Module 3: Multivariate Calculus

Partial Derivative

A partial derivative is a function's derivative that has two or more other variables instead of one variable. Because the function is dependent on several variables, the derivative converts into the partial derivative.

Partial Derivative in Sympy

In Python, the Sympy module is used to calculate the partial derivative in a mathematical function. This module uses symbols to perform all different kinds of computations. It can also be used to solve equations, simplify expressions, compute derivatives and limits, and other computations.

Sympy needs to be manually installed before it can be used. Therefore, cd to your computer terminal and run the following command to install the sympy package.

```
pip install sympy
```

Example-1

Now, let's use the following example to derive the partial derivative of the function

f(a, b, c) = 5ab - acos(c)+ a^2 + c^8b

part_deriv(function = f, variable = a)

The expected output after differentiating the function to its partial derivative is

2*a + 5*b - cos(c)

To evaluate the partial derivative of the function above, we differentiate this function in respect to a while b and c will be the constants.

```
from sympy import symbols,cos,diff
a,b,c=symbols('a b c', real=True)
f = 5*a*b - a*cos(c) + a**2 + c**8*b
ans=diff(f,a)
print(ans)
```

```
2*a + 5*b - cos(c)
```

Example-2

Find all first and second order partial derivative of the following function by using python

$$f(x,y) = x^4 - 3x^2y^3$$

$$f_x = \frac{\partial f}{\partial x} = 4x^3 - 6xy^3$$

$$f_y = \frac{\partial f}{\partial y} = 0 - 9x^2y^2 = -9x^2y^2$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(4x^3 - 6xy^3\right) = 12x^2 - 6y^3$$

$$f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(4x^3 - 6xy^3\right) = -18xy^2$$

$$f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(-9x^2y^2\right) = -18xy^2$$

$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(-9x^2y^2\right) = -18x^2y$$

Code:

```
from sympy import symbols,cos,diff
x,y=symbols('x y', real=True)
f = x**4-3*x**2*y**3
f_x=diff(f,x)
f_y=diff(f,y)
f_xx=diff(f,x,x)
f_xy=diff(f,x,y)
f_yy=diff(f,y,y)
f_xxx=diff(f,x,x,x)
print("f_x=",f_x)
print("f_y=",f_y)
print("f_xx=",f_xx)
print("f_xy=",f_xy)
print("f_yy=",f_yy)
print("f_xxx=",f_xxx)
```

Output:

```
f_x= 4*x**3 - 6*x*y**3
f_y= -9*x**2*y**2
f_xx= 6*(2*x**2 - y**3)
f_xy= -18*x*y**2
f_yy= -18*x**2*y
f_xxx= 24*x2*a + 5*b - cos(c)
```

Exercise question

1. Compute the first order derivative for the following function by using python:

(i) $f(x,y)=x^5+3x^3y^2+3xy^4$

(ii) $f(x,y)=\dfrac{x-y}{x+y}$

(iii) $f(x,y)=\sin\left(\dfrac{x}{1+y}\right)$

(iv) $w=\ln(x+2y+3z)$

(vii) $w=\sqrt{r^2+s^2+t^2}$

(viii) $u=xe^{-t}\sin\theta$

(ix) $f(x,y,z,t)=xyz^2\tan(yt)$

(x) $f(x,t)=\arctan\left(x\sqrt{t}\right)$

2. Compute the all first and second order derivative for the following function by using python:

(i) $f(x,y)=x^4-3x^2y^3$

(ii) $z=\dfrac{x}{(x+y)}$

(iii) $u=e^{-s}\sin t$

(iv) $f(x,y)=\ln(3x+5y)$

(v) $z=y\tan 2x$

(vi) $v=\sqrt{x^2+y^2}$

Thank You