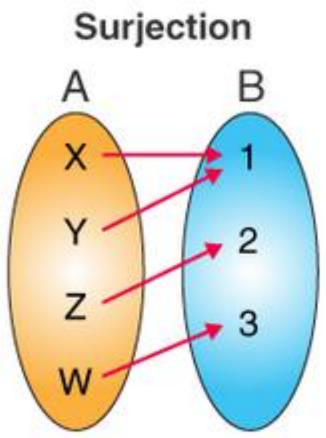


Fig.1

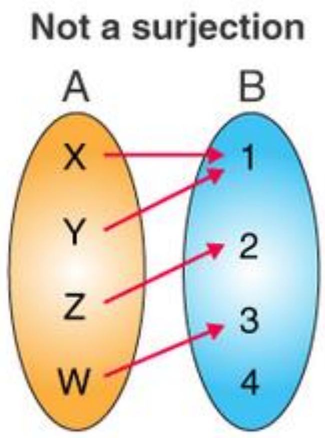


Fig.2

In the first figure, you can see that for each element of B, there is a pre-image or a matching element in Set A. Therefore, it is an onto function. But if you see in the second figure, one element in Set B is not mapped with any element of set A, so it's not an onto or surjective function.

Formally, a function $f{:}A \rightarrow B$ is onto if, for every element $b \in B$, there exists an element $a \in A$ such that $f(a)=b$.

Examples

1. Let A = {1, 5, 8, 9) and B {2, 4} And *f*={(1, 2), (5, 4), (8, 2), (9, 4)}. Then prove *f* is a onto function.

Solution:

From the question itself we get,

A={1, 5, 8, 9)

B{2, 4}

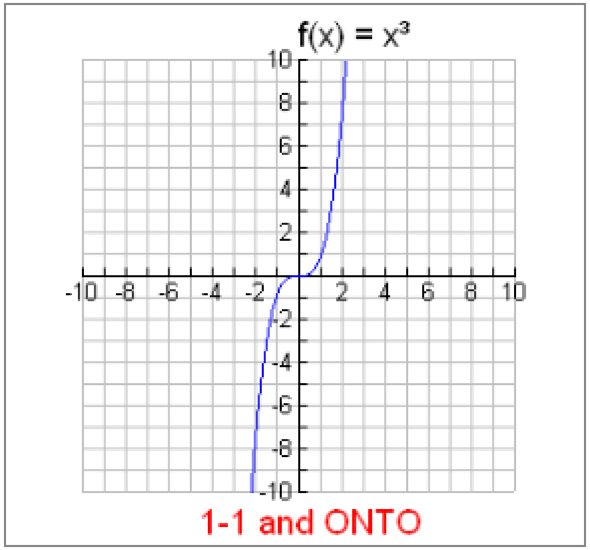
& f={(1, 2), (5, 4), (8, 2), (9, 4)}

So, all the element on B has a domain element on A or we can say element 1 and 8 & 5 and 9 has same range 2 & 4 respectively.

Therefore, f: A → B is a surjective function.

2. Is f (x) = x^3 one-to-one where f : R→R ?

Solution:



This function is One-to-One.

This cubic function possesses the property that each x-value has one unique y-value that is not used by any other x-element. This characteristic is referred to as being 1-1.

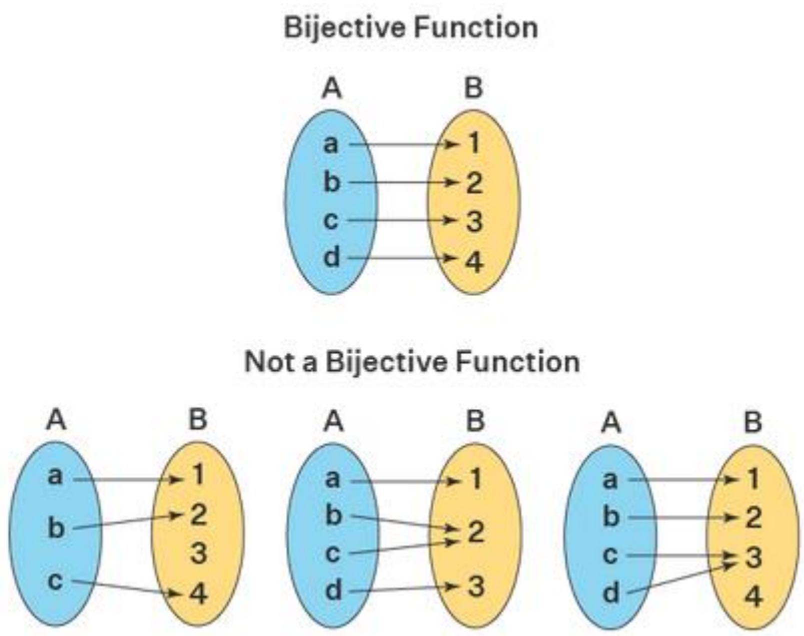
Also, in this function, as you progress along the graph, every possible y-value is used, making the function onto.

Applications

1. Let Set A be employees in an organization and Set B be the bank accounts of different banks. If we see that for each element of B, there is a pre-image or a matching element in Set A. Therefore, it is an onto function.

Bijective Function

A function is said to be bijective or bijection, if a function f: A → B satisfies both the injective (one-to-one function) and surjective function (onto function) properties.



From the above examples of bijective function, we can observe that every element of set B has been related to a distinct element of set A. The non-bijective functions have some element in set B which do not have a pre-image in set A, or some of the elements in set B is the image for more than one element in set A.

Injective Surjective and Bijective Functions Example-1

Explain why the real-valued exponential function $f(x) = x^2$ is neither injective nor surjective, from $f : \mathbb{R} \to \mathbb{R}$.

Solution

Notice from the graph of $f(x) = x^2$, that the range is only non-negative numbers. This means that negative numbers cannot be reached. The function $f(x) = x^2$ is not surjective, from $f : \mathbb{R} \to \mathbb{R}$. In particular there is no real, x, for which, for example, $f(x) = x^2 = -1$.

Also notice that $f(2) = 4$, and $f(-2) = 4$, but $2 \neq -2$, which means the function is not injective. A horizontal line at $y = 4$, meets the graph at both $x = 2$, and $x = -2$.

Example-2

The function from set {1,2,3,4} to set {10,11,12,13} defined by the formula $f(x) = x + 9$ is a bijection.

The Ceiling and Floor Function

The ceiling, $f(x) = \lceil x \rceil$, function rounds up x to the nearest integer.

The **ceiling function** , used to compute the ceiling of x, denoted, $f(x) = \lceil x \rceil$ gives the smallest integer *greater than or equal to* x.

For example, $\lceil 3.4 \rceil = 4$ and $\lceil 3.7 \rceil = 4$.

The floor $f(x) = \lfloor x \rfloor$, rounds down x to the nearest integer.

The **floor function** , used to compute the floor of x, denoted $f(x) = \lfloor x \rfloor$, gives the greatest integer *less than or equal to* x.

For example, $\lfloor 3.4 \rfloor = 3$ and $\lfloor 3.7 \rfloor = 3$.

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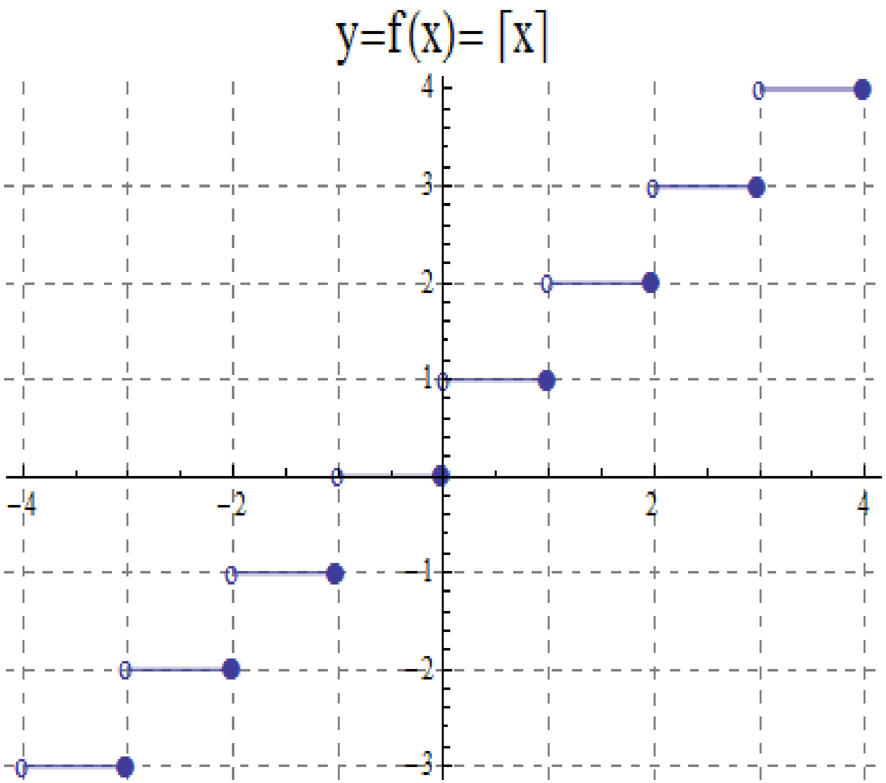
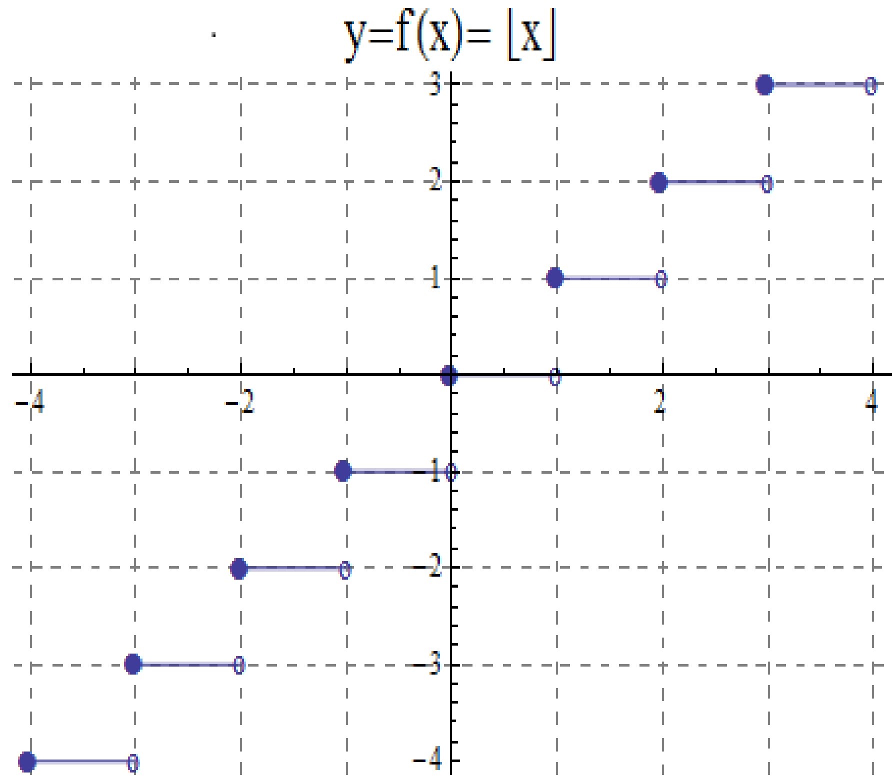
The Ceiling and Floor Function





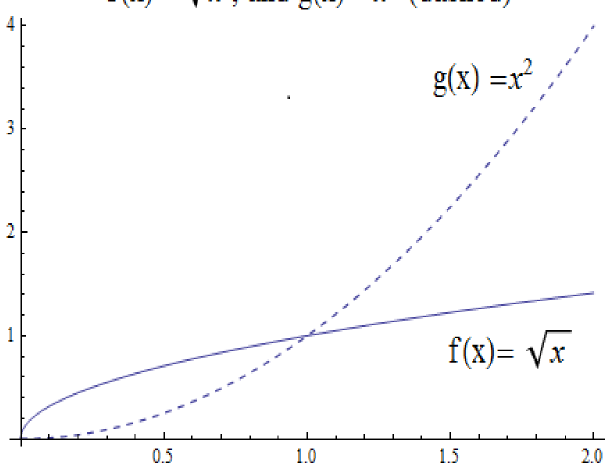
The Max Function

The function $h(x) = \max(f(x)), \; g(x))$ is evaluated at each x for which both $f(x)$ and $g(x)$ are defined by the function

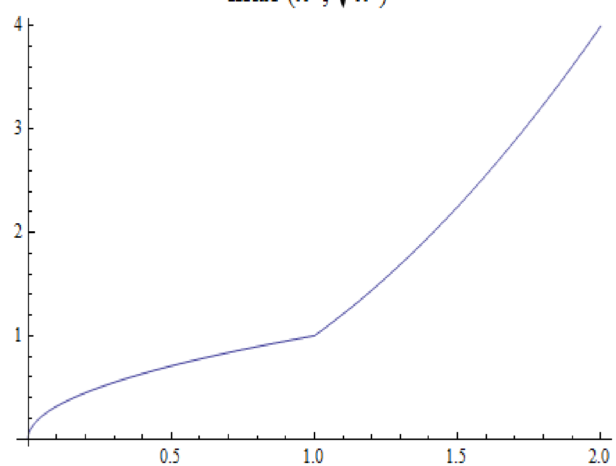
$$h(x) = \max(f(x), g(x)) = \begin{cases} f(x) & \text{if } f(x) \geq g(x) \\ g(x) & \text{if } f(x) < g(x) \end{cases}$$

So for example if $f(x) = \sqrt{x}$, and $g(x) = x^2$ then $h(x) = \max(f(x), g(x))$, has $h(1/4) = \max\left(\sqrt{\frac{1}{4}}, \; \left(\frac{1}{4}\right)^2\right) = max\left(\frac{1}{2}, \frac{1}{16}\right) = \frac{1}{2}$, and $h(4) = \max$ $(\sqrt{4}, \; 4^2) = \max(2, 16) = 16$. The graph of $h(x) = \max(\sqrt{x}, \; x^2)$ over the interval $(0, 2)$ is shown below.

$f(x)= \sqrt{x}$, and $g(x) = x^2$ (dashed)



$\max(x^2, \sqrt{x})$



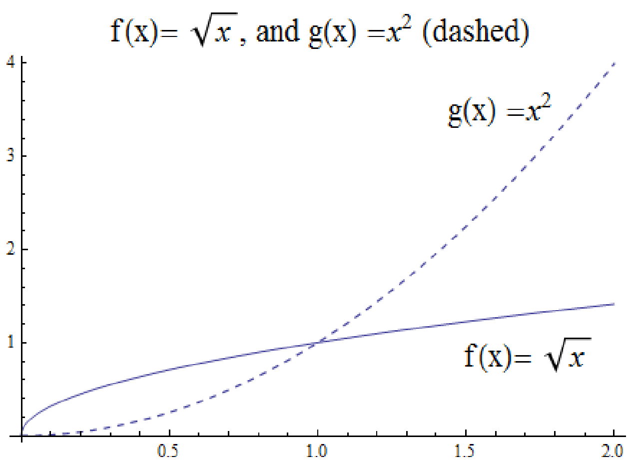
The Min Function

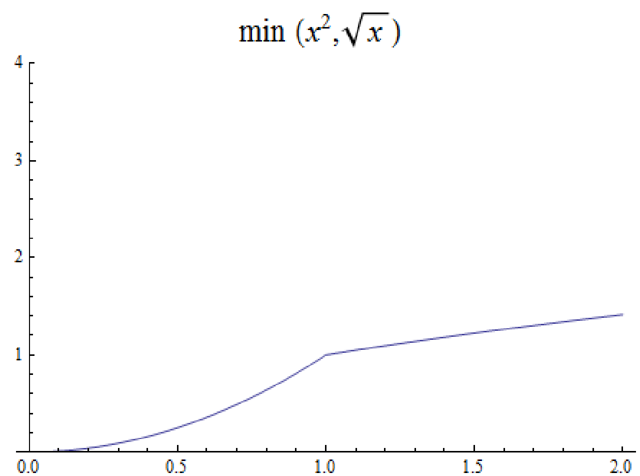
The function $h(x) = \min(f(x), g(x))$ is evaluated at each x for which both $f(x)$ and $g(x)$ are defined and is similar to the max function, but is defined by the minimum of $f(x)$, and $g(x)$ at each x.

$$h(x) = \min(f(x), g(x)) = \begin{cases} f(x) & \text{if } f(x) \leq g(x) \\ g(x) & \text{if } f(x) > g(x) \end{cases}$$

So for example if $f(x) = \sqrt{x}$, and $g(x) = x^2$ then $h(x) = \min(f(x), g(x))$, has $h(1/4) = \min\left(\sqrt{\frac{1}{4}},\ \left(\frac{1}{4}\right)^2\right) = \min\left(\frac{1}{2}, \frac{1}{16}\right) = \frac{1}{16}$, and $h(4) = \min\left(\sqrt{4},\ 4^2\right) = \min(2, 16) = 2$.

The graph of $h(x) = min(\sqrt{x},\ x^2)$ over the interval $[0, 2]$, is shown below





The Algebra of Functions

If two functions $f(x)$ and $g(x)$ have the same domain A, then we can combine these functions using the common algebraic operations of addition, subtraction, multiplication, and division.

a. $(f+g)(x) = f(x) + g(x)$

b. $(f-g)(x) = f(x) - g(x)$

c. $(f \cdot g)(x) = f(x) \cdot g(x)$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

The Algebra of Functions Example-1

Consider $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$ defined on $f,\ g : R \to R$.Form $(f+g)$, $(f-g)$, $(f \cdot\ g)$, and $\left(\frac{f}{g}\right)$, and determine their respective domains.

Solution

The common domain is $x \geq 0$, since the square root is real valued only for $x \geq 0$.

$(f+g)(x) = f(x) + g(x) = x^2 + 1 + \sqrt{x}$, for $x \geq 0$

$(f-g)(x) = f(x) - g(x) = x^2 + 1 - \sqrt{x}$, for $x \geq 0$

$(f \cdot\ g)(x) = f(x) \cdot\ g(x) = (x^2+1) \cdot\ \sqrt{x}$, for $x \geq 0$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+1\cdot}{\sqrt{x}}$, for $x > 0$.

Notice that the domain of $\frac{f}{g}$ is $x > 0$, because $g(0) = \sqrt{0} = 0$, and division by 0 is not defined.

Composition of Functions

Suppose $g : A \to B$ and $f : B \to C$, then the functions f and g, can be **composed** to obtain a function $h : A \to C$, denoted as follows,

$h(x) = (f \circ g)(x) = f(g(x))$ provided $x \in A$ and $g(x) \in B$.

Example-1

Consider $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$ defined on on $f, g : R \to R$. Show that $(g \circ f)(1) = 1$, $(g \circ f)(2) = 2$, $(g \circ f)(3) = 3$, and $(g \circ f)(x) = x$

Composition of Functions

Solution

$$f(1) = 1^3 + 1 = 2$$

$$f(2) = 2^3 + 1 = 9$$

$$f(3) = 3^3 + 1 = 28$$

$$f(x) = x^3 + 1$$

Therefore,

$$(g \circ f)(1) = g(f(1)) = g(2) = \sqrt[3]{2-1} = \sqrt[3]{1} = 1$$

$$(g \circ f)(2) = g(f(2)) = g(9) = \sqrt[3]{9-1} = \sqrt[3]{8} = 2$$

$$(g \circ f)(3) = g(f(3)) = g(28) = \sqrt[3]{28-1} = \sqrt[3]{27} = 3$$

$$(g \circ f)(x) = g(f(x)) = g(x^3 + 1) = \sqrt[3]{x^3 + 1 - 1} = \sqrt[3]{x^3} = x$$

Relations

Introduction

Relationships between elements of sets occur in many contexts. Every day we deal with relationships such as those between a business and its telephone number, an employee and his or her salary, a person and a relative, and so on. Relationships such as that between a program and a variable it uses, and that between a computer language and a valid statement in this language often arise in computer science

Relationships between elements of sets are represented using the structure called a relation,

which is just a subset of the Cartesian product of the sets. Relations can be used to solve problems such as determining which pairs of cities are linked by airline flights in a network, finding a viable order for the different phases of a complicated project, or producing a useful way to store information in computer databases.

Cartesian Product

Let A and B be any two non-empty sets. The Cartesian product (or) cross product of A and B denoted by $A \times B$ is set of all ordered pairs (a, b) where a belongs to A and b belongs to B.

$$A \times B = \{(x, y): x \in A, y \in B\}.$$

Note:

- If A and B are finite then the Cardinality of $|A \times B| = |A| \times |B|$.
- $|A| = m$, $|B| = n$ then total number of relations from A to B formed is given by 2^{mn}.

Example 1: Let $A = \{1,2,3\}$ and B= $\{p, q, r\}$, then

$$A \times B = \{(1, p), (1, q), (1, r), (2, p), (2, q), (2, r), (3, p), (3, q), (3, r)\}$$

and $|A \times B| = |A| \times |B| = 3 \times 3 = 9$.

Note:

Although we generally will not have $A \times B = B \times A$, but we will have $|A \times B| = |B \times A|$.

Example 2: Let $A = \{2,3,4\}$ and $B = \{4,5\}$. Find

i. $A \times B$

ii. $B \times A$

iii. $|A \times B|$

iv. $|B \times A|$

Solution:

i. $A \times B = \{(2,4), (2,5), (3,4), (3,5), (4,4), (4,5)\}$

ii. $B \times A = \{(4,2), (4,3), (4,4), (5,2), (5,3), (5,4)\}$

iii. $|A \times B| = 6$

iv. $|B \times A| = 6$.

Problem:

1. If $A = \{1, 2, 3, 4\}$, $B = \{2, 5\}$, and $C = \{3, 4, 7\}$, determine $A \times B$; $B \times A$; $A \cup (B \times C)$; $(A \cup B) \times C$; $(A \times C) \cup (B \times C)$.

Relation: A relation R from a non-empty set A to a non-empty set B is a subset of a Cartesian product $A \times B,$ i.e., if $(a, b) \in R$, we say that a is related b and we write aRb.

If $R_1 \subseteq A \times B,$ then R_1 is called relation from A to B.

Note:

- When a relation is defined on A i.e., $(A\ to\ A)$ then the relation is called binary relation.

Let $A = \{1,2,3\}$ and B= $\{p, q, r\}$

- Let $R_1 = \{(1,1),(2,2),(3,3)\}$.

  Then R_1 is not a subset of A $\times B$. Therefore, R_1 is not a relation from A to B.

- Let $R_2 = \{(1,p),(1,q),(1,r)\} \subseteq A \times B$. Therefore, R_2 is a relation from A to B.

- Let $R_3 = \{(1,p),(1,q),(1,r),(2,p),(2,q),(2,r),(3,p),(3,q),(3,r)\}$,

  since every set is a subset of itself, therefore, $R_3 \subseteq A \times B$. Therefore, R_3 is also a relation from A to B.

Different types of relations:

1. Reflexive relation:

A relation R defined on set A is called reflexive relation if

$aRa, or\ (a, a) \in R,\ \forall\ a \in A.$

Note:

- A relation in which no element is related to itself is called Irreflexive relation.
- In the relation if at least one element is not related itself then it is called Non-reflexive.

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Example

Let $A = \{1,2,3,4\}$ then

- $R_1 = \{(1,1), (2,2), (3,3), (4,4)\} \subseteq A \times A$, therefore R_1 is a relation and is a reflexive relation.
- $R_2 = \{(1,1), (2,2), (3,2), (4,4)\} \rightarrow$ Non-reflexive

  Since (3,3) is not in R_2
- $R_3 = \{(1,2), (2,3), (3,4), (4,1)\} \rightarrow$ Irreflexive
- $R_4 = \{(1,1), (1,2), (2,2), (3,3), (4,4), (4,3)\} \rightarrow$Reflexive

(Since it contains every element which is connected to itself.)

Symmetric relation:

Let R be a relation defined on set A, then R is called symmetric relation if $aRb \Rightarrow bRa$, i.e., whenever $(a, b) \in R$, then $(b, a) \in R$ for every $(a, b) \in R$.

Example

Let $A = \{1,2,3,4\}$ then

- $R_1 = \{(1,2), (2,1), (3,2), (2,3), (4,4)\} \rightarrow$ Symmetric.
- $R_2 = \{(1,2), (3,2), (2,3), (4,4)\} \rightarrow$ Not symmetric (Asymmetric).
- $R_3 = \{(1,1), (2,2), (3,3), (4,4)\} \rightarrow$ Symmetric.
- $R_4 = \emptyset \rightarrow$ Symmetric.

Note:

A relation which is not symmetric is called asymmetric relation.

Transitive relation:

Let R is a relation defined on a set A. Then R is called transitive relation

$$If\ aRb\ and\ bRc \Rightarrow aRc\ .$$

Example

Let $A = \{1,2,3,4,5\}$ then

- $R_1 = \{(1,2),(2,3),(1,3),(3,4),(4,5),(3,5)\} \rightarrow$ Transitive.
- $R_2 = \{(1,2),(2,3)\} \rightarrow$ Not Transitive.

Equivalence relation:

A relation R is defined on a set A. then R is called an equivalence relation. If it is reflexive, symmetric, and transitive.

Example

Let $A = \{1,2,3,4\}$ then

- $R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$ → equivalence relation.
- $R_2 = \{(1,1), (1,3), (2,2), (3,3), (3,2)\}$ → Not equivalence relation .

Example

1. Let $A = \{2,4,6,8\}$ and $B = \{1,2,3\}$ and let relations R_1, R_2, R_3, R_4 from A to B be defined as follows :

i. aR_1b if $a \leq b$

ii. aR_2b if $a > b$

iii. aR_3b if $a\ divides\ b$

iv. aR_4b if $b\ divides\ a$

Solution: Given that $A = \{2,4,6,8\}$ and $B = \{1,2,3\}$

$\therefore A \times B = \{(2,1), (2,2), (2,3), (4,1), (4,2), (4,3), (6,1), (6,2), (6,3), (8,1), (8,2), (8,3)\}$

i) $R_1 = \{(2,2), (2,3)\}$

ii) $R_2 = \{(2,1), (4,1), (4,2), (4,3), (6,1), (6,2), (6,3), (8,1), (8,2), (8,3)\}$

iii)$R_3 = \{(2,2)\}$

iv)$R_4 = \{(2,1), (2,2), (4,1), (4,2), (6,1), (6,2), (6,3), (8,1), (8,2)\}$

2. Let A and B be finite sets with $|B| = 3$. If there are 4096 relations from A to B then what is $|A|$?

Solution

We know that if $|A| = m$, $|B| = n$ then total number of relations from A to B formed is given by 2^{mn}

It is given that $|B| = 3 = n$

Thus, we have

$$2^{3m} = 4096$$

$$\log 2^{3m} = \log 4096$$

$$3m \log 2 = \log 4096$$

$$m = \frac{\log 4096}{3 \log 2}$$

$$m = 4 = |A|$$

Or

$$2^{3m} = 4096 = 2^{12} = 2^{3\times 4}$$

Hence $m = 4$

3. Let $A = \{1,2,3,4\}$ and $B = \{1,2,3\}$, R_1, R_2, R_3 are relations on A defined as follows

i) aR_1b if $a \leq b$

ii) aR_2b if $a > b$

iii)aR_3b if $a\ is\ odd\ b\ and\ b\ is\ even$[Homework].

4. *Let* $A = \{1, 2, 3\}$ *and* $B = \{2, 4, 5\}$. *Determine the following.*

(1) $|A \times B|$.

(2) *Number of relations from A to B.*

(3) *Number of binary relations on A.*

Solution: We have $|A| = m = 3$, $|B| = n = 3$. Therefore:

(1) $|A \times B| = mn = 9$.

(2) No. of relations from A to B is $2^{mn} = 2^9 = 512$.

(3) No. of binary relations on A is $2^{mm} = 2^{m^2} = 2^9 = 512$.

5. Show that the following relation R defined on the set of all integers Z is an equivalence relation. $\boldsymbol{R = \{(x, y)\colon\ x, y \in Z \text{ and } (x - y) \text{is an even number}\}}$ **.**

Solution:

It is given that xRy iff $x - y[= 2m]$ is an even number

We shall show that R is reflexive, symmetric, transitive.

Reflexive relation:

Let us consider xRx i.e., $x - x = 0 = 2(0)$ is an even number. Hence R is Reflexive

Symmetric Relation:

Let $xRy \Rightarrow x - y = 2m$ is even integer $m \in Z$

$\therefore\ yRx \Rightarrow y - x = -(x - y) = -2m = 2(-m)$ is also an even integer

That is $xRy \Rightarrow yRx$. Hence R is Symmetric

Transitive Relation:

Let $xRy \Rightarrow (x - y)$ is even. That is $x - y = 2m$(say), $m \in Z$.

$yRz = (y - z)$ is even. That is $y - z = 2n$(say), $n \in Z$.

Now $(x - y) + (y - z) = 2m + 2n$

$(x - z) = 2(m + n) = 2k$(say) $k = m + n \in Z$ is also even $\Rightarrow xRz$

That is $xRy, yRz \Rightarrow xRz$. Hence R is transitive.

Thus, we conclude that the relation R is equivalence relation.

6. Show that the following relation R defined on the set of all integers Z is an equivalence relation. $\boldsymbol{R = \{(x, y) : x, y \in Z \text{ and } (x - y) \text{ is a multiple of } 5\}}$ ***[homework].***

7. Show that the relation congruence modulo m, $\boldsymbol{a \equiv b (mod\ m)}$ ***on the set of all positive integers Z is an equivalence relation.***

Solution

By the definition of congruence modulo m $a \equiv b(mod\ m) \Rightarrow m\ divides\ (a-b) \Rightarrow (a-b) = km$ where m is fixed and $a, b, k \in Z$

We shall show that relation $R = \{(a, b) : (a-b) = km\}$ is an equivalence relation

Reflexive relation:

$aRa\ is\ a - a = 0$ and m divides 0. Hence R is reflexive.

Symmetric Relation:

$$aRb \Rightarrow (a-b) = km$$
$$\therefore bRa = (b-a) = -(a-b) = -km \Rightarrow m\ divides\ (b-a)$$

That is $aRb \Rightarrow bRa$. Hence R is Symmetric

Transitive Relation:

$aRb \Rightarrow (a-b) = km$ and $bRc = (b-c) = lm$ where $k, l \in Z$.

Now $a - c = (a-b) + (b-c) = km + lm = (k+l)m, \quad (k+l) \in Z$

$\therefore a - c = (k+l)m \Rightarrow m\ divides\ (a-c)\ or\ \ a \equiv c(mod\ m)$

That is $aRb, bRc \Rightarrow aRc$. Hence R is transitive.

Thus, we conclude that the relation R is equivalence relation.

Mathematical Induction:

Mathematical induction is a method of proof that has been useful in every area of mathematics. It is used to prove statements that assert something is true for every integer value of the natural numbers , you need mathematical induction.

Procedure:

We follow the following steps to establish that a given $S(n)$: statement/ expression/ condition true for all integers $n \geq 1$.

Basic Step:

Step 1: We verify that $S(n_0)$ is true for $n = n_0$.

In particular, we verify that $S(1)$ is true.

Induction Step:

Step 2: We assume that $S(n)$ is true for $n = k$, where k is integer $\geq n_0$[we replace n by k].

Step 3: Next we show that $S(n)$ is true for $n = k + 1$, i.e., we prove that $S(k + 1)$ is true.

Problems:

1. Prove by mathematical induction that

$$\mathbf{1+2+3+\cdots+n=\frac{n(n+1)}{2}}$$

for all n ≥ 1.

Proof:

For each positive integer $n \geq 1$, let the statement

$$S(n):\ \ 1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

Step 1. $S(1)$: $1=\frac{1(1+1)}{2}$. Thus $S(1)$ is true.

Step 2. (Inductive step): Suppose that

$$S(k):\ \ 1+2+3+\cdots+k=\frac{k(k+1)}{2}$$

is true for all positive integers $k \geq 1$.

Step 3. We want to prove that

$$S(k+1):\ \ 1+2+3+\cdots+k+(k+1)=\frac{(k+1)((k+1)+1)}{2}$$

We have

$$1+2+3+\cdots+k=\frac{k(k+1)}{2}$$

Add $(k+1)$ on both side

Then

$$\begin{aligned}1+2+3+\cdots+k+(k+1)&=\frac{k(k+1)}{2}+(k+1)\\&=\frac{k(k+1)+2(k+1)}{2}\\&=\frac{(k+1)(k+2)}{2}\\&=\frac{(k+1)((k+1)+1)}{2}\end{aligned}$$

Hence, $S(k+1)$: $1+2+3+\cdots+k+(k+1)=\frac{(k+1)((k+1)+1)}{2}$ is true and by the principle of mathematical induction, the statement

$S(n)$: $1+2+3+\cdots+n=\frac{n(n+1)}{2}$.

2. Use mathematical induction to prove that

$$\sum_{\mathbf{i=1}}^{\mathbf{n}} (\mathbf{2i-1}) = \mathbf{1+3+5+\cdots+(2n-1) = n^2}$$

for all integers n ≥ 1.

Proof:

For each positive integer $n \geq 1$, let the statement

$$S(n):\ \ 1+3+5+\cdots+(2n-1) = n^2$$

Step 1. $S(1)$: $1 = 1^2$. Thus $S(1)$ is true.

Step 2. (Inductive step): Suppose that

$$S(k):\ \ 1+3+5+\cdots+(2k-1) = k^2$$

is true for all positive integers $k \geq 1$.

Step 3. We want to prove that

$$S(k+1):\ \ 1+3+5+\cdots+(2k-1)+(2(k+1)-1) = (k+1)^2$$

We have

$$S(k): \; 1 + 3 + 5 + \cdots + (2k - 1) = k^2$$

Add $2(k+1) - 1$ on both side

Then

$$\begin{aligned} 1 + 3 + 5 + \cdots + (2k - 1) + (2(k+1) - 1) &= k^2 + (2(k+1) - 1) \\ &= k^2 + 2k + 2 - 1 \\ &= k^2 + 2k + 1 \\ &= (k+1)^2 \end{aligned}$$

Hence, $S(k+1)$: $1 + 3 + 5 + \cdots + (2k - 1) + (2(k+1) - 1) = (k+1)^2$ is true, and by the principle of mathematical induction, the statement

$$S(n): \; 1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

is true for all positive integers $n \geq 1$.

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3. Use mathematical induction to prove that

$$\sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{i^2} = \mathbf{1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}}$$

for all integers n ≥ 1.

Proof:

For each positive integer $n \geq 1$, let the statement

$$S(n):\ \ 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Step 1. $S(1)$: $1^2 = \frac{1(2)(3)}{6} = 1$. Thus $S(1)$ is true.

Step 2. (Inductive step): Suppose that

$$S(k):\ \ 1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

is true for all positive integers $k \geq 1$.

Step 3. We want to prove that

$$S(k+1):\ \ 1^2+2^2+3^2+\cdots+k^2+(k+1)^2$$

$$=\frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

We have

$$S(k):\ \ 1^2+2^2+3^2+\cdots+k^2=\frac{k(k+1)(2k+1)}{6}$$

Add $(k+1)^2$ on both side

Then

$$\begin{aligned}1^2+2^2+3^2+\cdots+k^2+(k+1)^2&=\frac{k(k+1)(2k+1)}{6}+(k+1)^2\\&=\frac{k(k+1)(2k+1)+6(k+1)^2}{6}\\&=\frac{(k+1)[k(2k+1)+6(k+1)]}{6}\\&=\frac{(k+1)(2k^2+k+6k+6)}{6}\\&=\frac{(k+1)(2k^2+7k+6)}{6}\\&=\frac{(k+1)(k+2)(2k+3)}{6}\\&=\frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}\end{aligned}$$

Hence,

$$S(k+1):\ \ 1^2+2^2+3^2+\cdots+k^2+(k+1)^2$$

$$=\frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

is true and by the principle of mathematical induction, the statement

$$S(n):\ \ 1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$

is true for all positive integers $n \geq 1$.

4. Use mathematical induction to prove

$$\sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{i^3} = \mathbf{1^3+2^3+3^3+\cdots+n^3} = \frac{\mathbf{n^2(n+1)^2}}{\mathbf{4}}$$

for all integers $\mathbf{n \geq 1}$.[homework]

5. Use mathematical induction to prove that $\mathbf{2^{2n} - 1}$ is divisible by 3 , that is $\mathbf{3|(2^{2n} - 1)}$ for all integers $\mathbf{n \geq 1}$.

Proof:

For each positive integer $n \geq 1$, let the statement

$$S(n):\ \ 3|(2^{2n} - 1)$$

Step 1.

$$S(1):\ \ 2^{2(1)} - 1 = 4 - 1 = 3$$

Thus $3|(2^{2(1)} - 1)$, and $S(1)$ is true.

Step 2. (Inductive step): Suppose that

$$S(k):\ \ 3|(2^{2k} - 1)$$

is true for all positive integers $k \geq 1$.

Step 3. We want to prove that

$$S(k+1):\ \ 3|(2^{2(k+1)} - 1)$$

But,

$$
\begin{aligned}
2^{2(k+1)} - 1 &= 2^{2k+2} - 1 \\
&= 2^{2k}.2^2 - 1 \\
&= 2^{2k}(3+1) - 1 \\
&= 2^{2k}.3 + (2^{2k} - 1)
\end{aligned}
$$

Now, $2^{2k}.3$ is divisible by 3, and $2^{2k} - 1$ is divisible by 3 by step 2. Hence, the sum of $2^{2k}.3$ and $2^{2k} - 1$ is divisible by 3. Therefore, $S(k+1)$: $3|(2^{2(k+1)} - 1)$ is true and $3|(2^{2n} - 1)$, and the statement

$$S(n):\ \ 3|(2^{2n} - 1)$$

is true for all integers $n \geq 1$.

6. Use mathematical induction to prove that $2n + 1 < 2^n$ for all integers $n \geq 3$.

Solution. For each positive integer $n \geq 3$, let the statement

$$S(n):\ \ 2n + 1 < 2^n$$

Step 1. $S(3)$: $2(3) + 1 = 6 + 1 = 7 < 2^3 = 8$. Thus $S(3)$ is true.

Step 2. (Inductive step): Suppose that

$$S(k):\ 2k + 1 < 2^k$$

is true for all positive integers $k \geq 3$.

Step 3. We want to prove that

$$S(k+1):\ 2(k+1) + 1 < 2^{k+1}$$

$$2(k+1)+1 = 2k+2+1$$
$$= 2k+1+2$$

since $S(k)$ is true

$2k+1 < 2^k$

Add 2 on both side

$2k+1+2 < 2^k+2$

$2(k+1)+1 < 2^k+2 < 2^k+2^k,$

$2(k+1)+1 < 2^k+2^k$, since $k \geq 3, \qquad 2 < 2^k$

$2(k+1)+1 < 2^k(1+1)$

$2(k+1)+1 < 2^k.2^1$

$2(k+1)+1 < \ 2^{k+1}$

Thus, the statement

$$S(k+1):\ \ 2(k+1)+1 < 2^{k+1}$$

is true. Hence

$$S(n):\ \ 2n+1 < 2^n$$

is true for all $n \geq 3$.

Strong Induction:

Recall the idea behind mathematical induction, prove that a proposition P(m) is true for a base step, say P_1. Then prove the inductive step. This involves proving that P_m implies P_{m+1}. Strong induction is a generalized version of induction where in the inductive step we assume $P_1,P_2,\ldots,P_m$ is true and show all these together imply P_{m+1} is true.

Strong version of Mathematical Induction:

Base Case: Prove the first statement P_1. (Or the first several P_r, if needed.)

Inductive Step: Given any integer $m \geq 1$, prove $P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_m$, implies P_{m+1}

Algorithmically:

Base Case: Prove that P_1, P_2 ,…, P_r, are all true.

Inductive Hypotheses: Assume, P_1, P_2 ,…, P_m are all true for some $m \geq 1$,

Inductive step: Assuming, P_1, P_2 ,…, P_m are all true for some $m \geq 1$, prove that P_{m+1} is true.

Problems:

1. **Consider the sequence $\{a_n\}$ defined as a recurrence relation, $a_1=5$, $a_2=13$, and $a_{n+2}=5a_{n+1}-6a_n$, for all $n\geq1$. Prove that $a_n=2^n+3^n$ for all $n\geq1$.**

Solution:

Base case: Consider the case n=1, $a_1=5=2^1+3^1$, and $a_2=13=2^2+3^2=4+9$.

Induction hypothesis: Assume $a_1, a_2, .. a_m$ are all given by $a_m=2^m+3^m$.

We want to show that $a_{m+1}=2^{m+1}+3^{m+1}$.

Well, $a_{m+1}=5a_m-6a_{m-1}$, using the definition of the recurrence relation.

By the inductive hypothesis, $a_m=2^m+3^m$ and $a_{m-1}=2^{m-1}+3^{m-1}$.

$a_{m+1}=5(2^m+3^m)-6(2^{m-1}+3^{m-1})$.

Consider then,

$$\begin{aligned}5(2^m+3^m)-6(2^{m-1}+3^{m-1}) &=5\times2^m+5\times3^m-6\times2^{m-1}-6\times3^{m-1}\\ &=5\times2^m+5\times3^m-3\times2\times2^{m-1}-2\times3\times3^{m-1}\\ &=5\times2^m+5\times3^m-3\times2^m-2\times3^m\\ &=(5\times2^m-3\times2^m)+(5\times3^m-2\times3^m)\\ &=(5-3)\times2^m+(5-2)\times3^m\\ &=2\times 2^m+3\times3^m\\ &=2^{m+1}+3^{m+1}\end{aligned}$$

So $a_{m+1}=2^{m+1}+3^{m+1}$ and therefore, $a_n=2^n+3^n$ for all $n\geq1$.

2. Consider the sequence $\{a_n\}$ defined as a recurrence relation, $a_1=1$, $a_2=8$, and $a_n=a_{n-1}+2a_{n-2}$, for all $n\geq3$. Prove that $a_n=3.2^{n-1}+2(-1)^n$ for all $n\geq1$. [Home work]

Python Programming

Python is a popular programming language. It was created by Guido van Rossum, and released in 1991. It is used for web development, software development, Mathematics, etc. Python can be used to handle big data and perform complex mathematics. Python works on different platforms (Windows, Mac, Linux, Raspberry Pi, etc). Python has syntax that allows developers to write programs with fewer lines than some other programming languages.

Starting with version 2.3, Python comes with an implementation of the mathematical set. A set is an unordered collection of objects, unlike sequence objects such as lists and tuples, in which each element is indexed. Sets cannot have duplicate members - a given object appears in a set 0 or 1 times.

Execute Python Syntax:

Python syntax can be executed by writing directly in the Command Line.

```
print("Hello, World!")
Hello, World!
```

Python Comments:

Comments can be used to explain Python code. Comments can be used to make the code more readable.

```
#This is a comment
print("Hello, World!")
Hello, World!
```

Python Variables:

Variables are containers for storing data values. Python has no command for declaring a variable.

A variable is created the moment you first assign a value to it.

```
x = 5
y = "John"
print(x)
print(y)
5
John
```

Topics :

1. Constructing sets
2. Set size and Membership testing
3. Adding and Removing items
4. Set operations
   4.1 Union
   4.2 Intersection
   4.3 Set Difference
   4.4 Disjoint sets
   4.5 Subset
5. Multiple sets
6. Frozenset

Sets in Python:

A set is created by placing all the elements inside curly braces **{}**, separated by comma or by using built-in function **set()**. A set is a collection which is unordered, unchangeable, and unindexed. The elements can be different types (integer, float, tuple, string, etc.).

1. Creating a set:

i. A set can be defined in two ways. One way to construct sets is to define a set with the built in set() function

```
s=set([1,2,3,4,5,6])
print(s)
{1,2,3,4,5,6}

s=set("hollow")
print(s)
{'o','l','w','h'}
```

Note: The resulting sets are unordered: the original order, as specified in the definition, is not necessarily preserved. Additionally, duplicate values are only represented in the set once.

ii. Alternately, a set can be defined with curly braces.

```
s={1,2,1,2,1,4,2,1,3,5,1,2,3,4}
print(s)
{1,2,3,4,5}
```

Note:

The difference between the two set definitions is:

```
s={'follow'}
print(s)
{'follow'}

s=set('follow')
print(s)
{'f','w','l','o'}
```

Empty set:

The only way to define empty set is set()

2. Set size and Membership test:

The len() function returns the number of elements in a set, and the in and not in operators can be used to test for membership.

Length of a Set:

```
S={1,2,3,4,5}
a=len(S)
print(a)
5
```

Membership:

```
S={1,2,3, 'India', 'Bharat'}
print('India' in S)
True
S={1,2,3, 'India', 'Bharat'}
print('India' not in S)
False
S={1,2,3, 'India', 'Bharat'}
print(5 in S)
False
S={1,2,3, 'India', 'Bharat'}
print(5 not in S)
True
```

3. Adding and removing elements to set:

We cannot access or change an element of set using indexing. We can add single element using the **add()** method and multiple elements using the **update()** method. In all cases duplicates are avoided.

```
S={1,2,3}
S.add(4)
print(S)
{1, 2, 3, 4}
```

```
S={1,2,3}
S.update((4,5,6))
print(S)
{1, 2, 3, 4,5,6}
```

Removing elements from a set:

A particular item can be removed from set using **discard()** and **remove()**. Using discard() if the item does not exist in the set, it remains unchanged but remove() will raise an error in such condition. We also have another operation for removing elements from a set, **clear()** which simply removes all elements from the set.

```
S={1,2,3, 'India', 'Bharat'}
S.discard(3)
print(S)
{1, 2, 'India', 'Bharat'}

S={1,2,3, 'India', 'Bharat'}
S.discard('India')
print(S)
{1, 2, 3, 'Bharat'}
```

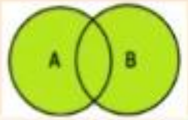
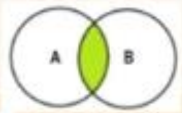
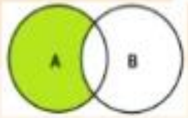
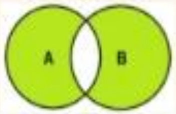
```
S={1,2,3, 'India', 'Bharat'}
S.remove(3)
print(S)
{1, 2, 'India', 'Bharat'}
```

```
S={1,2,3, 'India', 'Bharat'}
S.remove(5)
print(S)
KeyError                                  Traceback (most recent call last)
<ipython-input-51-de0e2e46e0e7> in <module>
      1 S={1,2,3, 'India', 'Bharat'}
----> 2 S.remove(5)
      3 print(S)
KeyError: 5
```

```
S={1,2,3, 'India', 'Bharat'}
S.discard(5)
print(S)
{1, 2, 3, 'India', 'Bharat'}
S={1,2,3, 'India', 'Bharat'}
S.clear()
print(S)
set()
```

4. Set operations:

Python allows us to perform all the standard mathematical set operations, using members of set. Set operations in Python can be performed in two different ways: by **operator** or by **method.**

Method	Operator	
union	|	
intersection	&	
difference	-	
symmetric_difference	^	

4.1 Union

```
S={1,2,3,4,5}
T={5,6,7,8}
ans=S|T (or) ans=S.union(T)
print(ans)
{1, 2, 3, 4, 5, 6, 7, 8}
```

4.2 Intersection

```
S={1,2,3,4,5}
T={5,6,7,8}
ans=S&T (or) ans=S.intersection(T)
print(ans)
{5}
```

4.3 Set Difference

```
S={1,2,3,4,5}
T={5,6,7,8}
ans=S-T (or) ans=S.difference(T)
print(ans)
{1, 2, 3, 4}
```

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4.4 Disjoint sets: If no single elements are common in two given sets then the sets. x1.isdisjoint(x2) returns True if x1 and x2 have no elements in common:

```
s1=set([1,2,3,4,5,6])
s2=set([5,6,7,8,9,10])
s3=set([5,7,8,11])
s4=set([9,5,2,13,15])
print(s1.isdisjoint(s2))
```

False

```
s1=set([1,2,3,4,5,6])
s2=set(['Amber','Rishi','Stella'])
print(s1.isdisjoint(s2))
```

True

```
s1=set([1,2,3,4,5,6])
s2=set([5,6,7,8,9,10])
s3=set([5,7,8,11])
s4=set([9,5,2,13,15])
print(s1.isdisjoint(s2,s3,s4))
```

Can only take one argument.

```
Traceback (most recent call last):
  File "C:/Users/HP/PycharmProjects/pythonProject/2.py", line 5, in <module>
    print(s1.isdisjoint(s2,s3,s4))
TypeError: isdisjoint() takes exactly one argument (3 given)

Process finished with exit code 1
```

4.5 **Subset:** To determine whether one set is subset of other, we use issubset() or <=.

x1.issubset(x2) and x1 <= x2 return True if x1 is a subset of x2:

```
s1=set([1,2,3,4,5,6])
s2=set([5,6,7,8,9,10])
s3=set([1,2,3])
print(s1.issubset(s2))
print(s3.issubset(s1))
print(s3<=s1)
```

```
False
True
True
```

5. Multiple sets

Union of more than two sets:

```
s1=set([1,2,3,4,5,6])
s2=set([5,6,7,8,9,10])
s3=set([5,7,8,11])
s4=set([9,5,2,13,15])
a1=s1.union(s2)
print(a1)
a2=s1|s2
print(a2)
a3=s1|s2|s3|s4
print(a3)
a4=s1.union(s2,s3,s4)
print(a4)
```

```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15}
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15}
```

Intersection of more than two sets:

```
s1=set([1,2,3,4,5,6])
s2=set([5,6,7,8,9,10])
s3=set([5,7,8,11])
s4=set([9,5,2,13,15])
a1=s1.intersection(s2)
print(a1)
a2=s1&s2
print(a2)
a3=s1&s2&s3&s4
print(a3)
a4=s1.intersection(s2,s3,s4)
print(a4)
```

```
{5, 6}
{5, 6}
{5}
{5}
```

Set difference of more than two sets:

```
a = {1, 2, 3, 30, 300}
b = {10, 20, 30, 40}
c = {100, 200, 300, 400}
print(a-b-c)
```

```
{1, 2, 3}
```

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6. Frozenset in Python:

Frozen set is just an immutable version of a Python set object. While elements of a set can be modified at any time, elements of the frozen set remain the same after creation.

```
# tuple of vowels
vowels = ('a', 'e', 'i', 'o', 'u')
fSet = frozenset(vowels)
print('The frozen set is:', fSet)
The frozen set is: frozenset({'o', 'i', 'u', 'e', 'a'})
```

```
#Adding an element in frozenset
s1=frozenset([1,2,3,4,5,6])
print(s1)
s1.add(9)
print(s1)
```

Methods that attempt to modify a frozenset fail:

Here add() ,won't work.

```
Traceback (most recent call last):
  File "C:/Users/HP/PycharmProjects/pythonProject/2.py", line 3, in <module>
    s1.add(9)
AttributeError: 'frozenset' object has no attribute 'add'
frozenset({1, 2, 3, 4, 5, 6})
```

frozenset() for Dictionary

When you use a dictionary as an iterable for a frozen set, it only takes keys of the dictionary to create the set.

```
person = {"name": "John", "age": 23, "sex": "male"}
fSet = frozenset(person)
print('The frozen set is:', fSet)
The frozen set is: frozenset({'sex', 'age', 'name’})
```

Frozenset operations

Like normal sets, frozenset can also perform different operations like copy, difference, intersection, symmetric_difference, and union.

```
# Frozensets
# initialize A and B
A = frozenset([1, 2, 3, 4])
B = frozenset([3, 4, 5, 6])
# copying a frozenset
C = A.copy() print(C)
# union
print(A.union(B))
```

```
# intersection
print(A.intersection(B))
# difference
print(A.difference(B))
# symmetric_difference
print(A.symmetric_difference(B))
frozenset({1, 2, 3, 4})
frozenset({1, 2, 3, 4, 5, 6})
frozenset({3, 4})
frozenset({1, 2})
frozenset({1, 2, 5, 6})
```

Similarly, other set methods like isdisjoint, issubset, and issuperset are also available.

```
# initialize A, B and C
A = frozenset([1, 2, 3, 4])
B = frozenset([3, 4, 5, 6])
C = frozenset([5, 6])
# isdisjoint() method
print(A.isdisjoint(C))  # Output: True
# issubset() method
print(C.issubset(B))  # Output: True
# issuperset() method
print(B.issuperset(C))  # Output: True
True
True
True
```

Exercise Questions:

1. Write a Python program to create the following sets:

   (i) A={1,2,3,4,5,6}, (ii) B={Hari, Prem, 102.55, 23, 12}

2. Write a code for add the following single element and multiple elements to the existing set A={2,3,4,5}:

   (i) 6, (ii) 7,8,9 and 10.

3. Write a code for removing the following elements from the set A={10, 15, 20, 25, 30}:

   (i) 25, (ii) 40, (iii) 10.

4. Check whether the following elements are member of the following set A={2,4,6,8,10}:

   (i) 3, (ii) 12, (iii) 6, (iv) Calculate the length of A.

5. Write a program to add all its elements of A and B into a single set C

   A={"Yellow", "Orange", "Black"} and B={"Blue", "Green", "Red“}.

6. Write a program to return a new set of identical items from two sets A and B

   A={10, 20, 30, 40, 50} and B={30, 40, 50, 60, 70}.

7. Write a Python program to update the first set with items that exist only in the first set A and not in the second set B, where A={10, 20, 30} and B={20, 40, 50}.

8. Write a Python program to remove items 10, 20, 30 from the following set A={10, 20, 30, 40, 50}.

9. Write a Python program to update set A by adding items from set B, except common items

   A={10, 20, 30, 40, 50} and B={30, 40, 50, 60, 70}.

10. Write a frozenset for any tuple and dictionary.