L1: The Geometry of Linear Equations

Ax: Combinations of Columns of A

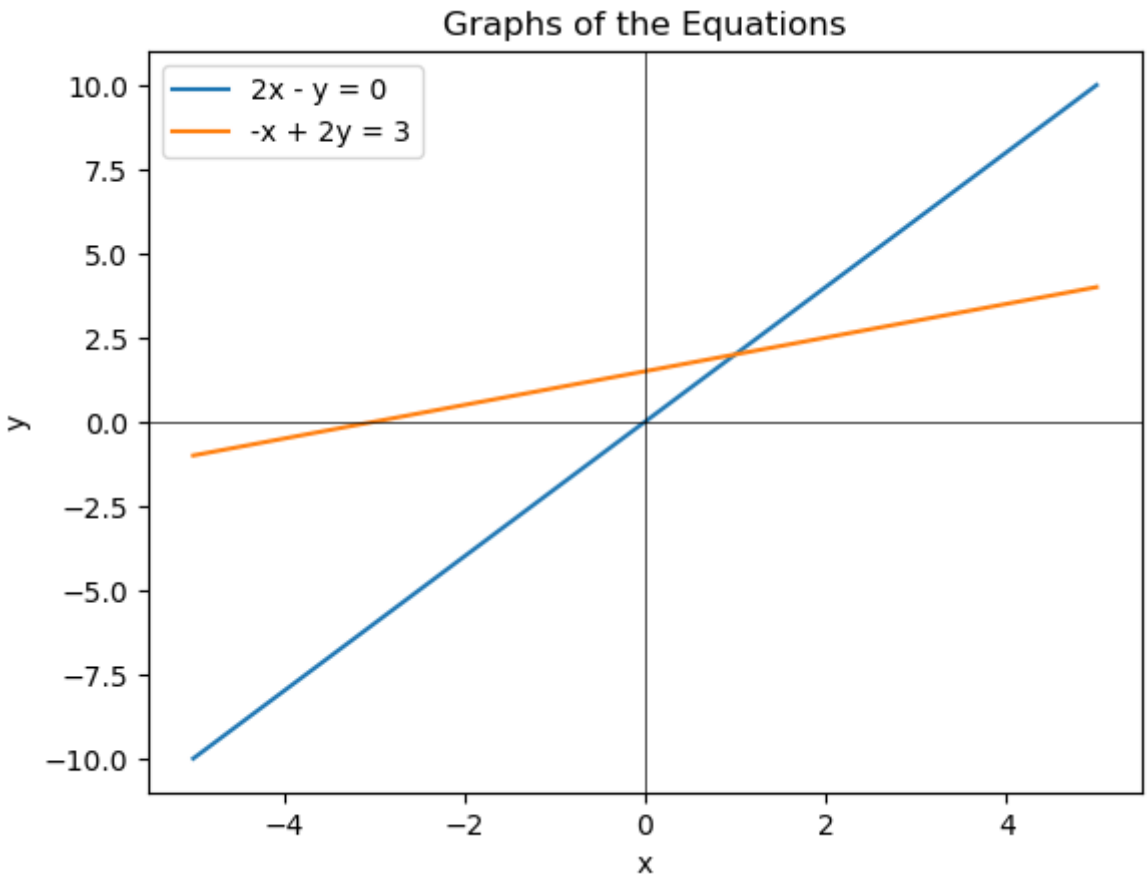
e.g 1

$$\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases}$$

We can view the equations through **row picture** and **column picture**

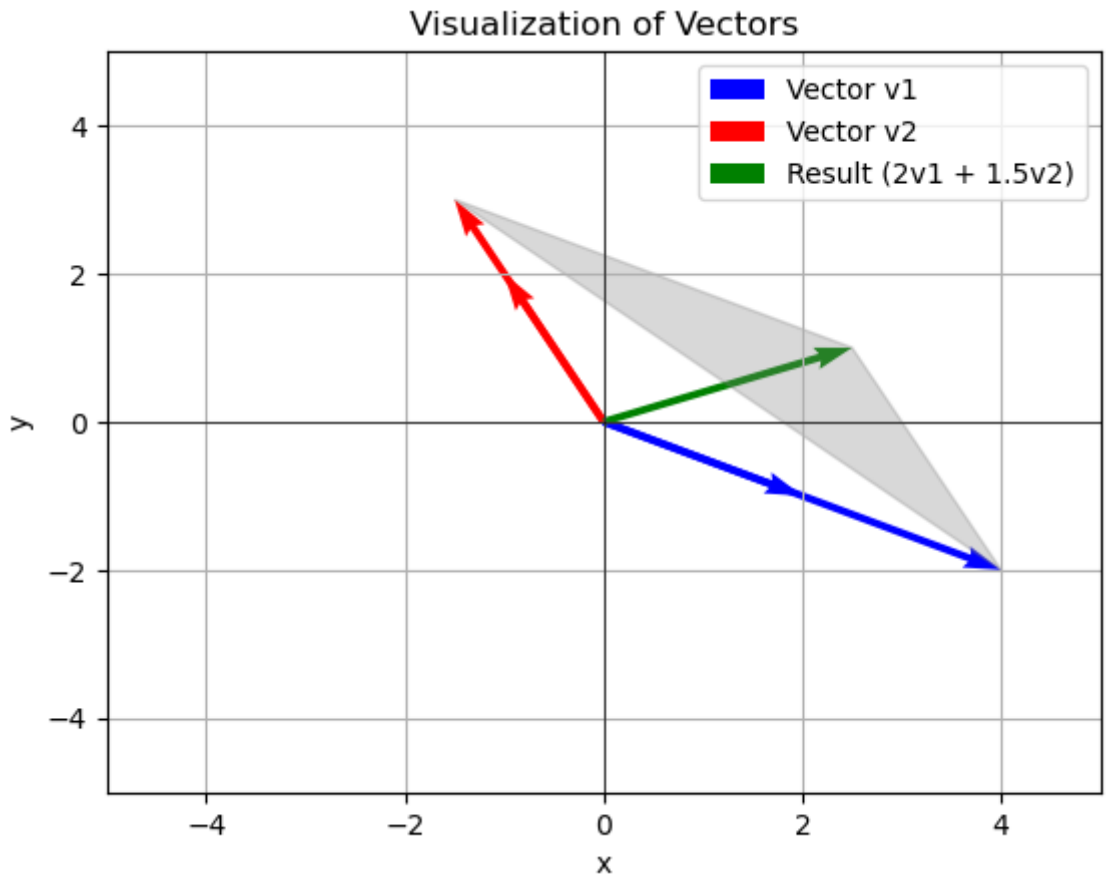
Row picture is by thinking of the equation being the intersection of two lines: $2x - y = 0$ and $-x + 2y = 3$

Below is the visualization:



column picture:

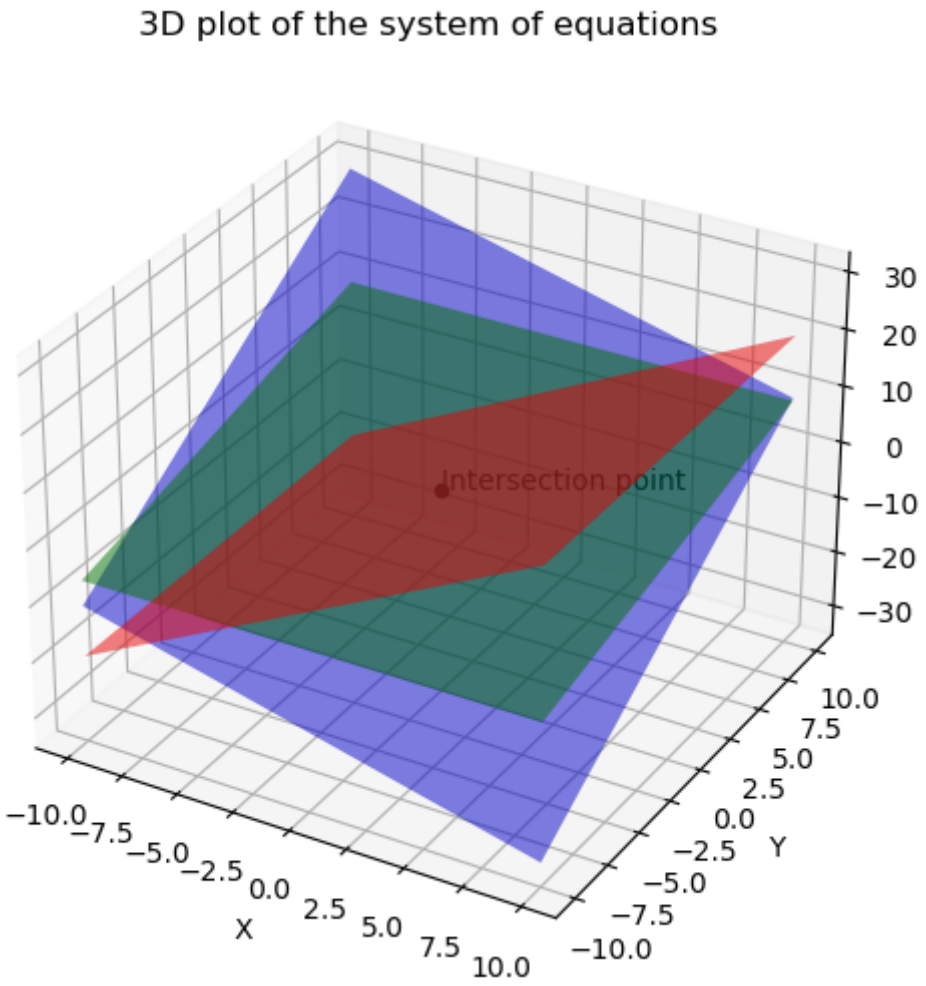
$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$



e.g 2

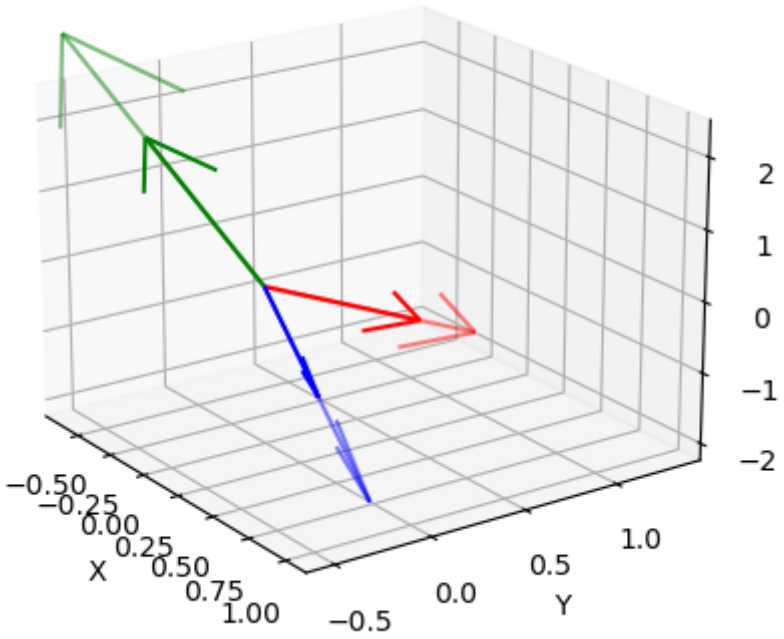
$$\begin{cases} 2x-y & = 0 \\ -x+2y-z & = -1 \\ -3y+4z & = 4 \end{cases}$$

below is a row picture:



column picture:

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$



Some questions here:

Can I solve $Ax = b$ for every b?

Do the linear combinations of columns fill 3-D space?

For these columns, the answer is yes.

But if the three vectors lie in the same plane, the answer will be wrong.