1.A R^n and C^n

1. addition
$(x_1, \cdots, x_n) + (y_1, \cdots, y_n)$
$= (x_1 + y_1, \cdots, x_n + y_n)$
2. 0, $0 = (0, \cdots, 0)$
3. additive inverse
$-x$ is a vector $\in F^n$
such that $x + (-x) = 0$
4. scalar multiplication
$\lambda(x_1, \cdots, x_n) = (\lambda x_1, \cdots, \lambda x_n)$

Def F^n prop

commutativity:
if $x, y \in F^n$, then $x + y = y + x$

1.B Vector Spaces

Vector Space

— a set V with the def of addition and scalar multiplication

addition: $u \in V$, $v \in V$, then $u + v \in V$
multiplication: $v \in V$, then $\lambda v \in V$

The following properties are required:

commutativity: $u + v = v + u$ for all $u, v \in V$
associativity: $(u+v) + w = u + (v + w)$
and $(ab)v = a(bv)$ for all $u, v, w \in V$ and $a, b \in F$
additive identity
there exists an element $0 \in V$ such that $v + 0$ for all $v \in V$
additive inverse: for every $v \in V$, there exists $w \in V$, such that $v + w = 0$

multiplicative identity: $1v = v$ for all $v \in V$

distributive properties:

$$a(u+v) = au + av \quad \text{and} \quad (a+b)v = av + bv$$

for all $a, b \in F$ and all $u, v \in V$

F^S denotes the set of functions from S to F

addition: $(f+g)(x) = f(x) + g(x)$

scalar multiplication: $(\lambda f)(x) = \lambda f(x)$

props of Vector Space

1. unique additive identity (0)
2. unique additive inverse $(-v)$
3. $0v = 0$ for all $v \in V$
4. $a0 = 0$ for all $a \in F$
5. $(-1)v = -v$ for $v \in V$

V — a vector space over F

1.C Subspaces.

conditions for Subspace:

- $0 \in U$
- addition closed
- multiplication closed

sum: $U_1 + \cdots + U_m = \{u_1 + \cdots + u_m,\ u_1 \in U_1, \cdots, u_m \in U_m\}$

direct sum: it's a sum satisfying this condition:

for $u \in U_1 + \cdots + U_m$, there exists unique $u_1 \in U_1, \cdots, u_m \in U_m$

such that $U = U_1 + \cdots + U_m$

conditions for direct sum:

$U_1 + \cdots + U_m$ is a direct sum iff the only way to write $u_1 + \cdots + u_m = 0$ is to let each u_i equal to 0

$U + W$ is a direct sum

$$\Longleftrightarrow U \cap W = \{0\}$$

(but don't extend this theorem to the case when involved subspaces are more than 2)