No need to show the parse tree, included for better understanding

CSE331: Automata and Computability
Quiz 03
Total Marks: 30

Obtained Marks

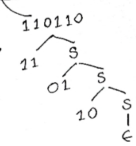
Name: **ID:**

Problem 1: CFG

Consider the following two languages.

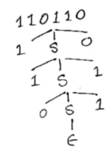
$L1 = \{w \in \{0, 1\}^*$: The length of w is even.$\}$
$L2 = \{w \in \{0, 1, \#\}^*: w = x\#1^n\#0^{2n+1}$ where $x \in L1$ and $n \geq 0\}$

a) Design a context-free grammar whose language is L1. **[Points 6]**

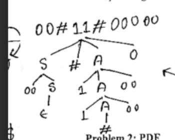


Solution 1: $S \to 00S \mid 01S \mid 10S \mid 11S \mid \epsilon$

Solution 2: ~~0n~~, $S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid \epsilon$



b) Design a context-free grammar whose language is L2. **[Points 4]**

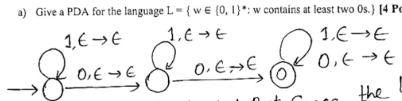


$W = X\#1^n\#0^{2n+1}$
$= X\#1^n\#0^{2n}0$

$S' \to S\#A0$
$S \to 00S \mid 01S \mid 10S \mid 11S \mid \epsilon$
$A \to 1A00 \mid \#$

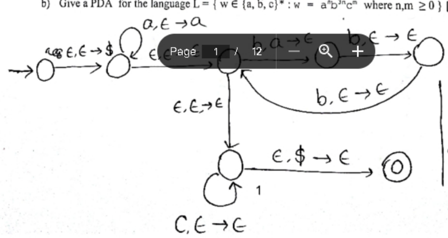
Problem 2: PDF

a) Give a PDA for the language $L = \{ w \in \{0, 1\}^*: w$ contains at least two 0s.$\}$ **[4 Points]**



you may push in the stack. But, since the language is regular, no need to use the stack.

b) Give a PDA for the language $L = \{ w \in \{a, b, c\}^* : w = a^nb^{3n}c^m$ where $n,m \geq 0 \}$ **[6 Points]**



Solution 2: You may also push three 'a' by reading one 'a' in the stack.

Problem 3 : Derivations, Parse Trees, and Ambiguity

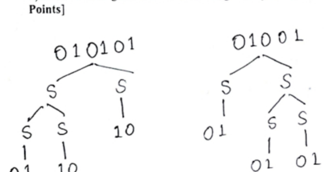
a) Consider the following context free grammar:

$$S \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10$$

i) Give a left derivation for the string 011010. **[2 Points]**

$S \to SS$
$\to 01S$
$\to 01SS$
$\to 0110S \to 011010$

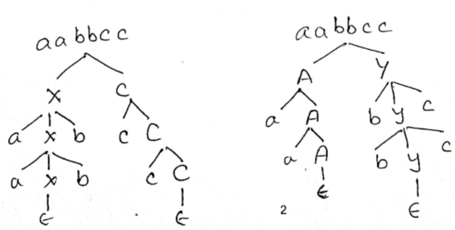
ii) Show that the grammar above is ambiguous by demonstrating two different parse trees for 010101. **[3 Points]**



b) Consider the following context free grammar:

$S \to XC \mid AY$
$X \to aXb \mid \epsilon$
$Y \to bYc \mid \epsilon$
$A \to aA \mid \epsilon$
$C \to cC \mid \epsilon$

Show that the grammar above is ambiguous by finding a length 6 string with two parse trees. **[5 Points]**



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Question 03

a) ... with '0' and the length of w is even.

ω starts with

$S \to 0AB$ $\qquad 0(\Sigma\Sigma)^*(0+1)$

$A \to 00A \mid 01A \mid 10A \mid 11A \mid \epsilon$

$B \to 0 \mid 1$

b) Every second letter in ω is b

$((a+b)\,b)^* \cancel{(\epsilon \mid a+b)}\ (\epsilon + a + b)$

$S \to AB$

$A \to XA \mid \epsilon$

$X \to PQ$

$P \to a \mid b$

$Q \to b$

$B \to a \mid b \mid \epsilon$

Solution 2:

$S \to abS \mid bbS \mid A$ $\qquad$ _b_b_b_

$A \to a \mid b \mid \epsilon$

c) the length of ω is divisible by ~~3~~ three.

$(\Sigma\Sigma\Sigma)^*$

$\to ((a+b)(a+b)(a+b))^*$

No need to write ~~it~~ in the answer script (Optional)

$S \to AS \mid \epsilon$

$A \to BBB$

$B \to a \mid b$

d) ω starts and ends with different letters.

$a(a+b)^*b + b(a+b)^*a$

$S \to aAb \mid bAa$

$A \to aA \mid bA \mid \epsilon$

e) the number of a is at least the number of b in ω

$S \to \underbrace{aSbS \mid bSaS}_{\text{ensuring \#'b's equal to \#'a's, means the amount of b's won't exceed the amount of 'a's}} \mid \overset{\text{more 'a's}}{aS} \mid \epsilon$

Similar Qs:

ω contain more ~~a's~~ 'a's than ~~b's~~ 'b's

$S \to XaX$

$X \to aXbX \mid bXaX \mid \cancel{aX}\ aX \mid \epsilon$

d) $L = \{\omega \in \{0,1\}^* : \omega = 0^n 1^{2n+3}, n \geq 0\}$

$\omega = 0^n 1^{2n+3}$

$= 0^n 1^{2n} 1^3$

$S \longrightarrow AB$

$A \longrightarrow 0A11 \mid \epsilon$

$B \to 111$

f) $w = 0^n 1^n$, where n is odd.

wrong ← Solution, since this doesn't ensure equal numbers of 0s & 1s.

$S \to AB$
$A \to 0X$
$X \to 00X \mid \epsilon$
$B \to 1Y$
$Y \to 11Y \mid \epsilon$

$S \to 0S11 \mid A$
$A \to 01$

g) $w = 0^i 1^j 2^k$, where $j \geq 2i + 3k$

$S \to ABC$
$A \to 0A11 \mid \epsilon$
$B \to 1B \mid \epsilon$
$C \to 111C2 \mid \epsilon$

* first try to solve $j = 2i + 3k$

$0^i 1^j 2^k$

$\to 0^i 1^{2i+3k} 2^k$

$\to \underbrace{0^i 1^{2i}}\, \underbrace{1^{3k} 2^k}$

the additional 1s ($j>$) will occur in this place.

h) $w = 0^i 1^j 2^k 3^m$, where $i = m$ and $j \geq 3k + 2$

$S \to 0S3 \mid X$
$X \to 111X2 \mid Y$
$Y \to 1Y \mid 11$

first try to solve $j = 3k+2$, $i = m$

$0^i 1^j 2^k 3^m$

$\to 0^i 1^{3k+2} 2^k 3^i$

$\to 0^i 1^{3k} 1^2 2^k 3^i$

the additional or more 1s ($j>$) will come from here.

i) $w = 0^i 1^j 2^k$, where $i > 2j + 3k$

$S \to 000S~~1~~2 \mid X$
$X \to 00X1 \mid Y$
$Y \to 0Y \mid 0$ → Since these can't have equal. we are not giving the ϵ transition

first try to solve $i = 2j + 3k$

$0^i 1^j 2^k$

$\to 0^{2j+3k} 1^j 2^k$

$\to 0^{3k} 0^{2j} 1^j 2^k$

we need $>$, here the # 0s ~~will be~~ are equal currently Hence, we have to increase the amount of 0s

J) A = w contains at least two 0s

$L = w : w = 0^{3i} \, v \, 1^{2i}, \; v \in A, \; i \geq 0$

at least two 0s

$\_\_ 0 \_\_ 0 \_\_$

any combinations of 0s and 1s

$S \to 000S11 \mid A$

$A \to X0X0X$

$X \to 0X \mid 1X \mid \epsilon$

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K) $L1 = \{ \omega \in \{0,1\}^* : \omega \text{ contains } 11 \}$

$L2 = \{ x\#y : x \in \{0,1\}^*, \; y \in L1, \; |x| = |y| \}$

0 1 1 $\boxed{10}$ 0 0 1 0 # 0 1 1 0 $\boxed{11}$ 0 1 0

$S \to XSX \mid A$

keep continuing the match until we are getting 11 in y.

$A \to 00B11 \mid 01B11 \mid 10B11 \mid 11B11$

$B \to XBX \mid \#$

$X \to 0 \mid 1$

Next, match the rest of the things.

we have found first 11 in y. Now to keep the length same we have to match 11 with all possible two length of 0,1 strings.

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L) $L = \{ \omega_1, \omega_2 \in \{0,1\}^* : \omega_1 \# \omega_2$ where number of 0s in ω_1 is equal to number of 1s in $\omega_2 \}$

What if at the same time $|\omega_1| = |\omega_2|$?

0 1 1 0 1 0 1 # 0 0 0 1 1 0 0 0 0 1 0

Note: here $1S \neq S1$

$S \to 0S1 \mid 1S \mid S0 \mid \#$

Since we are generating the 1s in ω_1.

Since we are only interested in the 0s in ω_1. And, we have to keep the

See the example: Has to produce/genera[te] the 1s in ω_1 and 0s in ω_2. Please [see] the question

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we ... count equal to 1s in ω_2. See, when we are having a 0 in ω_1, at the same time we are having a 1 in ω_2 → this is how we are equalizing the counts of 0 in ω_1 and 1s in ω_2

note, the ... doesn't ask for $|\omega_1| = |\omega_2|$

m) $L = \{\omega_1, \omega_2 \in \{0,1\}^* : \omega_1 \# \omega_2$ where length of ω_2 is double of $\omega_1\}$

$S \longrightarrow ASAA \mid \#$

$A \longrightarrow 0 \mid 1$

0 1 1 0 # 1 1 0 1 1 0 0 0

Regular Expression ... CFG

a) $\overbrace{\underbrace{(\underbrace{(ab)^*}_{B} + \underbrace{(a+b^3)}_{D}\underbrace{cb}_{E})}_{A}{}^*}^{S}$ → C

$S \to AS \mid \epsilon$

$A \to B \mid C$

$B \to abB \mid \epsilon$

$C \to DE$

$D \to a \mid bbb$

$E \to cb$

b) $\overbrace{(\overbrace{\underbrace{a^*b^*}_{P} + \underbrace{(ac + b^*c)b}_{Q}}^{R})^*}^{S}$

$S \to RS \mid \epsilon$

$R \to P \mid Q$

$P \to AB$

$A \to aA \mid \epsilon$

$B \to bB \mid \epsilon$

$Q \to Zb$

$Z \to X \mid Y$

$X \to ac$

$Y \to Bc$